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Analysis and Comparison of Time Replica and Time Linear Interpolation for Pilot Aided Channel Estimation in OFDM Systems

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Abstract

This paper analyzes and compares two time interpolators, *i.e.*, time replica and time linear interpolator, for pilot aided channel estimation in orthogonal frequency division multiplexing (OFDM) systems. The mean square error (MSE) of two interpolators is theoretically derived for the general case. The equally spaced pilot arrangement is proposed as a special platform for these two time interpolators. Based on this proposed platform, the MSE of two time interpolators at the virtual pilot tones is derived analytically; moreover, the MSE of per channel estimator at the entire OFDM symbol based on per time interpolator is also derived. The effectiveness of the theoretical analysis is demonstrated by numerical simulation in both the time-invariant frequency-selective channel and the time varying frequency-selective channel.

Keywords: OFDM, Channel Estimation, Time Replica, Time Linear Interpolation, Virtual Pilots

1. Introduction

Orthogonal frequency division multiplexing (OFDM) [1-3] has been widely used in high-speed wireless communication systems, such as broadband wireless local area networks (WLANs) [4], wireless metropolitan area networks (WMANs) [5] and worldwide interoperability for microwave access (WIMAX) [6], due to its advantages of transforming frequency-selective fading channels into a set of parallel flat fading sub-channels and eliminating inter-symbol interference [7].

Channel estimation is one of the most essential tasks in compensating distortion from channels and performing coherent detection in OFDM systems. Estimation is usually performed by using pilot tones [8, 9] and is based on inserting known pilot tones in each OFDM symbol, where interpolation in time-frequency grid [10] plays an important role in the estimation process. The usage of virtual pilot tones [11-13] and time interpolation can reduce the redundancy and guarantee a higher transmission bit rate. Among time interpolation methods, time replica [14, 15] is widely used in time-invariant or slow time-varying channel, which is simple to implement and also efficient for subcarrier usage; time linear interpolation [16-18] is widely used in slow or fast time-varying channel, because it is simple to realize and usually can give a satisfactory performance. However, some interesting questions are raised as follows: 1) what kind of time-varying channel is slow enough to utilize time replica? 2) Conversely, what kind of time-varying channel is so fast that we have to employ time linear interpolation instead of time replica? And 3) how much does time linear interpolation perform better than time replica by for a time-invariant channel?

To answer these questions above, this paper analyzes and compares the performances of time replica and time linear interpolator in both the time-invariant frequency-selective channel and the time varying frequency-selective channel. The MSE of both time interpolators is theoretically derived for the general cases. The equal spaced pilot arrangement is employed as a special platform for both time interpolators, where the positions of virtual pilot tones in one OFDM symbol correspond to those of pilot tones of its last and next OFDM symbols. Channel state information (CSI) [19] at pilot tones is estimated by least square (LS) estimator. CSI at virtual pilot tones in one OFDM symbol is obtained by either of time interpolators, where time replica is to completely replicate the CSI at pilot tones of its last OFDM symbol while time linear interpolator is to linearly interpolate values by using the estimated CSI at the corresponding pilot tones of both its last and next OFDM symbols. CSI at data tones is finally obtained by frequency linear interpolation [20].

This paper is organized as follows. In Section 2, the MSEs of two interpolators, *i.e.*, time replica and time linear interpolation, are theoretically derived for the general case. In Section 3, the equally spaced pilot arrangement is proposed as a special platform for analyzing these two time interpolators. In Section 4, based on the proposed platform, the MSE of two time interpolators at the virtual pilot tones is derived analytically; moreover, the MSE of channel estimators at the entire OFDM symbol based on these two time interpolators is also derived, respectively. Numerical results are reports in Section 5, followed by conclusion in Section 6.

Notation: $|g|^2$ denotes the modulus. $\|g\|^2$ is the 2-norm operation. $E_k\{g\}$ is the expectation operation on *k*. $E_{k,l}\{g\}$ means the expectation on both *k* and *l*. $Var_k\{g\}$ means the variance on *k*. $d_{m-i,m+j}(k)$ denotes the variation of the CSI of the *k*th tone from the $(m-i)$th OFDM symbol to the $(m+j)$th OFDM symbol. $d_m(k)$ denotes the variation of the CSI of the *k*th tone from the *m*th OFDM symbol to the $(m+1)$th OFDM symbol. $e_m^R(k)$ and $e_m^L(k)$ are the channel estimation errors of the *m*th OFDM symbol at the *k*th tone where time replica or time linear interpolation are employed for CSI estimation at the virtual pilot tones, respectively.

2. MSE of Two Time Interpolators

Assume that each OFDM symbol has *N* subcarriers where pilots occupy *P* subcarriers. Denote the set of pilot tones by I_P. By LS estimation, the CSI at pilot tones in the m^{th} OFDM symbol can be obtained as

$$\hat{H}_m(k) = \frac{Y_m(k)}{X_m(k)} \tag{1}$$

where $X_m(k)$ and $Y_m(k)$ are the transmitted and received pilots of the *m*th OFDM symbol, respectively. Assuming the pilot tones $X_m(k) = 1$ for convenience of analysis, we have

$$\hat{H}_m(k) = H_m(k) + W_m(k) \tag{2}$$

where $H_m(k)$ represents the true value and $W_m(k)$ is a complex-valued sample of additive white Gaussian noise (AWGN) process at the *m*th OFDM symbol, $W_m(k) \sim CN\left(0, s^2\right)$.

Assuming that along the time axis in **Figure 1**, the data tones in the m^{th} OFDM symbol correspond to the pilot tones in both the $(m-p)$th and the $(m+q)$th OFDM symbol, the CSI at the data tones in the *m*th OFDM symbol can be obtained by time interpolation by using the estimated CSI at the pilot tones of both the $(m-p)$th and the $(m+q)$th OFDM symbol, which is thus called the virtual pilot tones. Denote the set of virtual tones by I_{PP}. In this section, we will analyze and compare the MSE performance of two time interpolators: time replica and time linear interpolator.

2.1. Time Replica

Time replica at the virtual pilot tones in the *m*th symbol is to replicate the CSI at the pilot tones in the $(m-p)$th symbol,

$$\hat{H}_m(k) = \hat{H}_{m-p}(k), \qquad k \in I_{PP}\ . \tag{3}$$

By (2) and (3), the estimation error of time replica at the *k*th tone can be expressed as

$$\begin{aligned} e_m^R(k) &= \hat{H}_{m-p}(k) - H_m(k) \\ &= H_{m-p}(k) - H_m(k) + W_{m-p}(k). \end{aligned} \tag{4}$$

The MSE using time replica can thus be obtained as

$$\begin{aligned} x_R &= E_k\left\{\left\|e_m^R(k)\right\|^2\right\} = E_k\left\{\left\|H_{m-p}(k) - H_m(k)\right\|^2\right\} + s^2 \\ &= E_k\left\{\left\|d_{m-p,m}(k)\right\|^2\right\} + s^2. \end{aligned} \tag{5}$$

2.2. Time Linear Interpolation

However, if using time linear interpolation, the estimated CSI can be obtained as follows,

$$\hat{H}_m(k) = \frac{p}{p+q}\hat{H}_{m-p}(k) + \frac{q}{p+q}\hat{H}_{m+q}(k) \tag{6}$$

for $k \in I_{PP}$. By (2) and (6), the estimation error of time linear interpolation at the k^{th} tone can be expressed as

$$\begin{aligned} e_m^L(k) &= \frac{p}{p+q}\left(H_{m-p}(k) - H_m(k)\right) + \frac{q}{p+q}W_{m-p}(k) \\ &= \frac{p}{p+q}\left(H_{m+q}(k) - H_m(k)\right) + \frac{q}{p+q}W_{m+q}(k). \end{aligned} \tag{7}$$

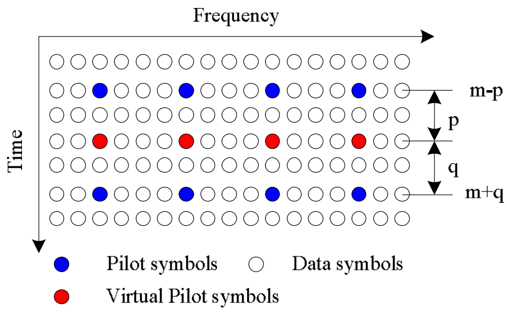

Figure 1. The virtual pilot tones in the *m*th OFDM symbol are time-interpolated by using the pilot tones at both the $(m-p)$th and the $(m+q)$th OFDM symbol.

Based on (7), the MSE of time linear interpolation can thus be obtained as

$$x_L = E_k\left\{\left\|e_m^L(k)\right\|^2\right\} = E_k\left\{\left\|\frac{pd_{m-p,m}(k)}{p+q} - \frac{qd_{m,m+q}(k)}{p+q}\right\|^2\right\} + \frac{p^2+q^2}{(p+q)^2}s^2. \tag{8}$$

2.3. Comparison

Subtracting (8) from (6), the difference between x_R and x_L can be expressed as

$$x_R - x_L = E_k\left\{\left\|d_{m-p,m}(k)\right\|^2\right\} + \frac{2pq}{(p+q)^2}s^2 - E_k\left\{\left\|\frac{pd_{m-p,m}(k)}{p+q} - \frac{qd_{m,m+q}(k)}{p+q}\right\|^2\right\}. \tag{9}$$

From (9), one can conclude that

1) In a time-invariant frequency-selective channel, x_L is always lower than x_R by $10\log\frac{(p+q)^2}{2pq}$ dB; while in a time-variant frequency-selective channel, the performance comparison depends on the specific channel variation;

2) Considering a real-valued channel variation, in low noise environment, when $d_{m-p,m}(k)d_{m,m+q}(k) < 0$ and $\left|d_{m,m+q}(k)\right| > \left|d_{m-p,m}(k)\right|$, $x_R < x_L$;

3) Considering a real-valued channel variation, in noisy environment, when $d_{m-p,m}(k)d_{m,m+q}(k) \ge 0$ or $d_{m-p,m}(k)d_{m,m+q}(k) < 0$ but $\left|d_{m,m+q}(k)\right| < \left|d_{m-p,m}(k)\right|$, $x_R > x_L$.

3. Special Case: Pilot Arrangement and Channel Estimators

Assume that each OFDM symbol has N subcarriers where pilots occupy P subcarriers and virtual pilots superimposed with data samples also occupy P subcarriers. **Figure 2** shows the proposed pilot arrangement as a platform, which is a special case but not loss of generality, where along frequency axis, the pilot spacing is $2L$ and the spacing between pilot and adjacent virtual pilot is L. From **Figure 2**, one can see that along time axis, the pilot spacing is 2 and the spacing between pilot and adjacent virtual pilot is 1. Also, by LS estimation, the CSI at pilot tones can be obtained by (1).

3.1. Time Interpolation at Virtual Pilot Tones

Denote the set of virtual tones by I_{PP}. The CSI at virtual pilot tones is obtained by time interpolation. In this special pilot arrangement, since the virtual pilot tones at the mth symbol corresponds to the pilot tones at the $(m-1)$th symbol, time replica at the virtual pilot tones in one symbol is to replicate the CSI at the pilot tones of its last symbol,

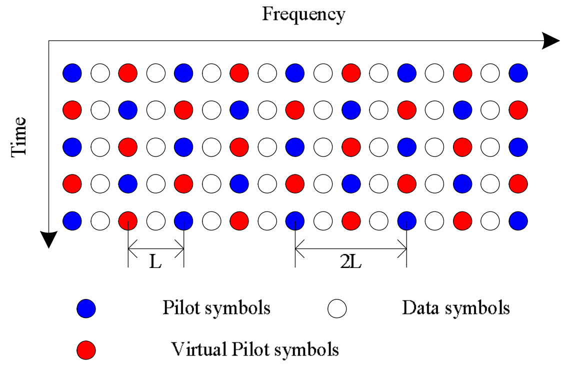

Figure 2. The proposed pilot arrangement as a special platform, where the pilot tones in one OFDM symbol correspond to the virtual pilot tones in its adjacent OFDM symbol.

$$\hat{H}_m(k) = \hat{H}_{m-1}(k), \quad k \in I_{PP}. \tag{10}$$

On the other hand, if using time linear interpolation, we can get

$$\hat{H}_m(k) = \frac{\hat{H}_{m-1}(k) + \hat{H}_{m+1}(k)}{2}, \quad k \in I_{PP}. \tag{11}$$

3.2. Frequency Interpolation at Data Tones

Denote the set of data tones as I_D. Using frequency linear interpolation [20], the CSI at the whole OFDM symbol can be expressed as

$$\hat{H}_m(k+l) = \begin{cases} \frac{L-l}{L}\hat{H}_m(k) + \frac{l}{L}\hat{H}_m(k+L) \\ \qquad \text{when } 1 \le k \le 1 + L(2P-2) \\ \hat{H}_m(k) \\ \qquad \text{when } k = 1 + L(2P-1). \end{cases} \tag{12}$$

where $(k+l) \in I_D$, $k \in I_P \cup I_{PP}$, $1 \le l \le L-1$. Note that the CSI for data tones located on the right side beyond the $(1+PL)$th pilot/virtual pilot tone is decided by the edge interpolation.

4. Performance Analysis for the Special Case

This section analyzes the performance of this special case in terms of the MSEs of time interpolators and the MSEs of the corresponding channel estimators.

4.1. MSE of Time Interpolators

For this special case, the MSE of time replica in (5) becomes

$$x_R = E_k\left\{\left\|d_{m-1}(k)\right\|^2\right\} + s^2. \tag{13}$$

On the other hand, the MSE of time linear interpolation in (8) becomes

$$x_L = E_k\left\{\left\|\frac{d_{m-1}(k)-d_m(k)}{2}\right\|^2\right\} + s^2. \tag{14}$$

So, based on (13) and (14), the difference between x_R and x_L can be obtained as follows,

$$x_R - x_L = E_k\left\{\left\|d_{m-1}(k)\right\|^2\right\} + \frac{1}{2}s^2 - E_k\left\{\left\|\frac{d_{m-1}(k)-d_m(k)}{2}\right\|^2\right\}. \tag{15}$$

And, from (15), one can conclude that 1) in a time-invariant frequency-selective channel, x_L is always lower than x_R by 3 dB; in a time-variant frequency-selective channel, the performance difference depends on the specific channel variation; 2) in most situations, as the general case in (9), $x_R > x_L$, *i.e.*, time linear interpolation is better than time replica.

4.2. MSE of Channel Estimation

4.2.1. Time Replica

By LS estimation on pilot tones, time replica on virtual pilot tones and frequency interpolation on data tones, the corresponding MSE of channel estimation can be expressed as

$$x_{LRL} = \frac{P}{N}x_P + \frac{P}{N}x_R + \frac{N-2P}{N}x_{RF}, \tag{16}$$

where x_P is the MSE of LS estimation and x_{RF} is the MSE of frequency interpolation when using time replica at virtual pilot tones.

As an average of both odd and even OFDM symbols, except for the right side $(L-1)$ tones using the edge interpolation, a half of other data tones with the index $(k+l)\in I_D$ have $k\in I_P$ while $(k+L)\in I_{PP}$ for frequency linear interpolation; for the remaining data tones, $k\in I_{PP}$ while $(k+l)\in I_P$ for frequency linear interpolation. Hence, using (12), we can get x_{RF} in (17), where $e_F(k+l) = \frac{L-l}{L}H_m(k) + \frac{l}{L}H_m(k+L) - H_m(k+l)$, $k\in I_P\cup I_{PP}$, $1\le k\le 1+L(2P-2)$, and $e_F(1+L(2P\text{-}1)+l) = H_m(1+L(2P\text{-}1)) - H_m(1+L(2P\text{-}1)+l)$, are the inherent errors by frequency interpolation, x_F is the inherent MSE of frequency interpolation.

By substituting (17) into (16), x_{LRL} can be expressed as the following (18),

$$\begin{aligned}
x_{RF} &= E_{k,l}\left\{\left\|\hat{H}_m(k+l) - H_m(k+l)\right\|^2\right\} = \\
&\frac{1}{2}\left(1-\frac{L-1}{N-2P}\right)E_{k,l}\left\{\left\|e_F(k+l) + \frac{L-l}{L}W_m(k) + \frac{l}{L}d_{m-1}(k)\right\|^2\right\} \\
&+\frac{1}{2}\left(1-\frac{L-1}{N-2P}\right)E_{k,l}\left\{\left\|e_F(k+l) + \frac{l}{L}W_m(k) + \frac{L-l}{L}d_{m-1}(k)\right\|^2\right\} \\
&+\frac{L-1}{N-2P}E_l\left\{\left\|e_F(1+L(2P-1)+l)\right\|^2\right\} = x_F \\
&+\frac{2L-1}{6L}\left(1-\frac{L-1}{N-2P}\right)s^2 \\
&+\frac{2L-1}{6L}\left(1-\frac{L-1}{N-2P}\right)E_k\left\{\left\|d_{m-1}(k)\right\|^2\right\},
\end{aligned} \tag{17}$$

$$\begin{aligned}
x_{LRL} &= \frac{P}{N}x_P + \frac{P}{N}x_R + \frac{N-2P}{N}x_F \\
&+\frac{(2L-1)(N-2P-L+1)}{6NL}\left[s^2 + E_k\left\{\left\|d_{m-1}(k)\right\|^2\right\}\right].
\end{aligned} \tag{18}$$

4.2.2. Time Linear Interpolation

By LS estimation on pilot tones, time linear interpolation on virtual pilot tones and frequency interpolation on data tones, the MSE of channel estimation can be expressed as

$$x_{LLL} = \frac{P}{N}x_P + \frac{P}{N}x_R + \frac{N-2P}{N}x_{LF}, \tag{19}$$

where x_{LF} is the MSE of frequency interpolation when using time linear interpolation at virtual pilot tones. Using (12), x_{LF} can be obtained as shown in (20).

Substituting (20) into (19), x_{LLL} can be expressed as the following (21),

$$\begin{aligned}
x_{LF} &= E_{k,l}\left\{\left\|\hat{H}_m(k+l) - H_m(k+l)\right\|^2\right\} = \\
&\frac{1}{2}\left(1-\frac{L-1}{N-2P}\right) \\
&E_{k,l}\left\{\left\|e_F(k+l) + \frac{L-l}{L}W_m(k) + \frac{l}{L}\frac{d_{m-1}(k)-d_m(k)}{2}\right\|^2\right\} \\
&+\frac{1}{2}\left(1-\frac{L-1}{N-2P}\right) \\
&E_{k,l}\left\{\left\|e_F(k+l) + \frac{l}{L}W_m(k) + \frac{L-l}{L}\frac{d_{m-1}(k)-d_m(k)}{2}\right\|^2\right\} \\
&+\frac{L-1}{N-2P}E_l\left\{\left\|e_F(1+L(2P-1)+l)\right\|^2\right\} =
\end{aligned}$$

$$x_F+\frac{2L-1}{6L}\left(1-\frac{L-1}{N-2P}\right)s^2 +\frac{2L-1}{6L}\left(1-\frac{L-1}{N-2P}\right)E_k\left\{\left\|\frac{d_{m-1}(k)-d_m(k)}{2}\right\|^2\right\}. \tag{20}$$

$$x_{LLL}=\frac{P}{N}x_P+\frac{P}{N}x_R+\frac{N-2P}{N}x_F +\frac{(2L-1)(N-2P-L+1)}{6NL} \left[s^2+E_k\left\{\left\|\frac{d_{m-1}(k)-d_m(k)}{2}\right\|^2\right\}\right]. \tag{21}$$

4.2.3. Comparison

Subtracting (21) from (18), the difference between x_{LRL} and x_{LLL} can be obtained as

$$x_{LRL}-x_{LLL}=\frac{P}{2N}s^2+\left(\frac{P}{N}+\frac{(2L-1)(N-2P-L+1)}{6NL}\right) \times\left[E_k\left\{\left\|d_{m-1}(k)\right\|^2\right\}-E_k\left\{\left\|\frac{d_{m-1}(k)-d_m(k)}{2}\right\|^2\right\}\right]. \tag{22}$$

From (22), one can notice that

1) Since $N >> P$, $\frac{P}{2N}s^2$ is negligible and the differential MSE using (22) is approximately independent with noise;

2) In a time-invariant frequency-selective channel, x_{LRL} is approximately equal to x_{LLL}; while in a time-variant frequency-selective channel, the performance comparison depends on the specific channel variation;

3) Considering a real-valued channel variation, in low noise environment, when $d_m(k)d_{m-1}(k)<0$ and $|d_m(k)|>|d_{m-1}(k)|$, $x_{LRL}<x_{LLL}$;

4) $|x_{LRL}-x_{LLL}|<|x_R-x_L|$.

5. Numerical Results

The OFDM system under consideration is with N = 512 subcarriers, and $2L$ = 8 equispaced pilot tones in each symbol. The length of cyclic prefix is 32. The interpolation distances p = q = 1. The modulation is QPSK. The pilot tones are all 1. For $0\le j\le 63$, in the odd OFDM symbols, the pilot is inserted at the $(1+8j)$th tone; while in the even OFDM symbols, the pilot is inserted at the $(5+8j)$th tone. The six-ray multipath Rayleigh fading channel is considered. The average power delay profile is selected as

$$l_l=\exp(-l)\Big/\sum_{l=0}^{5}l_l,\quad 0\le l\le 5. \tag{23}$$

Figure 3 shows the MSE performance of time interpolator and channel estimation in the time-invariant frequency-selective channel, where one can see that time linear interpolator generating less noise has a 3 dB lower MSE than time replica at the virtual pilot tones. However, for the corresponding channel estimation at the whole OFDM tones, time linear interpolator performs similarly to time replica due to a negligible noise.

Figure 4 shows the MSE performance in a time varying channel where the parameters are $E_k\{d_{m-1}(k)\}=0.001$, $Var_k\{d_{m-1}(k)\}=10^{-6}$, $E_k\{d_m(k)\}=-0.002$, and $Var_k\{d_m(k)\}=10^{-6}$, respectively. For interpolation at virtual pilot tones, when $SNR\le 25$ dB, time linear interpolator performs better than time replica due to better noise reduction; when SNR > 25 dB, time replica, which guarantees a more accurate interpolation in a low noise environment, performs better than linear interpolator. While for the corresponding channel estimation, when $SNR\le 25$ dB, time linear interpolator performs very similarly to time replica due to better noise reduction; when SNR > 25 dB, time replica also performs better than time linear interpolator.

Figure 5 shows the MSE performance in the time varying channel where the parameters are $E_k\{d_{m-1}(k)\}$

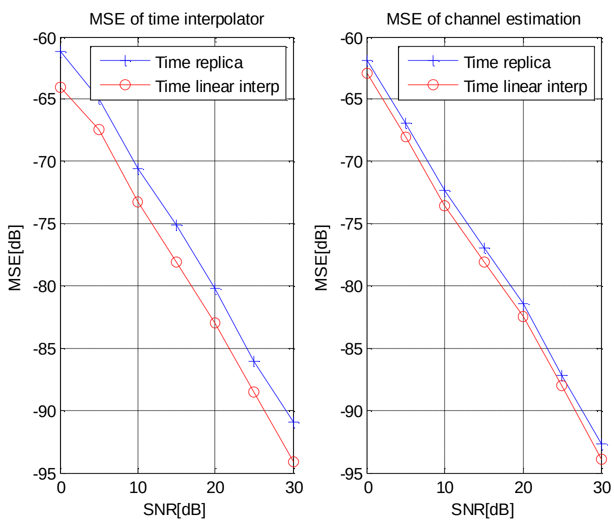

Figure 3. MSE of time interpolator and channel estimation in time-invariant frequency-selective channel.

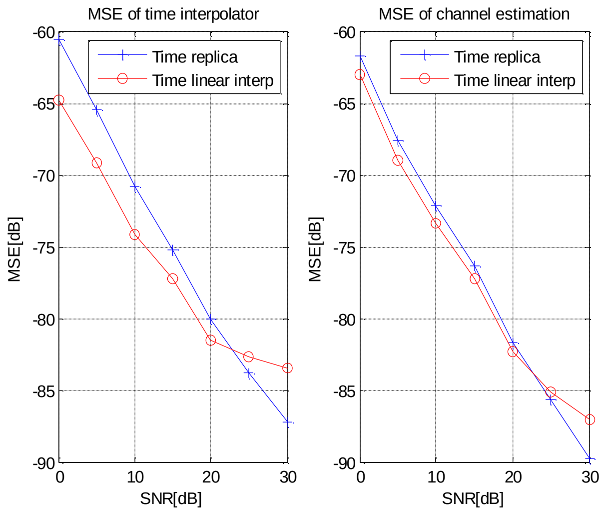

Figure 4. MSE of time interpolator and channel estimation in time-variant frequency-selective channel, where the expectation is equal to $E_k\{d_{m-1}(k)\} = 0.001$, the variance is equal to $Var_k\{d_{m-1}(k)\} = 10^{-6}$, the expectation $E_k\{d_m(k)\}$ $= -0.002$, and variance $Var_k\{d_m(k)\} = 10^{-6}$, respectively, 0 £ *k* £ 512.

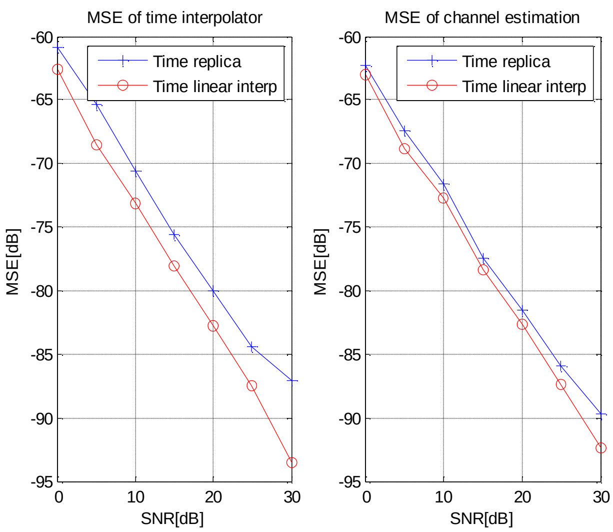

Figure 5. MSE of time interpolator and channel estimation in time-variant frequency-selective channel, where the expectation is equal to $E_k\{d_{m-1}(k)\} = 0.001$, the variance is equal to $Var_k\{d_{m-1}(k)\} = 10^{-6}$, the expectation $E_k\{d_m(k)\}$ $= 0.002$, and variance $Var_k\{d_m(k)\} = 10^{-6}$, respectively, 0 £ *k* £ 512.

$= 0.001$, $Var_k\{d_{m-1}(k)\} = 10^{-6}$, $E_k\{d_m(k)\} = 0.002$, and $Var_k\{d_m(k)\} = 10^{-6}$, respectively. Time linear interpolator always performs better than time replica for both interpolation at the virtual pilot tones and the corresponding channel estimation at the entire tones.

6. Conclusions

Time replica and time linear interpolation were analyzed and compared, especially under our proposed pilot arrangement. The MSEs of both time interpolators were derived analytically for both interpolations at the virtual pilot tones and their corresponding channel estimation at the entire OFDM symbol. Numerical simulation results were demonstrated to reach an agreement with theoretical analysis. From the given results, one can see that, in a time-invariant frequency-selective channel, when the interpolation distances $p = q = 1$, time linear interpolator has a 3 dB lower MSE than replica at the virtual pilot tones while they provide a similar performance at the entire OFDM symbol. Moreover, one can also see that, in a time varying frequency-selective channel, time linear interpolator outperforms time replica except the case, in a low noise environment, the CSI variation from the last OFDM symbol to the present symbol is negative to and has a smaller absolute value than that from the present symbol to the following symbol.

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