J. Mod. Phys., 2010, 1, 67-69
doi:10.4236/jmp.2010.11008 Published Online April 2010 (http://www.scirp.org/journal/jmp)

Tilted Bianchi Type VI_0 Cosmological Model in Saez and Ballester Scalar Tensor Theory of Gravitation

Subrata Kumar Sahu
Post Graduate Department of Mathematics, Lingaya's University, Faridabad, India
E-mail: *subrat_sahoo2002@rediffmail.com*
Received February 17, 2010; *revised March* 17, 2010; *accepted March* 20, 2010

Abstract

Tilted Bianchi type VI_0 cosmological model is investigated in a new scalar tensor theory of gravitation proposed by Saez and Ballester (Physics Letters A 113:467, 1986). Exact solutions to the field equations are derived when the metric potentials are functions of cosmic time only. Some physical and geometrical properties of the solutions are also discussed.

Keywords: Saez and Ballester Theory, Tilted Cosmological Model, Scalar Field

1. Introduction

In recent years, there has been a considerable interest in the investigation of cosmological models in which the matter does not move orthogonally to the hyper surface of homogeneity. These are called tilted cosmological models. The general behaviors of tilted cosmological models have been studied by King and Ellis [1], Ellis and King [2], Collins and Ellis [3], Bali and Sharma [4,5], Bali and Meena [6].

Bradely and Sviestins [7] investigated that heat flow is expected for cosmological models. Following the development of inflationary models, the importance of scalar fields (mesons) has become well known. Saez and Ballester [8] have developed a new scalar tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field, an anti-gravity regime appears. This theory suggests a possible way to solve the missing matter problem in non-flat FRW cosmologies. Sing and Agrawal [9], Reddy and Rao [10], Reddy *et al.* [11], Mohanty and Sahu [12,13], Adhav *et al.* [14], Tripathy *et al.* [15] are some of the authors who have studied the various aspects of Saez and Ballester [8] scalar tensor theory.

We derived the field equations for Bianchi type VI_0 metric in Section 2. We solved the field equations in Section 3. We mentioned some physical and geometrical properties of the solutions in Section 4 and also mentioned the concluding remark in Section 5.

2. Field Equations

Here we consider the Bianchi type VI_0 metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2qx} dy^2 + C^2 e^{2qx} dz^2 \quad (1)$$

where A, B and C are functions of cosmic time t only and q is a non-zero constant.

The field equations given by Saez and Ballester [8] for the combined scalar and tensor fields are

$$G_i^j - \omega V^n \left(V_{,i} V^{,j} - \frac{1}{2} g_i^j V_{,a} V^{,a} \right) = -T_i^j \quad (2)$$

and the scalar field satisfies the equation

$$2V^n V_{;i}^{i} + nV^{n-1} V_{,a} V^{,a} = 0 \quad (3)$$

where $G_i^j \equiv R_i^j - \frac{1}{2} g_i^j R$ is the Einstein tensor; n, an arbitrary exponent; and ω, a dimensionless coupling constant; T_i^j is the stress tensor of the matter. The energy momentum tensor for a perfect fluid distribution with heat conduction given by Ellis [2] as

$$T_i^j = (\rho + p) u_i u^j + p g_i^j + q_i u^j + u_i q^j \quad (4)$$

together with

$$g_{ij} u^i u^j = -1 \quad (5)$$

$$q_i q^i > 0 \quad (6)$$

and

$$q_i u^i = 0, \quad (7)$$

where p is the pressure, ρ is the energy density, q_i is the heat conduction vector orthogonal to u^i. The fluid vector u^i has the components

$\left(\frac{\sinh\lambda}{A},0,0,\cosh\lambda\right)$ satisfying Equation (5) and λ is the tilt angle. Here comma and semicolon denote ordinary and co-variant differentiation respectively.

With the help of Equations (3-7), the field Equation (2) for the metric (1) in the commoving co-ordinate system take the following explicit forms:

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}+\frac{q^2}{A^2}-\frac{\omega V^n V_4^2}{2}= -\left[(\rho+p)\sinh^2\lambda+p+2q_1\frac{\sinh\lambda}{A}\right] \quad (8)$$

$$\frac{C_{44}}{C}+\frac{A_{44}}{A}+\frac{C_4A_4}{CA}-\frac{q^2}{A^2}-\frac{\omega V^n V_4^2}{2}=-p \quad (9)$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{q^2}{A^2}-\frac{\omega V^n V_4^2}{2}=-p \quad (10)$$

$$\frac{A_4B_4}{AB}+\frac{B_4C_4}{BC}+\frac{C_4A_4}{CA}-\frac{q^2}{A^2}+\frac{\omega V^n V_4^2}{2}= (\rho+p)\cosh^2\lambda-p+2q_1\frac{\sinh\lambda}{A} \quad (11)$$

$$(\rho+p)A\sinh\lambda\cosh\lambda+q_1\cosh\lambda +q_1\frac{\sinh^2\lambda}{\cosh\lambda}=\frac{B_4}{B}-\frac{C_4}{C} \quad (12)$$

$$V_{44}+\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)V_4+\frac{n}{2}\frac{V_4^2}{V}=0 \quad (13)$$

Hereafterwards the suffix 4 after a field variable represents ordinary differentiation with respect to time.

3. Solutions

Equations (8-13) are six equations with eight unknowns A, B, C, p, ρ, V, λ and q_1, therefore, we require two more conditions.

First we assume that the model is filled with stiff fluid which leads to

$$p=\rho \quad (14)$$

We also assume that

$$A=B^n \quad (15)$$

In order to derive exact solutions of the field Equations (8-13) easily, we use the following scale transformations:

$$A=e^{n\beta},\quad B=e^{\beta},\quad C=e^{\gamma},\ dt=ABCdT \quad (16)$$

The field Equations (8-13) reduce to

$$\beta''-n\beta'^2-(n+1)\beta'\gamma'+\gamma''+q^2e^{2(\beta+\gamma)}-\frac{\omega v^n v'^2}{2}= -\left[(\rho+p)\sinh^2\lambda+p+\frac{2q_1\sinh\lambda}{e^{n\beta}}\right]e^{2[(n+1)\beta+\gamma]} \quad (17)$$

$$\gamma''-(n+1)\beta'\gamma'+n\beta''-n\beta'^2- q^2e^{2(\beta+\gamma)}-\frac{\omega v^n v'^2}{2}=-pe^{2[(n+1)\beta+\gamma]} \quad (18)$$

$$n\beta''-n\beta'^2-(n+1)\beta'\gamma'+ \beta''-q^2e^{2(\beta+\gamma)}-\frac{\omega v^n v'^2}{2}=-pe^{2[(n+1)\beta+\gamma]} \quad (19)$$

$$n\beta'^2+(n+1)\beta'\gamma'-q^2e^{2(\beta+\gamma)}+\frac{\omega v^n v'^2}{2}= \left[(\rho+p)\cosh^2\lambda-p+\frac{2q_1\sinh\lambda}{e^{n\beta}}\right]e^{2[(n+1)\beta+\gamma]} \quad (20)$$

$$(\rho+p)e^{n\beta}\sinh\lambda\cosh\lambda+q_1\cosh\lambda+ q_1\frac{\sinh^2\lambda}{\cosh\lambda}=(\beta'-\gamma')e^{-2[(n+1)\beta+\gamma]} \quad (21)$$

$$V''+\frac{n}{2}\frac{V'^2}{V}=0 \quad (22)$$

In view of Equation (14), Equation (17) and Equation (21), Equations (18,19), yield

$$\beta(=\gamma)=K_1T+K_2 \quad (23)$$

and

$$\alpha=(K_1T+K_2)^n \quad (24)$$

where $K_1(\neq 0)$, K_2 are arbitrary constants.

Thus the corresponding metric of our solution can be written as

$$ds^2=-T^{2n+2}dT^2+T^{2n}dX^2+T\left(e^{-2qx}dY^2+e^{2qx}dZ^2\right) \quad (25)$$

4. Some Physical and Geometrical Properties of the Solutions

On integration Equation (22) yields

$$V=\left(\frac{n+2}{2}K_3T+K_4\right)^{\frac{2}{n+2}} \quad (26)$$

where $K_3(\neq 0)$, K_4 are arbitrary constants.

Using Equations (23,24) and Equation (26) in Equations (19, 20), we get

$$p(=\rho)=\left[K_5^2+q^2e^{4(K_1T+K_2)}\right]e^{-(2n+4)(K_1T+K_2)} \quad (27)$$

where $K_5\left(=\left((2n+1)K_1^2+\frac{\omega K_3^2}{2}\right)\right)$ is a constant.

Substituting Equations (23),(26) and (27) in Equation (17) we get

$$\sinh^2\lambda = \frac{1}{2}\left[\left\{K_5^2 - q^4 e^{16(K_1T+K_2)}\right\}^{-1} e^{-(2n+4)(K_1T+K_2)} - 1\right] \tag{28}$$

Further substituting Equations (23), (27) and (28) in Equation (21), we get

$$q_1 = -e^{n(K_1T+K_2)}\left[K_5^2 + q^2 e^{4(K_1T+K_2)}\right]\times \left[\frac{1}{2}\left[\left\{K_5^2 - q^4 e^{16(K_1T+K_2)}\right\}^{-1} e^{-(2n+4)(K_1T+K_2)} - 1\right]^{\frac{1}{2}}\right]\times \left[e^{-(2n+4)(K_1T+K_2)} + \left\{K_5^2 - q^4 e^{16(K_1T+K_2)}\right\}\right] \tag{29}$$

The spatial volume for the model (25) is given by

$$\text{Vol.} = (K_1T + K_2)^{n+1} \tag{30}$$

From Equations (27-29) we find that the pressure, energy density, tilt angle, heat conduction vector of the fluid distribution are constants at time T=0 and gradually decreases in the course of evolution. Equation (26) shows that the scalar field V changes with time and at time T=0, the scalar filed is found to be a constant. Equation (30) implies the anisotropic expansion of the universe with time. It is interesting to note that the model does not admit singularity throughout evolution.

5. Conclusions

In this paper we have solved Saez and Ballester field equations for the tilted Bianchi VI_0 cosmological model. It is observed that the pressure, energy density, tilt angle, heat conduction vector of the fluid distribution are constants at time T=0 and gradually decrease with the increase of the age of the universe. It is interesting to note that the models we have constructed here is free from singularity at time T=0 and for $\omega = 0$ the Saez and Ballester [8] theory approaches general relativity. This supports the analysis that the introduction of scalar field avoids initial singularity.

6. Acknowledgements

The author would like to convey his sincere thanks and gratitude to the anonymous referee for his useful and kind suggestions for the improvement of this paper.

7. References

[1] A. R. King and G. F. R. Ellis, "Tilted Homogeneous Cosmological Models," *Communications in Mathematical Physics*, Vol. 31, 1973, p. 209.

[2] G. F. R. Ellis and A. R. King, "Was a Big Bang a Whimper," *Communications in Mathematical Physics*, Vol. 38, 1974, p. 119.

[3] C. B. Collins and G. F. R. Ellis, "Alternatives to the Big Bang," *Physics Reports*, Vol. 56, 1979, p. 65.

[4] R. Bali and K. Sharma, "Tilted Bianchi Type-I Models with Heat Conduction Filled with Disordered Radiations of Perfect Fluid in General Relativity," *Astrophysics and Space Science*, Vol. 271, 2000, p. 227.

[5] R. Bali and K. Sharma, "Tilted Bianchi Type-I Dust Fluid Cosmological Model in General Relativity," *Pramana*, Vol. 58, No. 3, 2002, p. 457.

[6] R. Bali and B. L. Meena, "Tilted Cosmological Models Filled with Disordered Radiations in General Relativity," *Astrophysics and Space Science*, Vol. 281, 2002, p. 565.

[7] J. M. Bradely and E. Sviestins, "Some Rotating Time-Dependent Bianchi Type VIII Cosmologies with Heat Flow," *General Relativity Gravitation*, Vol. 16, 1984, p. 1119.

[8] D. Saez and V. J. Ballester, "A Simple Coupling with Cosmological Implections," *Physics Letters A*, Vol. 133, 1986, p. 477.

[9] T. Sing and A. K. Agrawal, "Some Bianchi Type Cosmological Models in a New Scalar-Tensor Theory," *Astrophysics and Space Science*, Vol. 182, 1991, p. 289.

[10] D. R. K. Reddy and N. V. Rao, "Some Cosmological Models in Scalar-Tensor Theory of Gravitation," *Astrophysics and Space Science*, Vol. 277, 2001, p. 461.

[11] D. R. K. Reddy, R. L. Naidu and V. U. M. Rao, "Axially Symmetric Cosmic Strings in a Scalar Tensor Theory," *Astrophysics and Space Science*, Vol. 306, 2006, p. 185.

[12] G. Mohanty and S. K. Sahu, "Bianchi Type-I Cosmological Effective Stiff Fluid Model in Saez and Ballester Theory," *Astrophysics and Space Science*, Vol. 291, 2004, p. 75.

[13] G. Mohanty and S. K. Sahu, "Effective Stiff Fluid Model in Scalar-Tensor Theory Proposed by Saez and Ballester," *Communications in Physics*, Vol. 14, No. 3, 2004, p. 178.

[14] K. S. Adhav, M. R. Ugale, C. B. Kale and M. P. Bhende, "Bianchi Type VI Sring Cosmological Model in Saez and Ballester's Scalar Tensor Gravitation," *International Journal of Theoretical Physics*, Vol. 46, 2007, p. 3122.

[15] S. K. Tripathy, S. K. Nayak, S. K. Sahu and T. R. Routray, "String Cloud Cosmologies for Bianchi Type-III Models with Electromagnetic Field," *Astrophysics and Space Science*, Vol. 315, 2008, p. 105.

