Robust adaptive generalized function projective synchronization of nonidentical chaotic systems with time-varying parameters

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Abstract: In this paper, the generalized function projective synchronization (GFPS) of different chaotic systems with unknown time-varying parameters is investigated. Based on Lyapunov stability theory, the robust adaptive control law and the parameter update law are derived to make the states of two different chaotic systems asymptotically synchronized. The scheme is successfully applied to the generalized function projective synchronization between hyper-chaotic system and generalized Loren system. Moreover, numerical simulations results are used to verify the effectiveness of the proposed scheme.

Keywords: chaotic systems; function projective synchronization; adaptive control; time-varying parameters

1 Introduction

Since the idea of synchronizing two identical autonomous chaotic systems under different initial conditions was first introduced in 1990 by Pecora and Carroll [1], chaos synchronization has been widely studied in physics, secure communication, chemical reactor, biological networks and artificial neural networks. Up to now, different types of synchronization phenomena have been presented such as complete synchronization (CS) [2], phase synchronization[3], anti-phase synchronization [4], etc. Also many control schemes such as the OGY method [5], delayed feedback method [6], adaptive control method [7], active control [8] approach have been employed to synchronize chaotic systems with different initial conditions.

Among all kinds of chaos synchronization schemes, projective synchronization, characterized by a scaling factor that two systems synchronize proportionally, has been extensively investigated by many authors [9-10]. This is because it can obtain faster communication with its proportional feature. However, most of investigations have concentrated on studying the constant scaling factor. Recently, a new kind of synchronization-function projective synchronization (FPS) was introduced by Tang et al [11] and Du [12]. Function projective synchronization is the more general definition of projective synchronization. As compared with projective synchronization, function projective synchronization means that the drive and response systems could be synchronized up to a scaling function, which is not a constant. This characteristic could be used to get more secure communication in application to secure communications. This is because the unpredictability of the scaling function in FPS can enhance the security of communication.

On the other hand, chaotic systems are unavoidably exposed to an environment which may cause their parameters to vary within certain ranges such as environment temperature, voltage fluctuation, mutual interfere among components, etc. As a result, in the studies [13–16] on control and synchronization of chaos, the problem of parametric uncertainty is a very significant and challenging one. In these research studies the most common method used to solve the parametric uncertainties is adaptive control schemes in which the unknown system parameters are updated adaptively according to certain rules. For example, in [13–14], it is assumed that the parameters of the driving system are totally uncertain or unknown to the response system. and the parameters of the response system can be different from those of the driving system. Some studies suppose that the parameters of the driving and the response systems are identical but there are also some parametric uncertainties or perturbations [15–16].

Motivated by the aforementioned discussion, we will formulate the GFPS problem of different chaotic systems with unknown time-varying parameters. A robust adaptive synchronization method is proposed. By adding a compensator in the input vector, the uncertainties of the parameters in the Lyapunov function is eliminated. And by a parameter updating law, the nominal value of the unknown time-varying parameter can be estimated. Based on Lyapunov stability theory, this controller can achieve the robust adaptive synchronization of a class of chaotic system with time-varying unknown parameters. Some typical chaotic and hyper-chaotic systems are taken as examples to illustrate our technique.

The rest of this paper is organized as follows. In Section 2, the definition of GFPS is introduced. In Section 3, the general method of GFPS is studied. In Section 4, two numerical examples are used to confirm the effectiveness of the proposed scheme. The conclusions are discussed in Section 5.Throughout this paper, the notation P^{T} denotes

the transpose of a vector P, while for $x \in R^n$, the notation $\|x\| = \sqrt{(x^{\mathrm{T}} x)}$ stands for the Euclidean norm of the vector

2 The definition of GFPS

The drive system and the response system are defined as follows

$$\dot{x} = f(x,t) \tag{1}$$

$$\dot{y} = g(y,t) + u(x,y,t) \tag{2}$$

where $x, y \in R^n$ are the state vectors, $f, g : R^n \to R^n$ are continuous nonlinear vector functions, $u(x,y,t)$ is the control vector. We describe the error term

$$e(t) = x - \alpha(t) y \tag{3}$$

where $\alpha(t)$ is a continuously differentiable bounded function and $\alpha(t) \neq 0$ for all t.

Definition1(GFPS). For two different systems described by Eqs. (1) and (2), we say they are globally generalized function projective synchronous (GFPS) with respect to the scaling matrix $\Upsilon = \alpha(t) I_{n\times n}$ ($\alpha(t)$ is a nonzero scaling function and $I_{n\times n}$ is the identity matrix) if there exists a vector controller $u(x,y,t)$ such that all trajectories

$(x(t), y(t))$ in (1) and (2) with any initial conditions $(x(0), y(0))$ in $Q = R_x^n \times R_y^n \subset R^n \times R^n$ approach the manifold $M = \{x(t), y(t) : x = \alpha(t) y\}$ with $M \subset Q$ as time t goes to infinity, that is to say, $\lim_{t\to\infty} \|x - \alpha(t) y\| = 0$. This implies that the error dynamical system between the drive system and response system is globally asymptotically stable.

Remark1. From the definition of GFPS, we can find that the definition of generalized function projective synchronization includes generalized projective synchronization when the scaling function $\alpha(t)$ is equal to a constant α

3 Design of the scheme of GFPS

Consider a class of chaotic system with unknown time-varying parameters, which is described by

$$\dot{x} = Ax + f(x) + D(x)\theta(t) \tag{4}$$

where $x \in R^n$ is the state vector, $A \in R^{n\times n}$ and $f(x)$: $R^n \to R^n$ are the linear coefficient matrix and nonlinear part of system (1), respectively. $D(x) : R^n \to R^{n\times p}$ and $\theta(t) = (\Phi + \Delta\Phi(t)) \in R^p$ is the uncertain parameter vector. Here Φ is the nominal value of $\theta(t)$ and $\Delta\Phi(t)$ is the uncertainty or disturbance. Eq.(4) is considered as the drive system. The response system with a controller $u(x,y,t) \in R^n$ is introduced as follows:

$$\dot{y} = By + g(y) + u(x,y,t) \tag{5}$$

where $y \in R^n$ is the state vector, $B \in R^{n\times n}$ and $g(y)$: $R^n \to R^n$ are the linear coefficient matrix and a continuous continuous nonlinear vector function and $u(x,y,t)$ is the control vector.

Remark2. The class of nonlinear dynamical systems includes an extensive variety of chaotic systems such as Lorenz system, Rössler system, Duffing system, Chua's circuit, generalized Lorenz system, etc.

Remark3. When $B = A$ and $g = f + D(x)\theta(t)$, the synchronization mentioned above is the GFPS of identical chaotic systems.

Assumption1. The norm of $\Delta\Phi(t)$ is bounded, that is

$$\|\Delta\Phi(t)\| \le M \tag{6}$$

for all $t \in R^+$, where $M \in R^+$ is the supremum of the norm of $\Delta\Phi(t)$. The dynamic equation of synchronization error (3) can be obtained easily by Eqs.(4) and (5), which is expressed as follows:

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$$\dot{e} = \dot{x} - \dot{\alpha}(t)y - \alpha(t)\dot{y}$$
$$= Ax + f(x) + D(x)\Phi + D(x)\Delta\Phi(t)$$
$$-\dot{\alpha}(t)y - \alpha(t)By - \alpha(t)g(y) - \alpha(t)u(x,y,t) \tag{7}$$
$$u = \frac{1}{\alpha(t)}[e + Ax + f(x) + D(x)\hat{\Phi}(t)$$
$$+\beta(e,x,M) - \dot{\alpha}(t)y - \alpha(t)By - \alpha(t)g(y)] \tag{8}$$

we select the controller in the following form:

where $\beta(e,x,M)$ is a compensator to be designed to compensate the uncertainties. $\hat{\Phi}(t)$ is updated by the updating law. Then, Eq. (7) can be formulated as

$$\dot{e} = -e + D(x)\tilde{\Phi} + D(x)\Delta\Phi(t) - \beta(e,x,M) \tag{9}$$

where $\tilde{\Phi} = \Phi - \hat{\Phi}(t)$.

Note the synchronization of two chaotic dynamical systems is essentially equivalent to stabilizing their corresponding error dynamical system at the origin, that is to say, two chaotic systems synchronize if the zero solution of their error system is asymptotically stable. In this paper, the robust adaptive controller should satisfy that the solution of Eq. (9) stable at $e = 0$.

We choose the Lyapunov function

$$V(t) = \frac{1}{2}e^{\mathrm{T}}(t)e(t) + \frac{1}{2}\tilde{\Phi}^{\mathrm{T}}(t)\tilde{\Phi}(t) \tag{10}$$

If $V(t)$ is positive definite and $\dot{V}(t)$ is negative definite, the synchronization will be achieved.

Theorem1. Under the Assumption 1, for given synchronization scaling function $\alpha(t)$ and any initial conditions $x(0), y(0)$, then there is a compensator and a parameter updating law

$$\beta(e,x,M) = M\frac{D(x)D^{\mathrm{T}}(x)e}{\left\|e^{\mathrm{T}}(t)D(x)\right\|} \tag{11}$$

$$\dot{\hat{\Phi}} = D^{\mathrm{T}}(x)e(t) \tag{12}$$

such that the error system (9) is globally stable. Then, the GFPS of different chaotic systems is achieved under the control law (8), the compensator (11) and the parameter updating law (12).

$$\dot{V}(t) = -e^{\mathrm{T}}(t)e(t) + e^{\mathrm{T}}(t)D(x)\Delta\Phi(t) - e^{\mathrm{T}}(t)\beta(e,x,M)$$
$$\le -e^{\mathrm{T}}(t)e(t) + \left\|e^{\mathrm{T}}(t)D(x)\right\|\left\|\Delta\Phi(t)\right\| - e^{\mathrm{T}}(t)\beta(e,x,M)$$
$$\le -e^{\mathrm{T}}(t)e(t) + \left\|e^{\mathrm{T}}(t)D(x)\right\|M - e^{\mathrm{T}}(t)\beta(e,x,M)$$

We note that

$$\left\|e^{\mathrm{T}}(t)D(x)\right\|M - e^{\mathrm{T}}(t)\beta(e,x,M)$$
$$= \left\|e^{\mathrm{T}}(t)D(x)\right\|M - M\left\|e^{\mathrm{T}}(t)D(x)\right\|$$
$$= 0.$$

Thus we have

Proof. For Eqs. (9) and (12), select the Lyapunov function as Eq.(10).Then its derivative along the error dynamical system (9) is

$$\dot{V}(t) \le -e^{\mathrm{T}}(t)e(t)$$

then $\dot{V}(t) < 0$, when $\left\|e(t)\right\| \ne 0$.

Since $V(t)$ is positive definite and $\dot{V}(t)$ is negative definite, based on the Lyapunov stability theory, the error system (9) is asymptotically stable, this completes the proof.

4 Numerical simulations

In this section, one examples is presented to show the effectiveness of the proposed robust adaptive controller. Consider the generalized projective synchronization between hyper-chaotic Chen system

$$\begin{cases} \dot{x}_1 = \theta_1(t)(x_2 - x_1) + x_4 \\ \dot{x}_2 = -x_1x_3 + \theta_2(t)x_1 + \theta_3(t)x_2 \\ \dot{x}_3 = x_1x_2 - \theta_4(t)x_3 \\ \dot{x}_4 = x_2x_3 - \theta_5(t)x_4 \end{cases} \tag{13}$$

and the generalized Lorenz system

$$\begin{cases} \dot{y}_1 = y_2 - y_1 + 1.5y_4 + u_1 \\ \dot{y}_2 = -y_1y_3 + 26y_1 - y_2 + u_2 \\ \dot{y}_3 = y_1y_2 - 0.7y_3 + u_3 \\ \dot{y}_4 = -y_1 - y_4 + u_4 \end{cases} \tag{14}$$

Comparing system (13) and (14) with Eqs. (4) and (5), we get that

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad D(x) = \begin{bmatrix} x_2 - x_1 & 0 & 0 & 0 & 0 \\ 0 & x_1 & x_2 & 0 & 0 \\ 0 & 0 & 0 & -x_3 & 0 \\ 0 & 0 & 0 & 0 & -x_4 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 1 & 0 & 1.5 \\ 26 & -1 & 0 & 0 \\ 0 & 0 & -0.7 & 0 \\ -1 & 0 & 0 & -1 \end{bmatrix}, \quad f = \begin{pmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \\ x_2x_3 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ -y_1y_3 \\ y_1y_2 \\ 0 \end{pmatrix}$$

The unknown time varying parameter vector

$$\theta(t) = [35 + 0.2\sin t, 7 + 0.2\cos t, 12 - 0.2\sin t, 3 + 0.2\cos t, 0.5 - 0.2\sin t]^{\mathrm{T}}$$

If the system parameters are chosen to be $\theta_1(t) = 35$, $\theta_2(t) = 7$, $\theta_3(t) = 12$, $\theta_4(t) = 3$, $\theta_5(t) = 0.5$, then hyper-chaotic Chen system have chaotic attractor. The behavior of the hyper-chaotic Chen system with $\theta(t) = [35 + 0.2\sin t,$

$7+0.2\cos t, 12-0.2\sin t, 3+0.2\cos t, 0.5-0.2\sin t]^{\mathrm{T}}$ is shown in Figure.1 and we get the nominal value of the parameter vector $\Phi=[35,7,12,3,0.5]^{\mathrm{T}}$. Thus $\Delta\Phi=[0.2\sin t, 0.2\cos t, -0.2\sin t, 0.2\cos t, -0.2\sin t]^{\mathrm{T}}$ .The maximum norm of $\Delta\Phi$ can be derived as $M=\sqrt{0.1}$, and the initial estimated values of the unknown parameters Φ are $\hat{\Phi}(0)=[0,0,0]^{\mathrm{T}}$. The initial states of the drive system and response system are chosen as $x(0)=[-3,0,0,5]^{\mathrm{T}}$ and $y(0)=[0.8,1.2,-0.8,0.8]^{\mathrm{T}}$ respectively. By taking the scaling function $\alpha(t)=100+100\sin(2\pi t/120)$, according to the Theorem1, we conclude that the control vector u along with the parameter updating law given by Eqs. (8), (11) and (12) will achieve the generalized projective synchronization of the hyper-chaotic Chen system and generalized Lorenz system, as verified by the simulation results shown in Figure.2. Furthermore, Figure.3 depicts the time evolution of the controllers and Figure.4 shows the evolution of the estimated parameter

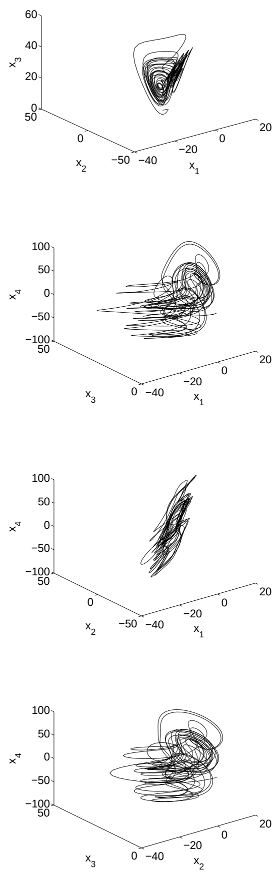


Figure 1. Chaotic behavior of hyper-chaotic Chen system with $\theta(t)$

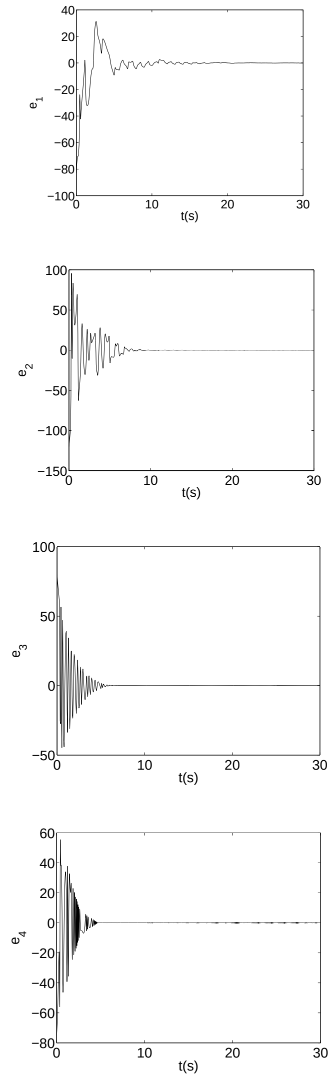
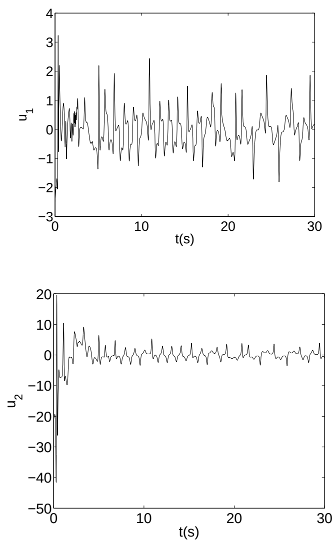


Figure 2. The time evolution of the GFPS errors.



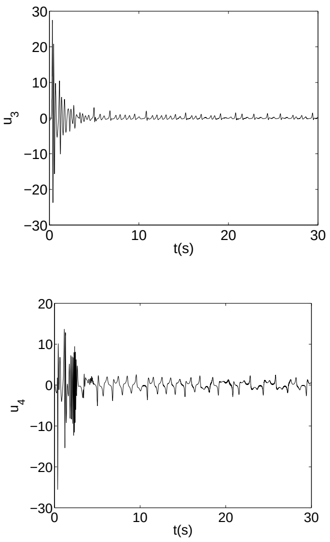
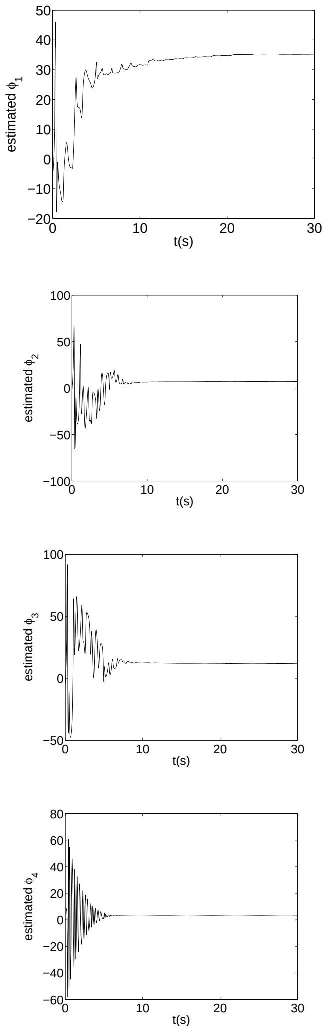


Figure 3. The time evolution of the controllers.



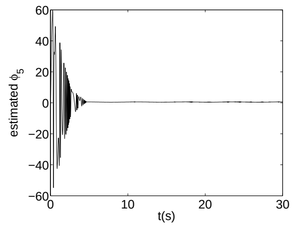


Figure 4. The time evolution of the estimated parameters.

5. Conclusions

In this paper, we have introduced the definition of GFPS and given the GFPS scheme of a class chaotic system with unknown time varying parameters. Based on the Lyapunov stability theory, a robust adaptive controller and the parameter update law are obtained for the stability of the error dynamics between the drive and the response systems. where the parameters are unknown and where there are also uncertainties in the parameters. if the uncertainties of the parameters are bounded. The simulation results show the effective performance of the proposed controller.

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