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REM Sleep Effects on Trauma, Depression, Anxiety

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Abstract

It is well established that during REM (Rapid Eye Movement) sleep, the brain processes emotions associated with stress and traumatic events. When significant traumatic exposures overwhelm this system or when REM sleep debt occurs, alterations arise that can result in posttraumatic stress disorder (PTSD), depression, and/or anxiety. A mathematical model is derived and calibrated to simulate this process. The model includes three independent variables: number of traumatic exposures, REM sleep debt (immediately before, during, and after traumatic exposure), and relative fitness related to aerobic exercise and cardiorespiratory fitness. The model was calibrated with laboratory REM sleep measurements and a known number of trauma expressions in a video. The number of memory intrusions was used to estimate relative risk of developing PTSD. After calibration, the model was applied to published data for military personnel on deployment. While exact inputs of REM debt and number of traumatic exposures are unknown for the military data, the mathematical model predicted results close to published relative risk for PTSD using reasonable estimates of these variables. Risks for depression and anxiety are often related to exposure to stress and trauma. Various data sources were used to calibrate and evaluate the benefits of aerobic exercise and the associated cardiorespiratory fitness on risks and treatment for mood, depression, PTSD, and anxiety. The model, supported by measured data in the literature, illustrates the importance of lifestyle choices early in life on health outcomes later in life.

Keywords

Trauma, PTSD, Depression, Anxiety, REM Sleep Debt, Traumatic Events, Mathematical Model, Mental Health, Aerobic Exercise, Relative Fitness, VO_{2max}

1. Introduction

Trauma is past emotional experiences interfering with current life. It's an imprint left by experiences on our brain and body. Bessel van der Kolk [1] in his book, ***The Body Keeps The Score***, reports that trauma leaves "traces on our minds and emotions, on our capacity for joy and intimacy, and even on our biology and immune systems." He goes on to say, "Trauma, whether it is the result of something done to you or something you yourself have done, almost always makes it difficult to engage in intimate relationships." We typically associate severe trauma with PTSD (Post Traumatic Stress Disorder) with experiences in war, but trauma also occurs in civilian life. Van der Kolk [1] reports that 1/5 Americans are sexually molested as a child, 1/4 are beaten as a child, and 1/3 of married couples engage in physical violence. Various studies indicate that PTSD occurs 5 to 12 percent over a lifetime in the general population [2]-[6]. Osgood *et al.* [7] reported 9.5 percent of a combat team surveyed 3 months post-deployment to Afghanistan met screening criteria for PTSD. Women have two to three times higher risk of developing PTSD than men [8]. Thirty to 40 percent of women 3 months after rape have PTSD [9] [10]. Trauma either directly or indirectly affects all of us.

Trauma associates with and causes much of the diagnosed depression, anxiety, white matter hyperintensities (WMH), and strokes that occur throughout life [11]. Brown [12] discusses how exposure to stress or traumatic events and embarrassment leads to depression. Thurston *et al.* [13] studied the relationship between sexual abuse in women and the development of WMH in parts of the brain later in life. They found that sexual abuse in young women was associated with increased levels of WMH later in life. Giese *et al.* [14] found a strong correlation between WMH and stroke. Gregory [15] developed an equation to predict the increased risk of WMH accumulation and stroke occurring as a function of age, traumatic events, and other risk factors usually associated with stroke patients. Unless treated, trauma early in life can result in major health risks later in life.

Exposure to trauma early in life as a child or teenager can increase the risk for PTSD, depression, anxiety, and even migraine headaches [16]. Migraines are up to three times more common in women than in men, and girls are more susceptible to the adverse effects of sexual abuse [17]. Traumatic experiences also lead to structural changes in the brain [18].

Kornerup *et al.* [19] reported that risk of stroke increases as the number of trauma exposures increases. Their work illustrates the long-term effects of trauma exposure and that multiple exposures increase this risk.

Memory intrusions are a characteristic of PTSD. Memory intrusions can also occur after watching traumatic footage used in trauma studies. The intrusion response increases with the degree of emotional response ([3] [20]. Multiple sensory stimulation (visual and sound) produces more intrusive memories than a single sensory input [3] [21] providing further evidence that signal strength of the traumatic emotion affects the intrusive memories that occur after trauma exposure.

Repantis *et al.* [22] state that "traumatic events build strong memories." They

also report a close link between REM sleep and the consolidation of emotional memories. Sleep or lack of sleep strongly affects consolidation of memories [23] [24].

Sleep consists of stages that occur in cycles. A typical young adult will have 4 or 5 cycles completed in a night of sleep. A typical sleep cycle takes about 90 minutes starting with stage 1 (transition from awake to sleep) followed by stage 2 then stages 3 and 4 (slow-wave sleep) then back to stage 2 followed by REM sleep. The cycle then repeats starting with stage 2. Slow-wave sleep provides the brain an opportunity to flush waste materials, such as amyloid β and tau when the glial cells shrink and, thus, is involved in long-term brain health. Slow-wave sleep has the longest durations during the first cycles, then decays down to only brief episodes during the latter part of the night. REM sleep durations start with short durations followed by longer episodes as sleep progresses.

Memory consolidation occurs during stages 2, slow-wave, and REM sleep. Van Rijn *et al.* [23] reported that the process of consolidation starts with slow-wave sleep during which non-adaptive memories are weakened and adaptive memories are strengthened. Walker [24] reports that stage 2 is also involved in the transfer process to clear the hippocampus of current memories. Van Rijin *et al.* [23] report that the second step occurs during REM sleep where adaptive memories are integrated with previous memories. Recently acquired memories are stored in the hippocampus and after normal processing important memories are transferred to neocortical structures [23].

Pace-Schott *et al.* [25] provide the following conclusions:

> Sleep, acting as a modulator of physiological stress and emotional memory, is crucial importance in maintaining day-to-day emotional homeostasis and long-term mental health. Sleep disturbance predating or acutely resulting from a traumatic event, particularly if it develops into chronic insomnia, may initiate positive feedback and allostatic mechanisms that impair emotional regulation and promote the pathophysiology of PTSD.

MRI scans can detect a history of emotional scarring or trauma in brain regions such as the hippocampus, amygdala, anterior cingulate cortex, and insular cortex, which engage in processing emotions and sleep [8] [26]. Nevertheless, it is difficult to put a number on this change in terms of risk assessment. Brain chemistry changes in association with stress or trauma.

Objective

The objective of this work is to provide a mathematical model to simulate the risk of developing trauma including the variable REM sleep debt and the number of exposures to traumatic events. A third variable, relative fitness, an age-independent nongender measure of cardiorespiratory fitness [27], will be added to complete a model to predict the risk of trauma, depression, and anxiety in response to exercise.

2. Methods

Data for calibration and testing is limited. Precise measurement of sleep components associated with individual traumatic events is usually missing and can only be estimated after the event through survey questions. People usually are not expecting to be exposed to traumatic events; thus, sleep information prior to trauma exposure is only an estimate and lacks information about sleep stages. Sleep debt, especially REM sleep debt may exist from prior sleep behavior. Estimates of sleep the night before the event often do not represent the total sleep debt condition.

Sleep measurements after trauma exposure are also difficult to obtain. Repantis *et al.* [22] report that it is difficult to find acutely traumatized individuals willing to accept polysomnography in the first hours after their trauma and initial treatment.

An alternative method exists where sleep measurements can be obtained in a laboratory setting and combined with an artificial exposure to trauma events. Ethically, this exposure must be weak enough not to cause long-term health risks. Nevertheless, this procedure produces intrusive memories that can be counted and related to symptoms from PTSD. Calibration for the model developed in this paper will be limited to this alternative with sleep measurements. Even this method may not address sleep changes that occur after artificial trauma exposure, but this process should provide the most reliable calibration of initial trauma exposure interacting with sleep measurements.

There is evidence of a transition period after trauma exposure where symptoms associated with trauma decrease. After this period, some people will be symptom-free. Others will continue to have PTSD with no further short-term change unless some form of treatment is administered (Figure 1). REM sleep is expected to be part of the memory consolidation process during this period.

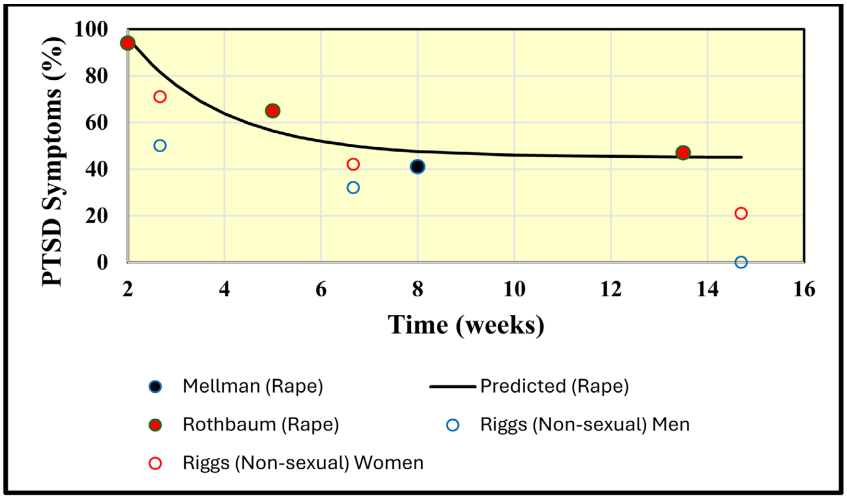

Figure 1. Transition from a two-week period with high percentage of PTSD symptoms to a leveling off for rape of women. Note that non-sexual trauma continues to decline within the 16-week measurement period. Data from Mellman *et al.* [9], Rothbaum *et al.* [10], and Riggs *et al.* [28].

Note that the reporting of PTSD data in Figure 1 does not start until about 2 weeks after the trauma occurrence. Also, note that sleep effects could be occurring or affecting recovery as much as 2 to 4 months after the trauma event. Another advantage of the detailed sleep measurements and exposure to trauma videos is that the intrusive memories tend to decay out of memory within one or two weeks. This short time period allows the measured sleep information to be more focused on the initial trauma.

Data shown in Figure 1 for real trauma occurring in a civilian environment illustrate much of the nature of trauma studies: limited number of measured points, differences associated with sex, and differences associated with type of trauma exposure. The data also illustrates a dynamic effect that occurs with time after exposure. Because of this effect, minor or weak trauma exposures, such as those used in controlled experimental studies, are expected to decay to no effect after a relatively short period of time following the study. The assumption in these studies is that there is a connection between the initial response and the observed diagnosed condition of PTSD or other outcomes associated with trauma. This limitation or assumption carries over to any mathematical model that is calibrated on data from these laboratory studies.

2.1. Development

The change in risk of developing trauma can be mathematically described with the following partial differential equation.

$$\frac{\mathrm{d}\tau}{\mathrm{d}t}=\frac{\partial\tau}{\partial n}+\frac{\partial\tau}{\partial RD}+\frac{\partial\tau}{\partial e}+\frac{\partial\tau}{\partial tr}+\cdots \tag{1}$$

where, τ = trauma risk (PTSD, depression, anxiety);

t = time (generally length of life or age);

n = number of traumatic events;

rd = REM debt;

e = aerobic exercise;

tr = treatment(s) (not evaluated in this paper).

There is some evidence that genetics also affects susceptibility to trauma [29]. Social and economic conditions also affect outcomes [19]. Thus, there may be other variables that need to be added later. In theory, the sum of all partial changes equals the total change over time.

2.2. Effect of Number of Trauma Exposures

The next step is to develop an equation for each partial change in risk and to integrate over the range of the independent variable to mathematically exit the partial or differential space. A derivation for the first partial term is presented next.

The change in accumulated trauma can be expressed as a function of existing trauma that an individual has times the change in trauma events:

$$\frac{\mathrm{d}\tau}{\mathrm{d}t}=C\tau\frac{\mathrm{d}n}{\mathrm{d}t} \tag{2}$$

where, t = time;

C = calibration coefficient that may vary with REM sleep debt;

τ = trauma risk;

n = number of traumatic events.

This equation implies that stress effects are cumulative [19] [29] and non-linear.

Next, a separation of variables to group all the trauma components leads to

$$\frac{\mathrm{d}\tau}{\tau} = C\mathrm{d}n \tag{3}$$

Note that the $\mathrm{d}t$ variables cancel each other.

Equation (3) can be integrated using boundary conditions for both sides:

$$\int_i^{T_\tau} \frac{\mathrm{d}\tau}{\tau} = C\int_0^N \mathrm{d}n \tag{4}$$

where, T_τ = final trauma risk;

i = initial or reference trauma;

N = number of traumatic events.

Integration results in

$$\ln T_\tau - \ln i = CN \tag{5}$$

Equation (5) can be rewritten as

$$\ln \frac{T_\tau}{i} = CN \tag{6}$$

The natural log can be removed by taking the exponential of both sides:

$$T_\tau = i\mathrm{e}^{CN} \tag{7}$$

If we now let the variable i have a value of 1.0 for a reference population, Equation (7) expresses the risk of developing trauma expressed as a risk relative to the reference conditions:

$$T_\tau = \mathrm{e}^{CN} \tag{8}$$

When the value for N is zero (no major traumatic encounters), the trauma level is one. This is the reference condition. Equation (8) does not contain a sleep variable: only the number of traumatic events.

The sum of logs results in a product in normal space. Thus, we should be able to express the solution of Equation (1) as a product of solutions to each partial term in Equation (1).

Gregory [15] used Equation (8) to model the effect of number of major traumatic events on the risk of having a stroke later in life as age increases. The results for data reported by Kornerup *et al.* [19] as part of the Copenhagen City Heart Study are shown in **Figure 2**. Kornerup *et al.* [19] presented their data in terms of relative risk for exposure expressed in ranges of trauma events: 1 - 2, 3 - 4, 5 - 6. Mean values were not given. The average of the range limits was used as an estimate of each mean. The value of C in Equation (8) is 0.074 based on the data from Kornerup *et al.* [19]. As calibrated, the values predicted with Equation (8) resulted

in an R^2 value of 0.969. There are only 4 points in terms of statistical significance. Nevertheless, with the high R^2, the equation has significance based on Student's test ($p < 0.01$). Number of traumatic events was obtained by a survey for lifetime exposure [19]. The average number of traumatic events was 2.6 for women and 2.4 for men in this study. With over 6,000 people in this study, these results indicate that the increasing risk of stroke associated with trauma exposure is both non-linear and that Equation (8) does a reasonable job of mathematically modeling this risk.

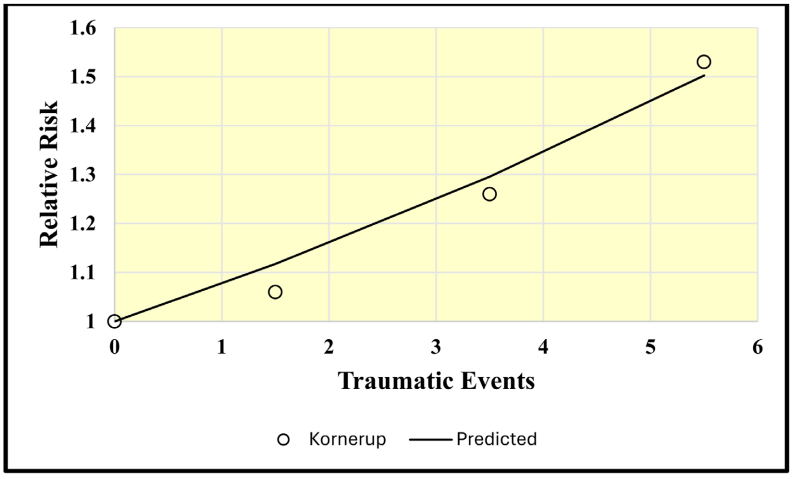

Figure 2. Relative risk of stroke for men and women starting at baseline of 59.1 (women) and 56.6 (men) years of age. Data from Kornerup *et al.* [19].

2.3. Effect of REM Sleep Loss on Short-Term Stress

There is evidence that short-term mental health symptoms can be generated through loss of sleep, especially REM sleep [30]. While these symptoms may not express PTSD, they certainly include anxiety, irritability, and difficulty concentrating. The second term in Equation (1) adds the effect of REM sleep debt.

If we replace ∂n with ∂rd in Equation (1) and use the same development procedure as used from Equation (2) through Equation (8), the following equation is obtained for the effects of REM debt:

$$T_\tau = \mathrm{e}^{KRD} \tag{9}$$

where, K = calibration coefficient;

RD = REM sleep debt before, during, and after trauma exposure.

The study by Dement [30] is interesting. It's an old study done shortly after the discovery of REM sleep. It was known then that dreaming occurs during REM sleep. All major sleep components were known by this time. REM could be determined in two ways: brainwave pattern that looks a lot like being awake and rapid eye movement. In this experiment, the participants were allowed to sleep except for REM sleep. When subjects began to experience REM sleep, they were awakened to stop REM and then allowed to return to sleep. Dement reported psychological disturbances. One subject quit the study in an apparent panic attack after three days (nights) of REM debt. Two others quit after four nights. They certainly

were mentally stressed; temporarily traumatized. Dement's discussion is not precise, but it implies that all eight subjects were stressed by the time the study was over. One subject went seven days before his night of recovery sleep.

Data in Figure 3 were calculated by setting relative risk of trauma for zero REM debt to 1 (first data point in Figure 3 then adding the increase from the fraction of people in the study who experienced experiment-ending trauma). There were only eight men (age ≈ 27 years) in this study. One person quit after three days of sleep causing REM debt. This produced a fractional increase of 0.125. This value added to 1 provided the second data point at three days of REM debt. The next point at four days of REM debt was calculated for three out of eight people having trauma symptoms (fractional increase = 0.375). Because all subjects in the end were stressed, the last point associated with day 7 was set at the upper boundary condition of 100 percent affected 2 (1 + 1). Seven of the subjects stopped after five nights. The stress risk for 7 out of 8 subjects is 1.875. Dement reported that he was not able to achieve a 100 percent loss of REM sleep. He estimated that the real REM debt was between 65 and 75 percent. The reported days of REM sleep debt were multiplied by 0.7 to estimate the real REM sleep debt used in Figure 3.

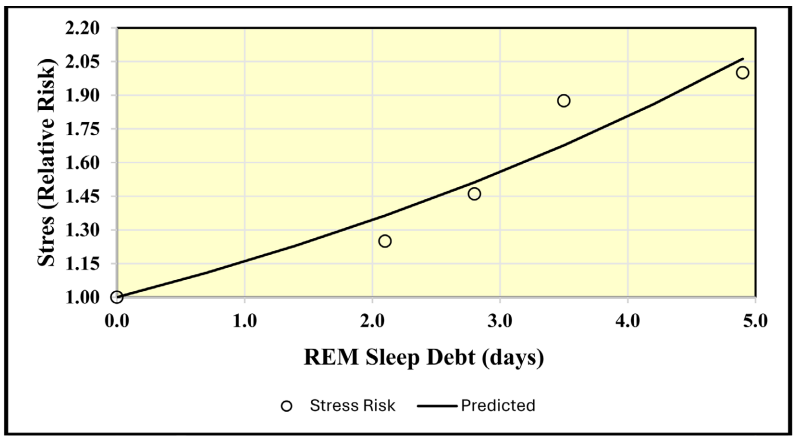

Figure 3. Short-term mental stress from exposure to REM sleep debt. Data calculated from [30].

While there are only five points in this dataset, only one calibration coefficient had to be determined. A value of 0.1477 was determined leading to an R^2 value of 0.916 ($p < 0.01$).

Next, it was observed that the total dream or REM time that occurred during the first night of recovery also seemed to vary with the number of nights of REM debt before recovery. This recovery represented individual response to the REM debt. The observed recovery above baseline amount was about 0.1 times the total previous REM debt. Dement reported that "the psychological changes disappeared as soon as the subjects were allowed to dream." In other words, dreaming during REM sleep removed or treated the traumatic symptoms accumulated during REM debt. Now, more than 50 years later it is known that dreaming about our troubling events during REM sleep results in memory consolidation that leads to

emotional healing [24].

If we combine the results so far, we obtain the following equation:

$$T_\tau = \mathrm{e}^{CN}\mathrm{e}^{KRD} \tag{10}$$

Simplification results in

$$T_\tau = \mathrm{e}^{CN+KRD} \tag{11}$$

The increase in trauma risk as predicted with Equation (11) is in general agreement with observations that sleep problems especially with REM sleep increase the risk associated with exposure to traumatic events [31]-[33].

Because REM debt can be removed by REM sleep, this variable can be temporary. It seems minor compared to the number of traumatic events. But many people have REM debt on a continuous basis. Most of the REM sleep occurs during the second half of the sleep period. Thus, if people use an alarm clock to get up in the morning, they have a degree of REM debt. Most high school students tend to go to bed late at night but must get up early to go to school. They, thus, have sleep debt, much of which is REM debt. People who have alcohol in their system at bedtime will delay the onset of REM sleep proportional to the amount of alcohol in the system causing REM sleep debt [34]. These people are often irritable the next day which we attribute to hangover. Caffeine in the system behaves much like alcohol causing a delay in the onset of REM sleep. Another common factor that limits REM sleep is obstructive sleep apnea. Most of us have or will have some REM debt as part of our daily routine.

There is indirect evidence that the C coefficient is a function of REM sleep debt. Alkalame *et al.* [33] ran a controlled experiment using intrusive memories as a measure of risk for developing future PTSD. They report that "Intrusive memories are a common experience following trauma exposure that can develop into a symptom of posttraumatic stress disorder (PTSD)." They used a circadian advancement over a two-day period to create REM debt. Twenty-seven healthy participants aged 25.4 years on average completed the experiment. The participants spent four nights in a sleep lab where the various sleep stages were monitored including REM sleep. On the fifth day, they watched a trauma film for 12 minutes including 11 distressing clips. The subjects were released from the lab but required to keep a written diary over the next three days of intrusive memories related to the trauma film.

The results of this experiment were presented in a graph of the number of intrusive memories as a function of REM sleep percentages expressed in standard deviations from mean REM percentage. Their average REM sleep percentage was 23.1 with a standard deviation of 4.03. Data were retrieved from their graph for five REM points associated with the average and standard deviation multiples. The maximum REM amount was used as reference for REM sleep needed and defined as the zero REM debt. The reduction in REM sleep for each of the other points was divided by this reference REM amount. This fraction was used as the REM debt as shown in **Figure 4**.

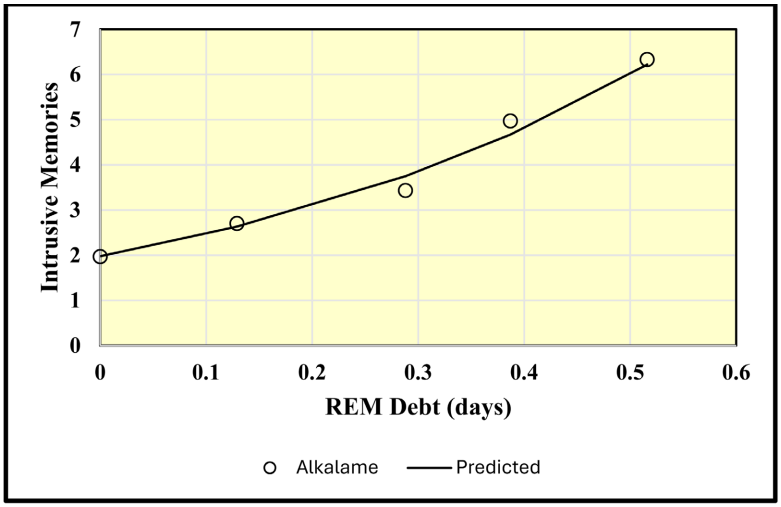

Figure 4. Measured and predicted intrusive memories as a function of REM debt. Data points extracted from Alkalame *et al.* [33].

The measurements made by Alkalame *et al.* [33] included intrusive memories from all 11 trauma events. The N value in Equation (11) was set to 11. The simplest formulation to include REM debt in this equation and satisfy the boundary condition of intrusive memories at zero REM Debt was to add a REM debt component to the C coefficient:

$$T_\tau = \mathrm{e}^{(C+jRD)N+KRD} \tag{12}$$

where, j = calibration coefficient for REM debt interacting with traumatic events.

The data from [33] were used to determine the j coefficient in Equation (12). Initially, the value of 0.1477 from the calibration of the Dement [30] data was used for coefficients j and K along with the value of 0.074 from the calibration with Kornerup *et al.* [19] data. These calibration values resulted in an R^2 value of 0.899, which was statistically significant ($p < 0.01$). There was no loss in degrees of freedom because the data of Alkalame *et al.* [33] were not used for calibration.

Nevertheless, the prediction for the first data point for zero REM debt overestimated the measured value by 14.6 percent. To correct this error, the value for coefficient C was reduced to 0.062. This change reduced the R^2 to 0.668 and reduced the significance level ($p < 0.05$).

Because the initial calibration with j set equal to K in Equation (12) was successful, the data of Alkalame *et al.* [33] were used to determine a new value for j and K. A value of 0.185 resulted in a new R^2 value of 0.977. Even with the loss of two degrees of freedom, the relationship was statistically significant ($p < 0.01$).

To determine how sensitive the initial data sets were to the new calibration, the new coefficients were used in the original data sets. Using 0.062 instead of 0.074 reduced the R^2 for Kornerup *et al.* data [19] from 0.968 to 0.901. The statistical significance remained at $p < 0.01$. This calibration improved the prediction for the first three points but under predicted the value for the upper data point (5.5) of traumatic events by 8.1 percent.

The 0.185 calibration for j and K was used to predict Dement's data. The R^2 value dropped from 0.916 to 0.537 ($p < 0.1$). Dement reported an uncertainty about the amount of REM debt. He estimated the real value of REM debt to be between 0.65 and 0.75. Changing the average of 0.7 used in the initial calibration to 0.65 with a value of 0.185 for K resulted in an R^2 of 0.767 ($p < 0.01$). With the lower value for percentage of REM debt adjustment, a new value for K of 0.16 resulted in the original R^2 value of 0.916 ($p < 0.01$). This value for K reduced the R^2 value in the Alkalame data set to 0.818 ($p < 0.01$).

The Dement data set is plagued with uncertainty and has a very small number of subjects in the study. Thus, it could be argued that the calibration of $C = 0.162$ and $K = j = 0.185$ with an R^2 value of 0.977 is the best calibration. The Alkalame *et al.* [33] data is based on intrusive memories that closely replicate the intrusive memory symptom of PTSD.

One other combination was tried. The 0.074 value was used for C based on data from Korornerup *et al.* [19]. With this value fixed, the $j = K$ coefficient was varied to maximize the R^2 value with the data set from Alkalame *et al.* [33]. A value of 0.162 resulted. This calibration set resulted in an R^2 value of 0.915 for the Dement data set, 0.968 for the Kornerup *et al.* [19] data set, and 0.951 for the Alkalame *et al.* [33] data set. All results were statistically significant ($p < 0.01$). Considering all three data sets, these coefficients were accepted as the best values for calibration.

Even with the limitations of the Dement data set, there appears to be evidence that short-term REM debt has a strong interaction with the number of traumatic events to predict intrusive memories. Better data sets with high REM debt interacting with the intrusive memory methodology are needed to better evaluate this interaction.

The average REM debt was estimated from an increase of 16.4 minutes of total sleep from day one to day two of the study by Alkalame *et al.* [33]. This increase in total sleep time is 3.6 percent. Dement [30] observed that the first night recovery from REM debt was about 10 percent of the total debt. If we assume REM recovery is 10 percent and that REM debt is about the same as total sleep debt, then the participants in the study by Alkalame *et al.* [33] had about 0.36 days of REM debt. This amount of about 0.4 days of REM debt will be used later as a reference condition for the general military population.

None of these data sets included an exercise or cardiorespiratory fitness measure; thus, the third term in Equation (1) was not added at this point in development.

3. Results

3.1 PTSD Predictions

Measurements of REM debt and number of traumatic exposures are usually not and may never be known for real mental health data. Nevertheless, indirect estimates are available to get a general sense of the performance of Equation (12).

Reduced length of sleep certainly is associated with reduced REM sleep. Gehrman *et al.* [35] reported an increase in odds ratio (OR) for 6-hours or less sleep

compared to 7 hours of sleep ($i = 1.0$) for PTSD for deployed military personnel. If this were just a one-day reduction, the REM debt would be about 1/7 or 0.143. As REM debt increases, the system adjusts, becomes more efficient, and increases the REM fraction at the expense of other sleep stages [34]. If we add 0.143 as new REM debt each day and subtract for the 10 percent recovery from REM debt times the previous day's REM debt [30], we eventually reach a steady state REM debt value of 1.4 days. This is only a rough estimate because the amount of sleep is less than but probably close to 6 hours.

The sleep assessments were done by survey before military deployment. Thus, it is unknown if this sleep information occurred during deployment and during combat. They also reported insomnia in some personnel. While the real sleep condition for deployed military personnel is unknown, it is assumed that the pattern during deployment remained the same as before except at times before and during combat when sleep is not a priority and sleep debt, especially REM debt is expected to increase.

Your author experienced lights out at 10 pm and lights on at 4 am for a limit of 6 hours of sleep during his time in Naval bootcamp. He also has heard Army personnel and Navy Marines report about training missions with no sleep for two days. At times, sleep can be very restricted, especially preparing for and during combat missions. To illustrate the severity of the restricted-sleep problem, consider that the military uses caffeine gum to partially counter fatigue [36] [37]. Instructions on a packet of caffeine gum state:

> Chew 1 stick for 5 minutes. If not alert, chew a second stick. Chew a 3rd stick if fatigue returns. Do not exceed 3 sticks in 3 hours.

Caffeine reduces fatigue, but it does not increase REM sleep and may even interfere with sleep when the opportunity occurs. The advantage of caffeine gum is that it is easy to carry, and it enters the blood more quickly than a cup of coffee [36]. The development and use of caffeine gum implies that extreme lack of sleep and decline in alertness are associated with military missions [36] [37].

The number of trauma exposures is also unknown. A Google search resulted in an estimate of 2 to 3 combat events per soldier but some soldiers may experience significantly more events during intense combat.

Gehrman *et al.* [35] analyzed data for two groups of military deployments: 2001-2004 and 2004-2007. The odds ratio (OR) for PTSD and confidence intervals (CI) for a sleep duration of less than 6 hours compared to 7 hours were 1.61 (CI: 1.08 - 2.39) for the first group and 1.81 (CI: 1.18 - 2.77) for the second group, resulting in an average OR of 1.71. Predictions from Equation (12) for relative risk are presented in **Table 1**. A value of 0.074 was used for C and 0.162 for j and K in Equation (12).

Predictions at a REM debt of 0.4 days highlighted in green were used in association with normal sleep of 7 hours as reference values for computing the relative risk for various REM debt values and number of trauma exposures. The computed relative risk values are shown in **Table 2**.

Table 1. Predictions of risk for PTSD from Equation (12), coefficient $C = 0.074$ and coefficients j and $K = 0.162$. An estimate of REM debt (0.4) in a normal population is shown in green. Predictions for an estimated REM debt associated with averaging 6 hours of sleep are shown in red.

	Number of Trauma Exposures									
REM Debt	**0**	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**5**
0.0	1.00	1.04	1.08	1.12	1.16	1.20	1.25	1.30	1.34	1.45
0.1	1.02	1.06	1.11	1.16	1.22	1.27	1.33	1.39	1.46	1.60
0.2	1.03	1.09	1.15	1.21	1.28	1.35	1.42	1.50	1.58	1.76
0.3	1.05	1.12	1.19	1.26	1.34	1.43	1.52	1.61	1.71	1.94
0.4	**1.07**	**1.14**	**1.23**	**1.31**	**1.41**	**1.51**	1.62	1.73	1.86	2.14
0.5	1.08	1.17	1.27	1.37	1.48	1.60	1.73	1.87	2.02	2.35
0.6	1.10	1.20	1.31	1.42	1.55	1.69	1.84	2.01	2.19	2.59
0.7	1.12	1.23	1.35	1.48	1.63	1.79	1.97	2.16	2.37	2.86
0.8	1.14	1.26	1.40	1.54	1.71	1.89	2.10	2.32	2.57	3.15
0.9	1.16	1.29	1.44	1.61	1.80	2.00	2.24	2.50	2.79	3.47
1.0	1.18	1.32	1.49	1.68	1.89	2.12	2.39	2.69	3.02	3.83
1.1	1.20	1.36	1.54	1.74	1.98	2.24	2.55	2.89	3.28	4.22
1.2	1.21	1.39	1.59	1.82	2.08	2.38	2.72	3.11	3.55	4.65
1.3	1.23	1.42	1.64	1.89	2.18	2.51	2.90	3.34	3.85	5.12
1.4	1.25	1.46	1.69	1.97	2.29	2.66	3.09	3.60	4.18	5.65
1.5	1.28	1.49	1.75	2.05	2.40	2.82	3.30	3.87	4.53	6.22
1.6	1.30	1.53	1.81	2.14	2.52	2.98	3.52	4.16	4.91	6.86
1.7	1.32	1.57	1.87	2.22	2.65	3.15	3.76	4.47	5.33	7.56
1.8	1.34	1.61	1.93	2.32	2.78	3.34	4.01	4.81	5.78	8.33
1.9	1.36	1.65	1.99	2.41	2.92	3.53	4.28	5.18	6.27	9.18
2.0	1.38	1.69	2.06	2.51	3.06	3.74	4.56	5.57	6.79	10.11
2.1	1.41	1.73	2.13	2.62	3.22	3.96	4.87	5.99	7.37	11.15
2.2	1.43	1.77	2.20	2.72	3.38	4.19	5.19	6.44	7.99	12.29
2.3	1.45	1.81	2.27	2.84	3.55	4.43	5.54	6.93	8.66	13.54
2.4	1.48	1.86	2.34	2.95	3.72	4.69	5.91	7.45	9.39	14.92
2.5	1.50	1.91	2.42	3.08	3.91	4.97	6.31	8.02	10.19	16.44
2.6	1.52	1.95	2.50	3.20	4.10	5.26	6.73	8.62	11.05	18.12
2.7	1.55	2.00	2.58	3.34	4.31	5.56	7.18	9.27	11.98	19.97
2.8	1.57	2.05	2.67	3.47	4.52	5.89	7.66	9.98	12.99	22.01
2.9	1.60	2.10	2.76	3.62	4.75	6.23	8.18	10.73	14.08	24.26
3.0	1.63	2.15	2.85	3.77	4.98	6.59	8.72	11.54	15.27	26.74

Table 2. Relative risks predicted with Equation (12) relative to the estimated risk at REM debt of 0.4 days.

	Number of Trauma Exposures									
REM Debt	**0**	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**5**
0.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.5	1.02	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.08	1.10
0.6	1.03	1.05	1.07	1.08	1.10	1.12	1.14	1.16	1.18	1.21
0.7	1.05	1.08	1.10	1.13	1.16	1.19	1.21	1.24	1.28	1.34
0.8	1.07	1.10	1.14	1.18	1.21	1.25	1.30	1.34	1.38	1.48
0.9	1.08	1.13	1.18	1.22	1.28	1.33	1.38	1.44	1.50	1.63
1.0	1.10	1.16	1.21	1.28	1.34	1.41	1.48	1.55	1.63	1.79
1.1	1.12	1.19	1.25	1.33	1.41	1.49	1.57	1.67	1.76	1.97
1.2	1.14	1.21	1.30	1.38	1.48	1.57	1.68	1.79	1.91	2.18
1.3	1.16	1.24	1.34	1.44	1.55	1.67	1.79	1.93	2.07	2.40
1.4	1.18	1.28	1.38	1.50	1.63	1.76	1.91	2.07	2.25	2.64
1.5	1.20	1.31	1.43	1.56	1.71	1.87	2.04	2.23	2.44	2.91
1.6	1.21	1.34	1.48	1.63	1.79	1.97	2.18	2.40	2.64	3.21
1.7	1.23	1.37	1.52	1.69	1.88	2.09	2.32	2.58	2.87	3.54
1.8	1.25	1.41	1.57	1.76	1.97	2.21	2.48	2.77	3.11	3.90
1.9	1.28	1.44	1.63	1.84	2.07	2.34	2.64	2.98	3.37	4.30
2.0	1.30	1.48	1.68	1.91	2.18	2.48	2.82	3.21	3.65	4.74
2.1	1.32	1.51	1.73	1.99	2.28	2.62	3.01	3.45	3.96	5.22
2.2	1.34	1.55	1.79	2.07	2.40	2.77	3.21	3.71	4.30	5.75
2.3	1.36	1.59	1.85	2.16	2.52	2.94	3.43	4.00	4.66	6.34
2.4	1.38	1.63	1.91	2.25	2.64	3.11	3.65	4.30	5.05	6.99
2.5	1.41	1.67	1.97	2.34	2.77	3.29	3.90	4.62	5.48	7.70
2.6	1.43	1.71	2.04	2.44	2.91	3.48	4.16	4.97	5.94	8.49
2.7	1.45	1.75	2.11	2.54	3.06	3.68	4.44	5.35	6.44	9.35
2.8	1.48	1.79	2.18	2.64	3.21	3.90	4.74	5.75	6.99	10.31
2.9	1.50	1.84	2.25	2.75	3.37	4.13	5.05	6.19	7.58	11.36
3.0	1.52	1.88	2.32	2.87	3.54	4.37	5.39	6.66	8.22	12.52

At a REM debt of 1.4 days, the predictions for two and three traumatic exposures (1.63, 1.91) are well within the 95 percent confidence interval for the two groups (1.08 - 2.77). Statistically, predictions are within the acceptable range of measured data. The mean relative risk at 2.5 trauma exposures is 1.76 compared to an average of 1.71 for the two measured deployment values. The difference in these two values is −2.92 percent error.

Gehrman *et al.* [35] reported another classification (PTSD with insomnia compared to PTSD without insomnia). The sleep amount associated with insomnia for this study is unknown. The predicted relative risk for less than 6 hours of sleep results in 15.6 percent error compared to the mean value from the two studies. If the REM debt is increased to 1.7 days (daily sleep estimate of 5.8 hours) highlighted in blue, the prediction for 2.5 trauma exposures results in −0.24 percent error. The difference in measured values for the two deployments is also close to the range in predictions for 2 trauma exposures and 3 trauma exposures. Because there is no reported sleep amount for insomnia, there is no statistical significance in this analysis. Nevertheless, the amount of sleep of 5.8 hours and predicted relative risks seem reasonable.

Gehrman *et al.* [35] present one more PTSD data set of interest in evaluating Equation (12). They report the following data for combat to no combat: group 1, relative risk 2.84 (CI: 2.04 - 3.96); group 2, relative risk 3.88 (CI: 2.76 - 5.46). The number of trauma exposures for non-combat is unknown but certainly smaller than that for combat. A value of 0.5 for number of noncombat exposures was assumed as shown in **Table 3**. If we add this number of trauma exposures to the 2 to 3 trauma exposures associated with combat, the most likely N values of interest are 2.5, 3, and 3.5.

Combat soldiers sometimes train without sleep for two days. They anticipate that a confrontation might last two days. After the confrontation is over, the REM debt will start to decay but tensions probably remain high with elevated heart rate and alertness that will interfere with sleep quality at least for a few days. Combat soldiers will often be faced with new traumatic exposures and high REM debt at the very time the emotional parts of the brain need REM sleep to process and reduce the severity of the traumatic memories.

Any previous REM sleep debt would be additive to the 2 days of no sleep during combat. It seems reasonable that combat soldiers might experience 2 to 3 days of REM debt. There are many ways to get the REM debt of 2.5 to 2.6 days. Five hours of sleep daily for a period of 23 days or about three weeks accumulate a REM debt of 2.6 days using the same process as was used to obtain the 1.4 value associated with 6 hours of sleep. Going 9 days with only 4 hours of sleep each day results in 2.6 days of REM debt. Insomnia for 1.6 hours of REM debt plus a night of no sleep also yields 2.6 hours of REM debt. Osgood *et al.* [7] state:

> Indeed, sleep difficulties are largely unavoidable in an operational contest (e.g., rocket fire, limited space, incoming aircraft, and shift changes.

It seems impossible to know exactly the number of traumatic events or the REM debt occurring before, during or after exposure. Matching the OR with predicted relative risk and working backwards to evaluate inputs is an alternative to the more conventional mathematical modeling procedure. Calibration of Equation (12) was achieved from more controlled data sets. The predicted PTSD relative risk for the < 6 hours of sleep was predicted well with this calibration. There is strong evidence that low sleep amounts occur in association with combat. There is insufficient evidence to reject Equation (12) as a model based on the available combat data.

Table 3. Predicted relative risk from Equation (12. The gray highlighted values closely match measured values from [35] and appear to be associated with high but unknown levels of REM debt.

	Number of Trauma Exposures									
REM Debt	**0**	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**5**
0.0		1.00	1.04	1.08	1.12	1.16	1.20	1.25	1.30	1.40
0.1		1.00	1.05	1.09	1.14	1.20	1.25	1.31	1.37	1.50
0.2		1.00	1.05	1.11	1.17	1.24	1.30	1.38	1.45	1.61
0.3		1.00	1.06	1.13	1.20	1.28	1.36	1.44	1.54	1.74
0.4		1.00	1.07	1.15	1.23	1.32	1.41	1.52	1.63	1.87
0.5		1.00	1.08	1.17	1.26	1.36	1.47	1.59	1.72	2.01
0.6		1.00	1.09	1.19	1.29	1.41	1.53	1.67	1.82	2.16
0.7		1.00	1.10	1.21	1.32	1.45	1.60	1.75	1.93	2.32
0.8		1.00	1.11	1.23	1.36	1.50	1.66	1.84	2.04	2.50
0.9		1.00	1.12	1.25	1.39	1.55	1.73	1.93	2.16	2.69
1.0		1.00	1.13	1.27	1.42	1.60	1.80	2.03	2.28	2.89
1.1		1.00	1.13	1.29	1.46	1.66	1.88	2.13	2.42	3.11
1.2		1.00	1.14	1.31	1.50	1.71	1.96	2.24	2.56	3.35
1.3		1.00	1.15	1.33	1.53	1.77	2.04	2.35	2.71	3.60
1.4		1.00	1.16	1.35	1.57	1.83	2.12	2.47	2.87	3.87
1.5		1.00	1.17	1.37	1.61	1.89	2.21	2.59	3.03	4.16
1.6		1.00	1.18	1.40	1.65	1.95	2.30	2.72	3.21	4.48
1.7		1.00	1.19	1.42	1.69	2.01	2.40	2.85	3.40	4.82
1.8		1.00	1.20	1.44	1.73	2.08	2.49	2.99	3.60	5.18
1.9		1.00	1.21	1.46	1.77	2.15	2.60	3.14	3.80	5.57
2.0		1.00	1.22	1.49	1.82	2.22	2.70	3.30	4.03	6.00
2.1		1.00	1.23	1.51	1.86	2.29	2.82	3.46	4.26	6.45
2.2		1.00	1.24	1.54	1.91	2.37	2.93	3.64	4.51	6.94
2.3		1.00	1.25	1.56	1.95	2.44	3.05	3.82	4.77	7.46
2.4		1.00	1.26	1.59	2.00	2.52	3.18	4.01	5.05	8.03
2.5		1.00	1.27	1.61	2.05	2.61	3.31	4.21	5.35	8.63
2.6		1.00	1.28	1.64	2.10	2.69	3.45	4.42	5.66	9.28
2.7		1.00	1.29	1.67	2.15	2.78	3.59	4.64	5.99	9.99
2.8		1.00	1.30	1.69	2.21	2.87	3.74	4.87	6.34	10.74
2.9		1.00	1.31	1.72	2.26	2.97	3.89	5.11	6.71	11.55
3.0		1.00	1.32	1.75	2.32	3.06	4.06	5.37	7.10	12.43

Overall, Equation (12) did well in predicting PTSD. It may be possible to have soldiers wear a smart watch to log daily REM sleep and to accumulate daily values of REM debt to obtain more precise data about risk of developing PTSD. Equation (12) provides a tool to estimate anticipated results.

3.2. Depression Predictions

Next, the measures for depression from Gehrman *et al.* [35] were analyzed. The combat-only PTSD average relative risk for the two groups is 3.36. The combat only average relative risk for depression is 3.44—a difference of only 0.08. The error in assuming that the depression risk for combat only is the same as that of PTSD is 2.4 percent. They both seem to be related to trauma predictions with Equation (12). This comparison supports the assumption that Gregory [15] made in predicting relative risk for stroke. He observed that the relative risks of trauma and depression are about the same and not independent. He, thus, did not consider depression as an independent variable for predicting risk for stroke. Based on these results, Equation (12) may be useful to predict the risk of developing depression.

There is one exception where depression can be directly related to the risk of developing PTSD. People with insomnia generally have much higher risk of developing depression. Gehrman *et al.* [35] observed an approximate doubling of the risk for depression.

In one study [38], as little as two weeks of insomnia in the past six months was associated with the development of depression. Franzen and Buysse [38] make a compelling case that insomnia and depression often occur together, and that insomnia poses significant risk for depression. They also state that insomnia may not cause depression. They state, "Insomnia may simply be a proxy for other causal factors". REM sleep debt takes away the healing process for trauma, and insomnia seems to amplify the effect of depression.

3.3. Anxiety

The relative risk of anxiety for people in combat was interesting and significantly different. Group 1 had a relative risk of 1.67, CI (1.00 - 2.79) compared to group 2 with a relative risk of 5.64, CI (2.74 - 11.61). The two groups were statistically different. Research [39] [40] has shown that purpose in life influences anxiety. The average value for anxiety from the two groups is 3.66.

The two military groups served during the presidency of George W. Bush. The first group served immediately after the attack on 9/11. People enlisted in the military with a strong purpose to bring justice to the people who planned and executed the 9/11 attacks. By the time the second group served, the military had bogged down and lost much of its original purpose and drive. The anxiety data reflects this change in attitude and purpose. The average of the relative risk for combat is 3.66, which is close to the 3.44 average for depression and 3.36 for PTSD. The maximum difference between the high and low of these averages is 10 percent.

It appears that the relative risk for combat and anxiety can be predicted from Equation (12) as was done for PTSD and then adjusted up or down based on attitude and purpose.

Equation (12) was certainly in the ballpark in predictions for PTSD. Combat data statistically remained the same for PTSD, depression, and anxiety except for the purpose effect on anxiety. While there is much uncertainty about REM debt and number of traumatic experiences, Equation (12) is a rational predictor of trauma risks.

3.4. Aerobic Exercise

The third term on the right side of Equation (1) considers the effect of aerobic exercise or physical activity on risk for trauma. A popular book, *Spark: The Revolutionary New Science of Exercise and the Brain,* [41] presents the case that exercise improves brain chemistry as well as our physical body. In this section, we will develop a relationship to consider relative fitness, a result of aerobic exercise, and risk of trauma.

An equation relating trauma to exercise was developed in an analogous manner to that used for the first two terms. The starting equation is

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = G\tau \frac{\mathrm{d}E}{\mathrm{d}t} \tag{13}$$

where, G = calibration coefficient;

E = measure of aerobic fitness, such as VO_{2max} or relative fitness.

Using separation of variables and integration from initial to final values of the variables yields

$$\ln \frac{T_\tau}{i} = G\left(E_f - E_i\right) \tag{14}$$

where, E_f = final aerobic fitness after executing an exercise plan;

E_i = initial or beginning aerobic fitness.

Converting from log space to normal space by taking the exponential of both sides results in

$$T_\tau = \mathrm{e}^{G\left(E_f - E_i\right)} \tag{15}$$

Relative fitness as proposed by Gregory [27] is independent of age and gender. It is unitless. These facts make relative fitness a more general measure of aerobic fitness than VO_{2max}, which has units and varies with gender and age. Thus, Equation (15) was rewritten in terms of relative fitness:

$$T_\tau = \mathrm{e}^{G\left(RF_f - RF_i\right)} \tag{16}$$

A high aerobic fitness as expressed in terms of relative fitness will improve health and reduce trauma. Thus, the coefficient G will have a negative value.

The next task is to test Equation (16). Dunn [42] *et al.* studied the effects of aerobic exercise as a treatment for sedentary people with depression. They measured physical activity at two levels compared to an initial sedentary condition to

evaluate the benefits of aerobic exercise on reducing depression. The study lasted 12 weeks. The median age was 35.9 years. They observed a reduction in mild to moderate major depressive disorder of 47 percent for their high exercise treatment and 30 percent for their low exercise treatment compared to their initial sedentary condition.

The two levels of physical activity were 7 Kcal/kg/week and 17.5 Kcal/kg/week. It would be ideal to have measures of VO_{2max} or distance walked or run per week. Without this information, it was assumed that their low-dose exercise level was 7 miles of walking or running per week and that the high dose was 17.5 miles per week (over three miles/day for five exercise days/week). Gregory [27] provided a table of exercise distances for walking and running from which relative fitness could be estimated. Building on measured VO_{2max} data for women [43] and men [44] and the mathematical model from Gregory [27], a relative fitness of 0.38 represents average sedentary conditions. Values for relative fitness vary from an upper limit of 1.0 for Olympic athletes down to 0.2 for low sedentary conditions. A value of 0.5 is the upper limit for sedentary conditions and the lower limit for endurance-trained individuals. From **Table 4** in Gregory [27], a sedentary person with a relative fitness of 0.38 will expend the equivalent of about 26 miles walking per week. Adding the distances of 7.0 and 17.5 to that of the sedentary condition facilitated a relative fitness estimate of 0.44 for the 7-mile estimated distance and 0.51 for the 17.5 estimated distance. The starting risk for depression for this study was set to 1.00 for the sedentary condition.

Equation (16) was programmed in an Excel spreadsheet to predict the risk of trauma and related mental health specifics, such as depression and anxiety. In this case the calibration was based on depression data. Predictions at relative fitness values of 0.51, 0.44, and 0.38 were divided by the prediction at 0.38. A value of 0.38 was used for RF_i. A value of -5.3 for G produced an R^2 value of 0.986 and was statistically significant ($p < 0.01$). These results seem to verify that Equation (16) is a valid model for evaluating the effects of aerobic fitness on recovery from depression.

A second data set by Wiklund *et al.* [45] evaluated the effect of maintaining cardiorespiratory fitness from teenage years (18.3 average age) to adult life (42.9 average age) on depression and anxiety. This study was limited to men in Sweden. They provided estimates of VO_{2max} (47.5 ml/kg/min with a standard deviation of 6.7 for age 18.3 years; 36.6 ml/kg/min with a standard deviation of 9.9 for age 42.9 years). They reported that "decreasing from moderate or high estCRF in adolescence to low estCRF in adulthood, compared to staying at a moderate or high level, was associated with a higher risk of depression and anxiety (HR: 1.24 95% CI 1.07-1.45 and 1.25 95% CI 1.06-1.49, respectively)." In other words, maintaining a moderate to high VO_{2max} from teenage years to adulthood has value in preventing depression and anxiety. The Gregory [27] equation for relating VO_{2max} to age and relative fitness was used to estimate the relative fitness values used in Equation (16). Predictions from Equation (16) are 1.16 for depression and anxiety com-

pared to the measured values of 1.24 and 1.25 resulting in a 6.45 percent error for depression and 7.20 percent for anxiety. The predicted value of 1.16 is well within the reported confidence intervals for depression and anxiety.

The study by Wiklund [45] certainly illustrates the value of maintaining aerobic or cardiorespiratory fitness through life in terms of risk for depression and anxiety. Their results also support the acceptance of Equation (16) as a model for predicting the effects of aerobic exercise and fitness on outcomes of depression and anxiety.

In another study of men in Sweden [46], it was found that low cardiorespiratory fitness at an average age of 18.3 years was associated with risk for receiving a disability pension for various causes (psychiatric, musculoskeletal, injuries, nervous system, circulatory, tumors) later in life. In other words, low aerobic fitness not only hurts the individual but also costs society to provide disability pensions to support the individual. The relative risk associated with a 40-year follow-up is shown in Figure 5.

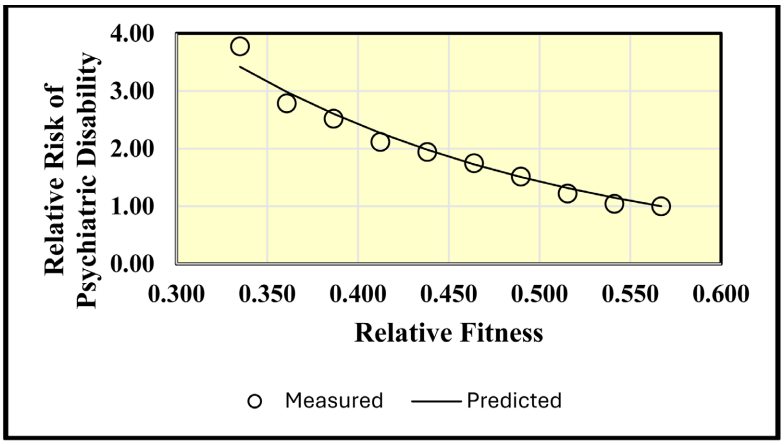

Figure 5. Prediction of the relative risk of being on a disability pension after 40 years as a function of 10 equally spaced units of relative fitness. Data from psychiatric cases after 40 years from Table 2 in the appendix of Henriksson *et al.* [46].

The line through the points was predicted with Equation (16). Henriksson *et al.* [46] provided an estimate of 41 for VO_{2max} as the lower fitness value. The relative fitness for males at age 18.3 from the Gregory [27] equation is 0.335. They did not provide the value for the upper limit. It was assumed that the risk of psychiatric disability payment is related to depression and anxiety. Thus, the calibration value of −5.3 was used and the unknown upper limit was determined from fitting Equation (16) to the data. A value of 0.567 was determined for the upper relative fitness, which would be associated with a VO_{2max} of 62.1 ml/kg/min. This value is near the midrange of endurance-trained men at age 18.3 from [44]. Equation (16) predictions matched the measured data with an R^2 of 0.968, which was significant ($p < 0.001$).

Equation (16) seems to work for both changing relative fitness as a treatment for depression and predicting future health risks associated with relative fitness

early in life. It allows an individual an opportunity to calculate the future health risks associated with depression early in life so that a healthy change can be made.

The curves shown in Figure 6 illustrate the risk of maintaining a low relative fitness compared to a midrange endurance-trained individual. Note that the curve for mood (depression) is remarkably similar to the curve in Figure 5.

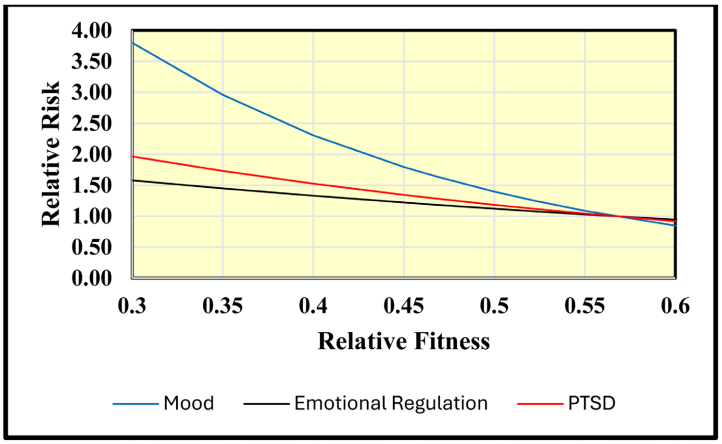

Figure 6. Predictions of mood (depression), PTSD, and Emotional Regulation as a function of relative fitness. Calibration is based on two measured points at a relative fitness of 0.467 and 0.522. Data from Mizzi *et al.* [47].

The only difference other than different studies is that the calibration coefficient for the mood curve is −5.0 compared to −5.3 for the depression curve in Figure 5 and the calibration from the data of Dunn *et al.* [42]: a 5.7 to 6 percent error depending on which calibration we use as a reference. Mizzi *et al.* [47] only provided two values for VO_{2max} from which relative fitness could be computed. Two points are insufficient to validate a relationship, but two points work well to calibrate once the relationship is known. Thus, because it was shown earlier that PTSD, depression, and anxiety seem to be related, Equation (16) was assumed to apply to consider the effect of aerobic exercise on PTSD and emotional regulation. The calibration for G for these curves is −2.53 for PTSD and −1.71 for emotional regulation.

The final equation to predict risk for PTSD, depression, anxiety, and emotional regulation is

$$T_\tau = \mathrm{e}^{(0.074+0.162RD)N+0.162RD+G\left(RF_f-RF_i\right)} \tag{17}$$

where, $G = -5.3$ for depression and anxiety;

$= -2.53$ for PTSD;

$= -1.71$ for emotional regulation;

$RF_i = 0.4$ mid sedentary relative fitness.

4. Discussion

A general mathematical model was presented as a starting point to mathematically predict the relative risk for trauma. The first two terms were integrated and com-

bined to obtain Equation (12), a model to predict trauma risk as a function of number of traumatic exposures and REM sleep debt. Predictions from Equation (12) agreed with measured data for PTSD from Gehrman *et al.* [35] for sleep duration <6 hours. Exact input data for REM debt was unknown for the prediction of conditions of insomnia and combat. As an alternative analysis, predicted values for PTSD were matched to measured data for conditions of insomnia and combat. The resulting REM debt requirement seemed reasonable for these extreme conditions.

It was observed that measured data for PTSD, depression, and anxiety were not statistically different (within the 95 CI for each of the 3 variables). The data for depression occurring with insomnia was an exception to this pattern.

The military study by Gehrman *et al.* [35] also contained one other data component, sex. The data for relative risk related to sex is provided in Table 4. Women are consistently at a higher risk of developing these mental health issues.

It appears that the relative risk for all three components of mental health in Table 4 is approximately the same (1.5). The relative risk of depression deviated the most with a 6.7 percent error above the average.

Table 4. Relative risk for women compared to men for three mental health components. Data from Gehrman *et al.* (2013).

	Group 1	Group 2	Average
PTSD	1.31	1.66	**1.49**
Depression	1.91	1.29	**1.60**
Anxiety	1.13	1.69	**1.41**
Average	**1.45**	**1.55**	**1.50**

The relative risk values in Table 4 are for military personnel on deployment with combat experience. The number of traumatic exposures in the men may be elevated compared to a general population accounting for the low 1.5 relative risk value. Olff [5] reports that "women have a two to three times higher risk of developing post-traumatic stress disorder (PTSD) compared to men." She reports the lifetime prevalence for PTSD as 10 - 12 percent for women and 5 - 6 percent for men supporting the relative risk of 2.0. She also reports that similar differences exist between the sexes for disorders such as major depression and anxiety, which agrees with the data in Table 4.

It is now well known from studies of rats and humans that stress causes changes that lead to volume reduction in the hippocampus as a person ages and experiences mental stress. These volume reductions are associated with various psychiatric disorders including major depression, PTSD, borderline personality disorder, schizophrenia, dissociative identity disorder, and antisocial personality disorder (Teicher *et al.* [18]). They also report that over 50 percent of current episodes of depression, addiction to illicit drugs, and suicide attempts are associated

with early adversity. In other words, problems later in life are often traceable back to traumatic exposure as a child or teenager.

In a later study (Teicher *et al.* [17]), male hippocampal volume was predicted by neglect but not abuse. Female hippocampal volume was predicted by abuse but not neglect occurring and peaking at ages 10 - 16 years of age. Their **Figure 1** expressed average severity as a function of age for both abuse and neglect. Dividing the severity value for females for abuse by the severity at the same age for males for neglect indicates that the relative risk for females compared to males begins at an early age. **Table 5** shows this information.

Table 5. Relative risk determined by dividing abuse rating for females by neglect rating for males as a function of age. Data from Figure 1 of Teicher *et al.* [17].

	Severity Rating		
Age (yrs)	**Male**	**Female**	**Relative Risk**
5.0	3.1	4.0	**1.3**
7.5	3.1	7.5	**2.4**
12.5	3.3	10.0	**3.0**
15.0	4.0	9.0	**2.3**

In an extensive literature review, Salk *et al.* [48] determined that the gender difference for depression "emerged earlier than previously thought with OR = 2.37 at age 12." They found that the difference peaked at an OR = 3.02 for ages 13 - 15.

Based on this information, young women start even before their teenage years at having a two to three times higher risk of developing PTSD and other related mental health issues. These mental health issues may not develop until later years, but the damage occurs early in life. Much of this stress exposure appears to be of a chronic nature [18]: physical abuse (38%), harsh corporal punishment (33%), parental verbal aggression (32%), familial physical abuse (31%), sexual abuse (14%). There is also evidence that young women but not men are sensitive to poor economic conditions [19]. They reported a relative risk of 2.28 for young women exposed to poor economic conditions.

The development of PTSD, depression, and anxiety is a complicated process that is affected by trauma exposure, REM sleep debt, exercise, and gender. Even with this complexity and input uncertainties, there is compelling evidence that Equations 12, 16, and 17 are reasonable mathematical descriptions of the final outcome of the process during which PTSD and related mental health manifestations of depression and anxiety develop. Individuals are unique: different genetics, different experiences, different environments, different social support groups, and different attitudes. Thus, some individuals can endure horrific exposures and not develop PTSD or depression. The opposite outcome occurs for others.

5. Conclusions

Equations 12 and 16 and support data provide evidence that trauma exposure, REM sleep debt, and aerobic exercise affect the risk for PTSD, depression, and anxiety. REM sleep debt is common in most societies but is a variable that can be managed. Aerobic exercise can also be managed and appears to have a strong effect on recovery from depression. A purpose-driven life helps to reduce anxiety.

Input data to use Equations (12), (16), or (17) is usually imprecise. Nevertheless, these equations provide a number that helps to quantify the effects of traumatic exposures, sleep, and aerobic exercise on the relative risk of developing mental health issues. To some degree, they help to evaluate ways to reduce risk either through prevention or treatment with changes in lifestyle.

While some researchers may disagree, it appears that PTSD, depression, and anxiety are linked. The data illustrated in **Table 4** for military personnel illustrate this observation. The relative risk as discussed with this data set from Gehrman *et al.* [35] supports this observation for relative risk for PTSD, depression, and anxiety with two exceptions. People with insomnia had double the normal depression without insomnia for this data set. The other exception is associated with purpose interacting with anxiety. Having a clear and strong purpose in life can greatly reduce anxiety. Lack of purpose can have devastating effects on anxiety.

A third exception exists in association with the relative fitness variable. The calibration for the mood (depression) curve in **Figure 6** is −5.0. This value is close to the −5.3 calibration for data from Dunn *et al.* [42]. Both are approximately double the value of −2.53 for the PTSD calibration. Observations from various data sets and treatment centers consistently show that insomnia significantly increases the relative risk of developing depression, more than doubling in some cases. Aerobic exercise increases relative fitness and helps people sleep better [49], which may explain the high benefit of exercise in treating depression.

In a general sense, it seems that many, maybe most cases of PTSD, depression, and anxiety have roots in stressful experiences in life. How well the brain processes these emotional stressors is governed by the amount of REM sleep, more specifically the amount of REM sleep debt at the time the exposure to stress occurs. Equations (12), (16), and (17) provide a way to numerically evaluate this interaction.

These equations have limitations. Equation (12) seems to do a reasonable job in predicting PTSD using REM sleep debt but fails to predict depression associated with insomnia. Equation (17) may, however, overcome some of these limitations by including exercise. There was a 47 percent difference in depression between sedentary and high exercise for data from Dunn *et al.* [42]. Data from Mizzi *et al.* [47] closely matched this relationship for depression. There was only a relatively weak effect of exercise for PTSD in their data. While the value of G for PTSD is only 1/2 of the value for depression, it does indicate that aerobic exercise has value for treating PTSD. This conclusion concurs with the results reported by Hegberg *et al.* [50]. It has also been observed that people with PTSD have less physical ac-

tivity compared to pre-PTSD time [50] [51]. Aerobic exercise affects insomnia through time awake after sleep onset (WASO) [49]. Further research is needed to test Equation (17) fully with measured REM sleep debt, number of trauma exposures, and relative fitness.

It is clear that gender affects risks for PTSD and related depression and anxiety. Equation (17) does not have a gender variable. Some researchers [13]-[19] have considered gender effects through weighted trauma exposures. Thurston *et al.* [13] calibrated their trauma exposure amounts based on measured accumulation of WMH. More research is needed to tease out gender effects and types of traumas for both male and female exposures to trauma.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Van Der Kolk, B. (2014) The Body Keeps the Score: Brain, Mind, and Body in the Healing of Trauma. Penguin Books.

[2] Breslau, N., Kessler, R.C., Chilcoat, H.D., Schultz, L.R., Davis, G.C. and Andreski, P. (1998) Trauma and Posttraumatic Stress Disorder in the Community: The 1996 Detroit Area Survey of Trauma. *Archives of General Psychiatry*, **55**, 626-632. https://doi.org/10.1001/archpsyc.55.7.626

[3] James, E.L., Lau-Zhu, A., Clark, I.A., Visser, R.M., Hagenaars, M.A. and Holmes, E.A. (2016) The Trauma Film Paradigm as an Experimental Psychopathology Model of Psychological Trauma: Intrusive Memories and Beyond. *Clinical Psychology Review*, **47**, 106-142. https://doi.org/10.1016/j.cpr.2016.04.010

[4] Kessler, R.C. (1995) Posttraumatic Stress Disorder in the National Comorbidity Survey. *Archives of General Psychiatry*, **52**, 1048-1060. https://doi.org/10.1001/archpsyc.1995.03950240066012

[5] Olff, M. (2017) Sex and Gender Differences in Post-Traumatic Stress Disorder: An Update. *European journal of Psychotraumatology*, **8**, Article 1351204. https://doi.org/10.1080/20008198.2017.1351204

[6] World Health Organization (2003) Mental Health in Emergencies: Mental and Social Aspects of Health of Populations Exposed to Extreme Stressors. World Health Organization.

[7] Osgood, J.M., Finan, P.H., Hinman, S.J., So, C.J. and Quartana, P.J. (2019) Combat Exposure, Post-Traumatic Stress Symptoms, and Health-Related Behaviors: The Role of Sleep Continuity and Duration. *Sleep*, **42**, zsy257. https://doi.org/10.1093/sleep/zsy257

[8] Olfati, M., Samea, F., Faghihroohi, S., Balajoo, S.M., Kuppers, V., Genon, S., Patil, K., Eickhoff, S.B. and Tahmasian, M. (2024) Prediction of Depressive Symptoms Severity Based on Sleep Quality, Anxiety, and Gray Matter Volume: A Generalizable Machine Learning Approach Across Three Datasets. Preprint. https://doi.org/10.1101/2023.08.09.23293887

[9] Mellman, T.A., Bustamante, V., Fins, A.I., Pigeon, W.R. and Nolan, B. (2002) REM Sleep and the Early Development of Posttraumatic Stress Disorder. *American Journal of Psychiatry*, **159**, 1696-1701. https://doi.org/10.1176/appi.ajp.159.10.1696

[10] Rothbaum, B.O., Foa, E.B., Riggs, D.S., Murdock, T. and Walsh, W. (1992) A Prospective Examination of Post-Traumatic Stress Disorder in Rape Victims. *Journal of Traumatic Stress*, **5**, 455-475. https://doi.org/10.1002/jts.2490050309

[11] Rosman, L., Sico, J.J., Lampert, R., Gaffey, A.E., Ramsey, C.M., Dziura, J., *et al.* (2019) Posttraumatic Stress Disorder and Risk for Stroke in Young and Middle-Aged Adults: A 13-Year Cohort Study. *Stroke*, **50**, 2996-3003. https://doi.org/10.1161/strokeaha.119.026854

[12] Brown, G.W. (2002) Social Roles, Context and Evolution in the Origins of Depression. *Journal of Health and Social Behavior*, **43**, 255-276. https://doi.org/10.2307/3090203

[13] Thurston, R.C., Jakubowski, K.P., Wu, M., Aizenstein, H.J., Chang, Y., Derby, C.A., *et al.* (2022) Sexual Assault and White Matter Hyperintensities among Midlife Women. *Brain Imaging and Behavior*, **16**, 773-780. https://doi.org/10.1007/s11682-021-00536-2

[14] Giese, A.-K., *et al.* (2020) White Matter Hyperintensity Burden in Acute Stroke Patients Differs by Ischemic Stroke Subtype. *Neurology*, **95**, e79-e88.

[15] Gregory, J.M. (2023) Mathematical Model for Stroke and White Matter Hyperintensities. *Journal of Behavioral and Brain Science*, **13**, 66-94. https://doi.org/10.4236/jbbs.2023.134006

[16] Tietjen, G.E. and Peterlin, B.L. (2011) Childhood Abuse and Migraine: Epidemiology, Sex Differences, and Potential Mechanisms. *Headache*: *The Journal of Head and Face Pain*, **51**, 869-879. https://doi.org/10.1111/j.1526-4610.2011.01906.x

[17] Teicher, M.H., Anderson, C.M., Ohashi, K., Khan, A., McGreenery, C.E., Bolger, E.A., *et al.* (2018) Differential Effects of Childhood Neglect and Abuse during Sensitive Exposure Periods on Male and Female Hippocampus. *NeuroImage*, **169**, 443-452. https://doi.org/10.1016/j.neuroimage.2017.12.055

[18] Teicher, M.H., Anderson, C.M. and Polcari, A. (2012) Childhood Maltreatment Is Associated with Reduced Volume in the Hippocampal Subfields CA3, Dentate Gyrus, and Subiculum. *Proceedings of the National Academy of Sciences*, **109**, E563-E572. https://doi.org/10.1073/pnas.1115396109

[19] Kornerup, H., Osler, M., Boysen, G., Barefoot, J., Schnohr, P. and Prescott, E. (2010) Major Life Events Increase the Risk of Stroke but Not of Myocardial Infarction: Results from the Copenhagen City Heart Study. *European Journal of Cardiovascular Prevention and Rehabilitation*, **17**, 113-118. https://doi.org/10.1097/HJR.0b013e3283359c18

[20] Ferree, N.K. and Cahill, L. (2009) Post-Event Spontaneous Intrusive Recollections and Strength of Memory for Emotional Events in Men and Women. *Consciousness and Cognition*, **18**, 126-134. https://doi.org/10.1016/j.concog.2008.11.008

[21] Pearson, D.G. (2012) Contextual Representations Increase Analogue Traumatic Intrusions: Evidence against a Dual-Representation Account of Peri-Traumatic Processing. *Journal of Behavior Therapy and Experimental Psychiatry*, **43**, 1026-1031. https://doi.org/10.1016/j.jbtep.2012.04.002

[22] Repantis, D., Wermuth, K., Tsamitros, N., Danker-Hopfe, H., Bublitz, J.C., Kühn, S., *et al.* (2020) REM Sleep in Acutely Traumatized Individuals and Interventions for the Secondary Prevention of Post-Traumatic Stress Disorder. *European Journal of Psychotraumatology*, **11**, Article 1740492. https://doi.org/10.1080/20008198.2020.1740492

[23] van Rijn, E., Eichenlaub, J.-B., Lewis, P.A., Walker, M.P., Gaskell, M.G., Malinowski,

J.E., *et al.* (2015) The Dream-Lag Effect: Selective Processing of Personally Significant Events during Rapid Eye Movement Sleep, but Not during Slow Wave Sleep. *Neurobiology of Learning and Memory*, **122**, 98-109. https://doi.org/10.1016/j.nlm.2015.01.009

[24] Walker, M. (2017) Why WE Sleep: Unlocking the Power of Sleep and Dreams. Scribner.

[25] Pace-Schott, E.F., Germain, A. and Milad, M.R. (2015) Sleep and REM Sleep Disturbance in the Pathophysiology of PTSD: the Role of Extinction Memory. *Biology of Mood & Anxiety Disorders*, **5**, Article No. 3. https://doi.org/10.1186/s13587-015-0018-9

[26] Ben Simon, E., Rossi, A., Harvey, A.G. and Walker, M.P. (2020) Overanxious and Underslept. *Nature Human Behaviour*, **4**, 100-110. https://doi.org/10.1038/s41562-019-0754-8

[27] Gregory, J.M. (2024) Modeling VO_{2max} with Relative Fitness and BMI. *Journal of Biosciences and Medicines*, **12**, 466-492. https://doi.org/10.4236/jbm.2024.1212037

[28] Riggs, D.S., Rothbaum, B.O. and Foa, E.B. (1995) A Prospective Examination of Symptoms of Posttraumatic Stress Disorder in Victims of Nonsexual Assault. *Journal of Interpersonal Violence*, **10**, 201-214. https://doi.org/10.1177/0886260595010002005

[29] Sareen, J. (2014) Posttraumatic Stress Disorder in Adults: Impact, Comorbidity, Risk Factors, and Treatment. *The Canadian Journal of Psychiatry*, **59**, 460-467. https://doi.org/10.1177/070674371405900902

[30] Dement, W. (1960) The Effect of Dream Deprivation: The Need for a Certain Amount of Dreaming Each Night Is Suggested by Recent Experiments. *Science*, **131**, 1705-1707. https://doi.org/10.1126/science.131.3415.1705

[31] Seelig, A.D., Jacobson, I.G., Smith, B., Hooper, T.I., Boyko, E.J., Gackstetter, G.D., Gehrman, P., Macera, C.A. and Smith, T.C. (2010) Sleep Patterns Before, During, and After Deployment to Iraq and Afghanistan. *Sleep*, **33**, 1615-1622. https://doi.org/10.1093/sleep/33.12.1615

[32] Ross, R.J., Ball, W.A., Sullivan, K.A. and Caroff, S.N. (1989) Sleep Disturbance as the Hallmark of Posttraumatic Stress Disorder. *American Journal of Psychiatry*, **146**, 697-707.

[33] Alkalame, L., Ogden, J., Clark, J.W., Porcheret, K., Risbrough, V.B. and Drummond, S.P.A. (2024) The Relationship between REM Sleep Prior to Analog Trauma and Intrusive Memories. *Sleep*, **47**, zsae203 https://doi.org/10.1093/sleep/zsae203

[34] Gregory, J.M., Xie, X. and Mengel, S.A. (2004) SLEEP (Sleep Loss Effects on Everyday Performance) Model. *Aviation, Space, and Environmental Medicine*, **75**, A125-A133.

[35] Gehrman, P., Seelig, A.D., Jacobson, I.G., Boyko, E.J., Hooper, T.I., Gackstetter, G.D., *et al.* (2013) Predeployment Sleep Duration and Insomnia Symptoms as Risk Factors for New-Onset Mental Health Disorders Following Military Deployment. *Sleep*, **36**, 1009-1018. https://doi.org/10.5665/sleep.2798

[36] Kamimori, G.H., Karyekar, C.S., Otterstetter, R., Cox, D.S., Balkin, T.J., Belenky, G.L., *et al.* (2002) The Rate of Absorption and Relative Bioavailability of Caffeine Administered in Chewing Gum versus Capsules to Normal Healthy Volunteers. *International Journal of Pharmaceutics*, **234**, 159-167. https://doi.org/10.1016/s0378-5173(01)00958-9

[37] Kamimori, G.H., McLellan, T.M., Tate, C.M., Voss, D.M., Niro, P. and Lieberman, H.R. (2015) Caffeine Improves Reaction Time, Vigilance and Logical Reasoning dur-

ing Extended Periods with Restricted Opportunities for Sleep. *Psychopharmacology*, **232**, 2031-2042. https://doi.org/10.1007/s00213-014-3834-5

[38] Franzen, P.L. and Buysse, D.J. (2008) Sleep Disturbances and Depression: Risk Relationships for Subsequent Depression and Therapeutic Implications. *Dialogues in Clinical Neuroscience*, **10**, 473-481. https://doi.org/10.31887/dcns.2008.10.4/plfranzen

[39] Boreham, I.D. and Schutte, N.S. (2023) The Relationship between Purpose in Life and Depression and Anxiety: A Meta-Analysis. *Journal of Clinical Psychology*, **79**, 2736-2767. https://doi.org/10.1002/jclp.23576

[40] Kashdan, T.B. and McKnight, P.E. (2013) Commitment to a Purpose in Life: An Antidote to the Suffering by Individuals with Social Anxiety Disorder. *Emotion*, **13**, 1150-1159. https://doi.org/10.1037/a0033278

[41] Ratey, J.J. (2008) Spark: The Revolutionary New Science of Exercise and the Brain. Little, Brown and Company.

[42] Dunn, A.L., Trivedi, M.H., Kampert, J.B., Clark, C.G. and Chambliss, H.O. (2005) Exercise Treatment for Depression: Efficacy and Dose Response. *American Journal of Preventive Medicine*, **28**, 1-8. https://doi.org/10.1016/j.amepre.2004.09.003

[43] Tanaka, H., Desouza, C.A., Jones, P.P., Stevenson, E.T., Davy, K.P. and Seals, D.R. (1997) Greater Rate of Decline in Maximal Aerobic Capacity with Age in Physically Active vs. Sedentary Healthy Women. *Journal of Applied Physiology*, **83**, 1947-1953. https://doi.org/10.1152/jappl.1997.83.6.1947

[44] Pimentel, A.E., Gentile, C.L., Tanaka, H., Seals, D.R. and Gates, P.E. (2003) Greater Rate of Decline in Maximal Aerobic Capacity with Age in Endurance-Trained than in Sedentary Men. *Journal of Applied Physiology*, **94**, 2406-2413. https://doi.org/10.1152/japplphysiol.00774.2002

[45] Wiklund, C.A., Lindwall, M., Ekblom, Ö., Nyberg, J., Åberg, M.I., Paulsson, S., *et al.* (2024) Change in Cardiorespiratory Fitness and Risk of Depression, Anxiety, and Cerebrovascular Disease. *American Journal of Preventive Medicine*, **67**, 849-858. https://doi.org/10.1016/j.amepre.2024.07.012

[46] Henriksson, P., Henriksson, H., Tynelius, P., Berglind, D., Lof, M., Lee, I., Shiro-ma, E.J. and Ortega, F.B. (2019) Fitness and Body Mass Index During Adolescence and Disability Later in Life: A Cohort Study. *Annals of Internal Medicine*, **170**, 230-239.

[47] Mizzi, A.L., McKinnon, M.C., Becker, S. (2022) The Impact of Aerobic Exercise on Mood Symptoms in Trauma-Exposed Young Adults: A Pilot Study. *Frontiers in Behavioral Neuroscience*, **16**, Article 829571. https://doi.org/10.3389/fnbeh.2022.829571

[48] Salk, R.H., Hyde, J.S. and Abramson, L.Y. (2017) Gender Differences in Depression in Representative National Samples: Meta-Analyses of Diagnoses and Symptoms. *Psychological Bulletin*, **143**, 783-822. https://doi.org/10.1037/bul0000102

[49] Gregory, J.M. (2022) WASO: Why Does It Increase with Age? *Journal of Behavioral and Brain Science*, **12**, 164-176. https://doi.org/10.4236/jbbs.2022.124009

[50] Hegberg, N.J., Hayes, J.P. and Hayes, S.M. (2019) Exercise Intervention in PTSD: A Narrative Review and Rationale for Implementation. *Frontiers in Psychiatry*, **10**, Article 133. https://doi.org/10.3389/fpsyt.2019.00133

[51] de Assis, M.A., de Mello, M.F., Scorza, F.A., Cadrobbi, M.P., Schooedl, A.F., da Silva, A.C., *et al.* (2008) Evaluation of Physical Activity Habits in Patients with Posttraumatic Stress Disorder. *Clinics*, **63**, 473-478. https://doi.org/10.1590/s1807-59322008000400010