Journal of High Energy Physics, Gravitation and Cosmology, 2025, 11(3), 784-794
https://www.scirp.org/journal/jhepgc
ISSN Online: 2380-4335
ISSN Print: 2380-4327

Temperature and Time in Quantum Wave Entropy

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How to cite this paper: Li, X.L. (2025) Temperature and Time in Quantum Wave Entropy. *Journal of High Energy Physics, Gravitation and Cosmology*, **11**, 784-794.
https://doi.org/10.4236/jhepgc.2025.113049

Received: April 15, 2025
Accepted: June 13, 2025
Published: June 16, 2025

Abstract

In quantum mechanics, particles have a new type of probabilistic property, which is quantum wave probability. The quantum wave probability corresponds to the quantum wave entropy. The action in classical mechanics corresponds to the quantum wave entropy. The least action principle corresponds to the stationary quantum wave entropy principle. Quantum wave entropy creates a bridge between dynamics and thermodynamics. Combining the Hamiltonian-Jacobian equation of classical mechanics and quantum wave entropy, we can derive the relationship between temperature and time. There is an inverse relationship between temperature and time. The phase of the wave function in quantum mechanics corresponds to the imaginary action. Combining the imaginary action and quantum wave entropy, we can derive the Wick rotation between temperature and imaginary time in quantum mechanics, thus explaining the physical meaning of the Wick rotation. Wick rotation is only applicable to the stationary state, not universally true. Imaginary time is only a mathematical representation and has no real physical significance.

Keywords

Quantum Wave Probability, Quantum Wave Entropy, Action, Least Action Principle, Stationary Quantum Wave Entropy Principle, Hamiltonian-Jacobian Equation, Imaginary Action, Temperature, Time, Imaginary Temperature, Imaginary Time, Wick Rotation

1. Introduction

In quantum mechanics, if we compare the time evolution operator with the partition function in thermodynamics, we can see that there is a mathematical correspondence between temperature and time. This is the mysterious relationship between temperature and imaginary time. This transition relationship is known as the Wick rotation. With the use of Wick rotation, we can convert the wave function representation of quantum mechanics into a thermodynamic representation [1].

$$e^{-\frac{iEt}{\hbar}} \leftrightarrow e^{-\frac{E}{\kappa_B T}}$$

$$T = -\frac{i\hbar}{\kappa_B t}$$

However, what is the physical significance of Wick rotation, is Wick rotation universally valid, what are the limitations on the use of Wick rotation, and is quantum mechanics really equivalent to thermodynamics, these questions have not been answered.

In the previous paper [2], the authors proposed a new concept of entropy, quantum wave entropy. The authors found that using the concept of quantum wave entropy, the Wick rotation can be derived. It is found that behind this Wick rotation formula, it actually represents the existence of quantum wave entropy. Quantum wave entropy is actually the physical property behind Wick rotation. This question can be explained by a reasonable reason. The author will explain this answer in detail.

2. Quantum Wave Entropy and Action

In the previous paper [2], the authors proposed a new concept of entropy, quantum wave entropy. In quantum mechanics, particles have a new probabilistic property. This new probabilistic property is inversely proportional to the wavelength of the particle wave. The shorter the wavelength of the particle, the greater the probability. Conversely, the longer the wavelength, the smaller the probability. For the new probability, there is a following relationship between wavelength and probability.

$$\mathrm{d}p = \alpha \frac{\mathrm{d}r}{\lambda} \tag{2.1}$$

The α is a proportionality constant. The λ is the wavelength of the particle. The $\mathrm{d}p$ actually represents the probability density within the length of $\mathrm{d}r$.

In quantum mechanics, a particle has intrinsic properties, such as the spin of a particle. For example, the spin of a particle has two possibilities, +1/2 and −1/2, that is, it has two degrees of freedom. Particles have this new probabilistic property, so particles have a new type of entropy property. The authors call this new type of entropy by quantum wave entropy. The density of quantum wave entropy is expressed as follow formula.

$$\mathrm{d}S = D\kappa_B \mathrm{d}p = D\kappa_B \alpha \frac{\mathrm{d}r}{\lambda} \tag{2.2}$$

The constant D is intrinsic degrees of freedom of the particle.

Define a new constant κ_p.

$$\kappa_p = D\alpha\kappa_B \tag{2.3}$$

The formula for quantum wave entropy is (2.4).

$$\mathrm{d}S = \kappa_p \frac{\mathrm{d}r}{\lambda} \tag{2.4}$$

Because quantum wave entropy depends on the intrinsic degrees of freedom of the particle. Only particles in quantum mechanics have intrinsic degrees of freedom. Particles in classical mechanics do not have intrinsic degrees of freedom, so particles in classical mechanics do not have the property of quantum wave entropy. Only particles in quantum mechanics have the property of quantum wave entropy.

Based on the concept of quantum wave entropy, we can get the Equation (2.5) [2].

$$T = \frac{h}{\kappa_p V} a \tag{2.5}$$

The T is the temperature of the particle in the case of quantum wave entropy, V is the velocity of the particle, a is the acceleration of the particle, and h is Planck constant. When the velocity is equal to the speed of light, the Formula (2.6) is obtained, which is the Unruh formula [3].

$$T = \frac{h}{\kappa_p C} a \tag{2.6}$$

We can also derive the Equation (2.7) [2].

$$\frac{\mathrm{d}S}{\mathrm{d}r} = \kappa_p \frac{mV}{h} \tag{2.7}$$

When the velocity is equal to the speed of light, the Formula (2.8) is obtained, which is the Verlinde entropy gravitational formula [4].

$$\frac{\mathrm{d}S}{\mathrm{d}r} = \frac{\kappa_p mC}{h} \tag{2.8}$$

We can also derive the black hole entropy Formula (2.9) [2] [5] [6].

$$S = \frac{\kappa_p R^2}{L_p^2} \tag{2.9}$$

The R is radius of black hole, L_p is Planck length.

For the momentum of a particle, there is the de Broglie formula.

$$\lambda = \frac{h}{P} \tag{2.10}$$

Take it into the definition formula of quantum wave entropy (2.4), we can obtain the formula for the relationship between particle momentum and quantum wave entropy (2.11).

$$P = \frac{h}{\kappa_p} \frac{\mathrm{d}S}{\mathrm{d}r} \tag{2.11}$$

We can deduce the following Equation (2.12) for the relationship between the action in classical mechanics and the quantum wave entropy [2] [7].

$$I=\int L\mathrm{d}t=\int(T-\phi)\mathrm{d}t=\frac{h}{\kappa_p}\int\mathrm{d}S=\frac{h}{\kappa_p}S \tag{2.12}$$

In order to distinguish between action and quantum wave entropy, the symbol I is used to identify the action, and the symbol S is used to identify quantum wave entropy. Readers need to pay attention to the distinction.

Therefore, we find that the action corresponds to the quantum wave entropy. Therefore, the least action principle (2.13) corresponds to the stationary quantum wave entropy principle (2.14).

$$\delta I=0 \tag{2.13}$$

$$\frac{h}{\kappa_p}\delta S=0 \tag{2.14}$$

In classical mechanics, the least action principle exists as a fundamental assumption. The existence of the least action principle seems very mysterious. We don't know why this principle works, and we can't find the physical meaning behind this principle. Now, starting from the concept of quantum wave entropy, we can find that the action actually corresponds to a kind of entropy. Entropy corresponds to probability. Therefore, we can conclude that the least action principle is actually the stationary entropy principle, which is the result of probability maximization. The evolutionary path represented by the least action principle is actually the path with the maximum probability. If the entropy increases, it is the fastest path to increase the probability. If the entropy decreases, it is the fastest path to a decrease in probability. This is the physical essence of the least action principle. Now, the least action principle is no longer mysterious, but is the result of a natural probabilistic evolution.

3. Relationship between Temperature and Time

In classical mechanics, the following formulas exist for the action [7] [8].

$$H=T+\phi=-\frac{\partial I}{\partial t} \tag{3.1}$$

$$L=T-\phi=\frac{\mathrm{d}I}{\mathrm{d}t} \tag{3.2}$$

$$P=\frac{\partial I}{\partial r} \tag{3.3}$$

Equation (3.1) is the Hamiltonian-Jacobian equation. For quantum wave entropy and action, there is a relational Formula (2.12), so we can use quantum wave entropy to express these formulas.

$$H=T+\phi=-\frac{h}{\kappa_p}\frac{\partial S}{\partial t} \tag{3.4}$$

$$L=T-\phi=\frac{h}{\kappa_p}\frac{\mathrm{d}S}{\mathrm{d}t} \tag{3.5}$$

$$P=\frac{h}{\kappa_p}\frac{\partial S}{\partial r} \tag{3.6}$$

In classical mechanics, H is the total energy E of the particle, and there is Equation (3.7).

$$E = -\frac{\partial I}{\partial t} \tag{3.7}$$

Combining Equations (3.4) and (3.7), we get Equation (3.8).

$$E = -\frac{h}{\kappa_p}\frac{\partial S}{\partial t} \tag{3.8}$$

In the case that E is a constant value, we can get (3.9).

$$Et = -\frac{h}{\kappa_p}S \tag{3.9}$$

And because in thermodynamics, there is the following Equation (3.10) between temperature T, energy E, and entropy [9].

$$E = TS \tag{3.10}$$

Combining Equations (3.9) and (3.10), so we get Formula (3.11).

$$T = -\frac{h}{\kappa_p t} \tag{3.11}$$

So, we find that in quantum wave entropy, temperature is inversely proportional to time. If you want to get Equation (3.11), you must satisfy both Equations (3.9) and (3.10). Equation (3.9) is a property of dynamics, and Equation (3.10) is a property of thermodynamics. So Equation (3.11) is a result of the combination of dynamics and thermodynamics. The key to the combination of the two lies in quantum wave entropy. Quantum wave entropy creates a bridge between dynamics and thermodynamics, combining the two so that the particle satisfies both Equations (3.9) and (3.10), so that Equation (3.11) can be derived. Without quantum wave entropy, we can't get the Equation (3.11).

A prerequisite for the existence of Equation (3.9) is that the total particle energy E must be a constant value. If E remains constant, then entropy S increases linearly over time. The negative sign is because the energy is negative and has no real physical significance. Therefore, Equation (3.9) actually represents the result of an increase in entropy. Because in thermodynamics there is another Equation (3.10), the entropy is increasing, and the energy E must remain constant, so the temperature can only decrease accordingly. So we get the Equation (3.11). So, we found that temperature is inversely proportional to time, which is actually the result of an increase in entropy. Therefore, this formula for the relationship between temperature and time does not actually have a mysterious physical meaning. The real physical meaning is actually entropy increase.

In quantum mechanics, the wave function of a free particle is

$$\psi = \mathrm{e}^{i(kr-\omega t)} \tag{3.12}$$

$i(kr-\omega t)$ indicates the phase of the particle wave. Using the de Broglie formula for particle waves, we can get.

$$kr=\frac{P}{\hbar}r=\frac{1}{\hbar}\frac{\partial I}{\partial r}r=\frac{1}{\hbar}\frac{\partial I}{\partial r}r=\int\frac{1}{\hbar}\frac{\partial I}{\partial r}\mathrm{d}r$$

$$\omega t=\frac{E}{\hbar}t=-\frac{1}{\hbar}\frac{\partial I}{\partial t}t=-\frac{1}{\hbar}\frac{\partial I}{\partial t}t=-\int\frac{1}{\hbar}\frac{\partial I}{\partial t}\mathrm{d}t$$

$$kr-\omega t=\frac{1}{\hbar}\int\left(\frac{\partial I}{\partial r}\mathrm{d}r+\frac{\partial I}{\partial t}\mathrm{d}t\right)=\frac{1}{\hbar}\int\mathrm{d}I=\frac{I}{\hbar}\tag{3.13}$$

$$i\left(kr-\omega t\right)=\frac{iI}{\hbar}\tag{3.14}$$

$$\psi=\mathrm{e}^{\frac{iI}{\hbar}}\tag{3.15}$$

So, we can see that the phase of the wave function actually corresponds to the action. But the action in the phase is an imaginary number, so it is actually an imaginary action. There is a correlation between the action and the quantum wave entropy in Formula (2.12). Therefore, there is actually a correlation between the phase of the wave function and the quantum wave entropy. Quantum wave entropy is defined by Formula (2.2). Quantum wave entropy itself is related to the wavelength of particle waves. Therefore, the correlation between the phase of the wave function and the quantum wave entropy is not an unexpected result.

We take Formula (2.12) into Formula (3.15) and we get the following formula.

$$\psi=\mathrm{e}^{\frac{i2\pi S}{\kappa_p}}$$

There is an unknown correlation between the quantum wave entropy and the phase of the wave function. The entropy in the phase can actually be seen as imaginary entropy. This is a topic that deserves in-depth study.

This inspired us to get a result. The imaginary action iI in quantum mechanics corresponds to the action I in classical mechanics

So, the above Formula (3.7) needs to be turned into Formula (3.16) in quantum mechanics.

$$E=-\frac{i\partial I}{\partial t}\tag{3.16}$$

Repeat the derivation process in Formula (3.11) above. When E is a constant value, so

$$Et=-iI$$

$$Et=-\frac{ihS}{\kappa_p}$$

$$E=TS$$

So we get Formula (3.17).

$$T=-\frac{ih}{\kappa_p t}\tag{3.17}$$

Formula (3.17) is the result in quantum mechanics. Because in the wave function Formula (3.15) of quantum mechanics, the action is an imaginary number, which is the imaginary action. Formula (3.17) is actually an imaginary number expression, not a real physical expression. The real physical expression is actually the Formula (3.11). Formula (3.17) is only a mathematical representation, not a real physical relationship. In real physical processes, the physical relationship between temperature and time is Formula (3.11). There is no such thing as imaginary time. Imaginary time is only a mathematical concept, and there is no imaginary time in real physical processes. Because the temperature in (3.17) is derived from the imaginary entropy, this temperature can be regarded as the imaginary temperature. Actually, time is not imaginary, but temperature is imaginary.

Formula (3.17) is the hypothetical Wick rotation in quantum mechanics. The Wick rotation was introduced as a hypothetical condition. But the physical meaning it represents has not been known. Now we find out that the physical meaning behind this assumption is actually quantum wave entropy. The establishment of Formula (3.17) actually contains three theories. First, the theory of action in classical mechanics. Second, the relationship between the action and the quantum wave entropy. Thirdly, phase in quantum mechanics is an imaginary action. Formula (3.17) can only be true if these three properties are satisfied.

We also find that in quantum mechanics, in the Formula (3.17) for the relationship between temperature and imaginary time, it is not the Boltzmann constant κ_B. It's actually a new constant κ_p. The relationship between the two constants is Formula (3.18).

$$\kappa_p = D\alpha\kappa_B \tag{3.18}$$

The new constant κ_p represents the property of quantum wave entropy. The constant D represents the intrinsic degrees of freedom of the particle. For example, spin is one of the intrinsic degrees of freedom. Different types of particles may have different intrinsic degrees of freedom. The constant α is a probability constant. So, different particles have different κ_p values. But for particles of the same type, the value of κ_p is always the same and is a constant.

We can see that the temperature included in Formula (3.17) is not the classical thermodynamic temperature. Although this temperature is defined by Formula (3.10), it is the same as the temperature definition formula in classical thermodynamics. But the entropy S in Formula (3.10) is not the entropy in classical thermodynamics, but a new type of entropy, which is the quantum wave entropy. The quantum wave entropy is defined by Formula (2.2). The probability distribution in Formula (2.2) is different from the Boltzmann probability distribution in classical thermodynamics. The probability distribution in Formula (2.2) is not a Boltzmann distribution. Readers need to be aware of the difference between the two probability distributions.

The probability distribution in Formula (2.2) is a special property of particle waves. Only particles in quantum mechanics have this special property. Therefore, quantum wave entropy is a type of entropy unique to quantum mechanics and does not exist in classical thermodynamics.

The temperature in Formula (3.17) can be referred to as the quantum wave temperature. Although quantum wave temperature is different from classical thermodynamic temperature, both have the same thermodynamic properties, both of which are defined by Formula (3.1). Both of these different types of temperatures are in accordance with the theory of thermodynamics. Therefore, we can find that the theory of thermodynamics has a very wide applicability. The probability in thermodynamic theory is not only the Boltzmann distribution, but also other different probability distributions. Different probability distributions can bring about different thermodynamic properties.

For each eigenstate of the particle, the energy E is a real number, no longer an imaginary number, and remains constant, so it satisfies Formula (3.11).

Throughout the above derivation, we found that quantum wave entropy plays a key bridging role. Quantum wave entropy creates a bridge between dynamics and thermodynamics. With this bridge, dynamics and thermodynamics are no longer independent of each other, but can be related to each other. When the two are connected, they can answer a lot of questions and bring a lot of new perspectives.

The use of Formula (3.10) may be questioned. Why use the formula $E = TS$ instead of the formula $\mathrm{d}E = T\mathrm{d}S$ [9]? If the formula $\mathrm{d}E = T\mathrm{d}S$ is used, the above derivation process cannot be established. There are two reasons for this. First, the precondition for energy E to satisfy Formula (3.9) is that the E must remain constant. Second, in quantum mechanics, the correspondence between temperature and imaginary time is obtained by analogy with the time evolution operator $\mathrm{e}^{-\frac{iEt}{\hbar}}$ and the partition function $\mathrm{e}^{-\frac{E}{\kappa_B T}}$. It must be a stationary state in order to get the time evolution operator of the form $\mathrm{e}^{-\frac{iEt}{\hbar}}$. So, for a stationary state, E is a constant value, $\mathrm{d}E = 0$. So, obviously, when we discuss this issue, we can't use the formula $\mathrm{d}E = T\mathrm{d}S$, we can only use the formula $E = TS$. Therefore, Formula (3.17) only applies to stationary states.

Note that in the derivation of Formula (3.17) above, the energy E satisfies Formula (3.9). So, here the energy E of the particle contains potential energy, not pure kinetic energy.

As discussed above, in quantum wave entropy, there is a radiant temperature for particles moving at an accelerated pace. The radiation temperature is Formula (2.5). In the case of uniformly accelerated particles, there is also an inverse temperature-time relationship.

$$T = \frac{h}{\kappa_p V} a = \frac{ha}{\kappa_p at} = \frac{h}{\kappa_p t} \tag{3.19}$$

In the derivation of this formula, the energy E is taken as a positive value. If we take the energy E as a negative value, we get a negative sign, which is the same as Formula (3.11). Although the derivation of Formula (3.19) uses the uniform acceleration condition of $V = at$. However, it can be seen that the final result in Formula (3.19) does not include acceleration a, and is not related to acceleration a. Therefore, Formula (3.19) can actually be applied to all cases of accelerated motion. Note that in the derivation of Formula (3.19), neither the energy E nor the entropy S of the particle contain a potential energy component. Readers need to be aware of the differences in different situations.

Formula (3.19) can also be used for experimental testing of quantum wave entropy. In contrast to the experimental instruments that are moving at an accelerated pace, all other particles are moving at an accelerated pace and have a radiation temperature. Therefore, the accelerated motion of the experimental instrument will detect a background radiation temperature that satisfies the Formula (3.19).

So, we can find that there is an inverse relationship between quantum wave temperature and time, and this seems to be a universal property. This is a topic that deserves further in-depth study.

In thermodynamics, for Boltzmann entropy, there exist the following partition function.

$$Z(T) = \sum e^{-\frac{E_n}{\kappa_B T}} \tag{3.20}$$

where E_n denotes the energy level. The lower the energy level, the greater the probability, and the more particles appear on the energy level. This partition function corresponds to the Boltzmann distribution.

In quantum mechanics, there exists the superposition of time evolution operators.

$$\hat{H}(t) = \sum \langle \varphi_n | e^{-\frac{iE_n t}{\hbar}} | \varphi_n \rangle \tag{3.21}$$

where E_n denotes the energy level also. The lower the energy level, the greater the probability, and the more particles appear on the energy level. This is very similar to the partition function. However, this property is not derived from the particle distribution, but from the superposition principle of wave functions and the Schrödinger equation.

Through the relationship between temperature and imaginary time, we can find that the above two formulas can be converted to each other and have a completely equivalent representational effect. It has been shown above that the relationship between temperature and imaginary time is derived from quantum wave entropy. So, we can find a result. In quantum mechanics, the evolution and superposition of wave functions, combined with the use of quantum wave entropy, is equivalent to Boltzmann entropy in thermodynamics.

In the partition function (3.20), the temperature T is a key physical quantity. All energy levels have the same temperature. However, the temperature of the system must first be determined before the partitioning function can be determined. However, the entropy and energy of the entire system must first be determined in order to calculate this temperature. However, the entropy and energy of the entire system need to be determined, and the particle distribution and total number of particles of the entire system must first be determined. So, Boltzmann entropy is actually a holistic perspective. It’s a holistic approach.

In contrast, in quantum mechanics, the evolution and superposition of wave functions do not require first determining the whole properties of the system. We can process each eigenstate separately to obtain the evolutionary properties of different eigenstates, and then superimpose them to obtain the whole properties of the system. In addition, different eigenstates have different quantum wave entropy and energy. We can determine the entropy and energy of different eigenstates separately, and then superimpose them to obtain the total entropy and energy. In Equation (3.21), the key physical quantity is imaginary time. All eigenstates have the same imaginary time. Through the relationship between temperature and imaginary time, the imaginary time is converted to temperature, which is equivalent to all eigenstates having the same temperature. Therefore, the quantum wave entropy and energy of different eigenstates can be superimposed to obtain the entropy and energy of the whole system, so we can get the thermodynamic properties of the whole system. So, it's really a way of decomposing and superimposing. So, quantum mechanics plus quantum wave entropy is a new thermodynamic approach. This new approach is very different from Boltzmann entropy approach. However, for a thermodynamic system, both methods can achieve the same thermodynamic results.

On the other hand, in quantum mechanics, the Schrödinger equation, the evolution of wave functions, and the superposition of wave functions all have a wide range of applicability and are not limited by equilibrium. The same applies to intermediate stages and non-equilibrium states in the evolutionary process. Quantum wave entropy is also not limited by equilibrium states. Therefore, we can find that by introducing the concept of quantum wave entropy in quantum mechanics, we are no longer limited by classical thermodynamics, and we can study the thermodynamic properties of various intermediate states and non-equilibrium states. The application of quantum wave entropy in quantum mechanics is a topic worthy of further study.

4. Conclusions

As can be seen from the above derivation process, using the concept of quantum wave entropy, we can simply deduce the Wick rotation in quantum mechanics. The existence of Wick rotation represents the existence of quantum wave entropy. Quantum wave entropy creates a bridge between dynamics and thermodynamics, thus correlating temperature and time, resulting in the Wick rotation. This reveals the physical meaning behind Wick rotation. Imaginary time is only a mathematical representation, and there is no real imaginary time. In real physical processes, the relationship between temperature and time is Formula (3.11). It is also found that there are also valid prerequisites for Wick rotation, which can only be applied to the stationary state. Wick rotation is not an unlimited way to use it everywhere.

In the process of derivation, we found a correlation between the phase of the wave function and the quantum wave entropy, and what is the physical meaning behind this correlation, which is worth further studying. The inverse relationship between quantum wave temperature and time seems to be a universal property. The new concept of quantum wave entropy can bring answers to many questions and bring a lot of new thinking, which needs to be further studied.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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