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Decision-Making during Control Pollutant Emissions from Pellet Burning with Tube Gas Heaters

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Abstract

The article is devoted to decision making regarding controlling the operation of tubular gas heaters (TGH) on wood pellets. Experimental results of the study of the operation of TGH on pellets are used for decision making. Experiments have shown the dependence of undesirable gas emissions, carbon oxides and nitrogen oxides in combustion products, on the parameters of the heater operation. The nature of the dependence is contradictory, it is not possible to simultaneously minimise emissions of carbon oxides and nitrogen, it is necessary to look for compromise solutions. The task was set to find such operating modes of pellet heaters that provide acceptable values of gas emissions at different power levels during heater operation. To solve the problem, we used expert judgements in the form of matrices of fuzzy pairwise comparison of separate results of heater operation with each other. The fuzzy decision selection functions were constructed, which extend not only to the set of experimental results, but also to the whole set of possible variation of the TGN operation parameters. For each selection function, their maxima are found, which provide the operation of TGN at different power modes with acceptable gas emissions values. These results can serve for three-stage control of the TGN.

Keywords

Green Energy Engineering, Wood Pellets, Tube Gas Heaters, Evolutionary Search, Stochastic Optimization, Binary Choice Relations

1. Introduction

Tube gas heaters (TGH) can be seen as the development of infrared gas tube heaters (IGTH). IGTHs have a long history of development and use. You can specify these articles [1]-[3] and this comprehensive scientific report [3]. These heaters are serially produced by a number of manufacturers in different countries, for example—ROBERTS GORDON [4]. The main components of such heaters are: automatic gas burner, tube emitter, infrared reflector and exhaust or supply fan. Technical solutions appeared, and due to the change in the heat exchange part, the field of application of gas tube heaters expanded, as reflected in [5]. Finally, pellet tube heaters appeared, and the gas burner was replaced by a pellet [6] [7]. The external view of a pellet burner unit with pellet bunker and control unit in an operating heat supply system and view of the experimental setup for testing pellet burner with tubular gas heater are shown in **Figure 1** and **Figure 2**. Experimental studies have shown that gas emissions from pellet gas burners depend significantly on the operating modes of tubular gas heaters. **Figure 3** and **Figure 4** show the operation of gas burners on pellets for two modes of operation. It is of interest to find such operating modes of tube heaters that would ensure minimum gas emissions.

In this paper, evolutionary search methods considering fuzzy experimental data and binary choice relations are used to solve the control problem of tube heaters. Evolutionary search methods have been successfully applied to find solutions to various optimisation problems in the presence of one or more criteria, for example, [8] discusses the use of evolutionary search to solve a multi-criteria problem. Evolutionary algorithms play a dominant role in solving problems with multiple conflicting objective functions. They aim at finding multiple Pareto-optimal solutions, thus in [9] a hybrid constrained evolutionary algorithm (HCEA) is proposed which uses two penalty functions simultaneously. Particle swarm optimisation (PSO) algorithms have been successfully used to solve various complex optimisation problems. However, the balance between diversity and convergence is still a problem that requires continuous study, so evolutionary particle swarm optimisation with dynamic search (EEDSPSO) has been proposed [10]. In [11], a decision-making approach for fuzzy Fermathean soft set based on a score matrix was proposed. A numerical example has been given to demonstrate the validity of the proposed approach. In [12], the proposed method is used to predict the output value in empirical applications where the observed value is a range or average of several values rather than a real fixed number. Stochastic optimisation plays an important role in the analysis, design and operation of modern systems [13]. A considerable number of papers have been devoted to stochastic optimisation, most notably [14] [15]. Evolutionary fuzzy systems are one of the greatest advances in the field of computational intelligence. They consist of evolutionary algorithms used to design fuzzy systems [16]. Modelling methods for fuzzy systems have received considerable development in the works [17] [18]. The work [19] uses the developed modification of genetic algorithm to optimise the performance of neural network.

In [20], the concept of trigonometric similarity measure (SM) for spherical fuzzy sets (SFS) is used, which has become very important in solving various pattern recognition and medical diagnosis problems. The approach to solve fuzzy nonlinear programming problems was presented in [21] [22]. In [23] proposed a multi-objective nonlinear programming problem to be solved as a linear programming problem. In [24] used evolutionary algorithm for multi-objective optimisation. In [25], binary choice relations were used for decision making . This direction was further developed, for example, in [26]-[28]. Decision making in complex systems by methods of self-organisation was developed in the works of Ivakhnenko O. G. [29] and his followers. In the works of Yudin D. Б. [30] [31], as well as in [32], computational methods of decision-making theory were considered, in which decision search problems are formulated in terms of binary relations, and the problems of nonlinear mathematical programming are transformed into generalised mathematical programming problems. Methods of evolutionary decision search in problems with binary choice relations were first developed in [33], then developed in [34] [35]. Finally, in [36] [37] a scheme for constructing an evolutionary selection mechanism for decision making in multi-criteria systems with a sample of fuzzy experimental results was proposed. It is of interest to use evolutionary search methods for decision making with several criteria to control the operation of a tubular gas heater on pellets, which determined the content of this work.

Figure 1. External view of a pellet burner unit with pellet bunker and control unit in an operating heat supply system.

Figure 2. External view of the experimental setup for testing pellet burner with tubular gas heater.

Figure 3. View of operating pellet gas burner at minimum output.

Figure 4. View of operating pellet gas burner at maximum output.

2. The Problem of Fuzzy Modeling of Pellet Burner

Mathematical modeling of a pellet burner for tube gas heater is considered. The basis for this mathematical modeling is the results of an experimental study of the operation of the pellet burner. The results of the study of the work of the pellet burner [33] are presented in the form **Tables 1-6**. There are 5 dimensional parameters and 3 dimensionless parameters (complexes) that characterize the operating pellet burner. Dimensional parameters are: burner area, S; useful area for primary air passage, S_P; primary air flow, L_P; total air flow, L; burner power, W. Outlet system functions of the heater: ash transfer by the time, Y_A; concentration CO at exhaust gases, Y_{CO}; concentration NO_x at exhaust gases, Y_{NOx}. A relationship was established between dimensionless complexes and parameters in the form

$$\Pi_1 = S_P/S; \Pi_2 = L_P/L; \Pi_3 = W/Y_A\Big/\left(L/S\right)^2; \phi\left(\Pi_1, \Pi_2, \Pi_3\right) = 0, \qquad (1)$$

where

$$\Pi_3 = d_1\left(\Pi_1\right)^{d_2}\left(\Pi_2\right)^{d_3}\left(1-\Pi_1\Pi_1\right)^{d_4}\left(1-\Pi_1\Pi_2\right)^{d_5} \qquad (2)$$

where parameters $d_1, \cdots, d_5$ are obtained from the condition of minimizing the relative error of the model (2) at the points of the training sequence, namely d_1 = 0.0116, d_2 = 1.465, d_3 = −1.029, d_4 = 6.34, d_5 = −0.14.

3. Materials and Methods

We will assume that the system is characterized by a set of parameters $v=\{v^1,v^2,\cdots,v^{N_v}\}, v\in\Omega_v$ and there are also initial parameters (functions, criteria) $w=\{w^1,w^2,\cdots,w^{N_w}\}, w\in\Omega_w$. We will assume that there is the set of experimental results in the form $u=\left\|v_j^{i_v},w_j^{i_w}\right\|, i_v=1,\cdots,N_v; i_w=1,\cdots,N_w; j=1,\cdots,N_f$, where N_f the number of experiments. The total number of experiments N_f was divided into three subgroups u_1,u_2,u_3, so that u_1 is the subgroup of the minimal heater power $W\in(1\text{-}6)\text{kW}$, u_2 is the average heater power $W\in(6\text{-}18)\text{kW}$, and u_3 is the maximal heater power $W\in(18\text{-}50)\text{kW}$, so it may be represent in form:

$$\begin{aligned} u_1&=\left\|v_j^{i_v},w_j^{i_v}\right\|, i_v=1,\cdots,N_v; i_w=1,\cdots,N_w; j=j_1,\cdots,N_{f_1}\\ u_2&=\left\|v_j^{i_v},w_j^{i_v}\right\|, i_v=1,\cdots,N_v; i_w=1,\cdots,N_w; j=j_2,\cdots,N_{f_2}\\ u_3&=\left\|v_j^{i_v},w_j^{i_v}\right\|, i_v=1,\cdots,N_v; i_w=1,\cdots,N_w; j=j_3,\cdots,N_{f_3} \end{aligned} \tag{3}$$

If we give the experimental results an expert assessment using fuzzy comparisons of the results with each other, then we will obtain a fuzzy correspondence matrix of experiments, which can be represented in the form

$$\begin{aligned} Z_1&=\left\|z_{ij}\right\|, i=1,\cdots,N_{f_1}; j=1,\cdots,N_{f_1}\\ Z_2&=\left\|z_{ij}\right\|, i=1,\cdots,N_{f_2}; j=1,\cdots,N_{f_2}\\ Z_3&=\left\|z_{ij}\right\|, i=1,\cdots,N_{f_3}; j=1,\cdots,N_{f_3} \end{aligned} \tag{4}$$

For expert evaluation the rating scale was used $z_{ij}\in\{0;0.3;0.4;0.5;0.6;0.8;1.0\}$, which make sense: {much worse; worse; slightly worse; comparable; slightly better; better; much better}. We also assume that the fuzzy binary relation $\tilde{R}_{S1}$ with the membership function $\mu_{R_{S1}}(z,z)$ is known. We assume that the known selection function $\Gamma_1(z)$ is such that $\Gamma_1(z(x_1))\ge\Gamma_1(z(x_2))\equiv z(x_1)\tilde{R}_{S1}z(x_2), \forall x_1,x_2\in\Omega_1$.

And we assume that the known selection function $\Gamma_2(z)$ with fuzzy binary relation $\tilde{R}_{S2}$ with the membership function $\mu_{R_{S2}}(z,z)$ is known, so that $\Gamma_2(z(x_1))\ge\Gamma_2(z(x_2))\equiv z(x_1)\tilde{R}_{S2}z(x_2), \forall x_1,x_2\in\Omega_2$.

And we assume that the known selection function $\Gamma_3(z)$ with fuzzy binary relation $\tilde{R}_{S3}$ with the membership function $\mu_{\tilde{R}_{S3}}(z,z)$ is known, so that $\Gamma_3(z(x_1))\ge\Gamma_3(z(x_2))\equiv z(x_1)\tilde{R}_{S3}z(x_2), \forall x_1,x_2\in\Omega_3$.

It is necessary to find a solution $x\in\Omega_1$ and for all $y\in\Omega_1$ so that

$$\Gamma_1(x)\ge\Gamma_1(y). \tag{5}$$

And it is necessary to find a solution $x\in\Omega_2$ and for all $y\in\Omega_2$ so that

$$\Gamma_2(x)\ge\Gamma_2(y). \tag{6}$$

And it is necessary to find a solution $x\in\Omega_3$ and for all $y\in\Omega_3$ so that

$$\Gamma_3(x)\ge\Gamma_3(y). \tag{7}$$

Algorithm with Mathematical Expectations

In the binary relations (5)-(7) we replace selection function $\Gamma_1(z)$, $\Gamma_2(z)$, $\Gamma_3(z)$ the sample mean values, which is calculated in the form

$$\begin{aligned}&\bar{\Gamma}_1(x,\theta)=1/n_i\sum_i^{n_i}\Gamma_1(x,\theta_i),\bar{\Gamma}_2(x,\theta)=1/n_i\sum_i^{n_i}\Gamma_2(x,\theta_i),\\&\bar{\Gamma}_3(x,\theta)=1/n_i\sum_i^{n_i}\Gamma_3(x,\theta_i).\end{aligned} \tag{8}$$

where θ_i-implementation of a random process, n_r -total number of realizations of a random process. We replace binary relation (5)-(7) with

$$\begin{aligned}&x\tilde{R}_{\bar{S}1}y\equiv\bar{\Gamma}_1(x,\theta)\ge\bar{\Gamma}_1(y,\theta),x\tilde{R}_{\bar{S}2}y\equiv\bar{\Gamma}_2(x,\theta)\ge\bar{\Gamma}_2(y,\theta),\\&x\tilde{R}_{\bar{S}3}y\equiv\bar{\Gamma}_3(x,\theta)\ge\bar{\Gamma}_3(y,\theta).\end{aligned} \tag{9}$$

The methods for solving the problems are based on the approach to the evolutionary search for $\tilde{R}_S$ -optimal solutions. For subset X, $X\subset\Omega$ we denote the function of choice in the form

$$S(X)=\left\{x\in X\,|\,\forall y\in\left[X\setminus S(X)\right],x\tilde{R}_S y\right\} \tag{10}$$

We shall assume that set $S(X)$ contains the concrete number of elements—N_{op}.

We shall that for the set Ω it was determined relation $\tilde{R}_G$ with membership function $\mu_{\tilde{R}_G}(x,y)$: $\Omega\times\Omega\to[0,1]$. Relation $\tilde{R}_G$ will be termed generation relation. For subset X, $X\subset\Omega$ we denote the function of generation in the form

$$G(X)=X\cup G_H(X) \tag{11}$$

$$G_H(X)=\left\{y\in\Omega\,|\,\exists x\in X,y\tilde{R}_G x,\mu_{\tilde{R}_G}(x,y)>0\right\} \tag{12}$$

We shall assume that set $G(X)$ contains the concrete number of elements—N_E.

The algorithm to search $\tilde{R}_S$ -optimal solution can be represented as

$$X_k=S\left(G\left(X_{k-1}\right)\right),k=1,2,\cdots \tag{13}$$

The iterate algorithm (13) is the general form of evolutionary search.

According to [34] [35] we will consider the decomposition

$$X_k=\bigcup_{j=1}^{N_B}X_{jk},X_{ik}\cap X_{jk}=\varnothing \tag{14}$$

The algorithm (13) takes the form

$$X_{jk}=S\left(G\left(X_{jk-1}\right)\right),k=1,2,\cdots;j=1,2,\cdots,N_B \tag{15}$$

These iterate algorithms (13), (15) are the general form of evolutionary search.

The evolutionary search algorithm converges to the most preferred solution of choice relation. This position has been theoretically and experimentally proven for clear choice relationships. For a fuzzy choice, this position is based on experimental results. Suppose that the solutions that passed the selection at some step of the iteration for all branches of the evolutionary search have the form $\left\{x_{lj}^i\right\}$, where i is the number of the variable value, for the selected l-th solution $l = 1,\cdots,N_l$ in the j-th branch of the search $j = 1,\cdots,N_B$. Average values for all selected solutions can be calculated as follows:

$$x_0^i = \frac{1}{N_B N_l}\sum_{j=1}^{N_B}\sum_{l=1}^{N_l} x_{lj}^i \tag{16}$$

At the same time, the values of the empirical dispersion will be

$$\sigma_i^2 = \frac{1}{N_B N_l - 1}\sum_{j=1}^{N_B}\sum_{l=1}^{N_l}\left(x_{lj}^i - x_0^i\right)^2 \tag{17}$$

The generation of new solutions at the next step of the iteration is performed with a normal distribution for each.

Parameter x^i and centers in $x_{lj}^i, j = 1,\cdots,N_B$, and variance σ_i^2. That is, the membership function μ_{R_G} for the fuzzy generation relation is the density function of the normal distribution:

$$\mu_{R_G}\left(y^i, x^i\right) = \frac{1}{\sigma_i\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{y^i - x^i}{\sigma_i}\right)^2\right] \tag{18}$$

4. Results

4.1. Isolation of Experimental Data for the Minimum Power of a Tubular Heater

The experimental data for the minimum power of a tubular heater was presented in Table 1.

Table 1. Experimental data for the minimum power of a tubular heater.

№	S	S_P	L	L_P	W	Y_A	Y_{CO}	Y_{NOx}
	m^2	m^2	m^3/h	m^3/h	kW	g/min	mg/m^3	mg/m^3
1	0.0025	0.00021	201	2.7	6.4	3.57	2765	89
2	0.0025	0.00021	168	4.1	9	7	2902	134
3	0.0025	0.00021	215	2.2	4.7	1.6	1429	146
4	0.0025	0.00021	178	2.5	5.3	1.8	812	201
5	0.0025	0.00021	167	2.8	4.5	0.7	2148	160
6	0.0025	0.00021	155	3	6	1.7	722	265
7	0.0025	0.00021	127	2.5	8.2	1.9	1099	134
8	0.0025	0.00021	123	3	9	1	450	188
9	0.0025	0.00021	210	2.75	3.9	1.3	2926	161
10	0.0025	0.00021	175	4.1	9	3.4	6663	56
11	0.0025	0.00021	172	4.3	7.5	5.6	2845	148
12	0.0025	0.00021	152	2.2	5	5	1826	116

All experimental data of Table 1 are divided into two arrays—training and test data, Table 2 and Table 3.

Table 2. The experimental data for the minimum power of a tubular heater—training sequence.

№	S	S_P	L	L_P	W	Y_A	Y_{CO}	Y_{NOx}
	m^2	m^2	m^3/h	m^3/h	kW	g/min	mg/m^3	mg/m^3
1	0.0025	0.00021	201	2.7	6.4	3.57	2765	89
2	0.0025	0.00021	215	2.2	4.7	1.6	1429	146
3	0.0025	0.00021	178	2.5	5.3	1.8	812	201
4	0.0025	0.00021	123	3	9	1	450	188
5	0.0025	0.00021	210	2.75	3.9	1.3	2926	161
6	0.0025	0.00021	175	4.1	9	3.4	6663	56
7	0.0025	0.00021	172	4.3	7.5	5.6	2845	148

Table 3. The experimental data for the minimum power of a tubular heater—test sequence.

№	S	S_P	L	L_P	W	Y_A	Y_{CO}	Y_{NOx}
	m^2	m^2	m^3/h	m^3/h	kW	g/min	mg/m^3	mg/m^3
1	0.0025	0.00021	168	4.1	9	7	2902	134
2	0.0025	0.00021	167	2.8	4.5	0.7	2148	160
3	0.0025	0.00021	155	3	6	1.7	722	265
4	0.0025	0.00021	127	2.5	8.2	1.9	1099	134
5	0.0025	0.00021	152	2.2	5	5	1826	116

4.2. Isolation of Experimental Data for the Average Power of a Tubular Heater

The experimental data for the average power of a tubular heater is presented in Table 4.

Table 4. Experimental data for the average power of a tubular heater.

№	S	S_P	L	L_P	W	Y_A	Y_{CO}	Y_{NOx}
	m^2	m^2	m^3/h	m^3/h	kW	g/min	mg/m^3	mg/m^3
1	0.01	0.00643	633.6	46.8	18	0.21	4500	257
2	0.0025	0.00021	165	4.3	18	10	7214	109
3	0.0025	0.00021	151	5.1	18	7	7844	125
4	0.0025	0.00021	201	2.8	11.3	4.9	1311	193

Continued

5	0.0025	0.00021	182	3.9	12.8	3.6	779	212
6	0.0025	0.00021	150	3.5	11.2	2.8	617	259
7	0.0025	0.00021	140	4	18	5.4	1144	240
8	0.0025	0.00021	111	3.4	11.3	1.9	246	151
9	0.0025	0.00021	105	3.8	15	3	438	190
10	0.0025	0.00021	97	4.1	15	4.8	1225	238
11	0.0025	0.00021	80	6.5	18	10.8	945	217

All experimental data in Table 4 was divided into two arrays—training and test data, Table 5 and Table 6.

Table 5. Experimental data for the average power of a tubular heater—the training sequence.

№	S	S_P	L	L_P	W	Y_A	Y_{CO}	Y_{NOx}
	m^2	m^2	m^3/h	m^3/h	kW	g/min	mg/m^3	mg/m^3
1	0.01	0.00643	633.6	46.8	18	0.21	4500	257
2	0.0025	0.00021	151	5.1	18	7	7844	125
3	0.0025	0.00021	201	2.8	11.3	4.9	1311	193
4	0.0025	0.00021	182	3.9	12.8	3.6	779	212
5	0.0025	0.00021	111	3.4	11.3	1.9	246	151
6	0.0025	0.00021	80	6.5	18	10.8	945	217

Table 6. Experimental data for the average power of a tubular heater—the test sequence.

№	S	S_P	L	L_P	W	Y_A	Y_{CO}	Y_{NOx}
	m^2	m^2	m^3/h	m^3/h	kW	g/min	mg/m^3	mg/m^3
1	0.0025	0.00021	165	4.3	18	10	7214	109
2	0.0025	0.00021	150	3.5	11.2	2.8	617	259
3	0.0025	0.00021	140	4	18	5.4	1144	240
4	0.0025	0.00021	105	3.8	15	3	438	190
5	0.0025	0.00021	97	4.1	15	4.8	1225	238

4.3. Isolation of Experimental Data for the Maximum Power of a Tubular Heater

The experimental data for the maximum power of a tubular heater was presented in Table 7.

Table 7. Experimental data for the maximum power of a tubular heater.

№	S	S_P	L	L_P	W	Y_A	Y_{CO}	Y_{NOx}
	m^2	m^2	m^3/h	m^3/h	kW	g/min	mg/m^3	mg/m^3
1	0.005	0.00286	572.4	25.2	33.5	2.1	510	293
2	0.005	0.00286	543.6	23.4	31.3	2.88	6734	207
3	0.005	0.00286	543.6	21.6	54.7	2.77	43	259
4	0.01	0.00643	651.6	54	32	0.47	694	205
5	0.01	0.00643	684	50.4	35.5	5.5	110	230
6	0.0025	0.00021	196	3	10	5	1019	210
7	0.0025	0.00021	136	4.5	22.5	10.5	853	257
8	0.0025	0.00021	128	7	22.5	11.3	783	261
9	0.0025	0.00021	85	5	22.5	10.3	830	203
10	0.0025	0.00021	168	5.1	18	35	1986	131

All experimental data in Table 7 was divided into two arrays—training and test data, Table 8 and Table 9.

Table 8. Experimental data for the maximum power of a tubular heater—the training sequence.

№	S	S_P	L	L_P	W	Y_A	Y_{CO}	Y_{NOx}
	m^2	m^2	m^3/h	m^3/h	kW	g/min	mg/m^3	mg/m^3
1	0.005	0.00286	572.4	25.2	33.5	2.1	510	293
2	0.005	0.00286	543.6	23.4	31.3	2.88	6734	207
3	0.005	0.00286	543.6	21.6	54.7	2.77	43	259
4	0.0025	0.00021	196	3	10	5	1019	210
5	0.0025	0.00021	136	4.5	22.5	10.5	853	257
6	0.0025	0.00021	128	7	22.5	11.3	783	261

Table 9. Experimental data for the maximum power of a tubular heater—the test sequence.

№	S	S_P	L	L_P	W	Y_A	Y_{CO}	Y_{NOx}
	m^2	m^2	m^3/h	m^3/h	kW	g/min	mg/m^3	mg/m^3
1	0.01	0.00643	651.6	54	32	0.47	694	205
2	0.01	0.00643	684	50.4	35.5	5.5	110	230
3	0.0025	0.00021	85	5	22.5	10.3	830	203
4	0.0025	0.00021	168	5.1	18	35	1986	131

4.4. Expert Evaluation the Rating Scale

For expert evaluation the rating scale was used $b_{ij} = \{0; 0.3; 0.4; 0.5; 0.6; 0.7; 1.0\}$; which make sense: {much worse; worse; slightly worse; comparable; slightly better; better; much better}. Two sets were identified for expert evaluation: 1) training sequence array, 2) testing sequence array. These heater comparison matrices are presented below

Comparison matrix for minimum power heaters training sequence array

0.5, 0.4, 0.2, 0.1, 0.5, 0.8, 0.5
0.6, 0.5, 0.3, 0.1, 0.7, 0.8, 0.6
0.8, 0.7, 0.5, 0.3, 0.7, 0.8, 0.7
0.9, 0.9, 0.7, 0.5, 0.9, 1.0, 0.9
0.5, 0.3, 0.3, 0.1, 0.5, 0.8, 0.5
0.2, 0.2, 0.2, 0.0, 0.2, 0.5, 0.2
0.5, 0.4, 0.3, 0.1, 0.5, 0.8, 0.5

Comparison matrix for minimum power heaters testing sequence array

0.5, 0.6, 0.3, 0.3, 0.4
0.4, 0.5, 0.3, 0.3, 0.4
0.7, 0.7, 0.5, 0.4, 0.5
0.7, 0.7, 0.6, 0.5, 0.6
0.6, 0.6, 0.5, 0.4, 0.5

Comparison matrix for average power heaters training sequence array

0.5, 0.7, 0.4, 0.3, 0.2, 0.3
0.3, 0.5, 0.3, 0.2, 0.1, 0.2
0.6, 0.7, 0.5, 0.4, 0.2, 0.4
0.7, 0.8, 0.6, 0.5, 0.3, 0.6
0.8, 0.9, 0.8, 0.7, 0.5, 0.7
0.7, 0.8, 0.6, 0.4, 0.3, 0.5

Comparison matrix for average power heaters testing sequence array

0.5, 0.3, 0.4, 0.3, 0.4
0.7, 0.5, 0.6, 0.3, 0.6
0.6, 0.4, 0.5, 0.3, 0.5
0.7, 0.7, 0.7, 0.5, 0.7
0.6, 0.4, 0.5, 0.3, 0.5

Comparison matrix for maximum power heaters training sequence array

0.5, 0.7, 0.2, 0.5, 0.5, 0.5
0.3, 0.5, 0.1, 0.3, 0.3, 0.3
0.8, 0.9, 0.5, 0.8, 0.8, 0.8
0.5, 0.7, 0.2, 0.5, 0.5, 0.5
0.5, 0.7, 0.2, 0.5, 0.5, 0.5
0.5, 0.7, 0.2, 0.5, 0.5, 0.5

Comparison matrix for maximum power heaters testing sequence array

0.5, 0.1, 0.5, 0.6
0.9, 0.5, 0.9, 0.9

0.5, 0.1, 0.5, 0.6
0.4, 0.1, 0.4, 0.5

4.5. Results for Choice Functions

There are presented results with choice function in the form (19)-(21).

$$\Gamma(x) = \prod_{i=1}^{5}\left(1 + a_{1i}\left(a_{2i} - r_i\right)^2\right) \tag{19}$$

$$r_1 = x_1^1 - x_2^1; r_2 = x_1^2 - x_2^2; r_3 = x_1^3 - x_2^3; r_4 = x_1^4 - x_2^4; r_5 = x_1^5 - x_2^5 \tag{20}$$

$$\Gamma(x_1) \ge \Gamma(x_2) \equiv x_1 \tilde{R}_S x_2 \tag{21}$$

Parameters a_{1i}, a_{2i} were obtained after evolutionary search the choice function for array 1 of experimental data and for array 2 of experimental data. The results of evolutionary search the choice function is presented in **Tables 10-12**.

Table 10. Parameters of the fuzzy choice function for the minimum power heater.

	i	a_{1i}	a_{2i}
a_{1i}, a_{2i}	1	−0.3071252	0.8014811
a_{1i}, a_{2i}	2	0.1532593	−0.3323147
a_{1i}, a_{2i}	3	0.4978446	−0.6202395
a_{1i}, a_{2i}	4	−0.4215357	−0.01208718
a_{1i}, a_{2i}	5	0.003904735	0.3095479

Table 11. Parameters of the fuzzy choice function for the average power heater.

	i	a_{1i}	a_{2i}
a_{1i}, a_{2i}	1	−0.3627225	0.2329837
a_{1i}, a_{2i}	2	−0.05706916	−0.3366217
a_{1i}, a_{2i}	3	0.3229559	−0.273395
a_{1i}, a_{2i}	4	−0.1807748	−0.1512893
a_{1i}, a_{2i}	5	−0.0007113587	−0.8040671

Table 12. Parameters of the fuzzy choice function for the maximum power heater.

	i	a_{1i}	a_{2i}
a_{1i}, a_{2i}	1	−0.4144383	0.1412199
a_{1i}, a_{2i}	2	−0.1136156	−0.1227887
a_{1i}, a_{2i}	3	0.2455468	−0.07456189
a_{1i}, a_{2i}	4	−0.07079072	−0.08849594
a_{1i}, a_{2i}	5	0.06467061	−0.443341

The choice function in the form (19)-(21) with specific values of parameters a_{1i}, a_{2i}, $i = 1, \cdots, 5$ was used to solve the problem of generalized mathematical programming: to find maximum of choice function

$\max \Gamma(x)$ with restrictions: $0.08 \le \Pi_1 \le 0.7; 0.01 \le \Pi_2 \le 0.1; 0.001 \le \Pi_3 \le 0.8$.

The results of determining the maxima of the selection functions for the three heater powers are shown below (Table 13).

Table 13. Values of parameters a_{1i}, a_{2i}, $i = 1, \cdots, 5$ as the result of solving mathematical programming problem.

	Dimensionless complex Π_1	Dimensionless complex Π_2	Dimensionless complex Π_3
minimum power heaters	0.4420147	0.03648748	0.02673138
average power heaters	0.4619097	0.04038155	0.02226561
maximum power heaters	0.5120847	0.04504298	0.01542043

For gas emission concentrations there are experimental dependencies [6] in the form (22):

For CO:

$$\Pi_4 = b_1 \cdot \left(\left(1 - \Pi_1^2\right)^{b_2} \Big/ \left(1 - \Pi_1 \cdot \Pi_2\right)^{b_3} \right) \cdot \left(b_4 + \left(\Pi_3 / \Pi_2 \right)^{b_5} \right) \tag{22}$$

where: $b_1 = 0.0256$, $b_2 = 5.945$, $b_3 = 63.4$, $b_4 = 1.95$, $b_5 = 0.48$.

For NOx:

$$\Pi_5 = a_1 + a_2 \cdot \left(\Pi_1\right)^{a_3} \cdot \left(\Pi_2\right)^{a_4} \cdot \left(\Pi_3\right)^{a_5} \tag{23}$$

where: $a_1 = 1.096$; $a_2 = 31.33$; $a_3 = 3.2155$; $a_4 = -01776$; $a_5 = 0.7470$.

Using dimensionless dependencies for harmful gases, the corresponding concentrations of harmful gases at different tube heater powers can be calculated in the form of a Table 14.

Table 14. Concentrations of gas emissions at optimum operation modes of heaters.

minimum power heaters $5\text{ kW} < W < 9\text{ kW}$	average power heaters $9\text{ kW} \le W < 18\text{ kW}$	maximum power heaters $18\text{ kW} \le W < 55\text{ kW}$
Optimum concentrations CO		
$\alpha_{CO} = 0.00307$	$\alpha_{CO} = 0.003001$	$\alpha_{CO} = 0.002205$
20.3 mg/m^3	20.04 mg/m^3	17.18 mg/m^3
Optimum concentrations NO_X		
$\alpha_{NO_X} = 1.579$	$\alpha_{NO_X} = 1.5809$	$\alpha_{NO_X} = 1.604$
242.1 mg/m^3	242.15 mg/m^3	243.9 mg/m^3

As it can be seen from the table of gas emission concentrations, at operation of heaters on all three modes of operation at selection of modes from the table of the most preferable modes the conditions for gas emissions are provided in the form of

$$\alpha_{CO} \leq 130\ \mathrm{mg/m^3} \quad \text{and} \quad \alpha_{NO_X} \leq 250\ \mathrm{mg/m^3} \tag{24}$$

Such conditions correspond, in particular, to the current Ukrainian requirements for natural gas combustion. Therefore, providing such conditions for combustion of wood pellets in tubular gas heaters should be considered quite acceptable.

5. Discussion and Conclusions

Three power modes of wood pellet fired tubular gas heaters (minimum power, medium power and maximum power) were determined based on the results of the experiments. The experiments showed that significant gas emission values were observed in each of the modes and it was not possible to minimise CO and NOx emissions simultaneously. The challenge was to find compromise solutions for all modes of heater operation that would provide the most preferred favourable gas emission values. Using fuzzy expert judgements of heater performance, the experimental modes were compared with each other in the form of a matching matrix. Using evolutionary search, fuzzy choice functions were obtained for the three modes of heater operation. For each fuzzy choice function, maxima were found on the entire set of possible parameters, not only on the set of experiments. The obtained dimensionless criteria at the points of maxima give the most favourable values for decision making in all three modes. At the same time, as shown by the final analysis, these modes ensure the operation of tubular gas heaters on pellets at quite acceptable gas emissions. The obtained results can be used to construct a three-stage control of the heater operation mode. To use the fuzzy selection procedure, which takes into account various aspects of the decisions to be made, it is advisable to construct several fuzzy selection functions and then solve a multi-criteria optimisation problem. For its formulation it is also convenient to use binary choice relations.

The use of matrices for pairwise comparisons of objects is a rather cumbersome procedure, and it should be further improved.

Indeed, the results presented in this article primarily pertain to the specific tube heater under investigation, as they are based on concrete experimental data. As a scientific contribution, the article proposes a decision-making methodology grounded in experimental results, which are interpreted through fuzzy modeling. The methodology involves constructing a choice function and employing expert assessments to establish preferences under multi-criteria conditions. The maxima of the choice function are then identified to support final decision-making. For other types of tube heaters, this methodology can certainly be applied, provided that all necessary research procedures are followed, to determine their own recommended control parameters.

The reliability of the obtained results is ensured as follows. All experimental data are divided into two sequences (sets): a training set and a validation set. The choice functions and their maxima are determined exclusively based on information from the training set, while the validation set is used to assess the reliability of the findings.

The application of evolutionary search methods for decision-making is based on our prior theoretical and experimental results, where the convergence of the developed evolutionary algorithms to the optimal solution, in terms of the binary preference relation, has been proven with probability one.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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