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Relic Black Holes, in Terms of a Quantum Number *n* & Torsion and Multi-Messenger Spin-Offs

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Abstract

Our idea for black holes is using Torsion to form a cosmological constant. Planck sized black holes allow for a spin density term canceling Torsion. Also, a solution to the early universe three-body problem at the start of the black holes, and number *n* selected. And we conclude with a generalized uncertainty principle which is then linked to a black hole versus white hole, linked by a worm hole problem. The spin-offs of connection to multi-messenger astronomy will be enumerated in the last part of this document.

Keywords

Inflation, Gravitational Waves

1. Part 1. Preliminaries, Recounting the Parameters of Black Hole Physics Used in This Essay, as Well as the Importance of a Quantum Number *n*

Following [1]-[3] using the substitutions outlined so we can re-do the introduction of black hole physics in terms of a quantum number n, to begin this first look at the references to the BEC condensate as given by [1]-[3] with respect to scaling.

i.e. the origins of the black holes have no hair theorem and a preview of what we will be trying to modify.

Our supposition has the no hair idea and starts off with a simple idea. We begin with the model as to how a black hole mass, M, could lose a loss of its essence. Here, M is a mass, T is temperature, and $\tilde{a}$ is a proportionality term, *i.e.* what we reference in the primordial era

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\tilde{a} \cdot T^4 \tag{1}$$

In terms of having T as temperature related to black hole mass, we use

$$T = \frac{\hbar c^3}{8\pi k_B G M} \tag{2}$$

This leads to, if indeed Equation (1) is observed

$$M^5(\text{loss}) = \left(\frac{-5}{64^2}\cdot\tilde{a}\right)\cdot\left(\frac{\hbar^4 c^{12}}{\pi^4 k_B^4 G^4}\right)\cdot t \tag{3}$$

As to how we can observe a violation of the black holes which have no hair idea we will need to do parameterization of a mass *M*, for black holes, in terms of the following inputs

2. First of All an Aside as to Muti Messenger Astrophysics, Which Is Relevant to Quantum Number *n*

Multi-messenger astrophysics is the observation of multiple signals received from the same astronomical event. Many types of cosmological events involve complex interactions between a variety of astrophysical processes, each of which may independently emit signals of a characteristic "messenger" type: electromagnetic radiation (including infrared, visible light and X-rays), gravitational waves, *i.e.* what we are doing is to set up the template as to how GW and gravitons as generated by primordial black holes may, if characterized by a quantum number n, lead to Electromagnetic spectrum. *I.e.* our mechanism will start with a new intro as to torsion, Black holes, and quantum number n while ending with possible Photonic traces. In CMBR *i.e.* we will initially be discussing the process of how GW and gravitons are related to primoridial black holes, of a quantum number *n*, and end up with speculations as to electromagnetic generation of signals which may be observable observationally.

3. Where Torsion May Allow for Understanding a Quantum Number *n*?

Following [1] [2], we do the introduction of black hole physics in terms of a quantum number *n*.

$$\begin{aligned} \sqrt{\Lambda} &= \frac{k_B E}{\hbar c S_{\text{entropy}}} \\ S_{\text{entropy}} &= k_B N_{\text{particles}} \end{aligned} \tag{4}$$

And then a BEC condensate given by [1] [3] as to

$$\begin{aligned} m &\approx \frac{M_P}{\sqrt{N_{\text{gravitons}}}} \\ M_{BH} &\approx \sqrt{N_{\text{gravitons}}}\cdot M_P \\ R_{BH} &\approx \sqrt{N_{\text{gravitons}}}\cdot l_P \\ S_{BH} &\approx k_B\cdot N_{\text{gravitons}} \\ T_{BH} &\approx \frac{T_P}{\sqrt{N_{\text{gravitons}}}} \end{aligned} \tag{5}$$

This is promising but needs to utilize [4] in which we make use of the following. First a time step

$$\tau \approx \sqrt{GM\delta r} \tag{6}$$

By use of the HUP [5] we use Equation (3) for energy [4] for radiation of a particle pair from a black hole,

$$|E| \approx \left(\sqrt{GM\delta r}\right)^{-1} \hbar \tag{7}$$

Here we assert that the spatial variation goes as

$$\delta r \approx \ell_P \tag{8}$$

This is of a Plank length, whereas we assume in Equation (7) that the mass is a Planck sized black hole

$$M \approx \alpha M_P \tag{9}$$

If so, we transform Equation (4) to be of the form for a "particle" pair as given in Carlip

$$|E| \approx \left(\sqrt{G\cdot(\alpha M_P)\cdot \ell_P}\right)^{-1} \hbar \tag{10'}$$

We argue that for small black holes that we are talking about intense radiation from a Planck sized black hole, so we approximate Equation (10) as the mass of a relic black hole. Now using the following normalization of Planck units, *i.e.* [6], as

$$G = M_P = \hbar = k_B = \ell_P = c = 1 \tag{11}$$

And, also reference the value of the initial energy, *E*, as given in reference [5]

$$E_{Bh} = -\frac{n_{\text{quantum}}}{2} \tag{12}$$

We then can use for a Black hole the scaling,

$$|E| \approx \left(\sqrt{G\cdot(\alpha M_P)\cdot \ell_P}\right)^{-1} \hbar \xrightarrow[G=M_P=\hbar=k_B=\ell_P=c=1]{} \left(1/M_{BH}\right)^{1/2} \approx \frac{n_{\text{quantum}}}{2} \tag{13}$$

We then reference Equation (5) to observe the following,

$$\begin{aligned} & M_{BH} \approx \sqrt{N_{\text{gravitons}}} M_P \\ & \Rightarrow \left(1/M_{BH}\right)^{1/2} \approx \frac{n_{\text{quantum}}}{2} \approx \frac{1}{\left(N_{\text{gravitons}}\right)^{1/4}} \\ & \Rightarrow n_{\text{quantum}} \approx \frac{2}{\left(N_{\text{gravitons}}\right)^{1/4}} \end{aligned} \tag{14}$$

This is a stunning result. *i.e.* Equation (5) is BEC theory, but due to micro sized black holes that we assume that the number of the quantum number, n associated goes way UP. Is this implying that corresponding increases in quantum number, per black hole, n, are commensurate with increasing temperature? We start off with the following table.

Table 1 from reference [2] assumes Penrose recycling of the Universe as stated in that document.

Table 1. Recycling values of black holes.

End of Prior Universe time frame	Mass (black hole): super massive end of time BH 1.98910^{+41} to about 10^{44} grams	Number (black holes) 10^6 to 10^9 of them usually from center of galaxies
Planck era Black hole formation Assuming start of merging of micro black hole pairs	Mass (black hole) 10^{-5} to 10^{-4} grams (an order of magnitude of the Planck mass value)	Number (black holes) 10^{40} to about 10^{45}, assuming that there was not too much destruction of matter-energy from the Pre Planck conditions to Planck conditions
Post Planck era black holes with the possibility of using Equation (1) and Equation (2) to have say 10^{10} gravitons/second released per black hole	Mass (black hole) 10 grams to say 10^6 grams per black hole	Number (black holes) Due to repeated Black hole pair forming a single black hole multiple time. 10^{20} to at most 10^{25}

The reason for using this table is because of the modification of Dark Energy and the cosmological constant [1]-[4] To begin this look at [2] which, which is akin, as we discuss later to [2] [8]

$$\rho_{\Lambda}c^2 = \int_0^{E_{\text{Plank}}/c} \frac{4\pi p^2 \mathrm{d}p}{(2\pi\hbar)^3} \cdot \left(\frac{1}{2}\cdot\sqrt{p^2c^2+m^2c^4}\right) \approx \frac{\left(3\times10^{19}\,\text{GeV}\right)^4}{(2\pi\hbar)^3} \xrightarrow[E_{\text{Plank}}/c\to10^{-30}]{} \frac{\left(2.5\times10^{-11}\,\text{GeV}\right)^4}{(2\pi\hbar)^3} \tag{15}$$

In [2], the first line is the vacuum energy which is completely cancelled in their formulation of application of Torsion. In our article, we are arguing for the second line. In fact by [2]

$$\frac{\Delta E}{c} = 10^{18}\,\text{GeV} - \frac{n_{\text{quantum}}}{2c} \approx 10^{-12}\,\text{GeV} \tag{16}$$

The term n (quantum) comes from a Corda expression as to energy level of relic black holes [7].

We argue that our application of [1] [2] will be commensurate with Equation (15) which uses the value given in [2] as to the following. *i.e.* relic black holes will contribute to the generation of a cut-off of the energy of the integral given in Equation (15) whereas what is done in Equation (15) by [1] [2] is restricted to a different venue which is reproduced below, namely cancellation of the following by Torsion

$$\rho_{\Lambda}c^2 = \int_0^{E_{\text{Plank}}/c} \frac{4\pi p^2 \mathrm{d}p}{(2\pi\hbar)^3} \cdot \left(\frac{1}{2}\cdot\sqrt{p^2c^2+m^2c^4}\right) \approx \frac{\left(3\times10^{19}\,\text{GeV}\right)^4}{(2\pi\hbar)^3} \tag{17}$$

Furthermore, the claim in [2] is that there is no cosmological constant, *i.e.* that

Torsion always cancelling Equation (17) which we view is incommensurate with **Table 1** as of [2]. We claim that the influence of Torsion will aid in the decomposition of what is given in **Table 1** and will furthermore lead to the influx of primordial black holes which we claim is responsible for the behavior of Equation (17) above.

4. Stating What Black Hole Physics Will Be Useful for in Our Modeling of Dark Energy. *I.e.* Inputs into the Torsion Spin Density Term

In [9] we have the following, *i.e.*, we have a spin density term of [1] [9]. And this will be what we input black hole physics into as to form a spin density term from primordial black holes.

$$\sigma_{Pl} = n_{Pl}\hbar \approx 10^{71} \tag{18}$$

5. Now for the Statement of the Torsion Problem as Given in [1] [2] [9]

The author is very much aware as to quack science as to purported torsion physics presentations and wishes to state that the torsion problem is not linked to anything other than disruption as to the initial configuration of the expansion of the universe and cosmology, more in the spirit of [9] and is nothing else. Hence, in saying this we wish to delve into what was given in [9] with a subsequent follow up and modification:

To do this, note that in [9] the vacuum energy density is stated to be

$$\rho_{vac} = \Lambda_{eff} c^4 / 8\pi G \tag{19}$$

Whereas the application is given in terms of an antisymmetric field strength $S_{\alpha\beta\gamma}$ [9].

In [2] due to the Einstein Cartan action, in terms of an SL (2, C) gauge theory, we write from [9]

$$L = -R/(16\pi G) + S_{\alpha\beta\gamma} S^{\alpha\beta\gamma} / 2\pi G \tag{20}$$

R here is with regards to Ricci scalar and Tensor notation and $S_{\alpha\beta\gamma}$ is related to a conserved current closing in on the SL (2, C) algebra as given by

$$J^{\mu} = J^{\mu} + 1/(16\pi G)\,\varepsilon^{\mu\alpha\beta\gamma} S_{\alpha\beta\gamma} \tag{21}$$

This is where we define

$$S_{\alpha\beta\gamma} = c_{\alpha} \times f_{\beta\gamma} \tag{22}$$

where c_{α} is the structure constant for the group SL (2, C), and

$$f_{\beta\gamma} \cdot \overline{g} = F_{\beta\gamma} \tag{23}$$

where

$$\overline{g} = (g_1, g_2, g_3) \tag{24}$$

Is for tangent vectors to the gauge generators of SL (2, C), and also for Gauge fields A_γ

$$F_{\beta\gamma} = \partial_\beta A_\gamma - \partial_\gamma A_\beta + \left[A_\beta, A_\gamma\right] \tag{25}$$

And that there is furthermore the restriction that

$$\partial_\rho\left(\varepsilon^{\rho\alpha\beta\gamma} S_{\alpha\beta\gamma}\right) = 0 \tag{26}$$

Finally in the case of massless particles with torsion present we have a space time metric

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 + a^2(\tau)\mathrm{d}^2\Omega_3 \tag{27}$$

where $\mathrm{d}^2\Omega_3$ is the metric of S^3.

Then the Einstein field equations reduce to in this torsion application, (no mass to particles) as

$$\left(\mathrm{d}a/\mathrm{d}\tau\right)^2 = \left[1 - \left(r_{\min}^4/a^4\right)\right] \tag{28}$$

With, if S is the so-called spin scalar and identified as the basic $\hbar$ unit of spin

$$r_{\min}^4 = 3G^2S^2/8c^4 \tag{29}$$

6. How to Modify Equation (28) in the Presence of Matter via Yang Mills Fields $F_{\mu\nu}^{\beta}$

First of all, this involves a change of Equation (20) to read

$$L = -R/(16\pi G) + S_{\alpha\beta\gamma}S^{\alpha\beta\gamma}/2\pi G + \left(1/4g^2\right)F_{\mu\nu}^{\beta}F_{\beta}^{\mu\nu} \tag{30}$$

And eventually we have a re-do of Equation (28) to read as

$$\left(\mathrm{d}a/\mathrm{d}\tau\right)^2 = \left[1 - \left(\beta_1/a^2\right) - \left(\beta_2/a^4\right)\right] \tag{31}$$

If $g = \hbar c$ we have $\beta_1 = r_{\min}^2, \beta_2 = r_{\min}^4$, and the minimum radius is identified with a Planck Radius so then

$$\left(\mathrm{d}a/\mathrm{d}\tau\right)^2 = \left[1 - \left(\left(\beta_1 = \ell_P^2\right)/a^2\right) - \left(\left(\beta_2 = \ell_P^4\right)/a^4\right)\right] \tag{32}$$

Eventually in the case of an unpolarized spinning fluid in the immediate aftermath of the big bang, we would see a Roberson Walker universe given as, if σ is a torsion spin term added due to [9] as

$$\left(\frac{\dot{\tilde{R}}}{\tilde{R}}\right)^2 = \left(\frac{8\pi G}{3}\right)\cdot\left[\rho - \frac{2\pi G\sigma^2}{3c^4}\right] + \frac{\Lambda c^2}{3} - \frac{\tilde{k}c^2}{\tilde{R}^2} \tag{33}$$

7. What [9] Does as to Equation (33) versus What We Would Do and Why

In the case of [1] we would see σ be identified as due to torsion so that Equation (33) reduces to

$$\left(\frac{\dot{\tilde{R}}}{\tilde{R}}\right)^2 = \left(\frac{8\pi G}{3}\right)\cdot[\rho] - \frac{\tilde{k}c^2}{\tilde{R}^2} \tag{34}$$

The claim is made in [2] that this is due to spinning particles which remain invariant so the cosmological vacuum energy, or cosmological constant is always cancelled.

Our approach instead will yield [9]

$$\left(\frac{\dot{\tilde{R}}}{\tilde{R}}\right)^2 = \left(\frac{8\pi G}{3}\right)\cdot[\rho] + \frac{\Lambda_{0\text{bserved}}c^2}{3} - \frac{\tilde{k}c^2}{\tilde{R}^2} \tag{35}$$

i.e. the observed cosmological constant $\Lambda_{0\text{bserved}}$ is 10^{-122} times smaller than the initial vacuum energy.

The main reason for the difference in Equation (34) and Equation (35) is in the following observation.

Mainly that the reason for the existence of σ^2 is due to the dynamics of spinning black holes in the precursor to the big bang, to the Planckian regime, of space time, whereas in the aftermath of the big bang, we would have a vanishing of the torsion spin term. *i.e.* Table 1 dynamics in the aftermath of the Planckian regime of space time would largely eliminate the σ^2 term.

8. Filling in the Details of the Equation (34) Collapse of the Cosmological Term, versus the Situation Given in Equation (35) via Numerical Values

First look at numbers provided by [9] as to inputs, *i.e.* these are very revealing

$$\Lambda_{Pl}c^2 \approx 10^{87} \tag{36}$$

This is the number for the vacuum energy and this enormous value is 10^{122} times larger than the observed cosmological constant. Torsion physics, as given by [9] is solely to remove this giant number.

In order to remove it, the reference [1] [9] proceeds to make the following identification, namely

$$\left(\frac{8\pi G}{3}\right)\cdot\left[-\frac{2\pi G\sigma^2}{3c^4}\right] + \frac{\Lambda c^2}{3} = 0 \tag{37}$$

What we are arguing is that instead, one is seeing, instead [9]

$$\left(\frac{8\pi G}{3}\right)\cdot\left[-\frac{2\pi G\sigma^2}{3c^4}\right] + \frac{\Lambda_{Pl}c^2}{3} \approx 10^{-122} \times \left(\frac{\Lambda_{Pl}c^2}{3}\right) \tag{38}$$

Our timing as to Equation (36) is to unleash a Planck time interval *t* about 10^{-43} seconds.

As to Equation (37) versus Equation (38) the creation of the torsion term is due to a presumed particle density of

$$n_{Pl} \approx 10^{98}\ \text{cm}^{-3} \tag{39}$$

Finally, we have a spin density term of $\sigma_{Pl} = n_{Pl}\hbar \approx 10^{71}$ which is due to innumerable black holes initially.

Future works to be commenced as to derivational tasks

We will assume for the moment that Equation (36) and Equation (37) share in common Equation (39).

It appears to be trivial, a mere round off, but I can assure you the difference is anything but trivial. And this is where **Table 1** really plays a role in terms of why there is a torsion term to begin with, *i.e.* will make the following determination, *i.e.*

The term of "spin density" in Equation (36) by Equation (39) is defined to be an ad hoc creation, as to [3]. No description as to its origins is really offered.

1st

We state that in the future a task will be to derive in a coherent fashion the following, *i.e.* the term of

$\left(\frac{8\pi G}{3}\right)\cdot\left[-\frac{2\pi G\sigma^2}{3c^4}\right]$ arising as a result of the dynamics of **Table 1**, as given in the manuscript.

2nd,

We state that the term $\left(\frac{8\pi G}{3}\right)\cdot\left[-\frac{2\pi G\sigma^2}{3c^4}\right]$ is due to initial micro black holes, as to the creation of a Cosmological term.

In the case of Pre Planckian space-time the idea is to do the following [9], *i.e.* if we have an inflaton field [9]-[17]

$$\begin{aligned} &\left|\mathrm{d}p_\alpha \mathrm{d}x^\alpha\right| \approx \frac{L}{l}\cdot\frac{h}{c}\cdot\left[\frac{\mathrm{d}l}{l}\right]^2 \\ &\xrightarrow[\alpha=0]{} \left|\mathrm{d}p_0 \mathrm{d}x^0\right| \simeq \left|\Delta E \Delta t\right| \approx \left(h/a_{init}^2 \phi(t)\right) \\ &\Rightarrow \frac{L}{l}\cdot\frac{h}{c}\cdot\left[\frac{\mathrm{d}l}{l}\right]^2 \approx \left(h/a_{init}^2 \phi(t_{init})\right) \end{aligned} \tag{40}$$

Making use of all this leads to [10] to making sense of the quantum number n as given by reference to black holes, [7] $E_{Bh} = -\frac{n_{\text{quantum}}}{2}$.

3rd

The conclusion of [1] states that Equation (40) would remain invariant for the life of the evolution of the universe. We make no such assumption. We assume that, as will be followed up later that Equation (38) is due to relic black holes with the suppression of the initially gigantic cosmological vacuum energy.

The details of what follow after this initial period of inflation remain a task to be completed in full generality but we are still assuming as a given the following inputs [9] [14]

$$\begin{aligned} &a(t) = a_{\text{initial}} t^\nu \\ &\Rightarrow \phi = \ln\left(\sqrt{\frac{8\pi G V_0}{\nu\cdot(3\nu-1)}}\cdot t\right)^{\sqrt{\frac{\nu}{16\pi G}}} \\ &\Rightarrow \dot{\phi} = \sqrt{\frac{\nu}{4\pi G}}\cdot t^{-1} \\ &\Rightarrow \frac{H^2}{\dot{\phi}} \approx \sqrt{\frac{4\pi G}{\nu}}\cdot t\cdot T^4\cdot\frac{1.66^2\cdot g_*}{m_P^2} \approx 10^{-5} \end{aligned} \tag{41}$$

A possible future endeavor can also make sense of [15] as well

1st CONCLUSION, how meeting conditions for applying Torsion to obtain the cosmological constant and DE modifies black hole physics in the early universe.

First of all, it puts a premium upon our **Table 1** as given and is shown in [9]. Secondly it means utilization of Equation (16) which takes into account the black hole energy equation given by Corda in [7] and it also means that the spin density term as given in Equation (18) is freely utilized.

We refer to black hole creation as given by torsion this way as a correction to [1] largely due to the insufficiency of black hole theory as eloquently given in [16] which we will cite their page 366 admonition as to the insufficiency of current theory.

Quote

Black holes of masses sufficiency smaller than a solar mass cannot be formed by gravitational collapse of a star; such miniholes can only form in the early stages of the universe, from fluctuations in the very dense primordial matter.

End of quote

Our torsion argument is directly due to this acknowledgement and is due to the sterility of much theoretical thinking, as well as the tremendously important Equation (12) which is due to Corda [7].

Furthermore, in order to obtain more details of Equation (12) being utilized for black holes, we state that a quantum state of the early universe will utilize [17] and its discussion, page 184, as to how Feynman visualized the quantization of the Gravitational field, *i.e.* Equations 9.121 and 9.122 of [17] for an early wavefunction path integral treatment for quantized gravity and its use for black holes. Corda himself [7] has alluded to a path forward in such treatment of how black holes can be modeled which leads to Equation (40).

In addition, we outlined the stunning result as given as of Equation (14) as far as a more than an inverse relationship between graviton number, per generated black hole (presumably primordial) and a quantum number n, attached to a black hole as due to [7]. What we see is that if we have small black holes, with BEC characteristics with small number of gravitons, per primordial black hole, that the quantum number n climbs dramatically. We need to obtain the complete dynamics of this relationship as it pertains to how very small black holes have high quantum number n, which we presume is commensurate with initially high temperatures.

The details of this development as well as its tie into the dynamics of **Table 1** as given and Torsion have to be fine tuned.

More work needs to be done so we can turn early universe gravitational generation and black hole physics into an empirical science.

2nd CONCLUSION, looking directly at a modification of the Black holes have no hair theorem, via the inputs of this document.

In [18] we have the essential black holes have no hair theorem which can be seen roughly as

Quote

The idea is that beyond mass, charge and spin, black holes don't have distinguishing features, no hairstyle, cut or color to tell them apart.

End of quote

How do we get about this? Note that in [19] there is a pseudo extension which we can chalk up to Hawking; but in order to apply a more direct treatment we go to what is given in [20].

i.e. we go to formula 65 of that reference. This will give a variation of the radius of a black hole, over the radius, according to a quantum number n AGAIN. Before we get there we will do some initial work up to that quantum number, n as used in formula 65 of reference [20].

i.e. using our Equation (14) for N and also the Planck scale normalization as given by

$\hbar = k_B = c = G = M_p = \ell_p = 1$, and if we take $\tilde{a}$ approximately scaled to 1 as well we have that if

$$\left|N\right| \approx \left|N_{\text{gravitons}}\right| \approx \left(\frac{5t}{64^2 \pi^4}\right)^{2/5} \tag{42}$$

Due to using [3]

$$M \approx \sqrt{N} M_p \tag{43}$$

M here being linked to the mass of a BEC black hole, and also using Equation (3) for the loss of a black hole, over time.

Also use

$$\left|N_{\text{gravitons}}\right|^{5/2} \times \left(M_p \equiv 1\right)^{5/2} \approx \left(\frac{5t}{64^2 \pi^4}\right) \tag{44}$$

Then use the last equation of Equation (14) to obtain, a quantum number associated with a graviton just outside a BEC primordial black hole

$$n_{\text{graviton quantum number}} \equiv n_{\text{graviton}} \approx \left[\frac{2 \cdot 64^{1/10} \pi^{1/5}}{5^{1/20} \cdot t^{1/20}}\right] \approx \frac{2.16245415907}{t^{1/20}} \tag{45}$$

Assuming Planck scale time, or close to it, and renormalization to have Planck time as set to 1.

This means then that the quantum number, n associated with a graviton with respect to a Planck sized black hole would be close to 2, initially.

If so then, and this is for primordial black holes, we then associate this graviton number, n for a graviton as linked to the following from [20], *i.e.* their Equation (65) so we have for the radius of a BEC black hole as deformed by this quantum number *n*, a small change

$$\frac{\Delta R_n}{R_n} \equiv \frac{\sqrt{n^2 + 2}}{3n} \tag{46}$$

If we use the value of $n = 2.16245415907$ for a graviton "quantum number" at about normalized Planck time, scaled to about 1, and we have according to [20] an ADM mass variance of M so then there is, due to gravitons, a rough change in initial Planck sized black holes

$$\Delta R_n = \left(\frac{\sqrt{n^2+2}}{3n}\right)\cdot R_n \approx \left.\left(\frac{\sqrt{n^2+2}}{3n}\right)\right|_{n\equiv 2.16245415907} \times R_n \tag{47}$$

where $n \geq (1-\varepsilon)\cdot(M/M_p)^2$ and we can compare our value of *R*, as given in Equation (5) with [20] having a different scale for *R*, as given in their Equation (60).

Needless to say, graviton number n, as specified, due to the processes within the primordial black hole we assert would lead to a violation of the black holes have no hair theorem, of [19].

We assert that this value of *n*, so obtained, as to gravitons would be as to the Corda result on Equation (12) the following

$$\begin{aligned} n(\text{black holes}) = {} & N(\text{graviton number per black hole}) \\ & \times n(\text{quantum number per graviton}) \end{aligned} \tag{48}$$

The left hand side of Equation (48) would be fully commensurate with Equation (12) of Corda's black hole quantum number.

The right hand side of Equation (48) would be commensurate with n being for a quantum number per graviton associated per black hole.

If there are a lot of gravitons, associated with a primordial black hole, this would commence with a very high initial quantum number, n (black holes) associated Cordas great result, as of [7].

Note that in future works, I told the onlookers that the original idea of my talk was to consider a black hole joined to a White Hole and to consider the generation of quantum number n, in the throat of a connecting worm hole between the black hole and white hole. In [7] we have a model along the lines I considered, and we ascertain that the Corda suggestion of n quantum number for back holes be compared to the quantum number, n, which may be derived from the energy condition in the [8] document. In doing so, we will ascertain if our value of n slightly larger than 2 is indeed feasible, and also optimal. The value of an energy, due to a quantum number, n, will be derived and compared with our value of n assumed in this early universe condition. In addition this will be done to give credence to [8] and to difficulty of forming primordial black holes.

9. Second Section. Now for Applications of the Generalized HUP and Its Applications to Black Hole Physics

Heavy Gravity is the situation where a graviton has a small rest mass and is not a zero mass particle, and this existence of "heavy gravity" is important since eventually, as illustrated by Will [9] [10] gravitons having a small mass could possibly be observed via their macroscopic effects upon astrophysical events. The second aspect of the inquiry of our manuscript will be to come up with a variant of the Heisenberg Uncertainty principle (HUP), in [11], with

$$\Delta x\Delta p \geq \frac{\hbar}{2} + \tilde{\gamma}\frac{\partial C}{\partial V} \tag{49}$$

As opposed to

$$\delta t\Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2}$$
$$\text{Unless} \quad \delta g_{tt} \sim O(1) \tag{50}$$

Which we claim in the Planckian regime will de evolve, as being effectively as being equivalent to

$$\Delta x\Delta p \geq \frac{\hbar}{\delta g_{tt}} \tag{51}$$

We will be comparing Equation (49) and Equation (50) as well as writing

$$\delta g_{tt} \sim a^2(t)\cdot\phi \ll 1 \tag{52}$$

The second term in Equation (27) comes directly from a simplified inflaton expression which is [12]-[14].

I.e. go to Equation (41).

In doing this, we adhere to the starting point of [14] [15]

$$\Delta l\cdot\Delta p \geq \frac{\hbar}{2} \tag{53}$$

We will be using the approximation given by Unruh,

$$\begin{aligned}(\Delta l)_{ij} &= \frac{\delta g_{ij}}{g_{ij}}\cdot\frac{l}{2}\\ (\Delta p)_{ij} &= \Delta T_{ij}\cdot\delta t\cdot\Delta A\end{aligned} \tag{54}$$

If we use the following, from the Roberson-Walker metric [14]-[17]

$$\begin{aligned}g_{tt} &= 1\\ g_{rr} &= \frac{-a^2(t)}{1-k\cdot r^2}\\ g_{\theta\theta} &= -a^2(t)\cdot r^2\\ g_{\phi\phi} &= -a^2(t)\cdot\sin^2\theta\cdot d\phi^2\end{aligned} \tag{55}$$

Following Unruh [14] [15], write then, an uncertainty of metric tensor as, with the following inputs

$$a^2(t) \sim 10^{-110},\ r \equiv l_P \sim 10^{-35}\ \text{meters} \tag{56}$$

Then, if $\Delta T_{tt} \sim \Delta\rho$

$$\begin{aligned}&V^{(4)} = \delta t\cdot\Delta A\cdot r\\ &\delta g_{tt}\cdot\Delta T_{tt}\cdot\delta t\cdot\Delta A\cdot\frac{r}{2} \geq \frac{\hbar}{2}\\ &\Leftrightarrow \delta g_{tt}\cdot\Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}\end{aligned} \tag{57}$$

This Equation (56) is such that we can extract, up to a point the HUP principle for uncertainty in time and energy, with one very large caveat added, namely if we use the fluid approximation of space-time [17] for the stress energy tensor as given in Equation (58) below.

$$T_{ii} = diag\left(\rho, -p, -p, -p\right) \tag{58}$$

Then

$$\Delta T_{tt} \sim \Delta\rho \sim \frac{\Delta E}{V^{(3)}} \tag{59}$$

Then,

$$\delta t \Delta E \ge \frac{\hbar}{\delta g_{tt}} \ne \frac{\hbar}{2} \quad \text{Unless } \delta g_{tt} \sim O(1) \tag{60}$$

How likely is $\delta g_{tt} \sim O(1)$? Not going to happen. Why? The homogeneity of the early universe will keep

$$\delta g_{tt} \ne g_{tt} = 1 \tag{61}$$

In fact, we have that from Giovannini [16], that if ϕ is a scalar function, and $a^2(t) \sim 10^{-110}$, then if

$$\delta g_{tt} \sim a^2(t) \cdot \phi \ll 1 \tag{62}$$

Then, there is no way that Equation (60) is going to come close to $\delta t \Delta E \ge \frac{\hbar}{2}$. *i.e.* it depends assuming time is for all purposes fixed at about Planck time to isolate V_0.

Equation (41) is crucial here, and it depends upon the scalar term in Equation (41) have a time dependence only, which means it is for near Planck time, almost a constant term. *I.e.* for the sake of argument, in the near Planckian regime, we can figure that Equation (62) will have as far as evaluation of the argument the following configuration, *i.e.* [15] [16]

$$a(t) \approx a_{\text{initial}} \cdot \left(t/t_P\right)^{\nu} \tag{63}$$

Given this we will be looking at, if we do the set up

$$\Delta x \Delta p \ge \frac{\hbar}{\delta g_{tt} = \left[a_{\text{initial}} \cdot \left(t/t_P\right)^{\nu}\right]^2 \left[\ln\left(\sqrt{\frac{8\pi G V_0}{\nu \cdot (3\nu - 1)}} \cdot t\right)^{\sqrt{\frac{\nu}{16\pi G}}}\right]} \tag{64}$$

Comparing this Equation (43) with Equation (27), we obtain then if $\hbar = c = t_p = k_B = l_P = G = 1$ the bound for V_0

$$V_0 \cong \left[\frac{\nu \cdot (3\nu - 1)}{8\pi}\right] \cdot \left[\exp\left(\frac{16\sqrt{\pi}}{\sqrt{\nu}} \cdot \frac{1}{a_{\min}^2 \cdot \left(t/t_p\right)^2}\right)\right] \cdot \left(\frac{1}{1 + 2\tilde{\gamma}\frac{\partial C}{\partial V}}\right)^2 \tag{65}$$

So then we are now doing an Evaluation of Equation (65) if we are near Planck time. Two limits.

1st, what if we have expansion of the scale factor initially at greater than the speed of light?

Set $\nu \approx 10^{88}$ and then we can obtain if we are just starting off inflation say $a_{\min}^2 \approx 10^{-44}$. Then

$$V_0 \cong \left[10^{176}\right] \cdot \left[\exp\left(16\sqrt{\pi}\right)\right] \cdot \left(\frac{1}{1+2\tilde{\gamma}\frac{\partial C}{\partial V}}\right)^2, \tag{66}$$

If we wish to have a Planck energy magnitude of the V_0 term, we will then be observing

$$V_0 \cong \left[10^{176}\right] \cdot \left[\exp\left(16\sqrt{\pi}\right)\right] \cdot \left(\frac{1}{1+2\tilde{\gamma}\frac{\partial C}{\partial V}}\right)^2, \xrightarrow[2\tilde{\gamma}\frac{\partial C}{\partial V}\approx\left[10^{88}\right]]{} o(1) \tag{67}$$

i.e. the system complexity will become effectively almost infinite, and this will be explained in the conclusion by use of

$$2\tilde{\gamma}\frac{\partial C}{\partial V} \approx \left[10^{88}\right] \Rightarrow V_0 \cong o(1) \tag{68}$$

On the other hand, if there is a very small value for $2\tilde{\gamma}\frac{\partial C}{\partial V}$ we can see the following behavior for Equation(66), namely

$$2\tilde{\gamma}\frac{\partial C}{\partial V} \approx o(1) \Rightarrow V_0 \cong \left[10^{176}\right] \tag{69}$$

i.e. low complexity in the measurement process will then imply an enormous initial inflaton potential energy.

2ndly, Now what if we have instead $\nu \approx 1$

$$V_0 \cong \left[\frac{1}{4\pi}\right] \cdot \left[\exp\left(\frac{16\sqrt{\pi}}{a_{\min}^2 \cdot \left(t/t_p\right)^2}\right)\right] \cdot \left(\frac{1}{1+2\tilde{\gamma}\frac{\partial C}{\partial V}}\right)^2 \tag{70}$$

The threshold if $2\tilde{\gamma}\frac{\partial C}{\partial V} \approx \left[10^{88}\right]$ *i.e.* a huge value for initial complexity would be effectively made insignificant in cutting down the initial inflaton leading to

$$\exp\left(\frac{16\sqrt{\pi}}{\sqrt{\nu}} \cdot \frac{1}{a_{\min}^2 \cdot \left(t/t_p\right)^2}\right) \xrightarrow[a_{\min}^2\approx 10^{-88}]{} V_0 \cong \exp\left(10^{88}\right) \tag{71}$$

i.e. we come to the seemingly counterintuitive expression that the initial inflaton potential would still be infinite if we used Equation (70) in Equation (66).

10. Future Developments for Applications of a Primordial HUP? Linking This to a Theory of Complex Initial and Final Structures. Black Holes Brought up

From Table 1 and information from [18] assuming Penrose recycling of the Universe as stated in that document. The limits in section four may give structural complexity data relevant to the following development. As given, see Table 1. This increase in complexity can be with work tied into the following for black hole physics [3] from Equation (1). References from [18]-[21] are to be generally reviewed as to inspiration as to what we say next. We will try to quantify all this in future research work to explain this in terms of the physics of phase transitions, in the universe and cyclic conformal cosmology. Finally the physics of initial transformations as given in Table 1 should have some linkage eventually to [22] as to the idea of Gravity breath, as given by Dr. Corda.

11. First Major Implication of This Use of the HUP Is to Investigate, *i.e.* Role of Complexity in Bridge from Black Hole Numbers as Given in Table 1

There are three regimes of black hole numbers given in Table 1. From Pre Planckian, to Planckian and then to post Planckian physics regimes. This is all assuming CCC cosmology. To start to make sense of this, we need to examine how one could achieve the complexity as indicated by Figure 1 in the Planckian era. To do this at a start, we will pay attention to a datum in reference [3], namely a Horizon, like a Schwarzschild black hole construction with [23]

$$L_A = \sqrt{\frac{3}{\Lambda}} \tag{72}$$

In what [23] deems as a corpuscular gravity one would have a "kinetic energy term" per graviton

$$\in_G \cong \frac{M_p}{\sqrt{\tilde{N}}} \tag{73}$$

And the mass of a black hole, scaling as [23]

$$M_{\text{black hole}} \cong \sqrt{\tilde{N}} M_p \approx \tilde{N} \in_G \tag{74}$$

This in [3] has the exact same functional forms as is given in Equation (27) so then we have $\tilde{N} = N$ and furthermore [23] also has

$$\in_G \cong \frac{M_p}{\sqrt{\tilde{N}}} \cong \frac{\hbar}{L_A} \approx \frac{M_p}{\sqrt{N}} \tag{75}$$

If so for Black holes, we have the following

$$\sqrt{\Lambda} \cong \frac{\sqrt{3} M_p}{\hbar \sqrt{N}} \tag{76}$$

Now as to what is given in [1] [2] as to Torsion, we have that as given in [18] that we can do some relevant dimensional scaling.

First look at numbers provided by [1] [2] as to inputs, *i.e.* these are very revealing, *i.e.* we go back to the argument as to the beginning of the document, namely $\Lambda_{Pl} c^2 \approx 10^{87}$

This is the number for the vacuum energy and this enormous value is 10^{122} times larger than the observed cosmological constant. Torsion physics, as given by [1] [2] is solely to remove this giant number.

Our timing is to unleash a Planck time interval t about 10^{-43} seconds. Also the creation of the torsion term is due to a presumed "graviton" particle density of $n_{Pl} \approx 10^{98}\ \text{cm}^{-3}$.

This particle density is directly relevant to the basic assumption of how to have relevant Gravitons initially created as to obtain the huge increase in complexity alluded to, in order to obtain the number of micro black holes in the Planckian era [1] [2].

***I.e.* assume that there are, then say initially up to 10^{98} gravitons, initially, and then from there, go to Table 1 to assume what number of micro sized black holes are available, *i.e.* Table 1 has say a figure of 10^{45} to at most 10^{50} micro sized black holes, presumably for 10^{98} gravitons being released, and this is meaning we have say 10^{50} black holes of say of Planck mass, to work with.**

12. Part 3, the Question of If There Is a Linkage to All This and Structure Formation in the Early Universe and the 3 Body Problem, and the Possible NLED Inputs, into Early Universe Conditions

We recall using that the stronger an early universe magnetic field is, the greater the likelihood of production of about 20 new domains of size $1/H$, with H early universe Hubble's constant, per Planck time interval in evolution. Which leads to statements as to the value of α in a gravitational potential proportional to $r^{-\alpha}$.

Part 1: We first of all recall that the scale f actor is affected by the NLED paradigm which in fact also is linked to the idea of "self reproduction" as given in [24], which is a different way as to outline how this affects the evolution of density in the early universe leading to equation for setting the value of α in a gravitational potential proportional to $r^{-\alpha}$. This α has real and complex values, unlike the Newtonian real value, *i.e.* the problem of the α in a gravitational potential proportional to $r^{-\alpha}$.

In order to review this, we need to look at [24] where we can use the following treatment of the Klein Gordon equation which we write as

$$\ddot{\varphi}_k + 3H\dot{\varphi}_k + \frac{k^2}{a^2}\varphi_k = 0, \quad \varphi_k \approx \frac{H\tau}{\sqrt{2k}} \cdot \left(1 + (ik\tau)^{-1}\right) \cdot \exp[-ik\tau], \quad \& \, \tau = -H^{-1} \cdot \exp[-H \cdot t] \tag{77}$$

Here, k is the value of wave number, and H is assumed, in the early universe to be a constant. The net result is that $k = 2\pi/\lambda$, with λ proportional to the "width" of a would be pre universe "bubble" as seen in [24] place of a singularity, and also that one would have, for a constant H, during this time as seen by

$$H = \sqrt{\frac{8\pi G}{3}\cdot\rho - \frac{\kappa}{a^2}},\ \rho = \text{'energy density'},\ \kappa = \text{'curvature'} \tag{78}$$

Further use of [1] will lead to the situation that

$$H \approx \sqrt{\frac{8\pi G}{3}}\cdot\sqrt{V(\varphi)-\frac{\dot{\varphi}^2}{2}} \Leftrightarrow \frac{\dot{\varphi}^2}{2} = \left(\frac{3}{8\pi G}\cdot\frac{\kappa}{a^2}+V(\varphi)\right)-\rho \tag{79}$$

Chaotic inflation uses that $V(\varphi)\approx\frac{k^2}{a^2}\cdot\varphi^2$ and the time derivative is $\mathrm{d}/\mathrm{d}\tau$, and $\varphi\equiv\varphi_k$, and if so,

$$\begin{aligned}\frac{\dot{\varphi}_k^2}{2} &= \left(\frac{3}{8\pi G}\cdot\frac{\kappa}{a^2}+\frac{k^2}{a^2}\cdot{\varphi_k}^2-\frac{16}{3}\cdot c_1\cdot B^4\right)\\ \&\ \Delta E &\approx \left(\frac{3}{8\pi G}\cdot\frac{\kappa}{a^2}+\frac{k^2}{a^2}\cdot\varphi_k^2-\frac{16}{3}\cdot c_1\cdot B^4\right)\end{aligned} \tag{80}$$

The last line of Equation (80) states that, if we apply it to the Pre Planckian to Planckian regime, that there will be a change in the energy, we then will call this shift in energy, as equivalent to a change in KINETIC energy,

$$\begin{aligned}&\left\langle\psi\left|\left[\text{Kinetic Energy}\approx\left(\frac{3}{8\pi G}\cdot\frac{\kappa}{a^2}+\frac{k^2}{a^2}\cdot\varphi_k^2-\frac{16}{3}\cdot c_1\cdot B^4\right)\right]\right|\psi\right\rangle\\ &\approx\left\langle\psi\left|\left(r\cdot\nabla\left[V\left(\text{Potential energy}\right)\approx c_2/r^\alpha\right]\right)\right|\psi\right\rangle\end{aligned} \tag{81}$$

In the Pre Planckian to Planckian space time, we will approximate, in the instant before time is initialized, formally, the mean value theorem with the results that we obtain

$$\begin{aligned}&\left(\frac{3}{8\pi G}\cdot\frac{\kappa}{a^2}+\frac{k^2}{a^2}\cdot\varphi_k^2-\frac{16}{3}\cdot c_1\cdot B^4\right)\approx-\alpha/r^\alpha\equiv-\alpha/\left(\text{Planck length}\right)^\alpha\\ &\Leftrightarrow\left[\alpha/\left(\text{Planck length}\right)^\alpha\right]\approx\frac{16}{3}\cdot c_1\cdot B^4-\left(\frac{3}{8\pi G}\cdot\frac{\kappa}{a^2}+\frac{k^2}{a^2}\cdot\varphi_k^2\right)\end{aligned} \tag{82}$$

Here, the magnetic field would be determined in part by the value of B, and the scale factor a, is given, and φ_k is given by Equation(82) This shows in part that α is no longer strictly real valued but is strongly influenced by the input from φ_k, *i.e.* which has real and imaginary components. What we should endeavour through judicious application of Equation (82) is to remove dependence upon the smallness of the third mass, and to examine if this can still, with a non-trivial third mass recover still much of the stability analysis. Later, at an appropriate time this question in terms of a serious application of the value of Equation (82) will be pursued, Secondly, as of [25] the section gives on page 154, entitled "6.4 Orbital changes in encounters with planets", which is a restricted 3 body problem, frequently is used as to the interaction of say comets (small mass) with a planet, circulating the Sun, where we have 2 "massive" masses, and the third body, in this case a comet, which gives usually parameters of how a hyperbolic orbit for a comet, should be reviewed again. This is meant to be in tandem with results as far as self reproduction of structure given in [26] which is how we started [24].

WHAT we will state is that in the early universe is that the scale factor used in Equation(82) will be closely aligned to the regime of when we apply Equation (5) in the earlier universe, which we will consider in future works.

13. Part 4. Looking at a Worm Hole Connecting a Black Hole and a White Hole, and the Possibility of a Quantum Number *n* Emerging

In doing this we should note that we are assuming as a future work that there would be black holes, in our initial configuration, plus a white hole in the immediate pre inflationary regime. Likely in a recycled universe. Reference [7] is what we will start off with [7] and its given metric as far as a black hole to white hole solution.

Namely

$$\mathrm{d}S^2 = -A(r,a)\mathrm{d}t^2 + B(r,a)^{-1}\mathrm{d}r^2 + g^2(r,a)\mathrm{d}\Omega^2 \tag{83}$$

We can perform a major simplification by setting, then

$$A(r,a) = B(r,a) = f(r,a) \tag{84}$$

In doing so, [7] gives us the following stress energy tensor values as give

$$\begin{aligned} T_t^t &= \frac{1}{8\pi}\cdot\left(\frac{1}{g}\cdot\left(f'g' + 2fg''\right) - \frac{1}{g^2}\cdot\left(1 - fg'^2\right)\right) \\ T_r^r &= \frac{1}{8\pi}\cdot\left(\frac{1}{g}\cdot\left(f'g'\right) - \frac{1}{g^2}\cdot\left(1 - fg'^2\right)\right) \\ T_\theta^\theta = T_\phi^\phi &= \frac{1}{8\pi}\cdot\left(\frac{1}{g}\cdot\left(f'g' + fg''\right) + \frac{1}{2}\cdot\left(f''\right)\right) \end{aligned} \tag{85}$$

In doing this, we will choose the primed coordinate as representing a derivative with respect to *r.*

Also in the case of black hole to white hole joining, we will be looking at a gluing surface as to the worm hole joining a black hole to white hole given as with regards to a gluing surface connecting a black hole to a white hole which we give as ξ. And $\tilde{\tilde{n}}$ is a quantum gravity index. Note that in [7] the authors often set it at 3, if so then for a black hole, to white hole to worm hole configuration they give

$$g(r,a) = \begin{cases} r^2 + a^2\left(1 - \dfrac{r^2}{\xi^2}\right)^{\tilde{\tilde{n}}}, & \text{when } (r \le \xi) \\ r^2, & \text{when } (r > \rho) \end{cases} \tag{86}$$

We then make the following connection to energy density in a black hole to white hole system, *i.e.*

$$\begin{aligned} &\rho_{\text{black hole white hole wormhole}} \equiv -T_r^r \\ &\approx \hbar\omega_{\text{black hole white hole wormhole}}\tilde{n}_{\text{black hole white hole wormhole}} \end{aligned} \tag{87}$$

This will lead to, if we use Planck units where we normalize *h* bar to being 1, of

$$\tilde{n}_{\text{black hole white hole wormhole}} = \frac{1}{8\pi} \cdot \left(\frac{1}{g} \cdot \left(f'g' \right) - \frac{1}{g^2} \cdot \left(1 - fg'^2 \right) \right) \cdot \frac{1}{\omega_{\text{black hole white hole wormhole}}} \tag{88}$$

If we are restricting ourselves to quantum geometry at the start of expansion of the universe, it means that say we can set these values to be compared to the inputs of quantum number n used to specify a quantum number *n*, and furthermore if

$$a \approx \ell_P = \text{Planck length} \xrightarrow[\text{Planck normalization}]{} 1 \tag{89}$$

We get further restrictions as to the quantum number in Equation (88) when we compare it to where we had a value of n given in the first section of our document.

Furthermore, it means that we can use this to model say, with additional work in a future project how a white hole (specified as in the prior universe.

14. What Sections We Call Part 1, Part 2 and Part 3 and Part 4 Are Saying and Future Prospects of Research

Part 3 is to the three body problem, and our scale factors assumed as part of the equations of state, of early GW formulation may indeed be modified.

Part 4 as given in XI. gives us the distinct way to investigate if there is a black hole to white hole pairing from a present to a prior universe, in terms of quantum number n associated with a black hole to white hole linked by a worm hole.

What we would have to do, find an optimal way to find functions f and g as to Part 3, and to see if that can be linked directly to the Part 1 derivation, and its quantum number as well as Table 1. In doing so we should be aware that the wormhole linkage would be repeated say 50 million times, with the energy density showing up in our analysis of how and why Torsion would be viable in the first place.

Finding optimal f and g functions will require serious matching condition work. Is it doable? Yes, but we should keep in mind something else, namely that we are also assuming the necessity of a pre Planckian negative energy density,

Considering that the Corda treatment of black hole energy also involves negative energy values, in [22] this is no surprise, but it means we need attendant data set analysis in order to make sense of the entire idea of a gluing parameter ξ have to be specified.

Finally the idea of transvesable wormholes has to be re-investigated to see if in this configuration it makes sense at all in this situation *i.e.* a 4 dimensional recent treatment of this idea is in [27].

If the bridge between a prior to a present universe, involves transversable wormholes, via linkage of a black hole to white hole, the author, namely me asserts that if the transversable wormhole is $\omega_{\text{black hole white hole wormhole}}$ approaching Planck frequency, of about 10^{45} or so Hertz, which has MAJOR inplications as far as High frequency GW. That in itself would be indicating, even if we have MASSIVE red shifting, that we are considering here 10^5 to say 10^{10} Hz GW in the present era.

15. A Final Pre Messenger Regime for Particle Production Consideration, the Re Acceleration of the Universe

When we quantize the gravitational field as an effective field theory, we find that it too comes in set "quanta," called gravitons. In short, we argue that to come up with a graviton based model of DE with reacceleration of the universe, and to have it commensurate with the modification of the $1/r$ potential, we are really coming up with a program of finding out if gravity can be quantized. In addition, what we have done is complimenting turbulence in the electroweak era. Which in turn is relatable to the question of whether micro black holes, could contribute to cosmology. This means keeping in mind, *i.e.* the diagram given by Abbott *et al.* [28] (2009) which shows the relation between GW frequency and GW energy density for different cosmological models What we are doing is to try to reconcile $-1 \le \omega_{\text{equation of state DE}} \le 1$ with the idea of graviton production from the early states of the universe. At first glance, this looks hopeless. *i.e.* the models are incommensurate with each other and we do have a huge problem. A way of having reconciliation may be to consider what is brought up on pages 114 to 115 of Li, Wang, and Wang, [29] as we have, then an examination of the equation of state for DE, that is commensurate with re acceleration of the universe as reading,

$$\omega_{\text{equation of state DE}} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \approx \frac{\dot{\phi}^2 - 2V_0 \exp\left(-\sqrt{\frac{2}{\tilde{m}}}\frac{\phi}{m_{\text{Plank}}}\right)}{\dot{\phi}^2 + 2V_0 \exp\left(-\sqrt{\frac{2}{\tilde{m}}}\frac{\phi}{m_{\text{Plank}}}\right)} \quad (90)$$

$$\text{for acceleration of universe } \tilde{m} > 1$$

Again, this looks like a very hard problem, but, what we need to accomplish is having the following identification made, which may allow for us to make a concrete bridge between formalisms which otherwise look like they have no linkage to each other, *i.e.* do the following, namely is there a way to link ϕ of Equation (90) with φ_k of Equation (82) so as to come up with an acceptable form of ϕ

$$\text{Here, we need for acceleration of universe } \tilde{m} > 1$$

$$V(\phi) = V_0 \exp\left(-\sqrt{\frac{2}{\tilde{m}}}\frac{\phi}{m_{\text{Plank}}}\right) \quad (91)$$

Then, we should try to reconcile the following, a way to link ϕ of Equation (90) with φ_k of Equation (82). In doing so we will be able to ask ourselves the details we are following up in Equation (5) plus the worm hole idea as brought up in [27].

Finally keep in mind this one, *i.e.* in the early universe a linkage as to Equation (82), and variations as to early space time. And the re acceleration of the Universe problem. *i.e.* starting with this for the regime of space time

$$V_0 \exp\left(-\sqrt{\frac{2}{\tilde{m}}}\frac{\phi}{m_{\text{Plank}}}\right) \leftrightarrow V \cong c_2 / r^{\alpha} \quad (92)$$

i.e. the claim to be investigated is the following. If we solve this correctly as far as black holes, in relic conditions can we tie this heavy graviton effects of space time into to the following?

$$\dot{a}^2 = \left[\frac{\tilde{\kappa}^2}{3} \left(\left[\rho + \frac{\rho^2}{2\lambda} \right] \right) a^2 + \frac{\Lambda \cdot a^2}{3} + \frac{m}{a^2} - K \right] \tag{93}$$

Maartens [30] also gives a 2nd Friedman equation:

$$\dot{H}^2 = \left[-\left(\frac{\tilde{\kappa}^2}{2} \cdot [p+\rho] \cdot \left[1 + \frac{\rho^2}{\lambda} \right] \right) + \frac{\Lambda \cdot a^2}{3} - 2\frac{m}{a^4} + \frac{K}{a^2} \right] \tag{94}$$

Also, an observer is in the low redshift regime for cosmology, for which $\rho \cong -P$, for red-shift values z from zero to 1.0 - 1.5. One obtains exact equality, $\rho = -P$, for z between zero to 5. The net effect will be to obtain, based on Equation (93), assuming $\Lambda = 0 = K$ and using $a \equiv [a_0 = 1]/(1+z)$ to get a deceleration parameter q as given in Equation (95).

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{2}{1 + \tilde{\kappa}^2 [\rho/m] \cdot (1+z)^4 \cdot (1+\rho/2\lambda)} \approx -1 + \frac{2}{2+\delta(z)} \tag{95}$$

Figure 1 is predicated upon a small 4 dimensional rest mass (stated in Equation (93) for a graviton behaving the same as dark energy… We will state in our discussions section as to what is needed to give experimental confirmation as to what is a current for a "massive" graviton which is appropriate for explaining in part, **Figure 1**.

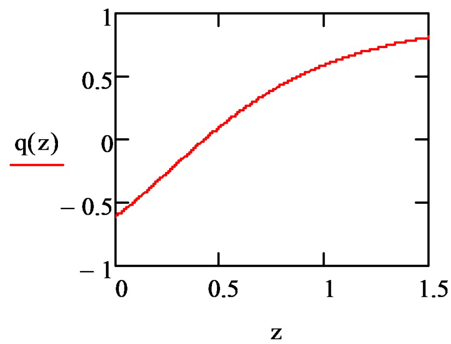

Figure 1. Re-acceleration of the universe based on Beckwith [31]; (note that $q(z) < 0$ if $z <$ 0.423.

This idea, with some revisions is similar to, for heavy gravity, at the start of Equation (5) to the following *i.e.*

Beckwith [31] used a higher-dimensional model of the brane world combined with KK graviton towers per Maartens [30]. The energy density ρ of the brane world in the Friedman equation is used in a form similar to Alves *et al.* [32] by Beckwith [31] for a non-zero graviton:

$$\rho \equiv \rho_0 \cdot (1+z)^3 - \left[\frac{m_g \cdot (c=1)^6}{8\pi G (\hbar = 1)^2} \right] \cdot \left(\frac{1}{14 \cdot (1+z)^3} + \frac{2}{5 \cdot (1+z)^2} - \frac{1}{2} \right) \tag{96}$$

Keep in mind the following, *i.e.*

Consider if there is then also a small graviton mass, *i.e.*, as stated by Beckwith [31]:

$$m_n(\text{Gravition}) = \sqrt{\frac{n^2}{L^2} + \left(m_{\text{gravition rest mass}} = 10^{-65}\text{ grams}\right)^2} = \frac{n}{L} + 10^{-65}\text{ grams} \quad (97)$$

Note that Rubakov (2002) [33] works with KK gravitons, without the tiny mass term for a 4-dimensional rest mass included in Equation (97). To obtain the KK graviton/DM candidate representation along RS dS brane world, Rubakov obtains his values for graviton mass and graviton physical states in space-time after using the following normalization: $\int \frac{\text{d}z}{a(z)} \cdot \left[h_m(z) \cdot h_{\tilde{m}}(z)\right] \equiv \delta(m - \tilde{m})$. Rubakov [33] (2002) uses J_1, J_2, N_1, N_2 which are different forms of Bessel functions. His representation of a graviton state is given by

$$-2 \cdot (m_{\text{Plank}})^2 \cdot \dot{H} = \dot{\phi}^2 \quad (98)$$

Equation (98), which is almost completely acceptable for our problem, since the rest mass of a graviton in four dimensions is so small. If so, then the wave function for a graviton with a tiny 4 dimensional space time rest mass can be written as [33].

$$h_m(z) = \sqrt{m/k} \cdot \frac{J_1(m/k) \cdot N_2([m/k] \cdot \exp(k \cdot z)) - N_1(m/k) \cdot J_2([m/k] \cdot \exp(k \cdot z))}{\sqrt{[J_1(m/k)]^2 + [N_1(m/k)]^2}} \quad (99)$$

Equation (99) is for KK gravitons having a TeV magnitude mass $M_Z \sim k$ (*i.e.*, for mass values at 0.5 TeV to above 1 TeV) on a negative tension RS brane. It would be useful to relate this KK graviton, which is moving with a speed proportional to H^{-1} with regards to the negative tension brane with $h \equiv h_m(z \to 0) = const \cdot \sqrt{\frac{m}{k}}$ as an initial starting value for the KK graviton mass. If Equation (98) is for a "massive" graviton with a small 4-dimensional gravition rest mass and if $h \equiv h_m(z \to 0) = const \cdot \sqrt{\frac{m}{k}}$ represents an initial state, then one may relate the mass of the KK graviton moving at high speed with the initial rest mass of the graviton. This rest mass of a graviton is $m_{\text{gravition}}(\text{4-Dim GR}) \sim 10^{-48}\text{ eV}$, opposed to $M_X \sim M_{\text{KK Gravition}} \sim 0.5 \times 10^9\text{ eV}$. Whatever the range of the graviton mass, it may be a way to make sense of what was presented by Dubovsky *et al.* [34], who argue for a graviton mass, using CMBR measurements, of $M_{\text{KK Gravition}} \sim 10^{-20}\text{ eV}$.

16. And Now for a Grand Slam, *i.e.* the Connection We Have Been Waiting for, *i.e.* Quantum *n*, Primordial Black Holes and Light Spectrum Issues

The key to doing this is to take into consideration Equation (82) which has a B (magnetic) field right in its description of scale and interaction issues. *i.e.* this is the point where we can take up the following [13]-[15]

$$\left\langle (\delta g_{uv})^2 (\hat{T}_{uv})^2 \right\rangle \geq \frac{\hbar^2}{V_{\text{Volume}}^2}$$
$$\xrightarrow[uv \to tt]{} \left\langle (\delta g_{tt})^2 (\hat{T}_{tt})^2 \right\rangle \geq \frac{\hbar^2}{V_{\text{Volume}}^2} \tag{100}$$
$$\& \ \delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$$

We assume that δg_{tt} is a small perturbation and look at $\delta t \Delta E = \frac{\hbar}{\delta g_{tt}}$ with

$$\Delta t_{\text{time}}(\text{initial}) = \hbar / (\delta g_{tt} E_{\text{initial}}) = \frac{2\hbar}{\delta g_{tt} \cdot g_{*s}(\text{initial}) \cdot T_{\text{initial}}} \tag{101}$$

This would put a requirement upon a very large initial temperature T_{initial} and so then, if $S(\text{initial}) \sim n(\text{particle count}) \approx g_{*s}(\text{initial}) \cdot V_{\text{volume}} \cdot \left(\frac{2\pi^2}{45}\right) \cdot (T_{\text{initial}})^3$ [35]

$$S(\text{initial}) \sim n(\text{particle count}) \approx \frac{V_{\text{volume}}}{g_{*s}^2(\text{initial})} \cdot \left(\frac{2\pi^2}{45}\right) \cdot \left(\frac{\hbar}{\Delta t_{\text{initial}} \cdot \delta g_{tt}}\right)^3 \tag{102}$$

And if we can write as given in

$$V_{\text{volume(initial)}} \sim V^{(4)} = \delta t \cdot \Delta A_{\text{surface area}} \cdot (r \leq l_{\text{Planck}}) \tag{103}$$

Then as to the follow-up to NLED and signals from primordial processes [36]

$$\alpha_0 = \sqrt{\frac{4\pi G}{3\mu_0 c}} B_0$$
$$\hat{\lambda}(\text{defined}) = \Lambda c^2 / 3 \tag{104}$$
$$a_{\min} = a_0 \cdot \left[\frac{\alpha_0}{2\hat{\lambda}(\text{defined})} \left(\sqrt{\alpha_0^2 + 32\hat{\lambda}(\text{defined}) \cdot \mu_0 \omega \cdot B_0^2} - \alpha_0 \right) \right]^{1/4}$$

Where the following is possibly linkable to minimum frequencies linked to E and M fields, and possibly relic Gravitons [36]

$$B > \frac{1}{2 \cdot \sqrt{10 \mu_0 \cdot \omega}} \tag{105}$$

We submit the following for future investigation, namely the n particle count as presented in Equation (103) is related directly to inputs into Equation (5) and that the quantum number as discussed is linkable to the discussion given in Equation (45) and Equation (46).

Furthermore, the frequency, as given in Equation (105) would be tied into Equation (14) via the n of that equation as well as specified by [37] on page 111, where we have

$$\omega = g_{rr} c k \tag{106}$$

Here g_{rr} is nearly zero, as given in Equation (100), and the entire frequency in terms of k, as a wave number as given as this construction would have this consideration, namely.

A black hole in a traditional sense has no frequency as we normally think of it, or a wave number because it is not a wave phenomenon, but the gravitational waves emitted by a black hole when it interacts with other massive objects can be described by a wave number, which is related to the wavelength of the gravitational wave it creates.

These details would be important as to obtain ideas as to data sets which would satisfy multimessenger astronomy namely the discussion as given in Mohanty, [38] namely a temperature, with scale factor as given in page 261

$$T \sim \frac{1}{g^{*} a} \tag{107}$$

With temperature T, as proportional to quantum number n as specified, whereas k as in Equation (106) may be tied into the details of Equation (99) of our manuscript.

Once our ideas of a candidate magnetic field are clarified, *i.e.* we can then examine some of the ideas of [39] which can make a connection analytically to muli messenger Astrophysics explicit [40].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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