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Classical Cosmology IV. The Photometric Maximum for Galaxies

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Abstract

The number of galaxies as a function of the redshift is derived in the framework of the generalized tired light hypothesis. Two new formulae are derived for the flux and the apparent magnitude as functions of the redshift. An application is made to four catalogs of galaxies: 2MRS, SDSS, Glade+ and Ultravista. The role of the limiting apparent magnitude of the position in redshift for the photometric maximum for galaxies is analysed.

Keywords

Galaxy Groups, Clusters, and Superclusters, Large Scale Structure of the Universe Cosmology

1. Introduction

Recent efforts in cosmology have been focused on two different targets. One is the determination of the cosmological parameters through the distance modulus for supernovae (SN). As an example, [1] analysed the difference for the cosmological parameters derived from Pantheon and Pantheon+ data. Another one involves the Hubble tension. The determination of the Hubble constant oscillates between a low value as derived by a revised Planck collaboration [2], $H_0 = (69.1 \pm 1.2)\,\mathrm{km \cdot s^{-1} \cdot Mpc^{-1}}$, and a high value, $H_0 = (72.9 \pm 1)\,\mathrm{km \cdot s^{-1} \cdot Mpc^{-1}}$, as measured with the Milky Way Cepheid variables [2]. The difference between these is referred to as the Hubble constant tension and takes the value of 2.43σ. Other cosmological problems, such as the number of galaxies as a function of the redshift, have been little analysed. We now pose some questions on this topic.

1) Can the number of galaxies as a function of the redshift be modeled in the framework of the tired light hypothesis?

2) Does the maximum of the galaxies as a function of the redshift depend on

the limiting apparent magnitude of the selected catalog?

In order to answer these questions, we review in Section 2 the Schechter luminosity function for galaxies, the pseudo-Euclidean universe in Section 3 and the generalized tired light theory in Section 4.1. The novel Section 4.2 derives the number of galaxies as a function of the redshift in the framework of the generalized tired light hypothesis. This result for the number of galaxies is then applied to four catalogs of galaxies in Section 5. Section 5.5 contains a discussion of how the influence of the limiting magnitude of the catalog shifts the position of the maximum for the number of galaxies.

2. Luminosity Function for Galaxies

The distance modulus is

$$m - M = 5\log(d) - 5, \tag{1}$$

where m is the apparent magnitude, M is the absolute magnitude and d is the distance in pc.

Let L, the luminosity of a galaxy, be defined in $[0,\infty]$. The Schechter LF of galaxies, Φ, see [3], is

$$\Phi\left(L;\Phi^*,\alpha,L^*\right)\mathrm{d}L = \frac{\Phi^*}{L^*}\left(\frac{L}{L^*}\right)^{\alpha}\exp\left(-\frac{L}{L^*}\right)\mathrm{d}L, \tag{2}$$

where α sets the slope for low values of L, L^* is the characteristic luminosity, and Φ^* is the number of galaxies per Mpc^3. The normalization is

$$\int_0^{\infty}\Phi\left(L;\Phi^*,\alpha,L^*\right)\mathrm{d}L = \Phi^*\Gamma(\alpha+1), \tag{3}$$

where

$$\Gamma(z) = \int_0^{\infty}\mathrm{e}^{-t}t^{z-1}\mathrm{d}t, \tag{4}$$

is the Gamma function. The average luminosity, $\langle L\rangle$, is

$$\left\langle\Phi\left(L;\Phi^*,\alpha,L^*\right)\right\rangle = L^*\Phi^*\Gamma(\alpha+2). \tag{5}$$

An equivalent form in absolute magnitude of the Schechter LF is

$$\Phi\left(M;\Phi^*,\alpha,M^*\right)\mathrm{d}M = 0.921\Phi^* 10^{0.4(\alpha+1)\left(M^*-M\right)}\exp\left(-10^{0.4\left(M^*-M\right)}\right)\mathrm{d}M, \tag{6}$$

where M^* is the characteristic magnitude.

3. The Pseudo-Euclidean Universe

In a Euclidean, non-relativistic and homogeneous universe the flux of radiation, f, expressed in units of $\frac{L_{\odot}}{\mathrm{Mpc}^2}$ units, where $L_{\odot}$ denotes the luminosity of the sun, is

$$f = \frac{L}{4\pi D^2}, \tag{7}$$

where D denotes the distance of the galaxy expressed in Mpc, and

$$D = \frac{cz}{H_0}. \tag{8}$$

The relation connecting the absolute magnitude, M, of a galaxy with its luminosity is

$$\frac{L}{L_\odot} = 10^{0.4(M_\odot - M)}, \tag{9}$$

where $M_\odot$ is the reference magnitude of the sun at the considered bandpass.

The flux, expressed in units of $\frac{L_\odot}{\mathrm{Mpc}^2}$, as a function of the apparent magnitude is

$$f = 7.957 \times 10^8 \mathrm{e}^{0.921 M_\odot - 0.921 m} \frac{L_\odot}{\mathrm{Mpc}^2}, \tag{10}$$

and the inverse relation is

$$m = M_\odot - 1.0857 \ln\left(0.1256 \times 10^{-8} f\right). \tag{11}$$

The joint distribution in z and f for galaxies, see formula (1.104) in [4] or formula (1.117) in [5], is

$$\frac{\mathrm{d}N}{\mathrm{d}\Omega \mathrm{d}z \mathrm{d}f} = 4\pi \left(\frac{c}{H_0}\right)^5 z^4 \Phi\left(\frac{z^2}{z_{crit}^2}\right), \tag{12}$$

where $\mathrm{d}\Omega$, $\mathrm{d}z$ and $\mathrm{d}f$ denote the differential of the solid angle, the redshift and the flux, respectively, and Φ is the Schechter LF. The critical value of z, z_{crit}, is

$$z_{crit}^2 = \frac{H_0^2 L^*}{4\pi f c^2}. \tag{13}$$

The number of galaxies in z and f as given by formula (12) has a maximum at $z = z_{pos\text{-}max}$, where

$$z_{pos\text{-}max} = z_{crit}\sqrt{\alpha + 2}, \tag{14}$$

which can be re-expressed as

$$z_{pos\text{-}max}(f) = \frac{\sqrt{2+\alpha}\sqrt{10^{0.4 M_\odot - 0.4 M^*}}\, H_0}{2\sqrt{\pi}\sqrt{f}\, c}, \tag{15}$$

or, on replacing the flux f with the apparent magnitude m,

$$z_{pos\text{-}max}(m) = \frac{1.772 \times 10^{-5}\sqrt{2+\alpha}\sqrt{10^{0.4 M_\odot - 0.4 M^*}}\, H_0}{\sqrt{\pi}\sqrt{\mathrm{e}^{0.921 M_\odot - 0.921 m}}\, c}. \tag{16}$$

The number of galaxies, $N(z, f_{min}, f_{max})$ comprised between a minimum value of the flux, f_{min}, and a maximum value of the flux, f_{max}, can be computed through the following integral

$$N(z) = \int_{f_{min}}^{f_{max}} 4\pi \left(\frac{c}{H_0}\right)^5 z^4 \Phi\left(\frac{z^2}{z_{crit}^2}\right) \mathrm{d}f. \tag{17}$$

4. The Generalized Tired Light Model

4.1. The Theory

We assume that the frequency ν of a photon decreases according to the following non-linear law

$$\frac{\mathrm{d}}{\mathrm{d}x}\nu(x) = -an_e\nu(x)^{\phi}, \tag{18}$$

where n_e is the number density of matter in $\frac{1}{\mathrm{m}^3}$ and a the attenuation coefficient in $Hz^{1-\phi}m^2$. We call the above ODE the generalized tired light (GTL) model. Imposing the initial condition $\nu(0)=\nu_0$, the solution is

$$\nu(x) = \frac{1}{\left(an_e x\phi - an_e x + \mathrm{e}^{-\phi\ln(\nu_0)}\nu_0\right)^{\frac{1}{-1+\phi}}}. \tag{19}$$

We now continue inserting

$$a = \frac{\nu_0^{1-\phi}H_0}{cn_e}, \tag{20}$$

where c is the speed of light. As a consequence, the redshift is

$$z = c^{-\frac{1}{-1+\phi}}\left(c+(-1+\phi)xH_0\right)^{\frac{1}{-1+\phi}} - 1. \tag{21}$$

The inversion of the above formula gives

$$x = -\frac{\left(-(z+1)^{\phi}+z+1\right)c}{H_0(-1+\phi)(z+1)}, \tag{22}$$

and the distance modulus in GTL is

$$m - M = 25 + \frac{5\ln\left(-\frac{\left(-(z+1)^{\phi}+z+1\right)c}{H_0(-1+\phi)(z+1)}\right)}{\ln(10)}. \tag{23}$$

More details can be found in [6].

4.2. The Number of Galaxies versus Redshift

Under the hypothesis of spherical symmetry, we use the symbol r for the distance. The flux of radiation, f, is introduced as

$$f = \frac{L}{4\pi r^2}, \tag{24}$$

and is here expressed in $\frac{L_\odot}{\mathrm{Mpc}^2}$. The joint distribution in distance, r, and flux, f, for the number of galaxies is

$$\frac{\mathrm{d}N}{\mathrm{d}\Omega\mathrm{d}r\mathrm{d}f} = \frac{1}{4\pi}\int_0^{\infty} 4\pi r^2\Phi\left(\frac{L}{L^*}\right)\delta\left(f - \frac{L}{4\pi r^2}\right)\mathrm{d}r, \tag{25}$$

where the factor ($\frac{1}{4\pi}$) converts the number density into the density for a solid angle and the Dirac delta function selects the required flux.

In the GTL the distance, r, is expressed by Equation (22) and the derivative of the distance with respect to the redshift is

$$\frac{\mathrm{d}r}{\mathrm{d}z} = \frac{(z+1)^{-2+\phi} c}{H_0}. \tag{26}$$

In the case of the Schechter LF, we have

$$\frac{\mathrm{d}N}{\mathrm{d}\Omega \mathrm{d}z \mathrm{d}f} = \frac{N}{D}, \tag{27}$$

where

$$\begin{aligned} N = {} & 2^{2+2\alpha} \pi^{\alpha+1} c^{5+2\alpha} \Phi^* L^{*-\alpha-1} f^{\alpha} H_0^{-5-2\alpha} (z+1)^{-6-2\alpha+\phi} \left((-1+\phi)^2 \right)^{-\alpha} \\ & \times \left(-1+(z+1)^{\phi} - z \right)^4 \left(\left(-1+(z+1)^{\phi} - z \right)^2 \right)^{\alpha} \mathrm{e}^{-\frac{4\pi f \left(-1+(z+1)^{\phi}-z\right)^2 c^2}{H_0^2 (-1+\phi)^2 (z+1)^2 L^*}}, \end{aligned} \tag{28}$$

and

$$D = (-1+\phi)^4, \tag{29}$$

where $\mathrm{d}\Omega$, $\mathrm{d}z$, and $\mathrm{d}f$ denote the differential of the solid angle, the red shift, and the flux, respectively.

The flux expressed in units of $\frac{L_\odot}{\mathrm{Mpc}^2}$ as a function of the apparent magnitude is

$$f = 7.957 \times 10^8 \mathrm{e}^{0.921 M_\odot - 0.921 m} \frac{L_\odot}{\mathrm{Mpc}^2}, \tag{30}$$

and the inverse relation is

$$m = M_\odot - 1.0857 \ln \left(0.1256 \times 10^{-8} f \right). \tag{31}$$

The derivative of the flux with respect to the apparent magnitude is

$$\frac{\mathrm{d}f}{\mathrm{d}m} = 7.3293 \times 10^8 \mathrm{e}^{0.921 M_\odot - 0.921 m}. \tag{32}$$

According to Equations (30) and (32) the number of galaxies as a function of the selected apparent magnitude and redshift is

$$\frac{\mathrm{d}N}{\mathrm{d}\Omega \mathrm{d}z \mathrm{d}m} = \frac{NN}{DD}, \tag{33}$$

where

$$\begin{aligned} NN = {} & 7.3293 \times 10^8 2^{2+2\alpha} \pi^{\alpha+1} c^{5+2\alpha} \Phi^* \left(10^{0.4 M_\odot - 0.4 M^*} \right)^{-1-\alpha} \left(7.9577 \times 10^8 \mathrm{e}^{0.921 M_\odot - 0.921 m} \right)^{\alpha} \\ & \times H_0^{-5-2\alpha} (z+1)^{-6-2\alpha+\phi} \left((-1+\phi)^2 \right)^{-\alpha} \left(-1+(z+1)^{\phi} - z \right)^4 \left(\left(-1+(z+1)^{\phi} - z \right)^2 \right)^{\alpha} \\ & \times \mathrm{e}^{-\frac{10^{10} \mathrm{e}^{0.921 M_\odot - 0.921 m} \left(-1+(z+1)^{\phi}-z\right)^2 c^2}{H_0^2 (-1+\phi)^2 (z+1)^2 10^{0.4 M_\odot - 0.4 M^*}}} \mathrm{e}^{0.921 M_\odot - 0.921 m}, \end{aligned} \tag{34}$$

and

$$DD = (-1+\phi)^4 . \tag{35}$$

The appearance of the photometric maximum can be evaluated using a fourth-order Taylor expansion about $z = 0.019$ of Equation (33)

$$\frac{dN}{d\Omega dz dm} = 35099 - 57450z - 2.3508\times10^8 (z-0.019)^2 + 4.2736\times10^9 (z-0.019)^3 , \tag{36}$$

see Figure 1.

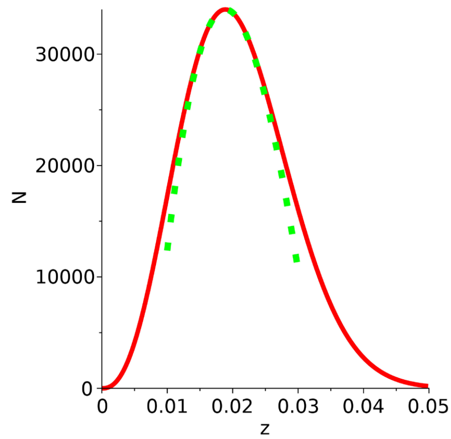

Figure 1. Behavior of $\frac{dN}{d\Omega dz dm}$, Equation (33), as a function of redshift, red full line, and a Taylor expansion around $z = 0.019$, green dotted line. The parameters are as in Table 1.

Table 1. Parameters for the 2MRS catalog.

$H_0 = 68.04\ \mathrm{km\cdot s^{-1}\cdot Mpc^{-1}}$	$M_\odot = 3.39$	$c = 299792\ \mathrm{km\cdot s^{-1}}$	$\phi = 3.07$
$\alpha = -0.79$	$M^* = -24.016$	$\Phi^* = 0.0037\ \mathrm{Mpc^{-3}}$	$m = 10.39$

The number of galaxies, $N(z, f_{min}, f_{max})$ comprised between a minimum value of flux and a maximum value of flux in GTL is

$$N(z) = \int_{f_{min}}^{f_{max}} \frac{N}{D} df , \tag{37}$$

when the fluxes are considered and

$$N(z) = \int_{m_{min}}^{m_{max}} \frac{NN}{DD} dm, \tag{38}$$

when the apparent magnitudes are considered.

5. Astrophysical Applications

In the following, we will process the galaxies of four catalogs. The merit function χ^2 is computed according to the formula

$$\chi^2 = \sum_{i=1}^{n} \frac{(T_i - O_i)^2}{T_i} , \tag{39}$$

where n is the number of bins, T_i is the theoretical value, and O_i is the experimental value represented by the frequencies.

5.1. The 2MRS Catalog

The release of the 2MASS Redshift Survey (2MRS), with its 44599 galaxies having $K_s < 11.75$ allows making a test on the radial number of galaxies because we have a small zone-of-avoidance, see Figure 1 in [7].

Figure 2 shows the number of observed galaxies of the 2MRS catalog for a given apparent magnitude and the theoretical curve for the pseudo-Euclidean universe.

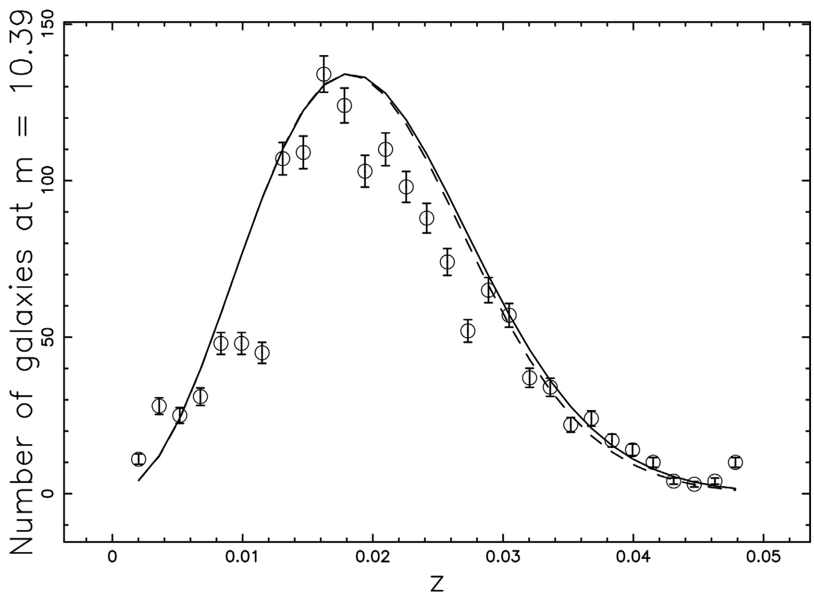

Figure 2. The galaxies of the 2MRS with $10.31 \le m \le 10.47$ or $1164793 \frac{L_\odot}{\text{Mpc}^2} \le f \le 1346734 \frac{L_\odot}{\text{Mpc}^2}$ are organized in frequencies versus heliocentric redshift, (empty circles); the error bar is given by the square root of the frequency. The maximum frequency of observed galaxies is at $z = 0.017$. The full line is the theoretical curve generated by $\frac{\mathrm{d}N}{\mathrm{d}\Omega \mathrm{d}z \mathrm{d}f}(z)$ as given by the application of the Schechter LF in the pseudo-Euclidean universe, which is Equation (12), and the dashed line denotes the number of galaxies in GTL, which is Equation (27). The parameters are the same as for Table 1, $\chi^2 = 150$ for pseudo-Euclidean universe and $\chi^2 = 154$ for the GTL.

Figure 3 gives the number of all the galaxies as a function of z, see equations (17) and (37), for the pseudo-Euclidean universe and for the GTL, respectively.

5.2. The SDSS Data

We processed the SDSS Photometric Catalogue DR 12, see [8], which contains 10450256 galaxies (elliptical + spiral) with photometric redshift. Figure 4 presents the number of galaxies that are observed in SDSS DR 12 as a function of the redshift for a given window in the flux, in addition to the theoretical curve for the GTL.

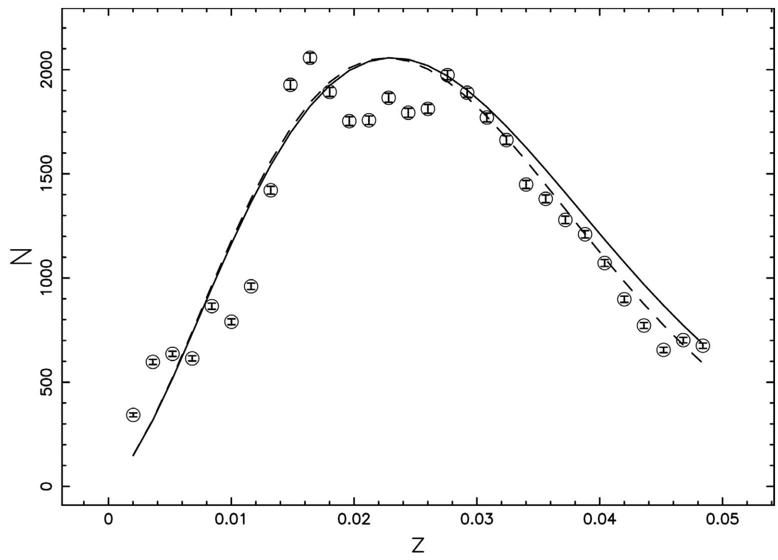

Figure 3. The galaxies of the 2MRS with $4.99 \le m \le 11.75$ or $360403 \frac{L_\odot}{\mathrm{Mpc}^2} \le f \le 182301584 \frac{L_\odot}{\mathrm{Mpc}^2}$ are organized in frequencies versus heliocentric redshift, (empty circles); the error bar is given by the square root of the frequency. The maximum frequency of observed galaxies is at $z = 0.0172$. The full line is the theoretical curve generated by the integral of $\frac{\mathrm{d}N}{\mathrm{d}\Omega \mathrm{d}z \mathrm{d}f}(z)$ as given by the application of the Schechter LF in the pseudo-Euclidean universe, which is Equation (17), and the dashed line denotes the number of galaxies in GTL, which is Equation (37). The parameters are the same as for Table 1, $\chi^2 = 1099$ for pseudo-Euclidean universe and $\chi^2 = 961$ for the GTL.

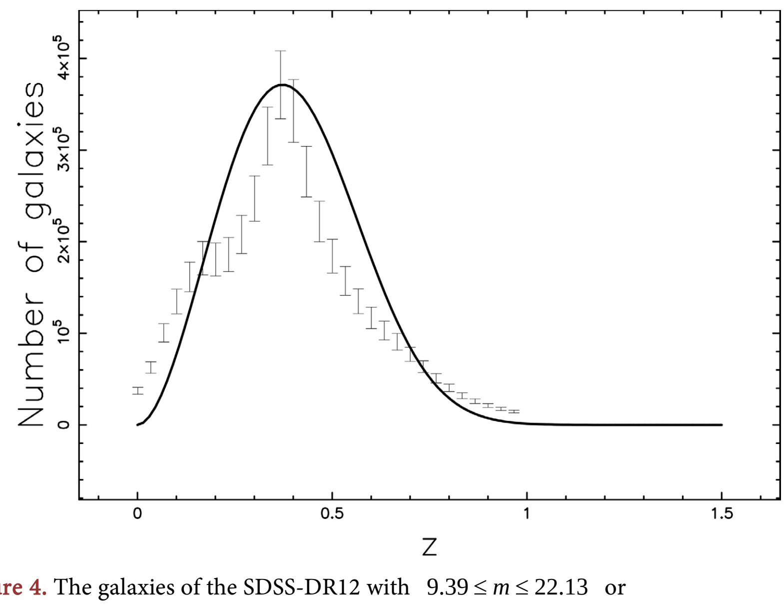

Figure 4. The galaxies of the SDSS-DR12 with $9.39 \le m \le 22.13$ or $400 \frac{L_\odot}{\mathrm{Mpc}^2} \le f \le 5\times10^7 \frac{L_\odot}{\mathrm{Mpc}^2}$ are organized in frequencies versus heliocentric redshift, (empty circles); the error bar is given by the square root of the frequency. The maximum frequency of observed galaxies is at $z = 0.378$. The full line is the theoretical curve for number of galaxies in GTL, which is Equation (37). The parameters are the same as for Table 2, and $\chi^2 = 671705$.

Table 2. Parameters for the SDSS-DR12 catalog.

$H_0 = 68.04\ \mathrm{km \cdot s^{-1} \cdot Mpc^{-1}}$	$M_\odot = 6.39$	$c = 299792\ \mathrm{km \cdot s^{-1}}$	$\phi = 3.07$
$\alpha = -0.90$	$M^* = -30$	$\Phi^* = 0.030\ \mathrm{Mpc^{-3}}$	

5.3. The GLADE Catalog

The catalog GLADE+ contains ≈ 22.5 million galaxies [9]. All the galaxies of the GLADE+ catalog are shown in Figure 5 together with the theoretical model.

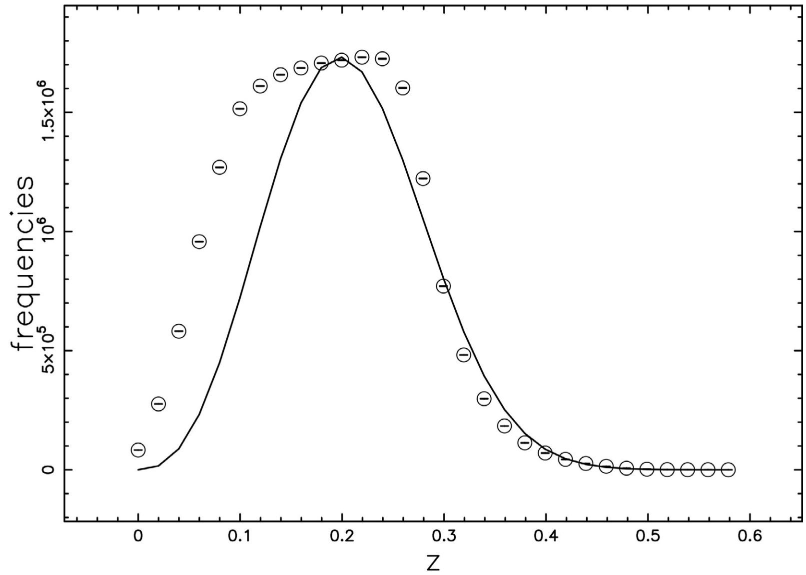

Figure 5. The galaxies of GLADE+ with $-2.6 \le m \le 20.22$ or $1000 \frac{L_\odot}{\mathrm{Mpc}^2} \le f \le 1.26 \times 10^{12} \frac{L_\odot}{\mathrm{Mpc}^2}$ are organized in frequencies versus heliocentric redshift, (empty circles); the error bar is given by the square root of the frequency. The maximum frequency of the observed galaxies is at $z = 0.219$. The full line is the theoretical curve for the number of galaxies in GTL, which is Equation (37). The parameters are the same as for Table 3, and $\chi^2 = 2752427$.

Table 3. Parameters for the GLADE+ catalog.

$H_0 = 68.04\ \mathrm{km \cdot s^{-1} \cdot Mpc^{-1}}$	$M_\odot = 5.47$	$c = 299792\ \mathrm{km \cdot s^{-1}}$	$\phi = 3.07$
$\alpha = -0.79$	$M^* = -19.5$	$\Phi^* = 0.030\ \mathrm{Mpc^{-3}}$	

5.4. Ultravista Catalog

The Ultravista catalog contains the photometric redshifts of 262,615 galaxies computed with the EAZY code [10]. All the galaxies of the Ultravista catalog are shown in Figure 6 together with the theoretical model.

5.5. Parametrization of the Maximum

We now analyse how the position in redshift of the maximum, z_{pos}, depends on the limiting magnitude of the catalog, m_u, see Figure 7, which has the parameters of the 2MRS catalog.

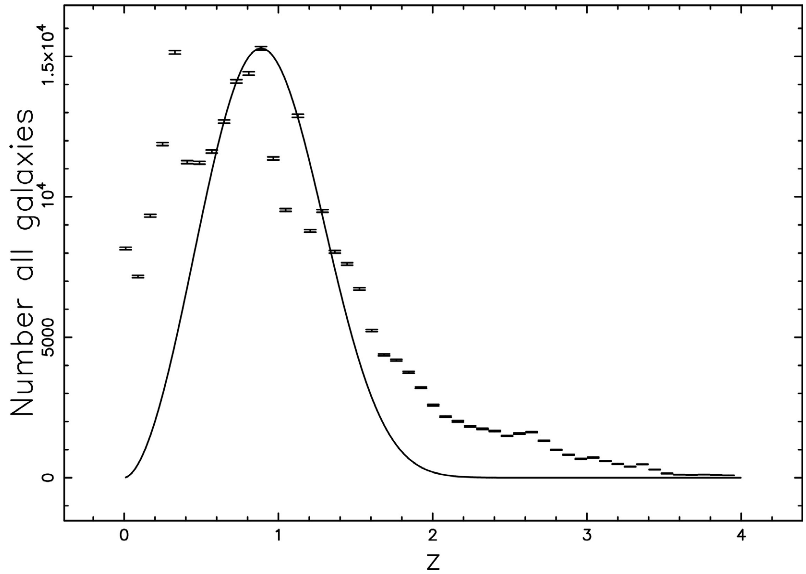

Figure 6. The galaxies of the Ultravista catalog with $17 \le m \le 22.5$ are organized in frequencies versus heliocentric redshift, (empty circles); the error bar is given by the square root of the frequency. The maximum frequency of the observed galaxies is at $z = 0.927$. The full line is the theoretical curve for the number of galaxies in GTL, which is Equation (37). The parameters are the same as Table 4, and $\chi^2 = 75703$.

Table 4. Parameters for the Ultravista catalog.

$H_0 = 68.04\ \mathrm{km \cdot s^{-1} \cdot Mpc^{-1}}$	$M_\odot = 5.47$	$c = 299792\ \mathrm{km \cdot s^{-1}}$	$\phi = 3.07$
$\alpha = -0.999$	$M^* = -17.4$	$\Phi^* = 0.030\ \mathrm{Mpc^{-3}}$	

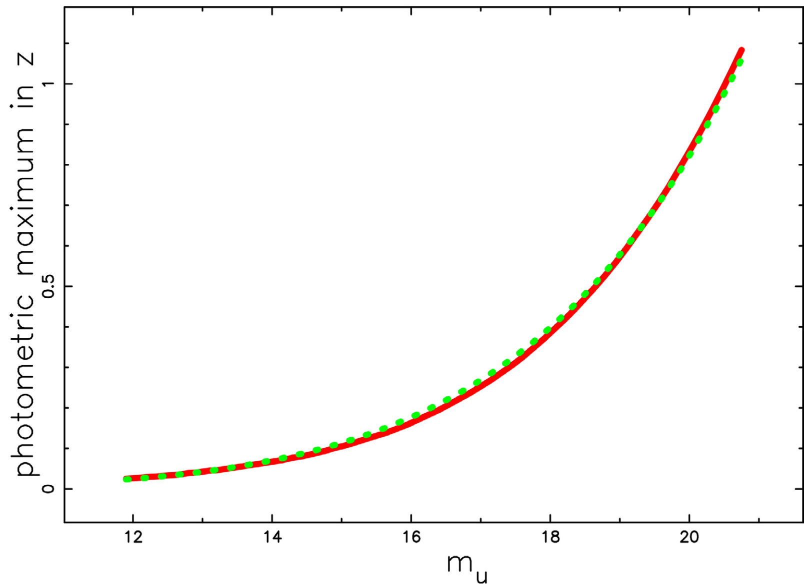

Figure 7. The red line marks the theoretical position in redshift of the maximum in the number of galaxies, z_{pos}, as function of the value of the limiting magnitude, m_u, when $4.99 \le m \le m_u$. The number of all the galaxies is evaluated with Equation (17) and other parameters for the 2MRS catalog as in Table 1. The fit with a power law as given by Equation (40) has parameters $C = 9.05 \times 10^{-10}$ and $\gamma = 6.88$, (green dotted points).

The above curve can be approximated with the following power law

$$z_{pos} = C m_u^{\gamma}, \tag{40}$$

where the two parameters, C and γ, can be found through a nonlinear fit. The second example analyses how the changes in the upper apparent magnitude modify the changes in the position of the photometric maximum for the data of the Ultravista catalog, see Figure 8.

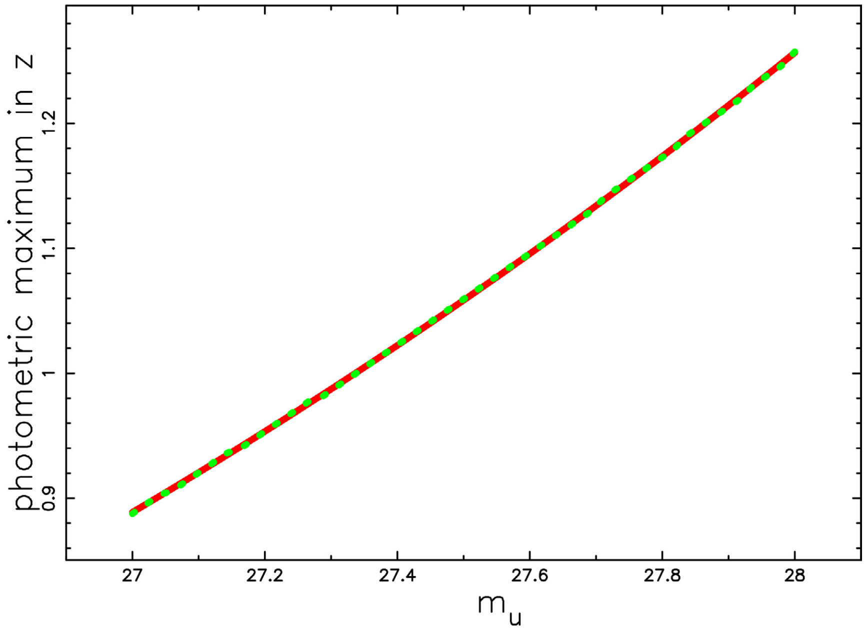

Figure 8. The red line marks the theoretical position in redshift of the maximum of the number of galaxies, z_{pos}, as a function of the value of the limiting magnitude, m_u, when $17 \le m \le m_u$. The number of all the galaxies is evaluated with Equation (17) and the other parameters of the Ultravista catalog are in Table 4. The fit with a power law as given by Equation (40) yields the parameters $C = 2.07 \times 10^{-14}$ and $\gamma = 9.52$, (green dotted points).

6. Conclusions

The number of galaxies: The theoretical number of galaxies in the framework of the GTL versus the redshift can be parametrized as a function of the observed flux, see Equation (27), or as a function of the selected magnitude, see Equation (33). The new derived formulae for all the galaxies are applied to four catalogs of galaxies: 2MRS, see Figure 3, SDSS DR 12, see Figure 4, GLADE+, see Figure 5 and Ultravista, see Figure 6.

Predictions: The role of the increasing limiting apparent magnitude is clearly outlined in Figure 7 with parameters of the 2MRS catalog and in Figure 8 with parameters of the Ultravista catalog. The position of the photometric maximum, z_{pos}, increases as $z_{pos} \propto m_u^{6.88}$ for the 2MRS catalog and $z_{pos} \propto m_u^{9.52}$ for the Ultravista catalog. As an example, the position of the photometric maximum for the Ultravista catalog is at $z_{pos} = 0.88$ for $m_u = 27$ and an increase of the limiting apparent magnitude by one shifts the maximum to $z_{pos} = 1.25$.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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