Applied Mathematics, 2025, 16(3), 219-234
https://www.scirp.org/journal/am
ISSN Online: 2152-7393
ISSN Print: 2152-7385

Research on Target Path Tracking Method for Smart Car Using Multi-Objective Bionic Game Theory

Chongzhi Song[1*], Xiang Zheng[1], Rong Wei[1], Lu Wang[2]

[1]College of Mechanical Engineering, Chaohu University, Hefei, China
[2]School of Mechanical Engineering, Anhui University of Technology, Maanshan, China
Email: *052102@chu.edu.cn

How to cite this paper: Song, C.Z., Zheng, X., Wei, R. and Wang, L. (2025) Research on Target Path Tracking Method for Smart Car Using Multi-Objective Bionic Game Theory. *Applied Mathematics*, **16**, 219-234.
https://doi.org/10.4236/am.2025.163011

Received: December 3, 2024
Accepted: March 22, 2025
Published: March 25, 2025

Abstract

To improve smart car drive performance and avoid side-slip during target path tracking, a linearized four wheel car model was adopted as a predictive control model, and target path tracking method was built based on multi-objective bionic game theory. Through calculating interaction factors, it established mapping factor indicators between design variables and objective functions, solving for the respective strategy spaces of each game participant, and ensuring that all game participants follow a common constraint protocol. The behavior and survival mechanism of side-blotched lizard were studied; Opportunism, egoism and collectivism were defined according to it's own color, and three behaviors were considered as set by the corresponding player. Based on the behavioral characteristics exhibited by each species during the evolutionary process, it established the mapping relationships among respective adaptation and objective functions to evaluate the three evolved lizard individuals nature adaptability. During evolution, three types of male lateral spotted lizards evolve games with their respective fitness functions as the objective. After each round of evolutionary games, the optimal genes of the three lizards were solved, and new chromosomes were constructed. Convergence criteria were used to determine convergence, and after multiple evolutionary iterations, the optimal chromosome, multi-objective solution was obtained. The biomimetic lizard evolutionary game algorithm was used to solve the smoothness index, and the simulation results showed the effectiveness of this algorithm. The test results show the method can track smart car quickly and steadily, and has good real-time performance.

Keywords

Smart Car, Target Path Tracking, Fuzzy Clustering,

Competitive-Cooperative Game, Predictive Control Model

1. Introduction

Smart car research is a key topic, and path tracking refers to the core technology of safe, accurate and stable driving of smart cars along the preset highway, and the strength of its performance plays a decisive role in the driving stability of smart cars [1]. To enhance the performance of path tracking, some scholars have adopted a variety of research strategies, such as applying the four-wheel robot path tracking method to smart car target tracking, pure + pursuit method was adopted in reference [2], the geometric shape method of circular look-ahead for tracking was adopted in reference [3], because of the constraints of car size, driving condition, turning radius and driving dynamics, the pure geometric path tracking method is not able to solve driving constraint problems of smart cars. At present, the control strategy is often used to study high precision control of smart car path tracking, on the basis of the deviation of car position, course and target path, incremental PID control algorithm was adopted [4], reflecting better path tracking performance. In the reference [5], a linear time-varying model predictive control strategy was used to transform three-DOF dynamic model into two programming problems, and the simulation results showed the control strategy has high accuracy, and good driving stability in middle, low speed. Reference [6] established a feedback fuzzy controller by using the feedback prediction error method, and achieved good simulation results when the speed less than 60 km/h.

The performance of path tracking is related to the driving condition and driving parameters of the cars, and all parameters are interconnected. Under the condition of normal driving, how to cooperatively calculate the interconnection between parameters and achieve high-precision control is current key research. Some research achievements have been made in solving multi-objective control by game theory. Xie Nenggang *et al*. [7]-[9] successfully applied game theory to engineering design, and gave the key solving techniques of game theory in multi-objective decision-making, the formulation of game rules, the division and solving steps of strategy space of each player. In game theory, the design system variables are mapped to the spatial strategy sets that can be selected by each player through fuzzy cluster analysis. Combined with bionic game theory, this paper gives a path tracking method for smart cars. Through calculating interaction factors, it established mapping factor indicators between design variables and objective functions, solving the respective strategy spaces of each game participant, and ensuring that all game participants follow a common constraint protocol. The behavior and survival mechanism of side-blotched lizard were studied; Opportunism, egoism and collectivism were defined according to it's own color, and the three behaviors were considered as set by the corresponding player. Based on the behavioral characteristics exhibited by each species during the evolutionary process, they establish mapping relationships between their respective adaptation functions and three objective functions to evaluate the ability of three evolved lizard individuals to adapt to nature. In the process of evolution, three types of male lateral spotted lizards evolve games with their respective fitness functions as the objective. After each round of evolutionary games, the optimal genes of the three lizards can be solved, and new chromosomes can be constructed. Convergence criteria are used to determine convergence, and after multiple evolutionary iterations, the optimal chromosome, i.e. multi-objective solution, can be obtained. The bio-mimetic lizard evolutionary game algorithm was used to solve the smoothness index, and the simulation results showed the effectiveness of this algorithm.

2. Path Tracking Control Model

Neglecting air resistance during the operation of smart cars, assuming tires and ground contact is point contact, the front wheel deflection angle is small, and there is no relative sliding, combined with the reference [10] [11], the four-wheel car model used in the paper as shown in **Figure 1**, where *XOY* is road coordinate system, *xoy* is local coordinate system of smart car, and the positive x-axis is defined as the direction of car travel.

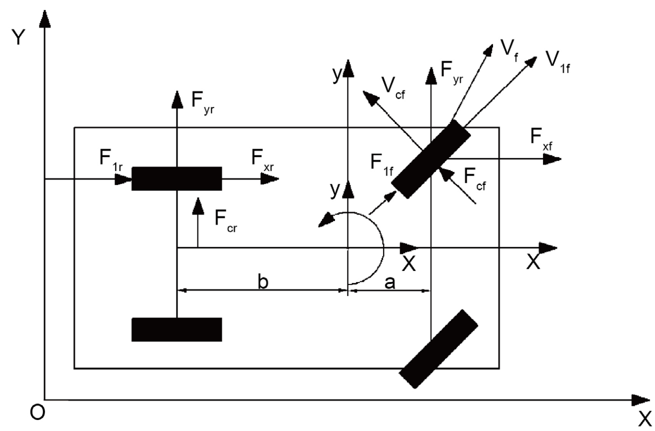

Figure 1. Dynamics of four-wheel.

The kinematics equations of the smart cars around *x-axis*:

$$m\ddot{x} = m\dot{y}\dot{\theta} + 2F_{xf} + 2F_{xr} \tag{1}$$

The lateral kinematics mechanics equation around *y-axis*:

$$m\ddot{y} = m\dot{x}\dot{\theta} + 2F_{yf} + 2F_{yt} \tag{2}$$

The transverse oscillation equation around *z-axis*:

$$I_z\ddot{\theta} = 2aF_{yf} - 2bF_{yr} \tag{3}$$

where m is car mass, a and b are the distances from car's center to front and rear axles. I_z is car inertia moment, θ is car heading angle, F_{xf} is *X-axis* force on the front wheel, F_{xr} is *X-axis* force on the rear wheels ; F_{yf} is *X-axis* force on the front wheel, F_{yr} is *X-axis* force on the rear wheels.

Combined with basic principles of horizontal pre-aiming error and path tracking error [11] to organize (1), (2), (3), the nonlinear dynamic model is obtained:

$$\begin{cases} m\ddot{y} = -m\dot{x}\dot{\theta} + 2\left[C_{cf}\left(\sigma_f - \dfrac{\dot{y}+a\dot{\theta}}{\dot{x}}\right) + C_{cr}\dfrac{b\dot{\theta}-\dot{y}}{\dot{x}}\right] \\ m\ddot{x} = m\dot{y}\dot{\theta} + 2\left[C_{lf}s_f\left(\sigma_f - \dfrac{\dot{y}+a\dot{\theta}}{\dot{x}}\right)\sigma_f + C_{lr}s_r\right] \\ I_z\ddot{\theta} = 2\left[aC_{ef}\left(\sigma_f - \dfrac{\dot{y}+a\dot{\theta}}{\dot{x}}\right)\right] \\ \dot{Y} = \dot{x}\sin\theta + \dot{y}\cos\theta \\ \dot{X} = \dot{x}\cos\theta - \dot{y}\sin\theta \end{cases} \tag{4}$$

where C_{ef} is lateral deflection stiffness, C_{er} is longitudinal stiffness; C_{lf} is longitudinal stiffness, C_{lr} is lateral deflection stiffness ; σ_f is tire deflection angle; s_f is the slip rate, s_r is the slip rate. Smart car path tracking has strong real-time performance, nonlinear dynamic tracking model is difficult to achieve real-time control, so the formula (4) is linearized. $\dot{\xi} = f(\xi, u)$, where ξ is state variable, $\xi = (\dot{y}, \dot{x}, \theta, \dot{\theta}, Y, X)$, u is control variable, $u = \sigma_f$, we can get:

$$\begin{cases} \dot{\xi} = A(t)\xi(t) + B(t)u(t) \\ y = C\xi(t) \end{cases} \tag{5}$$

$$A(t) = \begin{bmatrix} a_{11} & \dfrac{\partial f_{\dot{y}}}{\partial \dot{x}} & 0 & a_{14} & 0 & 0 \\ a_{21} & \dfrac{\partial f_{\dot{x}}}{\partial \dot{x}} & 0 & a_{24} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ a_{41} & \dfrac{\partial f_{\dot{\theta}}}{\partial \dot{x}} & 0 & a_{44} & 0 & 0 \\ \cos\theta_t & \sin\theta_t & a_{53} & 0 & 0 & 0 \\ -\sin\theta_t & \cos\theta_t & a_{63} & 0 & 0 & 0 \end{bmatrix}; \quad a_{11} = \frac{-2(C_{cf}+C_{cr})}{m\dot{x}_t},$$

$$a_{14} = -\dot{x}_t + \frac{2(bC_{cf}-aC_{cr})}{m\dot{x}_t}; \quad a_{21} = \dot{\theta} - \frac{2C_{cf}\sigma_{f,t-1}}{m\dot{x}_t}; \quad a_{24} = \dot{y}_t - \frac{2aC_{cf}\sigma_{f,t-1}}{m\dot{x}_t};$$

$$a_{41} = \frac{2(bC_{cr}-aC_{cf})}{I_z\dot{x}_t}; \quad a_{44} = \frac{-2(b^2C_{cr}+a^2C_{cf})}{I_z\dot{x}_t}, \quad a_{53} = \dot{x}_t\cos\theta_t - \dot{y}_t\sin\theta_t,$$

$$a_{63} = -\dot{y}_t\cos\theta_t - \dot{x}_t\sin\theta_t.$$

$$B(t) = \begin{bmatrix} c\dfrac{C_{cf}}{m} & b_{12} & 0 & \dfrac{2aC_{cf}}{I_z} & 0 & 0 \end{bmatrix}; \quad b_{12} = \frac{2C_{cf}\left(2\sigma_{f,t-1} - \dfrac{\dot{y}_t - b\dot{\theta}_t}{\dot{x}_t}\right)}{m};$$

$$C = (0,0,0,0,1,0)^{\mathrm{T}}$$

Model output equation

For formula (5), set state variable $x\left(k|t=\begin{bmatrix}\xi(k|t)\\ u(k-1|t)\end{bmatrix}\right)$, then the new state space expressed as

$$\begin{cases}x(k+1|t)=\tilde{A}(t)\xi(k|t)+\tilde{B}(t)\Delta u(k|t)\\ \eta(k|t)=\tilde{C}(k|t)x(k|t)\end{cases} \tag{6}$$

In the formula, $\tilde{A}(k)=\begin{bmatrix}A(k) & B(k)\\ 0_{1\times 6} & 1\end{bmatrix}$, $\tilde{B}(k)=\begin{bmatrix}B(k)\\ 1\end{bmatrix}$;
$\tilde{C}(k)=\begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0\end{bmatrix}^{\mathrm{T}}$; $\eta(k|t)$ is the output of tracking prediction horizon. The output can be defined:

$$Y(k+1|t)=\begin{bmatrix}\eta(k+1|k)\\ \eta(k+2|k)\\ \vdots\\ \eta(k+N_p|k)\end{bmatrix} \tag{7}$$

where N_p is system tracking prediction horizon. The input k is defined:

$$\Delta U(k+1|t)=\begin{bmatrix}\Delta u(k)\\ \Delta u(k+1)\\ \vdots\\ \Delta u(k+m-1)\end{bmatrix} \tag{8}$$

Therefore, the output equation k expressed:

$$Y(k+1|k)=S_\xi \xi(k)+S_u\Delta(k) \tag{9}$$

where $S_\xi=\left(C\tilde{A}(k),C\tilde{A}(k)^2,\cdots,C\tilde{A}(k)^{N_C},\cdots,C\tilde{A}(k)^{N_p}\right)^{\mathrm{T}}$

$$S_u=\begin{bmatrix}C\tilde{B}(k) & 0 & \cdots & 0\\ C\tilde{A}(k)\tilde{B}(k) & C\tilde{B}(k) & \cdots & 0\\ \vdots & \vdots & & \vdots\\ C\tilde{A}(k)^{N_c-1}\tilde{B}(k) & C\tilde{A}(k)^{N_c-2}\tilde{B}(k) & \cdots & C\tilde{A}(k)\\ C\tilde{A}(k)^{N_c}\tilde{B}(k) & C\tilde{A}(k)^{N_c-2}\tilde{B}(k) & \cdots & C\tilde{A}(k)\tilde{B}(k)\\ \vdots & \vdots & & \vdots\\ C\tilde{A}(k)^{N_p-1}\tilde{B}(k) & C\tilde{A}(k)^{N_p-2}\tilde{B}(k) & \cdots & C\tilde{A}(k)^{N_p-N_c-1}\tilde{B}(k)\end{bmatrix} \tag{10}$$

where N_c is system control time domain, $N_c\le N_p$.

Construction of model constraints

The manipulation stability directly affects the riding comfort, so in addition to the dynamic constraints, this paper also increases wheel deflection angle, side deflection increment constraint, to ensure the smart car's driving stability and comfort.

Front wheel deflection angle and its incremental constraint: when tire deflection angle is less than 5˚, there is a linear proportional relationship between lateral deflection force and lateral deflection angle. In this paper, the lateral deflection angle is: $-2^\circ \le \delta \le +2^\circ$. According to physical position, the range of deflection angle changes at any time, the deflection angle and incremental constraint expressed as:

$$\begin{cases} u_{\min}(k+t) < u(k+t) < u_{\max}(k+t) \\ \Delta u_{\min}(k+t) < \Delta u(k+t) < \Delta u_{\max}(k+t) \\ k = 0,1,\cdots,N-1 \end{cases} \tag{11}$$

Formula (11) can be converted into $U_{\min} \le A\Delta U + U_f \le U_{\max}$, where

$$A = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix},\ \Delta U = \begin{bmatrix} \Delta u_{\min}(t+1) \\ \Delta u_{\min}(t+2) \\ \vdots \\ \Delta u_{\min}(t+N) \end{bmatrix}_{N\times 1};\ U_t = \begin{bmatrix} u(t) & u(t) & \cdots & u(t) \end{bmatrix}_{N\times 1}^{\mathrm{T}}.$$

In the range of linear lateral deflection angle change, the product of tire side deflection angle and tire stiffness is tire's lateral force, and expressed as:

$$\begin{cases} F_{yf} = 2C_f\delta_f \\ F_{yr} = 2C_r\delta_r \end{cases} \tag{12}$$

where

$$\begin{cases} \delta_f = \sigma_f - \dfrac{\dot{y} + a\dot{\theta}}{\dot{x}} \\ \delta_r = \sigma_r - \dfrac{\dot{y} - b\dot{\theta}}{\dot{x}} \end{cases} \tag{13}$$

δ_r is rear wheel deflection angle, σ_f, σ_r are front and rear rotation angle. Taking u, ξ as control quantity and state quantity respectively, the formula (12) is linearized, it can be obtained:

$$\delta = E\xi(k,t) + Fu(k,t) \tag{14}$$

where $\delta = \begin{bmatrix} \delta_f & \delta_r \end{bmatrix}^{\mathrm{T}}$ is tire side deflection matrix; $F = \begin{bmatrix} -1 & 0 \end{bmatrix}^T$ is transfer equations matrix; E is the output matrix.

Spatial transformation of formulas (13): $\delta = \tilde{E}\xi(k,t)$, $\tilde{E} = \begin{bmatrix} E(t) & F(t) \\ 0 & I_m \end{bmatrix}$.

Construction of Model Predictive controller

Considering the complexity of driving process, when designing objective function, the relaxation factor is introduced, and the tracking objective function expressed:

$$\begin{aligned} & J\left(\xi(t), u(t-1), \Delta U(t)\right) \\ & = \sum_{i=1}^{N_p} \left\| \Delta\eta(t+i|t) \right\|_Q^2 + \sum_{i=1}^{N_c-1} \left\| \Delta u(t+i|t) \right\|_R^2 + \rho\varepsilon^2 \end{aligned} \tag{15}$$

where $\Delta\eta(t+i|t) = \eta(t+i|t) - \eta_r(t+i|t)$ is the actual driving and reference path difference, ρ is weight coefficient, ε is relaxation factor, Q and R are weighting matrix.

Based on comprehensive constraints and objective function, the smart car path control based on the dynamic can be transformed into an optimization problem, that is, each cycle needs to solve:

$$\min \Delta U, \varepsilon \sum_{i=1}^{P} \left\| \eta\left(k+i|k\right) - \eta\left(k+i|k\right) \right\|_Q^2 + \sum_{i=1}^{N} \left\| \Delta u\left(k+i|k\right) \right\|_R^2 + \rho\varepsilon^2$$

$$\begin{cases} \Delta U_{\min} \le \Delta U \le \Delta U_{\max} \\ \Delta U_{\min} \le A\Delta U + U_f \le \Delta U_{\max} \\ \delta_{\min} \le \delta \le \delta_{\max} \\ \varepsilon \ge 0 \end{cases} \tag{16}$$

3. The Main Ideal of Multi-Objective Bionic Game Theory

Barry Sinervo and Tim Stephens' research [12] [13] found that the male side spotted lizard's throat appears in three colors: blue, yellow, and orange, as shown in **Figure 2**; During the evolution of biological populations, different throat colors exhibit different behaviors. The male spotted lizard with yellow throat is weaker in physique than those with blue and orange throats, and its movements are secretive. When competing for sexual power with females, it often plays the role of an adulterer, commonly known as the "mistress". The male spotted lizard with orange throat has a robust physique, hot tempered and aggressive personality, large habitat, and numerous female partners. The male spotted lizard with blue throat has a physical fitness between the yellow and the orange. When faced with foreign enemy, it can form a defend group with its companions to defend its habitat. During the evolution of biological populations, each male spotted lizard with blue throat only has one female partner and follows the principle of monogamy. In the entire evolutionary biological populations, the male spotted lizard with orange throat often engages in wars with the male spotted lizard with blue throat in search of more female mating rights. the male spotted lizard with blue throat lizard selflessly fights against the invasion for partner and territory. The male spotted lizard with orange throat has wide territory and numerous mates, it can't fully consider territory and numerous mates. This provides an opportunity for the male spotted lizard with yellow throat lizard to launch a surprise attack. The male spotted lizard with yellow throat lizard will run to places beyond its reach and secretly mate with mates.

Figure 2. Three male side spotted lizard (orange, blue, and yellow).

During the biological evolution, Barry Sinervo and other scholars found that the populations of three lizards are mutually constrained. When the male spotted lizard with blue throat dominates in reproduction, the male spotted lizard with orange throat adds dramatically; When the male spotted lizard with orange throat has an advantage in reproduction, the male spotted lizard with yellow throat lizard's population adds dramatically, thereby suppressing the reproductive speed of the male spotted lizard with orange throat. When the male spotted lizard with yellow throat has an advantage in reproduction, the male spotted lizard with blue throat lizard's population adds dramatically, thereby suppressing the reproductive speed of the male spotted lizard with yellow throat. The three lizard species repeat in a cycle in this way, mutually constraining each other, in order to prevent the population of single throat lizards from reproducing too quickly.

Research results show that these three male side spotted lizards inherit their own behavior patterns and pass on this evolved behavior pattern to the next generation through gene combinations. Although the behavior of the three lizards with different throats during reproduction varies greatly, they ultimately achieved "fittest survival" in Darwin's theory of evolution, and achieved co-evolution.

A bio-mimetic lizard evolutionary algorithm is proposed based on the phenomenon of population reproduction and male side spotted lizard evolution. The algorithm's main idea is: ① Through calculating interaction factors, it established mapping factor indicators between design variables and objective functions, solving for the respective strategy spaces of each game participant, and ensuring that all game participants follow a common constraint protocol. ② The behavior and survival mechanism of side-blotched lizard were studied; Opportunism, egoism and collectivism were defined according to it's own color, and the three behaviors were considered as set by the corresponding player. ③ Based on the behavioral characteristics exhibited by each species during the evolutionary process, establish mapping relationships between their respective adaptation functions and three objective functions to evaluate the ability of three evolved lizard individuals to adapt to nature. ④ In the process of evolution, three male side spotted lizard evolve games with their respective fitness functions as the objective. ⑤ After each round of evolutionary games, the optimal genes of the three lizards can be solved, and new chromosomes can be constructed. Convergence criteria are used to determine convergence, and after multiple evolutionary iterations, the optimal chromosome, multi-objective solution, can be obtained.

4. Multi-Objective Bionic Design

In the construction of the evolutionary game algorithm for the three objective bio-mimetic male side spotted lizard, three core skills must be considered: ① Establishing a segmentation method for the chromosome genes of the male side spotted lizard population, as well as the mutual mapping relationship between design variables and subsets of design variables. ② Establish adaptive functions for three lizards. ③ Structure of evolutionary game algorithm for bio-mimetic lizards.

4.1. Chromosomal Genes Segmentation Method

For the tri-objective functions, each function has three types of male side spotted lizard bio-mimetic objects to select, namely male spotted lizard with orange throat lizard, male spotted lizard with yellow throat lizard, and male spotted lizard with blue throat lizard. Based on evolutionary game behavior characteristics of three male lizards, establish mutual mapping relationship between the adaptive and objective functions . Design variable subsets that can obtain tri-objective functions S_1, S_2, S_3 $S_1 \cup S_2 \cup S_3 = X$; $S_a \cap S_b = 0$ ($a,b = 1,2,3$; $a \neq b$).

4.2. Construction of Adaptation Functions for Bio-Mimetic Objects (Figure 3)

For tri-objective functions, each function has three types of male side spotted lizard bio-mimetic objects to choose, Based on evolutionary behavior characteristics, establish the mutual mapping relationship adaptive and objective functions.

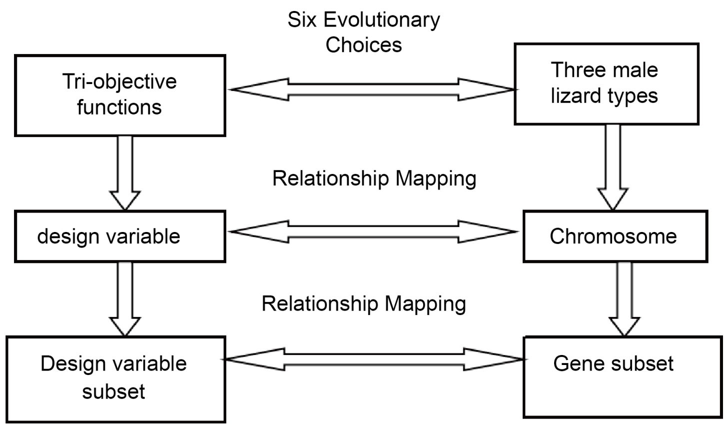

Figure 3. Correspondence between Objective Function and Chromosomes of Three Lizards.

Bio-mimicry of male spotted lizard with orange throat lizard-Oligopolism.

Compared with other species,male spotted lizard with orange throat lizard has a more aggressive personality and is like to conflict and competitive with other lizard subspecies. It exhibits a typical selfish, oligopolism, and exclusive characteristics during evolution. Therefore, this paper biometric male spotted lizard with orange throat lizard as an oligopoly, and constructs. The adaptive function is:

$$u_i = f_i \tag{17}$$

Bio-mimicry of male spotted lizard with blue throat lizard—Cooperativism.

For monogamous nature of male spotted lizard with blue throat lizard, it is able to form alliances and partnerships with other similar lizards when invasive species invade. It has a gentle personality and does not actively engage in conflicts with other subspecies of lizards, demonstrating typical cooperative behavior during evolution. The species collectivist behavior not only pursues maximum self-interest, but also does not harm the other species benefits. The fitness function reflects the collective interests during evolution, and the function construction expressed:

$$u_i = w_{ii}\frac{f_i}{\overline{f}_i} + \sum_{j=1(j\neq i)}^{3} w_{ij}\frac{f_j}{\overline{f}_j}, \quad \sum_{j=1}^{3} w_{ij} = 1 \tag{18}$$

In the formula, $\overline{f}$ is a standardized reference value adopted to avoid differences in weight levels among various objective functions, usually taken as function's initial value, or as function value during loop iteration. w_{ii} defined as selfish factors and $w_{ij}(j\neq i)$ altruistic factors, w_{ii} directly reflects degree of cooperative behavior. The value smaller, the cooperative higher.

Bio-mimetic study of male spotted lizard with yellow throat lizard—Opportunism.

The male spotted lizard with yellow throat lizard has a mysterious and uncooperative behavior; Even when the other species compete mating rights, the male spotted lizard with yellow throat lizard will not take advantage of attacking losing side, but quietly leave to unguarded areas and secretly mate with female lizards. Therefore, this species exhibits opportunism in evolution. However, this opportunism is done without harming other male lizards, only achieve individual needs. The article take this behavioral as opportunism, and its adaptive function expressed:

$$u_i = f_i, \quad f_j \le \overline{f}_j, \quad j = 1,2,3, \quad j \neq i \tag{19}$$

4.3. The Structure of Bio-Mimetic Lizards Evolutionary Algorithm

This paper gives a bio-mimetic lizard evolutionary algorithm based on tri-male spotted lizards in their reproductive evolution. The algorithm is constructed as follows:

(1) Firstly, analyze and calculate the impact factor on tri-objective functions, establish initial samples and features using fuzzy clustering, and establish design variables subset using fuzzy clustering.

(2) Secondly, analyze and calculate corresponding relationships and tri-objective functions (All in $P_3^3 = 6$ choices). Construct tri-objective adaptive functions based on evolutionary behavior, establish map relation between design variables and chromosome space, and map relation between design variable subsets and corresponding gene segments.

(3) Randomly generate initial chromosome vectors in chromosome gene space $\boldsymbol{s}^{(0)} = \left(\boldsymbol{s}_1^{(0)} \quad \boldsymbol{s}_2^{(0)} \quad \boldsymbol{s}_3^{(0)}\right)$, and encode the chromosome vector body according to the encoding method in genetic algorithm, requiring a one-to-one mapping relationship between chromosomes and individual design variables.

(4) Note $\overline{\boldsymbol{s}}_1^{(0)}, \overline{\boldsymbol{s}}_2^{(0)}, \overline{\boldsymbol{s}}_3^{(0)}$ as corresponding complements, $\boldsymbol{s}_1^{(0)}, \boldsymbol{s}_2^{(0)}, \boldsymbol{s}_3^{(0)}$ in $\boldsymbol{s}^{(0)}$, three lizards types engage in a cyclic evolutionary game.

The flow chart of lizard evolutionary algorithm proposed is shown in Figure 4. If design variables and goals few, $\boldsymbol{X}$ (design variables set)can be divided as corresponding strategy for each game participant $\boldsymbol{S}_1, \cdots, \boldsymbol{S}_m$, based on the size of the influencing factors. If there are many design variables or objectives, during fuzzy clustering analysis, similar variables or identical properties grouped together based on characteristics, which could reduce computing complexity and complexity.

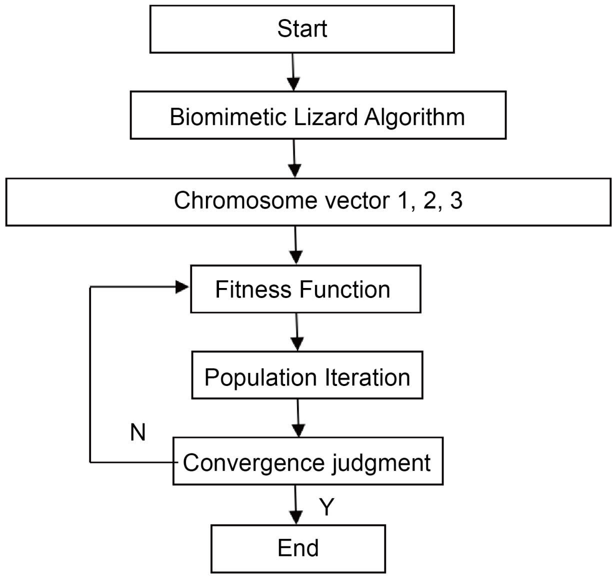

Figure 4. Flow chart of bio-mimetic lizard evolutionary game algorithm.

5. Calculation Results and Experimental Analysis

To verify the application of multi-objective bionic game in smart car path tracking, the author combined Fortran and MATLAB software to simulate, and the smart car parameters were shown in Table 1.

Table 1. Smart car parameters.

Description	symbol	Value
Total weight /kg	m	1300
Front wheel's lateral stiffness /N·rad^{-1}	C_f	90000
Rear wheel's lateral stiffness /N·rad^{-1}	C_r	90000
The car's moment inertia about the Z axis /kg·m^2	I_z	1627
The distance from center to front axle /m	a	1
The distance from center to rear axle /m	b	1.45
Wheel radius /m	R	0.35
Wheel rotational inertia /kg·m^2	I_w	2.1
Front wheel width /m	L_f	1.436
Rear wheel width /m	L_r	1.436

5.1. Comparison and Analysis of Calculation Results

In this paper, the double shift trajectory at three kinds of speed of 30 km/h, 60 km/h and 80 km/h is tracked and verified, the N_p is set to 20, the N_c is set to 5, and simulation results were shown in **Figures 5-10**.

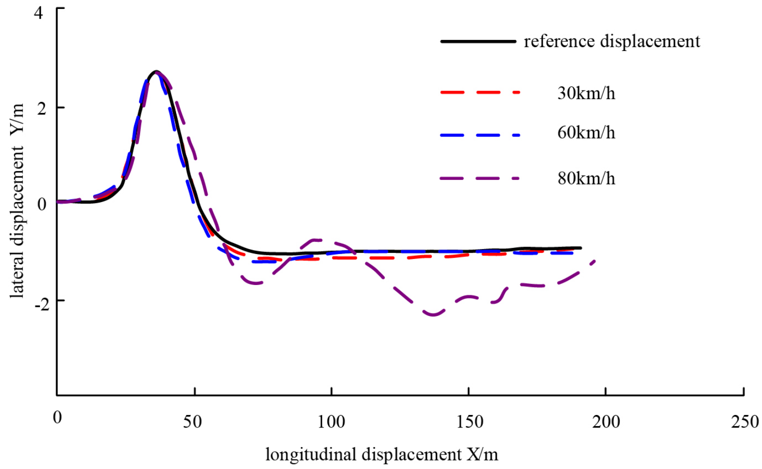

Figure 5. Actual lateral displacement.

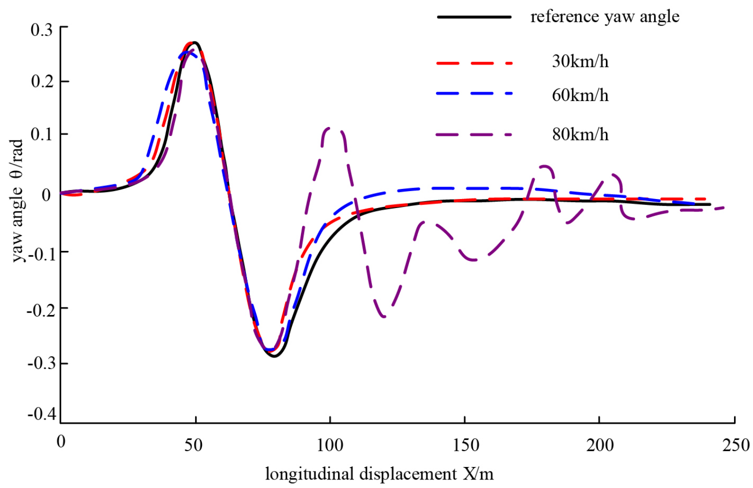

Figure 6. Yaw angle.

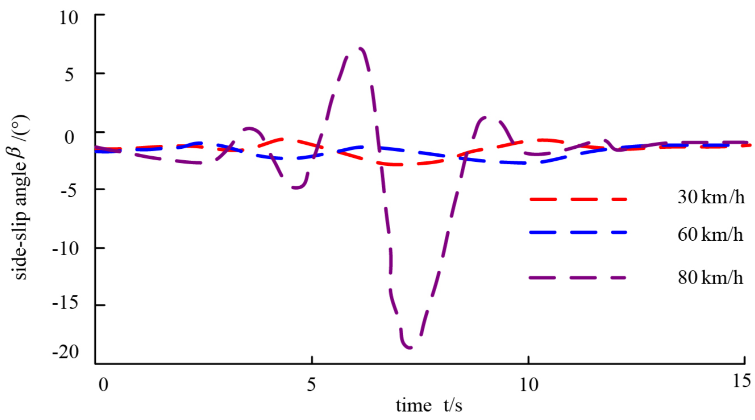

Figure 7. Side-slip angle.

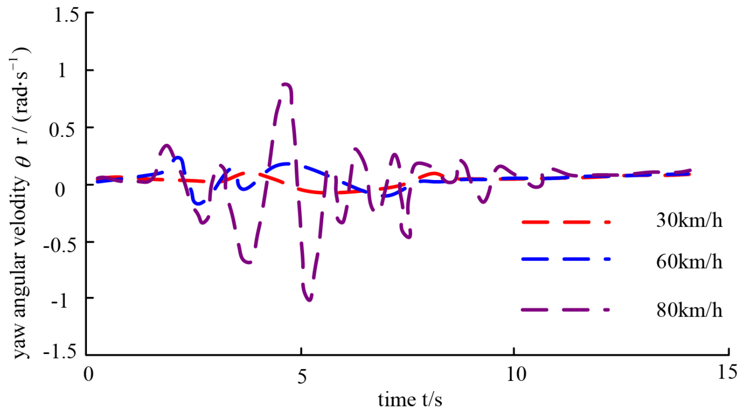

Figure 8. Yaw angular velocity.

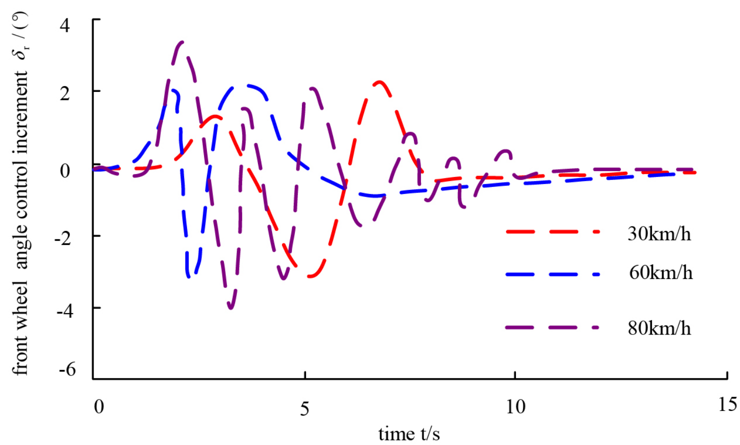

Figure 9. Front wheel angle control.

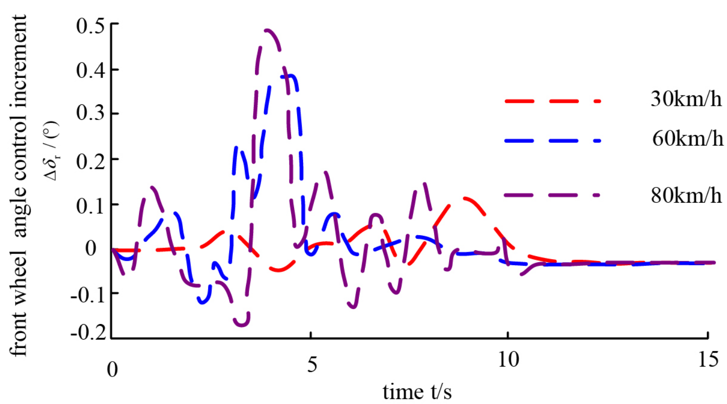

Figure 10. Front wheel angle control increment.

From the simulation results **Figures 5-10**, under low speed, the bio-mimetic lizards evolutionary algorithm can realize the tracking reference path well, the transverse pendulum angle can track the expectation well, the variation of slip-angle, the transverse pendulum angle velocity, the front wheel angle control amount and the incremental control are all in the controllable range. When speed achieves 80km/h, path tracking shows a large deviation. Once speed increases, the lateral acceleration also increases, which results in tire lateral deflection nonlinear, in the following research, the yaw force can be introduced to control the yaw angular velocity of cars, in order to achieve high-speed tracking.

5.2. Comparative Analysis of Car Experiments

Through partial replacement of the experimental retrofit car and body quality adjustment, the asphalt tarmac surface is used as the test pavement, using the CA-YD-103 model acceleration sensor, the speed meter (as shown in Figure 11), the VG400CC-200 all-solid-state vertical gyroscope (as shown in Figure 12), The data is obtained through the WAVEBOOK/512H high-speed portable data acquisition system of American QUATRONIX company, and the data acquisition device is shown in Figure 13, and the platform hardware controller (ECU) integrates sensors, racks and other related equipment through the LMS SCADAS multi-channel data acquisition system. Using CAN bus to debug and compile the program online.

Figure 11. Car speed meter.

Figure 12. VG gyroscope.

Figure 13. Data acquisition device WAVEBOOK.

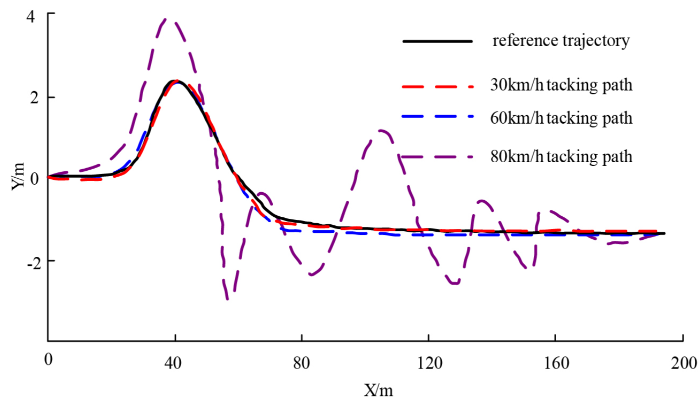

Figure 14. The comparison of car trajectory.

Figure 13 shows the driving trajectory of the test cars at different speeds, as **Figure 14** shows, when speed under 30 km/h and 60 km/h, the controller can achieve the target path tracking very well, when driving at 80 km/h speed, due to large speed caused by the phenomenon of side-slip, but in the end can still drive smoothly, The driving status is controllable.

6. Conclusion

To improve car's driver-less performance, avoid the side-slip of cars in target path tracking, using the four-wheel car model after linearization as the tracking control model, a method of smart car target path tracking based on multi-objective bionic game theory is proposed. Combining the constraint relationship between front wheel deflection angle, side deflection increment constraint and tire side deflection dynamics, taking the path tracking state model in time domain as the objective function, the competitive cooperative game method is used to simulate, and the results show the validity of the algorithm. Combined with the experimental modification car, the experimental results show that the method can track the smart cars quickly and stably, the distance deviation and azimuth deviation are in a reasonable range with a good real-time performance, which provides a new research for the target path tracking of smart cars.

Acknowledgements

Project was supported by Anhui Provincial Department of Science and Technology Project (202423a11020012), Major Natural Science Research Project of Anhui Provincial Universities (Grant No.2023AH040300), Anhui Province University Science and Engineering Teachers' Internship Program in Enterprises (Grant No.2024jsqygz90), the "Discipline Construction Quality Improvement Project" of Chaohu University (kj22zkfp01) and the Research Start-up Fund Project of Chaohu University (KYQD-2023065).

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Ren, D., Zhang, J., Zhang, J. and Cui, S. (2011) Trajectory Planning and Yaw Rate Tracking Control for Lane Changing of Intelligent Vehicle on Curved Road. *Science China Technological Sciences*, **54**, 630-642. https://doi.org/10.1007/s11431-010-4227-6

[2] Duan, J.M., Yang, C. and Shi, H. (2016) Path Tracking Based on Pure Pursuit Algorithm for Smart Cars. *Journal of Beijing University of Technology*, **42**, 1301-1306.

[3] Akeb, H. and Hifi, M. (2013) Solving the Circular Open Dimension Problem by Using Separate Beams and Look-Ahead Strategies. *Computers & Operations Research*, **40**, 1243-1255. https://doi.org/10.1016/j.cor.2012.11.025

[4] Tan, B.C. and Wang, B. (2017) Path Following Incremental PID Control for Unmanned Car. *Journal of Xi'an Technological University*, No. 10, 44-48.

[5] Wang, Y., Cai, Y.F., Chen, L., *et al*. (2017) Design of Smart Car Path Tracking Controller Based on Model Predictive Control. *Automobile Technology*, No. 10, 44-48.

[6] Yin, X.L., Li, L. and Jia, X.C. (2011) Predictive Fuzzy Control of Unmanned car Path Tracking. *Journal of North University of China* (*Natural Science Edition*), **32**, 135-138.

[7] Xie, N.G., Sun, L.S., Bao, J.H., *et al*. (2007) Muti-Objective Game Design of Arch Dam Shape. *Acta Mechanica Solid Sinica*, **28**, 200-206.

[8] Xie, N., Chen, Y.W., Sun, L.S., *et al*. (2008) Multi-Objective Bionics Design Method Based on Mixed-Behavior Game. *Chinese Journal of Theoretical and Applied Mechanics*, **40**, 229-237.

[9] Xie, N.G., Sun, L.S., Bao, J.H., *et al*. (2007) Multi-Objective Optimization Design of Arch Dam Shape Based on Unselfish Cooperation Game Model. *Journal of Hydraulic Engineering*, **38**, 1109-1114.

[10] Duan, J.M., Tian, X.S., Xia, T., *et al*. (2017) Research on Target Path Tracking Method of Smart Car Based on Model Predictive Control. *Automobile Technology*, No. 8, 6-11.

[11] Zhang, W., Yu, C.X., Teng, Y.B., *et al*. (2017) Research on UUV Path Tracking Control Based on Model Predictive Control. *Chinese Journal of Scientific Instrument*, **38**, 2659-2666.

[12] Sinervo, B., Chaine, A., Clobert, J., Calsbeek, R., Hazard, L., Lancaster, L., *et al*. (2006) Self-Recognition, Color Signals, and Cycles of Greenbeard Mutualism and Altruism. *Proceedings of the National Academy of Sciences*, **103**, 7372-7377. https://doi.org/10.1073/pnas.0510260103

[13] Stephens, T. (2006) Greenbeards with Blue Throats Shed New Light on the Evolution of Altruistic Behavior. http://currents.ucsc.edu/05-06/05-08/lizards.asp