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Probabilistic Assessment of PV-DG for Optimal Multi-Locations and Sizing Using Genetic Algorithm and Sequential-Time Power Flow

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Abstract

This paper presents an optimized strategy for multiple integrations of photovoltaic distributed generation (PV-DG) within radial distribution power systems. The proposed methodology focuses on identifying the optimal allocation and sizing of multiple PV-DG units to minimize power losses using a probabilistic PV model and time-series power flow analysis. Addressing the uncertainties in PV output due to weather variability and diurnal cycles is critical. A probabilistic assessment offers a more robust analysis of DG integration's impact on the grid, potentially leading to more reliable system planning. The presented approach employs a genetic algorithm (GA) and a determined PV output profile and probabilistic PV generation profile based on experimental measurements for one year of solar radiation in Cairo, Egypt. The proposed algorithms are validated using a co-simulation framework that integrates MATLAB and OpenDSS, enabling analysis on a 33-bus test system. This framework can act as a guideline for creating other co-simulation algorithms to enhance computing platforms for contemporary modern distribution systems within smart grids concept. The paper presents comparisons with previous research studies and various interesting findings such as the considered hours for developing the probabilistic model presents different results.

Keywords

Photovoltaic Distributed Generation, Probability, Genetic Algorithm, Radial Distribution Systems, Time Series Power Flow

1. Introduction

Since 2005, the term smart grid has gained significant attention. A "smart grid" is an electricity network that combines advanced communication, control, and automation technologies to enhance the power system's efficiency, reliability, and sustainability [1] [2].

Various methods have been devised in the past to identify the optimal allocation and sizing of photovoltaic distributed generation (PV-DG) systems. These approaches are generally classified into analytical techniques and optimization programming methods [3]. Analytical approaches were proposed by authors, such as in [4], where was proposed to identify the optimal placement of DG systems, aiming to minimize power losses in distribution and transmission networks. Theoretical analysis was conducted to determine the optimal allocation for installing DG on radial test systems with three distinct load distributions such as a uniform distribution, a central concentration, and a uniformly increasing distribution load. This analysis was carried out for case studies involving constant loads values over time and DG. Another case study was conducted in [4] involving varied load over time and DG.

Optimization algorithms were presented in various articles. In [5], the author presented GA and Particle Swarm Optimization (PSO) approaches for defining the optimal allocation and capacity of DG, considering multiple objective functions. In [6], the Ant Colony Search (ACS) algorithm is employed to find the optimal allocation of DG and reclosers, focusing on system reliability [7].

Most research in the field has focused on different types of DGs, assuming that the PV-DG generation profile remains unchanged. But in the experimental field, there are extremely complicated frequent fluctuations compared with a determined 24-hour profile. This study evaluates both deterministic and probabilistic multiple integration of PV-DG 33 bus distribution system, taking into account the substantial impact on reducing power losses and enhancing voltage profiles. The proposed framework comprises a novel stochastic analysis through the parameterization of the daily profile of PV based on one-year measurements of solar radiation in Cairo, Egypt. Finally, a comparative study with other research studies and the uncertainty in PV output over time is considered based on historical data.

The objective of this research is to introduce appropriate optimization frameworks for evaluating the optimal sizing and allocations of multiple PV-DG in a 33-bus radial distribution system toward employing a dynamic PV-DG model for more accurate results. Section 2 presents the background of the problem statement about minimizing the power loss in the radial distribution system and probabilistic model of PV-DG and finally GA algorithm. Section 3 introduces the proposed methodology and problem formulation. Section 4 presents the results and outcomes of this study. Finally, Section 5 presents the conclusions.

2. Problem Statement

The primary objective of integrating PV-DG is to reduce the overall power loss, which is the cumulative loss across all branches of the network. The overall power loss in power systems is expressed by Equation (1), which is commonly referred to as the “exact loss formula” [8]-[10].

$$P_L = \sum_{i=1}^{N}\sum_{j=1}^{N}\left[\alpha_{ij}\left(P_iP_j + Q_iQ_j\right) + \beta_{ij}\left(Q_iP_j - P_iQ_j\right)\right] \quad (1)$$

where $\alpha_{ij} = \frac{r_{ij}}{V_iV_j}\cos\left(\delta_i - \delta_j\right)$, $\beta_{ij} = \frac{r_{ij}}{V_iV_j}\sin\left(\delta_i - \delta_j\right)$, and $r_{ij} + jx_{ij} = Z_{ij}$ are the ijth element of [Zbus] matrix with [Zbus] = [Ybus]$^{-1}$.

P_i, Q_i represents real and reactive power of bus i and P_j, Q_j at bus j.

r_{ij} represents line resistance between buses i and j.

V_i, V_j defines voltage magnitude of buses i and j.

δ_i, δ_j represents voltage angle of buses i and j.

N is the overall number of buses.

Also, the real and reactive power are represented as [11]:

$$P_i = \sum_{j=1}^{nb}\left|Y_{ij}V_iV_j\right|\cos\left(\theta_{ij} + \delta_j - \delta_i\right) \quad (2)$$

$$Q_i = -\sum_{j=1}^{nb}\left|Y_{ij}V_iV_j\right|\sin\left(\theta_{ij} + \delta_j - \delta_i\right) \quad (3)$$

The main goal of implementing PV-DG is to minimize the power loss, which is the sum of losses in all the network branches. The optimization problem is centered on reducing this cumulative active power loss throughout the system as its objective function [12] [13].

Power flow equations can be solved for a radial distribution network using real power, reactive power, voltages at the sending and receiving ends of a branch. Where, P_i, Q_i, and V_i represent the real power, reactive power, and voltage at the sending end, respectively, while P_{i+1}, Q_{i+1}, and V_{i+1} represents the corresponding quantities at the receiving end. The following equation illustrates these relationships. The quadratic terms in the equations account for the losses on the branches. Therefore, the power loss on a branch is calculated as follows [14]:

$$Loss_i = \frac{r_i\left(P_i^2 + Q_i^2\right)}{V_i^2} \quad (4)$$

The overall system losses are determined by summing up the losses across all branches, In this method, the active and reactive power flow (P_i and Q_i) from bus i to bus $i+1$ is calculated as follows:

$$Total\_Loss = \sum_{i=0}^{n-1} Loss_i = \sum_{i=0}^{n-1}\frac{r_i\left(P_i^2 + Q_i^2\right)}{V_i^2} \quad (5)$$

This research presents an optimization approach aimed at reducing total power losses by using them as the objective function, while considering the time-dependent variations in PV-DG generation. Where, P_{losses} denotes the active power losses in the circuit for $t = 1$ to $t = 24$ hours.

$$f = \min\sum_{t=1}^{24} P_{losses_i} \quad (6)$$

Considering a basic distribution system consisting of two buses, a source, a load, and a DG interconnected by a transmission line, as illustrated in **Figure 1**. Then, the following equation can be derived.

$$P_i = P_{DGi} - P_{Di} \quad (7)$$

where, P_{DGi} represents the real power injected by the DG at node i, and P_{Di} denotes the load demand at node i. By combining (5) and (6) Equation (8) can be obtained [8].

$$P_{DGi} = P_{Di} + \frac{1}{\alpha_{ii}}\left[\beta_{ii}Q_i - \sum_{j=1, j\neq i}^{N}\left(\alpha_{ij}P_j - \beta_{ij}Q_j\right)\right] \tag{8}$$

The equation above determines the optimal DG size for minimizing losses at each bus$_i$. Any other DG size installed at bus ii would lead to increased losses [8].

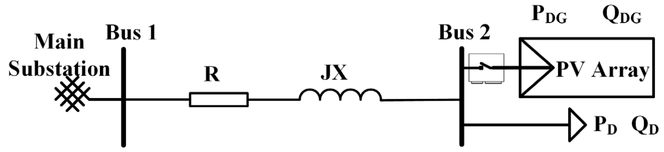

Figure 1. Single line diagram of the 2-bus system.

DG systems can typically be categorized into various types based on their capability to produce or consume active and reactive power. These include: systems that generate either active power or reactive power exclusively; systems capable of generating both active and reactive power; and systems that generate active power while consuming reactive power [15] [16].

The reactive power output of the DG is represented by (9) [10].

$$Q_{DGi} = aP_{DGi} \tag{9}$$

where, $a = \tan\left(\cos^{-1}\left(PF_{DGi}\right)\right)$, and PF_{DG} denotes the power factor of the DG. The reactive power injected at bus ii, is described by (10), respectively [10],

$$Q_i = Q_{DGi} - Q_{Di} = aP_{DGi} - Q_{Di} \tag{10}$$

2.1. Solar PV-DG Probabilistic Modelling

The implementation of PV-DGs into the power system largely effects the power flow, power quality, and dynamic performance of the test system. Since the output of PV-DG is affected by real-time changing of solar radiation values, traditional deterministic power flow methods are insufficient for accurately capturing and assessing these effects [17]. So, approaches for computing probabilistic load flow such as Monte Carlo simulation method are required. The authors in [18] presented the probabilistic density function (PDF) following the integration of PV-DG into the grid. A probabilistic power flow methodology using the stochastic formation approach was proposed in [19], using the principles of uncertainty quantification concept. The Beta probability distribution functions used to estimate hourly solar irradiance are derived from three years of historical data collected from the study site [20]. This data is then used to create a representative frequency distribution of irradiance and wind speed measurements for a typical day in each season.

The research in [17] explored the integration capacity of large-scale PV systems into the grid, PV connection points, and the output of multiple PV power plants using techniques such as probability density distribution, sensitivity evaluation, and over-limit probability assessment. In summary, the research investigated how large-scale PV systems can be connected to the grid and how their output fluctuates and impacts grid performance using statistical and analytical methods. The researchers of the PV probability model in [21] and [17] stated that the solar radiation follows the Beta distribution and the normal distribution. As a result, the probability density function for solar radiation (G) can be presented as in Equation (11):

$$f(G)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\left(\frac{G}{G_{\max}}\right)^{\alpha-1}\left(1-\frac{G}{G_{\max}}\right)^{\beta-1} \tag{11}$$

where $G_{\max}$ is the maximum solar radiation value, and α, β are the Beta distribution fitting parameters.

Statistical models are crucial for accurately capturing the variability in PV-DG outputs. These models, represented by PDFs, enable the creation of random variables required for Monte Carlo simulations (MCS). A sequential Monte Carlo simulation was used to assess the distribution system's probabilistic three-phase power flow. This simulation incorporates statistical data on solar resources, advancing through time in a step-by-step manner to account for the time-dependent characteristics of variables [22].

Figure 2 shows the complete diagram of the model that combines the PV module with the inverter. The model parameters are specified at an irradiance level of 1 kW/m^2, making it particularly accurate at higher power output levels. When a given irradiance level is input, the panel's output is adjusted by a factor that depends on the PV module temperature [22]. The following equation was used in [22] to calculate the output power of the PV-DG based on solar radiation and PV module temperature values.

$$P_{pv}=P_{mpp}f_t(T_c)G. \tag{12}$$

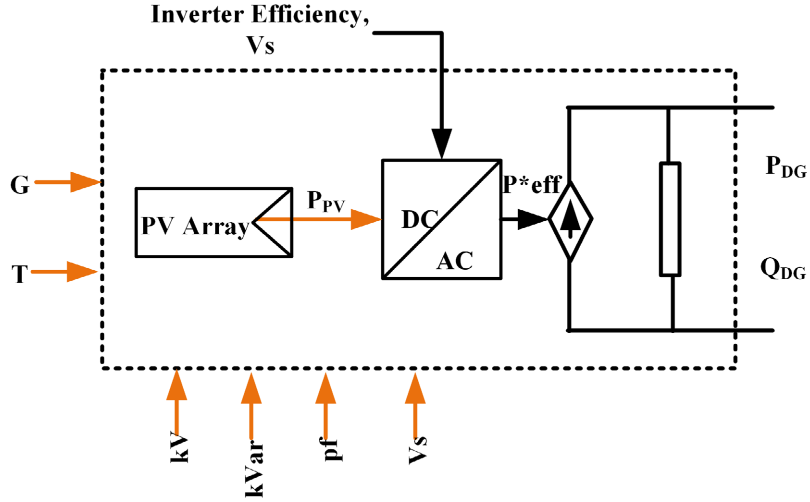

Figure 2. General diagram for PV model for Co-simulation.

2.2. Genetic Algorithm

The core goal is to find the optimal allocation sizing of PV-DG systems in the test system. The losses are the key factor, deeply impacting both economic and technical performance. To achieve the optimal integration of PV-DG units must be strategically positioned to minimize power losses across the system without breaching voltage thresholds, as highlighted in [23]. Consequently, the main objective function to be minimized is defined in Equation (13).

$$\text{Minimize } P_{Loss} = \sum_{i=1}^{N} P_{Li} \tag{13}$$

where, P_{Li} indicates the i^{th} line losses, while N signifies the total count of lines in the test system.

To enhance the voltage profile as a secondary objective in the radial power system, the aim is to minimize the total voltage deviations at the load buses. This objective function is typically formulated mathematically and expressed as follows [23]:

$$\text{Minimize } \sum_{i=1}^{N} \left| V_i - V_{ref} \right| \tag{14}$$

In this context, V_i denotes the voltage at load bus i, while V_{ref} represents the reference voltage at load bus i, typically set to 1.0 per unit (pu).

Strategic deployment of PV-DG units and distribution static compensators (DSTATCOM) in a test improves the voltage profile significantly. This optimized configuration supports efficient provision of real and reactive power, thereby minimizing power losses and strengthening voltage stability. The Total Voltage Variation (TVD) across the network is formulated as [24]:

$$TVD = \begin{cases} 0, \text{ if } 0.95 \le V_t \le 1.05 \\ \sum_{t=1}^{N} \left| V_{ref} - V_t \right|, \text{ else} \end{cases} \tag{15}$$

A multi-objective function combines multiple goals to be optimized simultaneously, all within predefined operating constraints. This multi-objective function, described in [24], was designed to minimize losses while also enhancing the voltage profile and maximizing the voltage stability index. The mathematical formulation for identifying the optimal allocation of DG and DSTATCOM was as follows:

$$\text{Minimize}\left(F\right) = \text{Min}\left(\beta_1 \Delta P_{TL}^{\frac{DG}{DST}} + \beta_2 \Delta TVD^{\frac{DG}{DST}} + \beta_3 \left(\frac{1}{\Delta VSI^{\frac{DG}{DST}}} \right) \right) \tag{16}$$

where, β_1, β_2 and β_3 represent the weighting factors associated with minimizing power loss, minimizing TVD, and maximizing voltage stability index (VSI), respectively.

3. Methodology and Problem Formulation

The suggested method is excuted using MATLAB within a co-simulation framework with OpenDSS. Initial simulations utilized a PV-DG production curve, and for each power flow solution computed with OpenDSS, the GA minimized the fitness function. This approach allows for sequential-time power flow simulations, essential for examining smart grid challenges arising from the integration of renewable resources, energy storage, and electric vehicles, which modify load profiles. Precise time-based modeling of system behavior is vital to achieving reliable and accurate results.

This section outlines a method for identifying the optimal sizing and allocation of multiple PV-DG units within a 33-bus test system. The approach employs a stochastic load flow analysis with integrated PV-DG units to derive the final solution. Also, PV probability model is proposed using the solar radiation measurements based on the Beta distribution These models, represented by PDFs, allow for the generation of random variables necessary for Monte Carlo Simulations (MCS). A 33-bus radial distribution test system serves as the test model. In this study, buses with PV-DG units are treated as PQ buses, operating at a unity power factor and a 0.95 power factor.

The proposed approach comprises three main parts, the first one handles GA optimization, loading solar radiation data, calculating Beta distribution parameters, and optimizing the allocation and sizing of PV-DG in the test system. The fitness function runs the optimization using OpenDSS, performing power flow simulations, adjusting the PV-DG generator output based on probabilistic irradiance, and calculating various metrics such as power losses and voltage deviations. Finally, Quadratic Voltage Deviation function calculates the quadratic deviation of node voltages from acceptable voltage limits. Figure 3 presents a flow chart for the proposed approach according to the following steps:

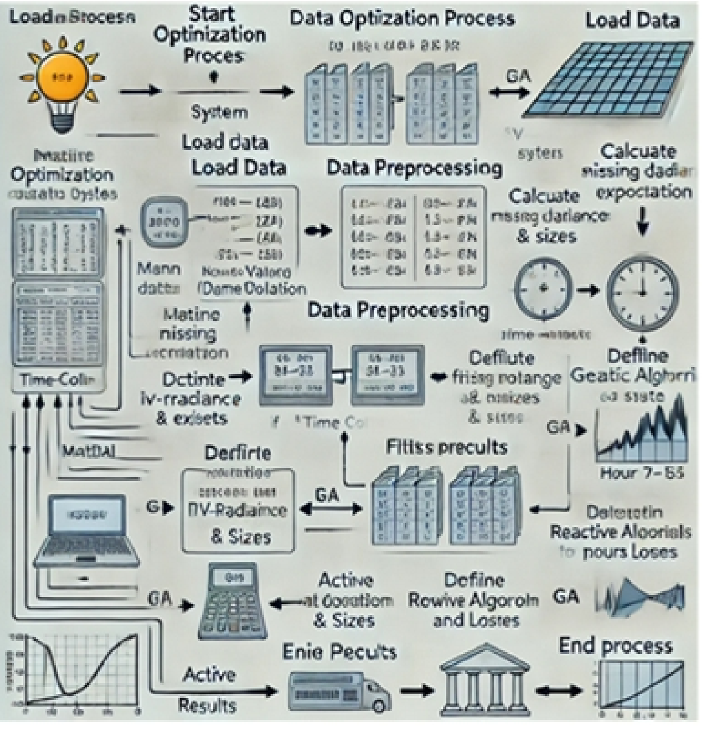

Figure 3. Flow chart of the proposed approach.

- Loading solar radiation data from an Excel file and interpolating Missing Data
- Calculating Beta Parameters: initializes a matrix to store the Beta distribution parameters for each hour.
- Define and Set Up GA: parameters are configured: Number of variables, Constraints (e.g., bounds for DG placement, power factor, and size). GA-specific options like population size and fitness plotting.
- Integrate OpenDSS: OpenDSS is used to simulate the 33-bus test system data.
- Loop for Each Hour: adjust PV-DG output based on probabilistic irradiance. Solve the power flow.
- The fitness function evaluates system performance based on these metrics.
- Display Results.

3.1. Proposed Probabilistic Modelling of Solar Radiation and PV-DG

A year's worth of hourly solar radiation data (in W/m^2) measured with a PV generation unit is used to create a dataset for a probabilistic PV solar radiation model. The data is collected using a Solar-Log Base100 system, a sensor box equipped with a reference cell for measuring solar radiation. To ensure consistency, the data was recorded as a time series with 5-minute interval at local standard time for every day of the year. This dataset, referred to as the "reference solar irradiance data," is displayed in Figure 4 and initially recorded as a time series with 5-minute intervals for the entire year. The analysis begins by importing the data from Excel into MATLAB, where missing values are identified and filled through interpolation. After completing this step, the data is preprocessed to calculate the average and variance of solar irradiance, which is then used to estimate the output of the PV system.

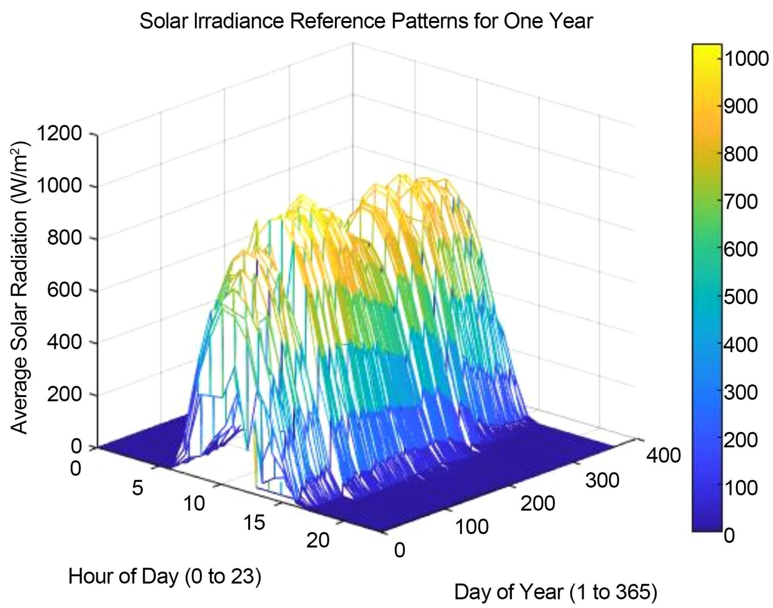

Figure 4. Initially recorded as a time series with 5-minute intervals.

The proposed approach implements a comprehensive approach to analyze solar radiation data using statistical methods and probabilistic modeling. It employs concepts from the Beta distribution, seasonal statistics, and linear modeling of PV output, which are critical in energy management and forecasting in photovoltaic systems.

The mean (μ) and variance (σ^2) of the solar radiation data are calculated as:

$$\mu = \frac{1}{n}\sum_{i=1}^{n} G_i \tag{17}$$

$$\sigma^2 = \frac{1}{n-1}\sum_{i-1}^{n}\left(G_i - \mu\right)^2 \tag{18}$$

where G_i represents the solar radiation data points.

The Beta distribution is parameterized by shape parameters α\alpha$\alpha$ and β\beta$\beta$, which are calculated from the mean and variance of the data as follows:

$$\beta = \frac{(1-\mu)\mu}{\sigma^2(1+\mu)} \tag{19}$$

$$\alpha = \frac{\beta\mu}{1-\mu} \tag{20}$$

The probabilistic model for solar irradiance is based on Beta distribution. Which is a continuous range of probabilities outlined by the range [0, 1], making it suitable for modelling proportion such as solar irradiance. The probability density function (PDF) of the Beta distribution is defined as:

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 \le x \le 1 \tag{21}$$

where Γ\GammaΓ is the gamma function, which generates the factorial function.

x is the normalized irradiance (between 0 and 1).

α and β are the shape parameters of the beta distribution.

Random samples are generated from the Beta distribution using the betarnd function in MATLAB, which follows the Beta distribution:

$$X \sim Beta(\alpha, \beta) \tag{22}$$

The PV output is modelled as a linear function of solar radiation:

$$P_{PV} = \eta \cdot G \tag{23}$$

where, P_{PV} is the PV output.

η is the efficiency of the PV system (15% in the script).

G is the solar irradiance.

The probability of solar radiation exceeding a specified threshold (G_{th}):

$$P(G > G_{th}) = \frac{1}{n}\sum_{i=1}^{n} 1(G_i > G_{th}) \tag{24}$$

where I, is a reflection symbol that returns one if the condition is true and zero otherwise.

The proposed method generates a normalized irradiance profile over a 24-hour period using the Beta PDF:

$$MyIrrad(t) = f(x_t; \alpha, \beta) \tag{25}$$

where x_t is a normalized time point for each hour (from 0 to 1).

The model uses the beta distribution to probabilistically represent the solar irradiance over a day. It adjusts the PV output based on the generated irradiance profile, which reflects varying conditions throughout the day.

3.2. Genetic Algorithm Formulation

When minimizing power losses is the only goal, an overproduction of active power from PV-DG can cause voltage levels to exceed permissible limits. To counteract this, Reactive Losses are introduced as a secondary objective as in Equation (27), and the Cumulative Voltage Deviation (CVD) as presented in Equation (28) is considered as a tertiary objective. To streamline the technique and procedure, a restriction function is incorporated in the approach as a Quadratic Penalty Factor (QPF), as outlined in Equation (29). Minimizing this factor then becomes the fourth objective. In this approach, all variables are calculated for 24 hours based on the probability of solar radiation

$$f_1 = \min \sum_{t=1}^{24} \sum_{i=1}^{N} P_{Li} \tag{26}$$

$$f_2 = \sum_{i=1}^{N} Q_{Li} \tag{27}$$

$$f_3 = CVD = \sum_{i=1}^{N} \left| V_i - V_{rated} \right| / N \tag{28}$$

$$f_4 = QPF = \begin{cases} \left(V_i - V_{\min}\right)^2 & V_i \geq V_{\min} \\ 0 & V_{\min} \leq V_i \leq V_{\max} \\ \left(V_i - V_{\max}\right)^2 & V_i \leq V_{\max} \end{cases} \tag{29}$$

where; Q_{Li} denotes the reactive power loss at the at i^{th} line, and N represents all lines considered in the test system.

V_i represents the voltage at the i^{th} bus. While, V_{rated} is the rated voltage for the test system, typically set to 1 pu. V_{lim} refers to the voltage limits, either minimum or maximum, for the system.

Equation (30) represents the comprehensive objective function that must be minimized.

$$f = \min \sum_{i=1}^{24} \left(\omega_1 f_{1i} + \omega_2 f_{2i} + \omega_3 f_{3i} + \omega_4 f_{4i} \right) \tag{30}$$

Each factor of ω is assigned a unique weight, and the summition of all these weights must equal 1.

The GA begins by generating an initial population, which includes the PV-DG locations, active power, and power factor, then examines the objective function using the power flow solver in OpenDSS, considering the full distribution system structure. The fitness function can be adjusted for improved outcomes, depending on whether the goal is to enhance the voltage profile or reduce losses. In this study, various weight combinations for each component of Equation (28) were tested through trial and error to determine the optimal configuration. The weights applied are 0.5 for P_{Loss}, 0.1 for Q_{Loss}, 0.2 for CVD, and 0.2 for the voltage limits.

3.3. The Power Flow Balance and Generation Equations

The constraints of the optimization methods are essential for accurately modeling the active and reactive power flows within the test system, as outlined in Equations (31) and (32) [12].

$$P_{Gi} = P_{Di} + \sum_{j=1}^{nb} |V_i| \left| V_j \left[G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij} \right] \right| \tag{31}$$

$$Q_{Gi} = Q_{Di} + \sum_{j=1}^{nb} |V_i| \left| V_j \left[G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij} \right] \right| \tag{32}$$

where V_i and V_j denote the voltage magnitudes at buses, P_{Gi} and refers to the active power generated at bus i, while P_{Di} indicates the active power demand at that same bus. Similarly, Q_{Gi} represents the reactive power generated at bus i, and Q_{Di} represents the reactive power demand. The terms G_{ij} and B_{ij} are the conductance and susceptance values of the line connecting buses i and j, which characterize the line's capacity to conduct and store electrical power, respectively.

The constraints set for the optimization include voltage limits across all buses within the network, along with specific operational limits for the PV-DG units. Equation (33) defines the upper and lower voltage boundaries, indicating the allowable range for voltage levels to ensure stable system operation.

$$V_{min} \le V_i \le V_{max} \tag{33}$$

where $V_{\min}$ and $V_{\max}$ denote the minimum and maximum permissible voltage magnitudes, respectively.

Determining the optimal size of PV-DG units involves adhering to specific operational limits, defined by the minimum $PVDG_{(\min)}$ and maximum $PVDG_{(\max)}$ boundaries, as outlined in Equation (24).

4. Numerical Results and Evaluation

In this study, the widely recognized 33-bus test system is chosen as the test model. Comprehensive network information, including line resistance and reactance, as well as the loads associated with each node, can be found in sources [25] [26]. **Figure 5** shows the schematic representation of the 33-bus radial test system. The proposed approach is evaluated using the 33-bus radial test network, which has an overall load of 3.72 MW and 2.3 MVAr [8] [16]. The base voltage for the 33-bus system is 12.66 kV, with a base apparent power of 10 MVA.

In this study, comprehensive network data—including line resistances, reactances, and node-connected loads—can be found in source [25]. The total load for the 33-bus test system is 3.72 MW and 2.3 MVar, as noted in [8] and [16]. The system utilises a base voltage of 12.66 kV and apparent power of 10 MVA. The overall load is 3.715 + j2.3 MVA, in [27]. For additional reference, source [28] uses base values of 100 MVA and 12.66 kV, with total real and reactive power loads listed as 3715 kW and 2300 kvar, respectively.

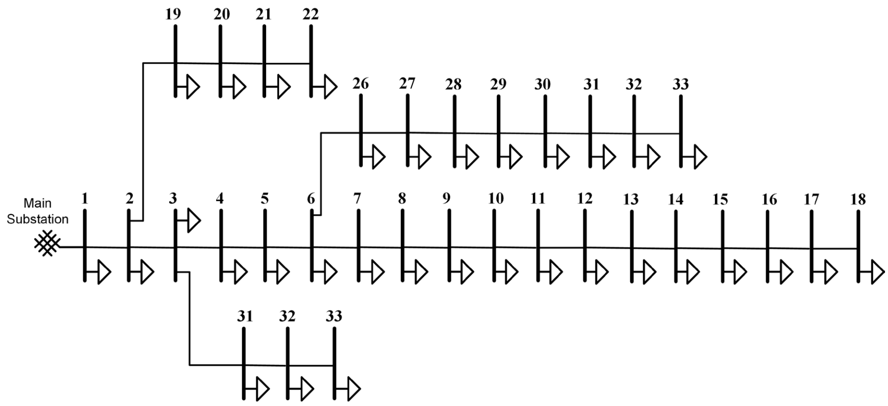

Figure 5. Schematic representation of the 33-bus radial test system.

4.1. Base Case Study

The load flow results for the base scenario, where no DG units are integrated, are detailed in **Table 1** for the 33-bus test system. **Table 1** provides a comprehensive comparison, presenting active and reactive power losses, minimum bus voltage levels, and cumulative active and reactive power metrics from this study as well as from comparable research sources. In this investigation, the active power loss for the system is determined to be 211.5 kW, with the lowest observed voltage being 0.9022 per unit at bus 18.

To validate the proposed algorithm's performance, its results are benchmarked against those from prior studies. In particular, study [24] reports an active power loss of 210.98 kW, with a minimum voltage of 0.9037 per unit. Additionally, the findings in this research and [28] indicate a base case real power loss of 210.98 kW and a reactive power loss of 143.13 kvar. In contrast, findings from [29] reported slightly lower losses, measuring 201.99 kW for real power and 134.77 kvar for reactive power. The worst or minimum voltage is 0.91337 pu at the 18th bus in the base case. This discrepancy arises because researchers in [24], [28] used resistance and reactance values of 1.7114 Ω and 1.2351 Ω for the 7th line, while studies in [29], [26] reported these values as 0.7114 Ω and 0.2351 Ω, as clarified in [28] with more references for this difference.

Table 1. Results for IEEE 33-bus test system.

Without DG	This research	[28]	[16]	[29]	[8]
real power loss (kW)	211.5	210.98	211.20	201.99	211.20
reactive power loss (kvar)	138.6	143.13	4.1259	134.77	-
Minimum voltage (p.u.) @bus	0.9022 @18	0.9038@18	0.9037 @18	0.91337 @18	
Pi/p (kW)	3715	3715	3700	3715	3720
Qi/p (kvar)	2300	2300	2300	2300	2300

4.2. Constant PV-DG Peak Power

In the first case, where a constant PV-DG production is considered, A comparative analysis are presented with other research studies using loss sensitivity factor (LSF), Exhaustive load flow (ELF), and GA methods, in case of unity power factor of PV-DG as in **Table 2**. **Table 2** provides a summary of results, including the optimal DG placement, corresponding optimal DG size, and the total power loss within the test system. The findings indicate that placing multiple DG units of optimal size at the ideal location results in a substantial reduction in power losses.

The optimal placement of a single PV-DG is at bus 6 with a size of 2600 kW, resulting in a system loss of 111.052 kW compared with 2601 kW and 111.1 kW for size and losses, respectively in [16]. While GA presented 2835 kW for size

Also, for the first case, a comparison of GA with other research studies for two cases of unity and 095 lag power factor of PV-DG are presented in **Table 3**. As discussed earlier, placing Type 1 DG compensates for active currents within the distribution lines, while Type 2 DG addresses reactive currents. This setup enables compensation for local active and reactive loads at the bus level, leading to a reduction in real power losses attributed to both the active and reactive components of branch currents. The proposed algorithm efficiently determines both the optimal locations and sizes of DG in a single step, unlike a two-stage approach. Here, installing one DG implies the combined installation of one Type 1 DG and one Type 2 (capacitor) DG. Detailed DG location findings are provided in **Table 3**.

At unity pf for 1 PV-DG Best Active Power Losses: 111082.136198 Best Reactive Power Losses: 76589.923719 k Var. At unity pf for 2 PV-DG the Active Power Losses: 87147.304554 Best Reactive Power Losses: 54536.497028. At unity pf for 3 PV-DG the Active Power Losses: 72.7142 kW and the Reactive Power Losses: 45.3521 kvar.

At 0.95 pf for 1 PV-DG the Best Active Power Losses: 78234.756217 Best Reactive Power Losses: 56115.774219. At 0.95 pf for 2 PV-DG the Best Active Power Losses: 45182.094978 Best Reactive Power Losses: 26406.109248. At 0.95 pf for 3 PV-DG the active Power Losses: 28.4468 kW and Best Reactive Power Losses: 15.7403 kvar.

Table 2. DG allocation and sizing using different techniques for 33-bus test system.

Cases	Techniques		[16]		Loss (kW)	This research		Loss (kW)
No DG					211.2			
1 DG	LSF	Bus	18		146.82	18		145.9
		Size	743			813.6		
	ELF	Bus	6		111.1	6		111.052
		Size	2601			2600		
	GA	Bus	-		-	6		111082
		Size	-			2434		
2 DG	LSF	Bus	18	33	100.69	18	33	
		Size	720	900		720	900	

Continued

	ELF	Bus	12	30		87.63	11	30		87.535
		Size	1020	1020			1050	1050		
	GA	Bus	-	-			13	30		87.1473
		Size	-	-			821	1112		
3 DG	LSF	Bus	18	33	25	85.07	18	33		
		Size	720	810	900		720	810		
	ELF	Bus	13	24	30	74.27	12	24	30	73.64
		Size	900	900	900		950	950	950	
	GA	Bus	-	-	-		13	24	30	72.7148
		Size	-	-	-		775	1002	1017	

Table 3. Results of 33 bus test system with DG's of 2 types.

Cases	[30]				This research (GA)			
	optimal Type1 location	optimal Size (MW)	Optimal Type2 location	Optimal Size (MVAr)	optimal Type1 location	optimal Size (MW)	Optimal Type2 location	Optimal Size (MVAr)
1 DG	6	2.5174	30	1.2508	6	2.434	6	2.604
2 DG	13	0.8399	12	0.4524	13	0.821	13	0.817
	30	1.1402	30	1.0411	30	1.112	30	1.247
3 DG	6	1.1736	3	0.8079	13	0.775	13	0.769
	14	0.6033	14	0.3351	24	1.002	24	1.025
	31	0.6798	30	0.9923	30	1.017	30	1.146

4.3. Probabilistic PV-DG Model

Assuming constant PV-DG generation does not adequately capture the typical behavior of a radial distribution system over time while incorporating variable PV-DG output offers a more realistic representation of actual conditions. Additionally, dynamically adjusting PV-DG output to minimize losses over a 24-hour period provides more accurate insights into the optimal DG placement and the maximum DG capacity required to reduce system power losses. Therefore, this research examines two distinct cases.

A year of 5 min averaging solar irradiance data gathered within a PV energy generation in Cairo, Egypt is utilized as raw data for developing the probabilistic model. In this article, this data is used as a reference for solar radiation data. The annual hourly average of the data collected is presented in Figure 5. The measured irradiance data (after interpolation and normalization) forms the foundation for the distribution parameters in the Beta distribution. The Alpha and Beta values as shown in Figure 6, reflect how the irradiance changes throughout every hour of the day as illustrated in Figure 6. The probability distribution functions (PDF) used to estimate hourly solar irradiance are based on one year of historical data collected from the study site. This data is then employed to generate frequency distribution of irradiance measurements for a typical day.

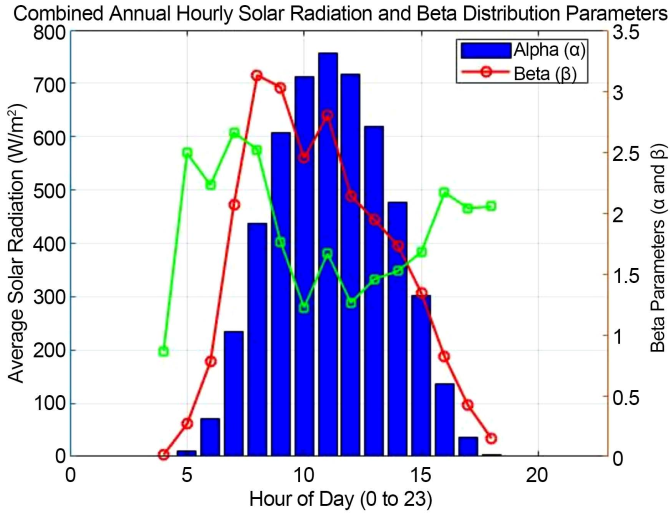

Figure 6. Annual hourly solar radiation and beta distribution parameters.

Figure 6 gives an overview of how the Beta distribution parameters change throughout the day, allowing to assess variability and predictability in solar irradiance. **Figure 7** zooms in on the probability of different irradiance values for specific hours of the day (10 to 15), helping in modelling the solar generation more accurately by visualizing the Beta-distributed solar radiation for those peak hours. **Figure 7** plots the probability density functions (PDFs) of solar radiation modeled by Beta distributions for each hour between 10 and 15. The normalized solar irradiance (values between 0 and 1), which represents the intensity of the solar radiation relative to the maximum possible value for each hour. The probability density for each irradiance value (*i.e.*, how likely a certain irradiance level is at each hour.

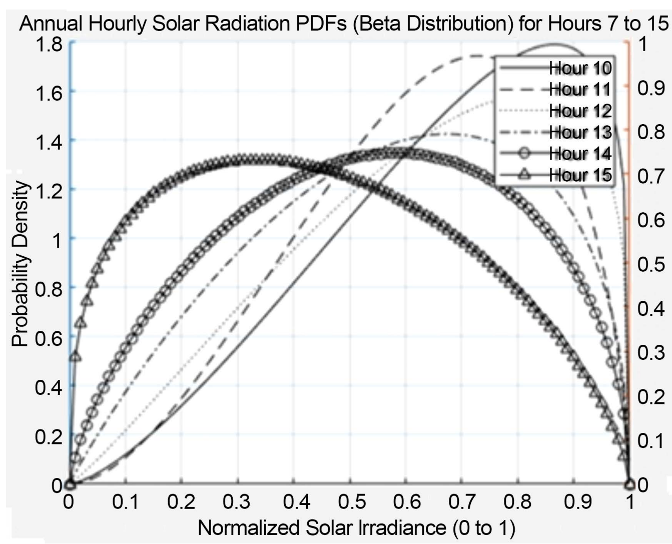

Figure 7. Solar radiation probability PDF for 6 hours from 10 AM to 15 PM.

Table 4 compares optimal solutions for DG allocation and sizing in a 33 bus test system across different methods: GA based approach from a prior study [29], This research and the probabilistic and deterministic approaches developed in the current study, and Artificial Bee Colony algorithm from (ABC) in [26]. Where, in Case 1: Minimization of active power losses under voltage constraint. Case 2: A multi-objective optimization problem aimed at minimizing active and reactive power losses while maximizing voltage profile enhancement, under voltage constraint. In OpenDSS, the PV system element has two operational modes, Mode 1 and Mode 2, which define how the PV system interacts with the grid. These modes govern the control of active and reactive power. Mode 1: Fixed Power Factor Mode, where the PV system operates with a fixed power factor. Mode 2: Voltage Control Mode, where the PV system dynamically adjusts its reactive power (Q) to regulate the voltage at the point of interconnection.

The results show consistency in single PV-DG allocation, where Bus 6 is consistently identified as the optimal location across all methods and approaches. The sizes and placements of the second DG vary widely between methods, likely due to differences in optimization criteria and assumptions about network behavior. Deterministic methods in this research tend to favor larger DG sizes, particularly for the second DG at Bus 30, which might indicate a focus on minimizing losses or improving voltage profiles. Probabilistic methods produce more conservative DG sizes, reflecting consideration of uncertainties in PV generation and load variability.

Table 4. Outputs of optimum solutions for different cases.

DG	Cases	[29] (GA)		This research (GA)					[26] ABC
					Probabilistic		Deterministic		
		Case 1	Case 2	PV	Mode 1	Mode 2	Mode 1	Mode 2	
1 DG	Bus	6	6	6	6	6	6	6	6
	kW	2685.3	3117.9	2322	2367	2424	2585	2665	2577.5
2 DG	Bus	6	6	6	6	6	13	13	6
	kW	2553	2955.7	1915	2329	2439	853	898	1970.7
	Bus	30	29	29	15	15	30	30	15
	kW	169.0	155.2	424	159	142	1151	1218	575.7

5. Conclusion

This study introduced an efficient multiple PV-DG topology aimed at minimizing active power losses and enhancing voltage profiles. The stochastic nature of PV generation was captured using a PDF derived from one year of measurements to represent the uncertainty of PV-DG in the distribution system. Probabilistic techniques and the GA algorithm were employed to determine the optimal sizing and placement of PVDGs. More analysis is needed for Voltage Control Mode, the PV system dynamically adjusts its reactive power (Q) to regulate the voltage at the point of interconnection, playing a critical role in maintaining grid stability. This capability is especially beneficial in modern power systems with high penetration of distributed energy resources, where voltage fluctuations are more prevalent. However, further exploration is needed to optimize the coordination of multiple PV systems operating in voltage control mode. Areas such as the impact of voltage control on system losses, interactions with other voltage regulation devices, and real-time control strategies require deeper investigation to fully harness the potential of this mode while avoiding adverse effects like voltage instability or over-regulation. The paper presents comparisons with previous research studies and various interesting findings such as the considered hours for developing the probabilistic model presents different results. Also, other distributions could be explored and compared and justification for chosen weights in the multi-objective function needs to be more rigorously established, including comparison of performance for alternative weights.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix: Data for 33-Bus Test System

From	to	Line Data		Cap	Load data		
		R (ohms)	X (ohms)		Bus ID	PL (kW)	OL (kVA)
					1	0	0
1	2	0.0922	0.0470	5064	2	100	60
2	3	0.4930	0.2511	3798	3	90	40
3	4	0.3660	0.1864	3798	4	120	80
4	5	0.3811	0.1941	3798	5	60	30
5	6	0.8190	0.7070	3798	6	60	20
6	7	0.1872	0.6188	3798	7	200	100
7	8	1.7114	1.2351	3798	8	200	100
8	9	1.0300	0.7400	3798	9	60	20
9	10	1.0440	0.7400	3798	10	60	20
10	11	0.1966	0.0650	3798	11	45	30
11	12	0.3744	0.1238	3798	12	60	35
12	13	1.4680	1.1550	3798	13	60	35
13	14	0.5416	0.7129	3798	14	120	80
14	15	0.5910	0.5260	3798	15	60	10
15	16	0.7463	0.5450	3798	16	60	20
16	17	1.2890	1.7210	3798	17	60	20
17	18	0.7320	0.5740	3798	18	90	40
2	19	0.1640	0.1565	5064	19	90	40
19	20	1.5042	1.3554	3798	20	90	40
20	21	0.4095	0.4784	3798	21	90	40
21	22	0.7089	0.9373	3798	22	90	40
3	23	0.4512	0.3083	3798	23	90	50
23	24	0.8980	0.7091	2532	24	420	200
24	25	0.8960	0.7011	2532	25	420	200
6	26	0.2030	0.1034	2532	26	60	25
26	27	0.2842	0.1447	2532	27	60	25
27	28	1.0590	0.9337	2532	28	60	20
28	29	0.8042	0.7006	2532	29	120	70
29	30	0.5075	0.2585	2532	30	200	600
30	31	0.9744	0.9630	2532	31	150	70
31	32	0.3105	0.3619	2532	32	210	100
32	33	0.3410	0.5302	2532	33	60	40
Substation voltage = 12.66 kV, MVA base = 10 MVA							