Scientific Research Publishing

Journal of Power and Energy Engineering, 2025, 13(2), 1-22
https://www.scirp.org/journal/jpee
ISSN Online: 2327-5901
ISSN Print: 2327-588X

Optimal Planning of Multiple PV-DG in Radial Distribution Systems Using Loss Sensitivity Analysis and Genetic Algorithm

A. Elkholy

Photovoltaic Cells Department, Electronics Research Institute, Cairo, Egypt
Email: elkholy.aly@gmail.com, aly_elkholy@eri.sci.eg

How to cite this paper: Elkholy, A. (2025) Optimal Planning of Multiple PV-DG in Radial Distribution Systems Using Loss Sensitivity Analysis and Genetic Algorithm. *Journal of Power and Energy Engineering*, **13**, 1-22.
https://doi.org/10.4236/jpee.2025.132001

Received: November 19, 2024
Accepted: February 15, 2025
Published: February 18, 2025

Abstract

This paper introduces an optimized planning approach for integrating photovoltaic as distributed generation (PV-DG) into the radial distribution power systems, utilizing exhaustive load flow (ELF), loss sensitivity factor (LSF), genetic algorithms (GA) methods, and numerical method based on LSF. The methodology aims to determine the optimal allocation and sizing of multiple PV-DG to minimize power loss through time series power flow analysis. An approach utilizing continuous sensitivity analysis is developed and inherently leverages power flow and loss equations to compute LSF of all buses in the system towards employing a dynamic PV-DG model for more accurate results. The algorithm uses a numerical grid search method to optimize PV-DG placement in a power distribution system, focusing on minimizing system losses. It combines iterative analysis, sensitivity assessment, and comprehensive visualization to identify and present the optimal PV-DG configurations. The present-ed algorithms are verified through co-simulation framework combining MATLAB and OpenDSS to carry out analysis for 12-bus radial distribution test system. The proposed numerical method is compared with other algorithms, such as ELF, LSF methods, and Genetic Algorithms (GA). Results show that the proposed numerical method performs well in comparison with LSF and ELF solutions.

Keywords

Photovoltaic Systems, Distributed Generation, Multiple Allocation and Sizing, Power Losses, Radial Distribution System, Genetic Algorithm

1. Introduction

Smart grid has the potential to enable numerous functions that improve the electric grid's overall performance in many areas, such as monitoring and managing grid performance over large areas, assessing the grid's maximum capacity in real-time, managing power flow, detecting and responding to disruptions or outages, and helping customers optimize electricity use through advanced communication and control technologies [1] [2].

In the context of the smart grid, the grid infrastructure is undergoing upgrades by adding power electronic equipments such as flexible alternating current transmission system (FACTS), high-voltage direct current (HVDC), and solid-state transformers. Consequently, advanced simulation tools are necessary to analyze the potential positive or negative impacts of devices on the planning, operation, control, and protection of modern distribution networks. Inverter-interfaced photovoltaic (PV) systems, including rooftop PV and residential wind generation systems, are experiencing increased penetration, leading to a shift in distribution networks from passive unidirectional power flow systems to active bidirectional power flow systems [3].

Designing appropriate optimization frameworks for evaluating and improving power system resilience has been a focal point since the inception of resilience requirements. Sophisticated optimization methods are needed because of the complicated situation and inclusion of several interconnected infrastructures for various energy resources [4]. The article in [4] presents an in-depth review and evaluative analysis of existing approaches for power system resilience, highlighting areas for improvement and suggesting future paths for developing universally recognized definitions, metrics, evaluation techniques, and improvement strategies. Also, developing mathematical and computationally effective approaches for resilience assessment methods is critical for forming robust power systems. In [4], several methods were proposed for evaluating power system resilience, including sequential and non-sequential Monte Carlo simulations, contingency, and machine learning techniques.

Sensitivity factors can be obtained using sensitivity analysis to identify critical bus locations for the optimal installation of distributed generation (DG) in a radial system [5]. Loss sensitivity factor was employed in [6] for DG to ensure its suitable deployment and minimize the loss, along with the development of an optimal power flow-based formulation to determine the optimal settings of DGs. In [7], the loss sensitivity approach was used to define the optimal size of DG, and accurate loss formula was employed to find the optimal location of DG for minimizing the losses. In [8], the loss sensitivity factor was presented based on a matrix of branch current with bus injection and Voltage.

In [9], a simple search algorithm was introduced for determining the optimal capacity and location of PV-DG within a power system, utilizing losses and expenses functions as the objective criteria. However, this approach can be time-intensive because it involves searching for both ideal location and size.

The installation of multiple PV-DG units in radial networks for reducing system losses was presented in [10]. An improved analytical method was proposed, which utilized expressions to evaluate the optimal sizes and locations of different DG types. Also, the loss sensitivity factor (LSF) and exhaustive load flow (ELF) approaches were discussed. Finally, a strategy for obtaining the ideal power factor was presented for DG capable of producing both real and reactive power [10].

An analytical approach was presented in [7] to define the optimal size and an approach to determine the optimal location for DG in to minimize losses. The analytical statement and approach employed the exact loss equation. That approach was tested on different configurations and sizes to verify its applicability in distribution systems such as the 30- and 33-bus loop test systems, and the 69-bus radial system.

The objective of this research is to introduce appropriate optimization frameworks for evaluating the optimal sizing and allocations of multiple PV-DG in a 12-bus radial distribution system towards employing a dynamic PV-DG model for more accurate results. Section 2 presents the theoretical background about minimizing the power loss in radial distribution system based on LSF and GA algorithms. Section 3 introduces the proposed methodology and problem formulation. Section 4 presents the results and outcomes of this study. Finally, Section 5 presents the conclusions.

2. Optimum DG Planning Based on Minimizing Losses

Power system networks continuously lose electrical energy due to resistance, with distribution systems losing more than transmission systems. Distribution systems have a higher R/X ratio, leading to significant voltage drops and power losses along feeders. Utilities worldwide face the challenge of reducing these losses, and well-placed DGs can help. DGs near load centers reduce line losses, especially when feeders are heavily loaded. However, incorrect DG sizing and placement can increase losses and worsen voltage profiles, making optimal planning essential. 12-bus radial test system with the layout illustrated in **Figure 1** was analyzed in [11]-[13].

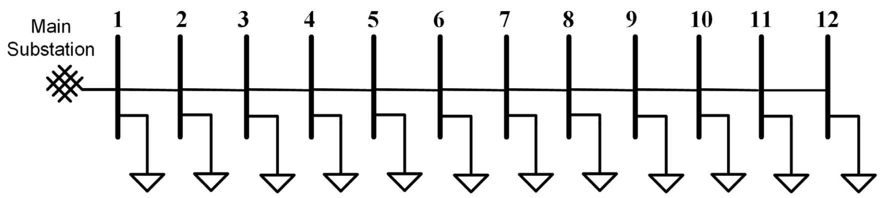

Figure 1. Single line diagram of the 12-bus radial distribution network.

The distribution system's typical power loss as a function of DG size at each bus is shown in a three-dimensional graph, as in **Figure 2**. The figure illustrates that, as the DG size value increases, the losses decrease until reaching a minimum value, representing the optimum DG size for that location. The main observation that can be obtained from **Figure 2** is that it is not recommended to penetrate excessive DG in the system [7].

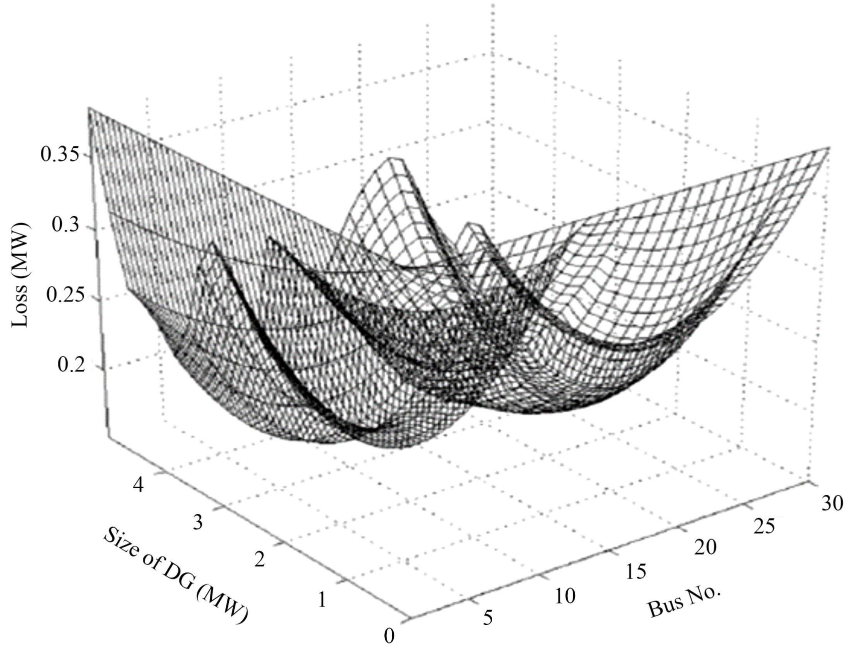

Figure 2. Effect of size and location of DG on system loss.

Power flow algorithms developed particularly for distribution systems, such as linear power flow, were used in various research studies and expressed as [14]:

$$\hat{Y}V = I \tag{1}$$

where, $\hat{Y}$ represents the modified admittance matrix.

The power flow is addressed using a numerical method as outlined in [9]. To formulate the injected real and reactive power to a bus, it is necessary to express those variables. The voltage at the i^{th} bus can be represented as follows [15]:

$$V_i \angle \delta_i = |V_i| \left(\cos \delta_i + j \sin \delta_i \right) \tag{2}$$

And the self-admittance at bus i is expressed as:

$$Y_{ii} = |Y_{ii}| \angle \theta_{ii} = |Y_{ii}| \left(\cos \theta_{ii} + j \sin \theta_{ii} \right) \tag{3}$$

Similarly, the mutual admittance between buses i and j can be expressed as [15]:

$$Y_{ij} = |Y_{ij}| \angle \theta_{ij} = |Y_{ij}| \left(\cos \theta_{ij} + j \sin \theta_{ij} \right) \tag{4}$$

Assuming a total number of n buses are contained in the test system. The injected current at i bus is considered as,

$$I_i = \sum_{k=1}^{n} Y_{ik} V_k \tag{5}$$

It is generally accepted that the injected current at bus is considered positive, while the current leaving the bus is negative. Consequently, both the reactive and real power injected to a bus are also supposed to be positive. The complex power at bus i can be defined as follows:

$$P_i - jQ_i = V_i^* I_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k \tag{6}$$

Therefore, by rearranging equation (6) and substituting with the other variables, the real and reactive power are represented as [15]:

$$P_i = \sum_{j=1}^{nb} \left| Y_{ij} V_i V_j \right| \cos\left(\theta_{ij} + \delta_j - \delta_i\right) \tag{7}$$

$$Q_i = -\sum_{j=1}^{nb} \left| Y_{ij} V_i V_j \right| \sin\left(\theta_{ij} + \delta_j - \delta_i\right) \tag{8}$$

The total real power loss was stated as in (9), commonly referred as "exact loss formula" [7] [16] [17]

$$P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} \left[\alpha_{ij} \left(P_i P_j + Q_i Q_j \right) + \beta_{ij} \left(Q_i P_j - P_i Q_j \right) \right] \tag{9}$$

where $\alpha_{ij} = \frac{r_{ij}}{V_i V_j} \cos\left(\delta_i - \delta_j\right)$, $\beta_{ij} = \frac{r_{ij}}{V_i V_j} \sin\left(\delta_i - \delta_j\right)$, and $r_{ij} + jx_{ij} = Z_{ij}$ are the ijth element of [Zbus] matrix with [Zbus] = [Ybus]$^{-1}$.

P_i, Q_i represents real and reactive power of bus i and P_j, Q_j at bus j.

r_{ij} represents line resistance between buses i and j.

V_i, V_j defines voltage magnitude of buses i and j.

δ_i, δ_j represents voltage angle of buses i and j.

N is the total number of buses.

The main goal of implementing PV-DG is to minimize the total power loss, which is the sum of losses in all the network branches. The optimization problem is centered on reducing this cumulative active power loss throughout the system as its objective function [18]. The real and reactive power flow (P_{ij} and Q_{ij}) from bus i to bus j is presented as follows [19]:

$$P_{ij,t}^{loss} = r_{ij} \left[\frac{\left(P_{ij,t}^{line}\right)^2 + \left(Q_{ij,t}^{line}\right)^2}{\left(V_0\right)^2} \right] \tag{10}$$

where, V_{nom}, V_0 Nominal and base voltage magnitude in the distribution system.

2.1. Loss Sensitivity Factor Method and Priority List

The Sensitivity Factor method was presented in the linearization of original nonlinear equation round the initial operating point, this procedure reduces the number of possible solutions. The Loss Sensitivity Factor method was used to tackle the capacitor allocation in the distribution system as well as the DG. By incorporating the sensitivity factor, the solution space can be narrowed down to a select few buses that rank highest on the priority list. It is important to note that the number of buses given priority will impact the optimal solution for certain systems. To determine the optimal size of the DG at all buses based on the priority list, the DG is defined then its size is gradually increased until the minimum system losses are achieved [7].

Sensitivities are widely utilized in various industries for real-time control purposes. In electric network analysis, the most commonly used sensitivities are power transfer distribution factors (PTDF) and loss factors (LF). PTDF quantifies the impact on power flow at each line when one MW is transferred between network buses. To linearize flow in the line based on bus injections, shift factors can be employed. Essentially, shift factors represent the sensitivity of line flows to changes in injections at the buses. LF represents the degree of sensitivity of system losses in response to a modification in the injection at a specific bus. Essentially, the loss factor at a particular bus indicates the extent to which system losses will be altered when the injection at that bus is adjusted by one (1) MW. Loss factors are commonly employed in linear analysis as a means to approximate the impact of various transfers or transactions on system losses.

An analytical technique was presented to determine the optimal size and placement of a single DG. This technique utilizes a sensitivity factor to narrow down the search region in many buses that are with the highest priority. In order to accurately estimate system loss, an accurate formula is used. By using this approach, the load flow only needs to be done twice, which helps in quickly identifying the optimal placement. Each case is formulated by considering 30% of the whole number of buses for the priority list [7] [15].

The relationship between total power loss and injected power forms a parabolic curve. At the point of minimum losses, the percentage of losses change relative to the injected power is zero. The sensitivity factor for real power loss concerning real power injection from a DG is provided in [7].

$$\frac{\partial P_L}{\partial P_i} = 2\sum_{j=1}^{N}\left(\alpha_{ij}P_j - \beta_{ij}Q_j\right) = 0 \tag{11}$$

It follows that [7].

$$\alpha_{ij}P_i - \beta_{ii} + \sum_{j=1,\, j\neq i}^{N}\left(\alpha_{ij}P_j - \beta_{ij}Q_j\right) = 0$$

$$P_i = \frac{1}{\alpha_{ii}}\left[\beta_{ii}Q_i + \sum_{j=1,\, j\neq i}^{N}\left(\alpha_{ij}P_j - \beta_{ij}Q_j\right)\right] \tag{12}$$

where, P_i represents the real power generation at bus i, which is the difference between real power production and demand at that bus.

$$P_i = \left(P_{DGi} - P_{Di}\right) \tag{13}$$

where, P_{DGi} defines the real power generation at bus i, and P_{Di} represents the load demand at the same bus. By merging (12) and (13), the resultant equation is in (14) [7].

$$P_{DGi} = P_{Di} + \frac{1}{\alpha_{ii}}\left[\beta_{ii}Q_i - \sum_{j=1,\, j\neq i}^{N}\left(\alpha_{ij}P_j - \beta_{ij}Q_j\right)\right] \tag{14}$$

The above equation provides the optimal size of DG for minimizing losses at each bus i. Any other DG size placed at bus *i* would result in higher losses. The optimal size of DG for each bus can be calculated using Equation (6) based on the base case load flow, which is the load flow without DG [7]. DG can generally be divided into four main types according to their capability to generate and consume active and reactive power, such as: Type 1: generate active power only, generate reactive power only, Type 2: generate active and reactive power, and finally generate active power and consume reactive power [10] [13].

2.2. Genetic Algorithm

The primary objective is to identify and size the PV-DG systems linked to the test system. The system losses play a pivotal role in influencing costs and technical challenges within the network. PV-DG units integrated into the feeders need to optimize the reduction of system power losses while avoiding violations of voltage limits, as indicated in [20]. The main objective function to minimize is thus expressed as (15):

$$\text{Minimize } P_{Loss} = \sum\nolimits_{i=1}^{N} P_{Li} \tag{15}$$

where P_{Li} denotes the power losses in the i line, and N indicates the total number of lines of the test system.

To optimize the voltage profile as a second objective of the radial power system by minimizing the sum of voltage deviations at load buses, the objective function can be formulated mathematically. This objective function is typically expressed as [20]:

$$\text{Minimize } \sum\nolimits_{i=1}^{N} \left| V_i - V_{ref} \right| \tag{16}$$

where V_i represents the voltage at load bus i, V_{ref} defines the reference value at load bus i, typically set to 1.0 pu.

The strategic placement of DG and Distribution Static Compensators (DSTATCOM) in an uncompensated system markedly improves the voltage profile. This optimization facilitates the effective delivery of the necessary real and reactive power, leading to a reduction in power losses and an enhancement in voltage stability. The Total Voltage Variation (TVD) in the network can be expressed as follows [21]:

$$TVD = \begin{cases} 0, & \text{if } 0.95 \le V_t \le 1.05 \\ \sum\nolimits_{t=1}^{N} \left| V_{ref} - V_t \right|, & \text{else} \end{cases} \tag{17}$$

3. Methodology and Problem Formulation

This section presents an approach for determining the optimal sizes and placements of multiple PV-DGs in radial distribution system. The presented methodology involves stochastic load flow analysis with the PV-DG integrated to obtain the final solution. A standard 12-bus radial test system, as depicted in **Figure 1**, is used as a test system and Appendix presents all related data in. In this research, the buses with PV-DG are presented as PQ buses with unity power factor for type 1 DGs and a 0.95 power factor for type 2 DGs.

Customised scripts for determining the optimal locations for PV-DGs system are developed in MATLAB, and a COM interface is used to interface with the OpenDSS Simulator. The 12-bus test system data is implemented in OpenDSS and developed based on the interface between MATLAB and OpenDSS. After each distributed load flow, the data and results are exported to MATLAB, where they are analysed, stored, and compared along with the outcomes of previous load flows to provide recommendations on the optimal placement to install PV-DG systems in order to minimize losses.

Power flow equations can be solved for power system by utilizing real and reactive power, and the voltages at both the sending and receiving buses. Where, P_i, Q_i, and V_i represent the real and reactive power, and voltage at the sending end, respectively, while P_{i+1}, Q_{i+1}, and V_{i+1} represent the corresponding variables at the receiving end. The following equation illustrates these relationships. The quadratic terms in the following equation account for the branch's losses [22]:

$$Loss_i = \frac{r_i\left(P_i^2 + Q_i^2\right)}{V_i^2} \tag{18}$$

In this approach, the overall system loss is the aggregate of all branch losses represented by:

$$Total\_Loss = \sum_{i=0}^{n-1} Loss_i = \sum_{i=0}^{n-1} \frac{r_i\left(P_i^2 + Q_i^2\right)}{V_i^2} \tag{19}$$

3.1. The Power Flow Balance and Generation Equations

The constraints of the optimization approaches are crucial for representing the active and reactive power flows within the test system as labelled (20) and (21) [18].

$$P_{Gi} = P_{Di} + \sum_{j=1}^{nb} |V_i|\left|V_j\left[G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}\right]\right| \tag{20}$$

$$Q_{Gi} = Q_{Di} + \sum_{j=1}^{nb} |V_i|\left|V_j\left[G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}\right]\right| \tag{21}$$

where V_i and V_j represent the voltage values at the buses, P_{Gi} and P_{Di} denote the active power generated and demanded, Q_{Gi} and Q_{Di} indicate the generated and required reactive power and G_{ij} and B_{ij} are the line conductance and susceptance.

The defined constraints include voltage thresholds for all buses in the network, as well as limitations on the PV-DG units. Equation (22) establishes the maximum and minimum voltage boundaries, specifying the range of the voltage levels

$$V_{\min} \le V_i \le V_{\max} \tag{22}$$

where $V_{\min}$ and $V_{\max}$ represent the min. and max. voltage magnitudes, respectively.

The optimal size of PV-DG units requires compliance with certain operational boundaries, specified by the minimum $PVDG_{(\min)}$ and maximum $PVDG_{(\max)}$ limits, as in (23).

$$PVDG_{(\min)} \le PVDG \le PVDG_{(\max)} \tag{23}$$

3.2. Proposed Numerical Method

In the proposed approach, the sensitivity analysis is implemented through a process of iterative simulation rather than explicit mathematical equations. The following is the process steps, along with the implicit equations involved in the sensitivity analysis as shown in Figure 3. The initial steps are to initialize OpenDSS, load test system, retrieve all bus names in the circuit, solve the power flow for the test system without any DG, and finally calculate the initial system loss. At this stage, it is important to initialize cell arrays and matrices to store loss information, optimal locations, and optimal sizes for each bus.

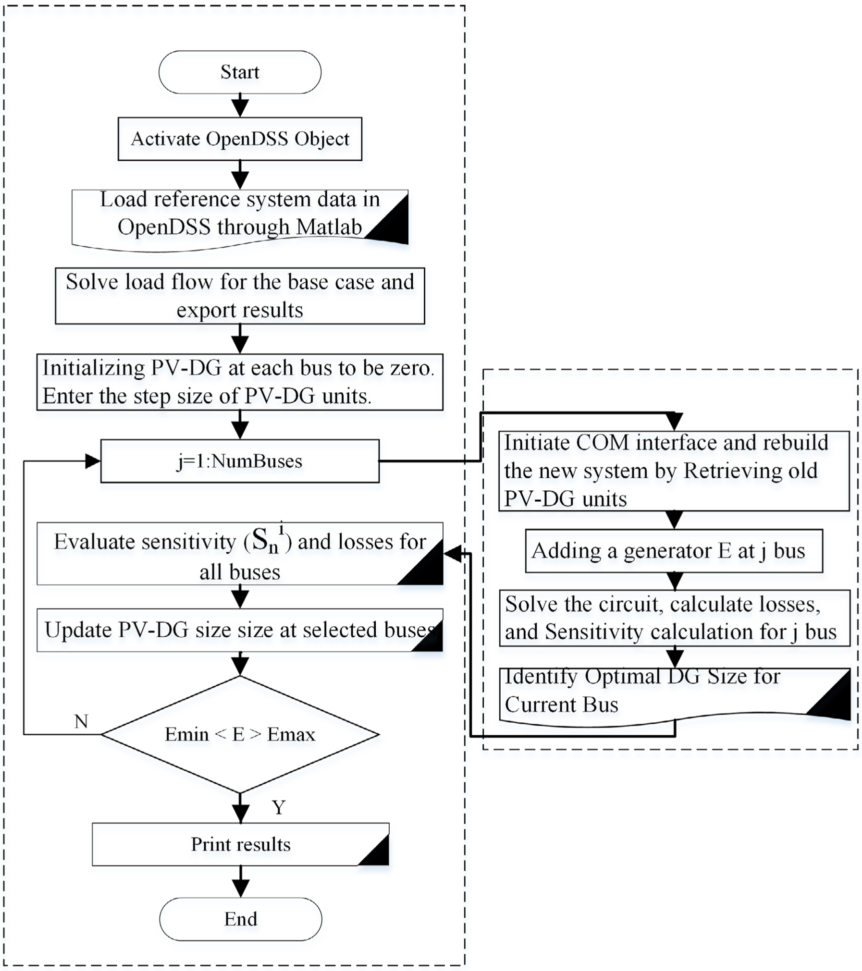

Figure 3. Chart of numerical method for PV-DG in distribution system.

The sensitivity analysis can be employed at this step. Where to start with setting parameters for DG step size and iteration control. The primary calculation performed by OpenDSS during each iteration is the power flow solution, which uses the following fundamental equations:

$$P_i + jQ_i = V_i * I_i^* \tag{24}$$

where P_i is the real power, Q_i is the reactive power, V is the voltage, and

I^* is the complex conjugate of the current.

At this stage of analysis, DGs can be add to the circuit based on previous iterations based on adding a small DG to each bus sequentially and calculate the resulting system losses. The system losses are calculated using the following equation:

$$Total\_Loss = \sum_{i=0}^{n-1} Loss_i = \sum_{i=0}^{n-1} \left(I_i^2 * R_i \right) \tag{25}$$

where P_{loss} is the total real power loss, I_i is the current through element iii, and R_i is the resistance of element i.

The losses are recalculated for each DG size at each bus. The placement of a DG changes the power injection at the bus:

$$P_i = P_{load} - P_{PV\text{-}DG} \tag{26}$$

where P_i is the net power at the bus, P_{load} is the load power, and $P_{PV\text{-}DG}$ is the DG power.

The sensitivity analysis is performed by systematically varying DG sizes and solving the power flow to observe the impact on system losses. This process inherently leverages the power flow and loss equations implemented within OpenDSS to achieve the sensitivity analysis.

Where to compute the sensitivity of each bus as the percentage reduction in system losses per unit DG added. Then, sort the buses based on sensitivity values, select the top buses with the highest sensitivity, and finally increment the DG size for the top sensitive buses and update the system loss values. A developed approach that incorporates based on continuous sensitivity analysis is proposed. The sensitivity index is estimated as follows:

$$S_n^i = \frac{\left(\text{Old Loss}\right)^{i-1} - \left(\text{New Loss with PVDG at node } n\right)^i}{E} * 100 \tag{27}$$

where, n represents the node for which the sensitivity is determined, i indicates the iteration, $(i-1)$ represents the previous iteration value of power loss when another PV-DG is installed at node n. E is the step size for PV-DG increment.

3.3. Loss Sensitivity Factor Method

The sensitivity factor method works by linearizing the original nonlinear equation near the initial operating point, effectively narrowing the solution space. The Loss Sensitivity Factor (LSF) at bus (i) is calculated from Equation (1) in relation to the active power injection at that specific bus, as follows [11]:

$$\alpha_i = \frac{\partial P_L}{\partial P_i} = 2\sum_{j=1}^{N} \left(\alpha_{ij} P_j - \beta_{ij} Q_j \right) \tag{28}$$

Figure 3 shows the flow chart of LSF method for multiple DG placement. Similar to IA method, the procedure to find the optimal locations and sizes of multiple DG units using the LSF is described in detail as follows. The objective of the analysis is to find multiple locations in the distribution system where small size DG units can be placed to minimize the system losses.

Step 1: Enter the number of DG step size to be installed.

Step 2: Initialize OpenDSS and load the circuit.

Step 3: Perform base case load flow (without any DG units) and calculate initial losses.

Step 4: Initialize variables and arrays to track the impact of placing DG units at different buses, as well as to store the results of each iteration.

Step 5: Iteratively determine the optimal DG placement and size: For each bus, the approach adds a small DG unit (with a predefined size) and recalculates the system losses. This is done to evaluate the sensitivity of each bus to the placement of PV-DG units. Sensitivity is calculated as the reduction in losses per unit of PV-DG power added. Rank the buses based on their sensitivity values. This ranking helps identify the buses that are most effective in reducing system losses when a DG unit is placed there. Select the bus with the highest sensitivity and increase the DG size at that bus.

Step 6: Check Termination Conditions: The voltage at a particular bus is over the upper limit; the total size of DG units is over the total load plus loss; the maximum number of DG units is required; the new iteration loss is greater than the previous iteration loss.

The script iteratively places and sizes DG units at the most sensitive buses in the distribution system to minimize losses, while checking for voltage constraints and loss reduction. It follows a heuristic approach based on sensitivity analysis.

3.4. Exhaustive Load Flow Iterative Method

ELF method, known as a repeated load flow solution, demands excessive computational time since all buses are considered in calculation; however, it can lead to a completely optimal solution. Its numerical results are presented in Section III. Overall, the algorithm effectively demonstrates an ELF approach by exhaustively searching for optimal DG placements and sizes based on minimizing system losses in a power distribution system.

The proposed approach utilizes an algorithm to optimize the placement and sizing of PV-DGs in electrical distribution systems using co-simulation. The algorithm uses a combinatorial approach to try different combinations of buses for DG placement. It leverages all possible combinations for placing 1 to 3 DGs creating a matrix whose rows consist of all possible combinations of the buses taken at a time. In the second step, for each combination of buses, it iteratively searches through different DG sizes to find the configuration that minimizes system losses. Then, It calculates the sensitivity of system losses to changes in DG sizes, which helps in determining the impact of DG placement on the overall system performance as a third step. The algorithm continuously resets and recompiles the circuit for each new configuration, ensuring that the calculations reflect the current state of the system with the added DGs. Finally, by iterating over possible configurations and sizes, the algorithm employs a heuristic search method to approximate the optimal solution for DG placement and sizing.

The Key Concepts in the Algorithm are as following

1) Combinatorial Optimization: a subfield of mathematical optimization that consists of finding an optimal object from a finite set of objects. Iterates through all combinations of buses and DG sizes to find the best configuration.

2) Iterative Search: Systematically evaluates each possible configuration through nested loops.

3) Simulation and Recalculation: Resets and recalculates the power system configuration for each DG placement scenario.

4) Heuristic Search: it explores all combinations exhaustively, akin to a brute-force heuristic approach.

3.5. Genetic Algorithm Formulation

When minimizing power losses is the sole objective, an excessive injection of active power from PV-DG can lead to voltage levels rising beyond acceptable limits. To address this, it's important to introduce reactive power Loss as a secondary target as presented in (25) and cumulative voltage deviation (26) as a tertiary goal. Minimizing Quadratic Penalty Factor (QPF) becomes the fourth objective as described in equation (27).

$$f_2 = \sum_{i=1}^{N} Q_{Li} \tag{29}$$

$$f_3 = CVD = \sum_{i=1}^{N} \left|V_i - V_{rated}\right| / N \tag{30}$$

$$f_4 = QPF = \begin{cases} \left(V_i - V_{\min}\right)^2 & V_i \geq V_{\min} \\ 0 & V_{\min} \leq V_i \leq V_{\max} \\ \left(V_i - V_{\max}\right)^2 & V_i \leq V_{\max} \end{cases} \tag{31}$$

where; Q_{Li} represents the reactive power loss at i^{th} line, and N defines all considered lines of the test system.

V_i considers the voltage of the bus.

V_{rated} refers to the rated voltage for the test system, set as 1 pu.

V_{lim} is the minimum or maximum voltage limit.

Equation (28) outlines the entire objective function that needs to be minimized.

$$f = \min \sum_{i=1}^{24} \left(\omega_1 f_{1i} + \omega_2 f_{2i} + \omega_3 f_{3i} + \omega_4 f_{4i}\right) \tag{32}$$

Each factor of ω have an exclusive weight, and the combined sum of all weights should be equal to 1.

The arranged minimisation challenge is achieved with a GA. The GA is appropriate to tackling such issues; it operates as a heuristic search algorithm driven by natural selection and particular evolution. The approach begins with 600 population size and a randomly created group of people, and at each step, this group forms a new generation through crossover and mutation processes, evolving iteratively until an optimal solve is identified.

Determining the size of the PV-DGs and their optimal locations necessitates calculating the objective function (32) during a specified time period and taking into account the applicable restrictions (20) through (23). The algorithm's functions from (29) to (31) include cumulative voltage deviations are objectives and to be determined. The GA establishes preliminary population, which include the PV-DG location, active power, and power factor. Subsequently, it computes the objective function by utilizing the power flow solver capability in OpenDSS, which takes into account the complete structure of the distribution system. In this study, various weights were tested for each component of equation (32) through trial and error to find the optimal configuration. The weights used are 0.5 for P_{Loss}, 0.1 for Q_{Loss}, 0.2 for CVD, and 0.2 for the voltage limits.

4. Numerical Results and Evaluation

The proposed method is executed in MATLAB within a co-simulation setting integrated with OpenDSS using COM interface. This section details the results. Initial tests used PV-DG production curve. For each power flow solution computed with OpenDSS, the GA minimizes the fitness function. This process enables sequential-time power flow simulations, which are crucial for analyzing smart grid challenges, such as those caused by integrating renewable resources, storage, and electric vehicles, which alter the load profile. Accurate time-based modelling of system behaviour is critical for obtaining reliable results.

This paper explores the challenge of placing and sizing multiple PV-DG systems to reduce losses in radial distribution systems. A proposed numerical method is presented in this paper and contentious loss sensitivity factor LSF method. The results contain three major subsections. The first subsection presents the results based on the proposed numerical method to define the optimal PV-DG size for each bus. The second subsection presents the optimal sizes of PV-DG for all busses in the system considering the sensitivity for all buses in each step and after adding PV-DG to previous bus which can be called optimal planning. Finally, this paper presents the optimum places and capacities of multiple PV-DG for 12 bus test system, as well as a comparative study with other research studies concerning the LSF, ELF, and GA algorithm.

The 12-bus radial distribution test system has a total load of 435 kW and 405 kVAr is used in this research. A 12-bus radial distribution system, with the structure illustrated in Figure 1 and data in Appendix, was examined in [11]-[13] [23].

4.1. Proposed Numerical Method Results

The proposed numerical algorithm is used to calculate the optimal sizes of PV-DGs at all buses of the system. The power loss at all buses without PV-DG is illustrated in Figure 4. This figure highlights the system baseline losses before integrating PV-DG units. Figure 5 displays the optimal sizes of DGs at all buses in the 12-bus distribution test system. For particular locations within the test system, this figure provides the value of PV-DG sizes at all buses that results in minimizing total loss. The DG sizes for the 12-bus test system for every single bus range from 0.222 to 0.4 MW, as illustrated in Figure 5. However, it is critical to determine the place where the overall power loss is lowest.

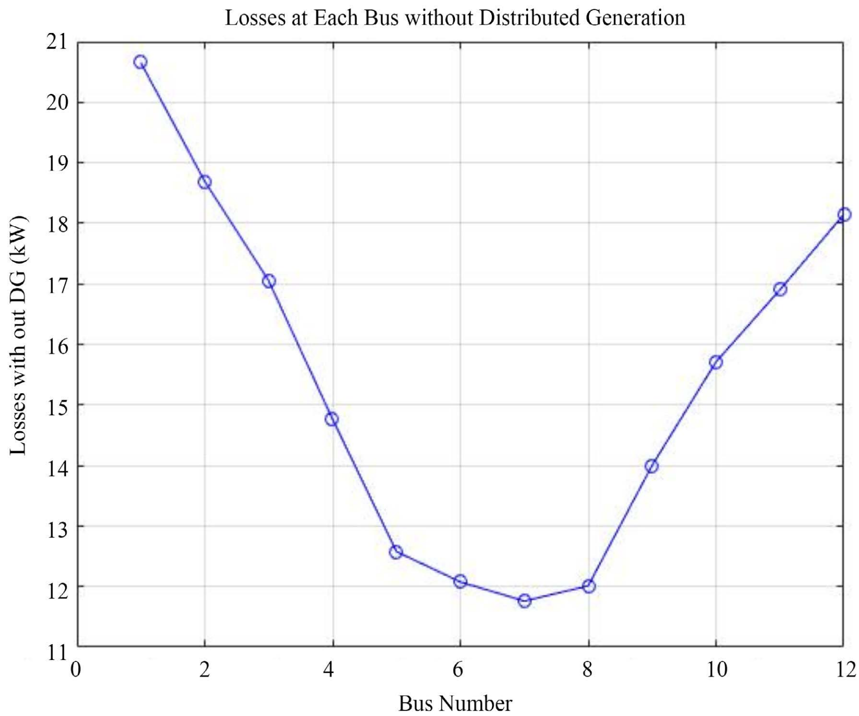

Figure 4. Losses of 12 bus distribution test system.

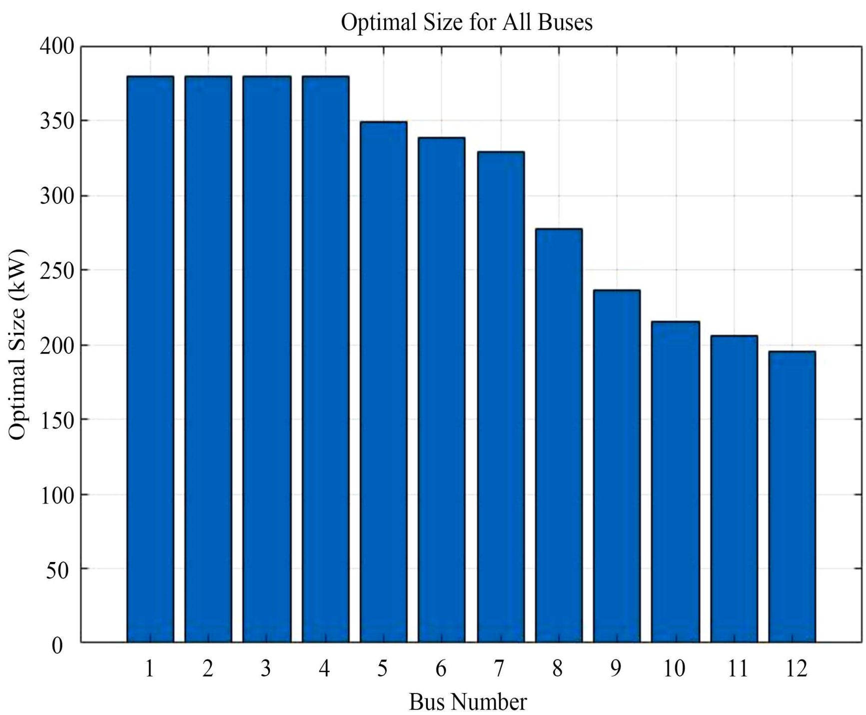

Figure 5. Optimum size of DG at all buses for 12 bus distribution system.

Figure 6 illustrates the overall power losses for the 12-bus test system, along with increasing PV-DG sizes at all buses within this system. The figure also displays the precise loss values. It can be observed, that the loss trends are effectively captured using the proposed solution, which proves to be sufficiently accurate to determine the sizes that result in minimizing the power losses. It is observed that the approximate loss patterns of the system, with optimally sized DG at all buses, closely match the actual losses in all scenarios.

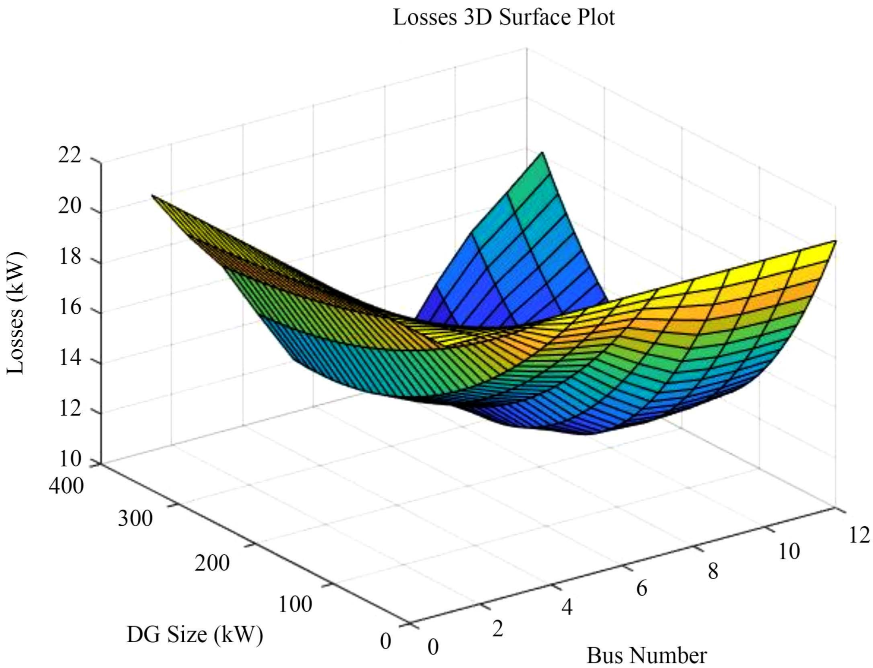

Figure 6. Losses, DG size of 12 bus distribution test system.

4.2. LSF Method Results

The algorithm iteratively identifies the optimal placement and size of DGs by calculating sensitivity values for each bus. Buses with higher sensitivities (*i.e.*, greater reduction in system losses per unit DG added) are prioritized for DG placement. This process continues until no further significant reduction in system losses is observed or the bus voltages reach the maximum allowable limit.

Figure 7 is a crucial plot that provides a visual summary of the effectiveness of the DG placement algorithm. It highlights how iterative placement of DGs, based on sensitivity analysis, leads to a reduction in system losses. The convergence of the plot indicates the point at which further iterations yield diminishing returns, marking the near-optimal state of the system. This figure is essential for validating the performance of the optimization algorithm and demonstrating its impact on improving the efficiency of the power distribution network.

The plot typically shows a significant reduction in losses during the initial iterations, followed by a gradual reduction as the algorithm converges. The point at which the plot starts to flatten out indicates that adding more DGs does not result in significant further reductions in losses. This is a sign of convergence, where the system has reached a state where optimal or near-optimal DG placement has been achieved. The figure, indicating that it depicts the reduction of power losses over multiple iterations of PV-DG placement based on sensitivity analysis.

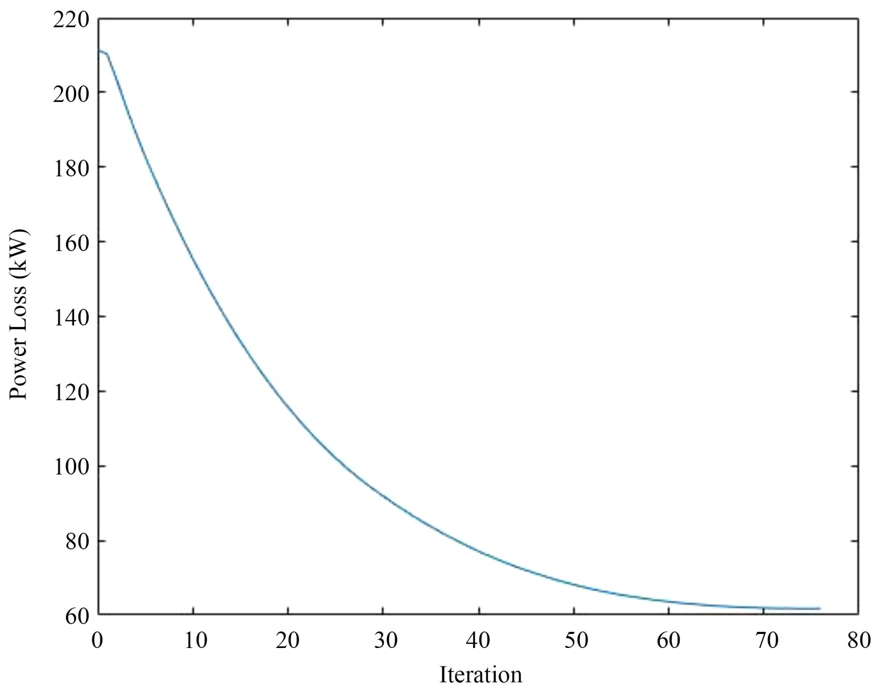

Figure 7. Power loss reduction for iterations in sensitivity-based DG placement.

Figure 8 represents a bar chart showing the optimal DG sizes at each bus after the iterative sensitivity-based optimization process. This plot provides a clear visualization of how much PV-DG size is assigned to each bus. Figure 8 is an important visualization that shows the outcome of the sensitivity-based DG placement algorithm. It highlights the optimal DG sizes at each bus, providing a clear picture of how PV-DG generation resources are allocated across the power distribution network. This figure helps in understanding the distribution of DG capacities and the effectiveness of the optimization process in reducing system losses.

Figure 9 visualizes the relationship between bus numbers, the optimal PV-DG sizes at each bus, and the corresponding system losses. The figure provides a detailed view of how different DG sizes at various bus locations influence the overall system losses. It helps identify the optimal DG size that minimizes system losses across the network. The visual pattern in the mesh may reveal trends or specific buses where DG setup plays a crucial role in reducing losses. The scatter plot in Figure 10 highlights which buses are most sensitive to DG installation, meaning that small changes in DG size at these locations result in significant loss reductions. The lines connecting the points in the plot emphasize the trend between these variables, making it easier to visualize how the sensitivity of each bus correlates with DG size and overall system performance.

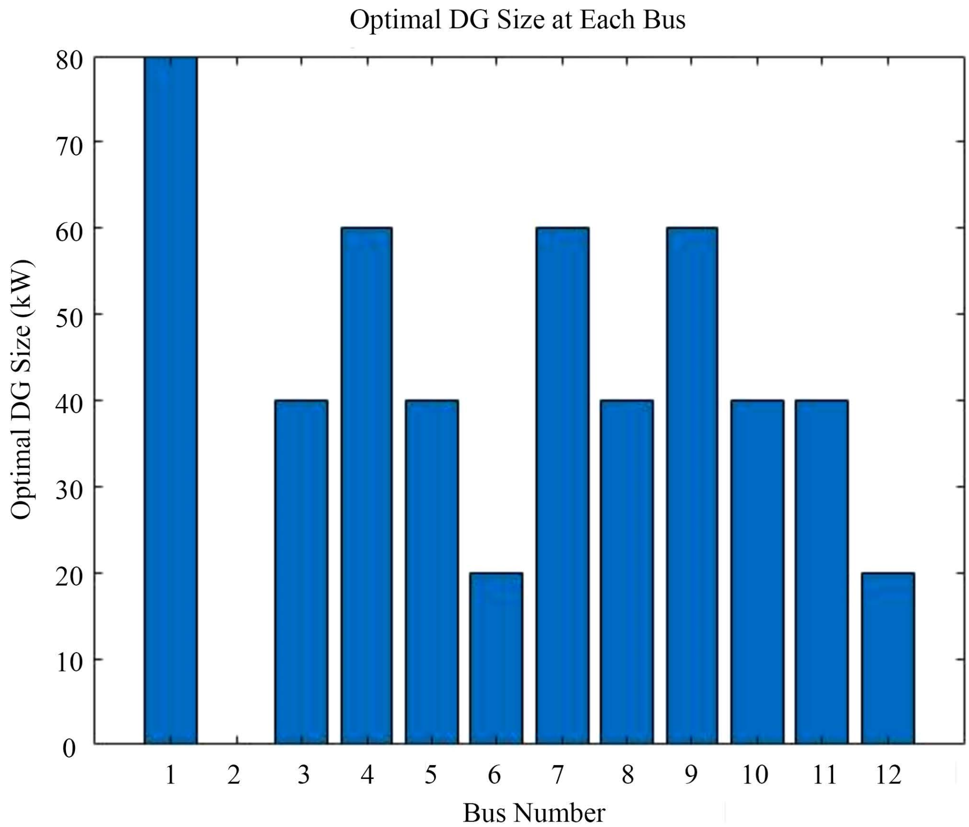

Figure 8. Optimal DG Sizes at estimated Buses based Sensitivity Optimization.

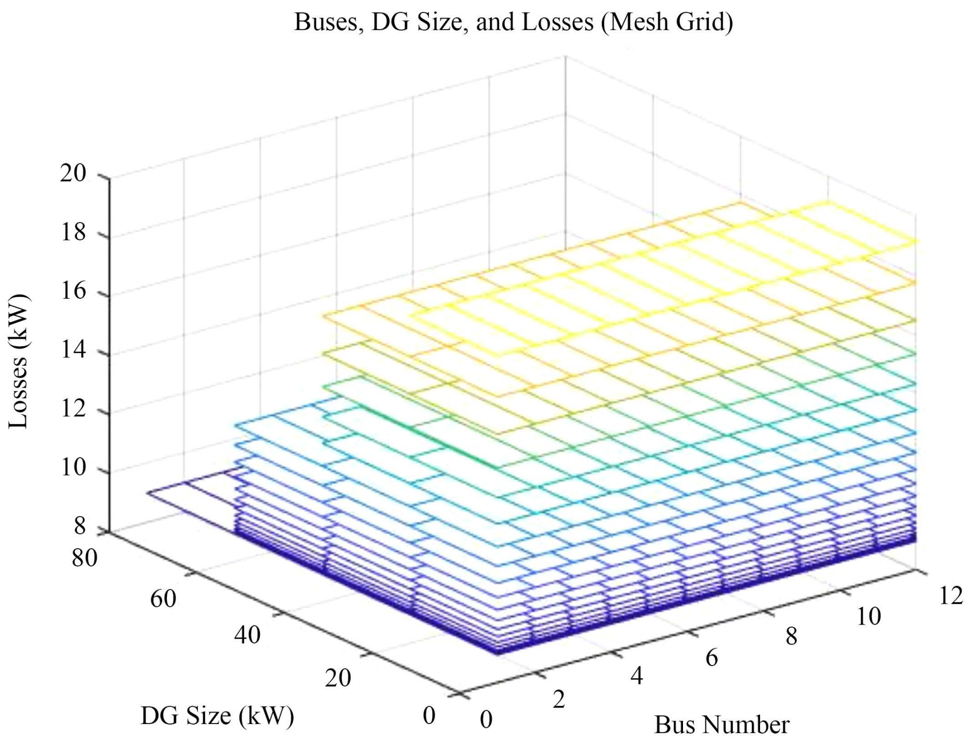

Figure 9. Losses at each bus after sensitivity-based optimization.

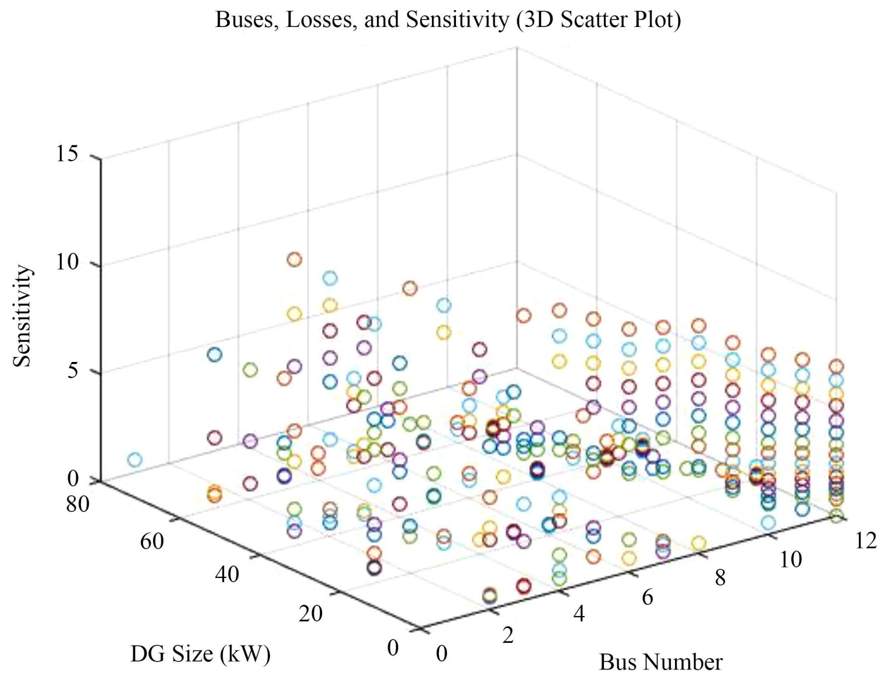

Figure 10. Sensitivity at each bus-based optimization.

4.3. Results Comparisons

The overall optimal location is at bus 9, with an overall optimal size of 234.6939 kW. This leads to a decrease in power losses to 10.7133 kW, as presented in Table 1 for only a single PV-DG. Table 2 presents multiple PV-DG units in 12 bus test system. Also, the table compares with other research studies in cases of 2 and 3 PV-DG and in cases of type 1 and type 2.

Table 1. 12 bus test system optimisation results

Test System	Numerical	Loss sensitivity	ELF	GA	PSI	VSM
	This research				[11]	[12]
DG location	9	9	9	9	9	9
DG size (kW)	234.694	260	260	237	234.9	235
real power loss (kW)	10.7113	10.713	10.7127	10.7139	-	10.77397

At power factor of unity, for single PV-DG, the best active power Losses are 10713.941301 W, and the best reactive power losses are 2711.656269. However, at power factor of 0.95 lagging, for one PV-DG, the best active power losses are 5892.779127 W, and the best reactive power losses are 786.838996 var. At power factor of unity, for the case of double PV-DG, the best active power losses are 9355.135275 W, and the best reactive power losses are 2241.741826 var. Also, in case of double PV-DG, and at power factor of 0.95 lagging, the active power losses are 3951.711861 W, and reactive power losses are 114.199227 var. Finally, in case of triple PV-DG, and power factor of unity lagging, the active power losses are

9095.274137 W, and the best reactive power losses are 2131.877167 var. while in 0.95 pf, the best active power losses are 3587.692807 W, and the best reactive power losses are −39.401425 var.

Figure 11 shows the single-line diagram of 12-bus system with 3 multiple PV-DG generators at locations 4, 7 and 10 to illustrate the difference with the results in Figure 8, which presents the sizes of the generation units considering the connection of the units at all locations of the 12-bus test system.

Table 2. Results of 12 bus system with multiple DG's.

	[24]		LSF		This research (GA)			
	Optimal Type location	*Optimal Size* (kW)	*Optimal Type location*	*Optimal Size* (kW)	*Optimal Type* 1 *location*	*Optimal Size* (kW)	*Optimal Type* 2 *location*	*Optimal Size* (kVAr)
1 DG	9	307	9	260	9	237	9	275
2 DG	7	239	7	170	7	184	7	212
	10	187.3	10	170	10	138	10	160
3 DG			4	140	4	113	4	130
			7	140	7	133	7	154
			10	140	10	139	10	160

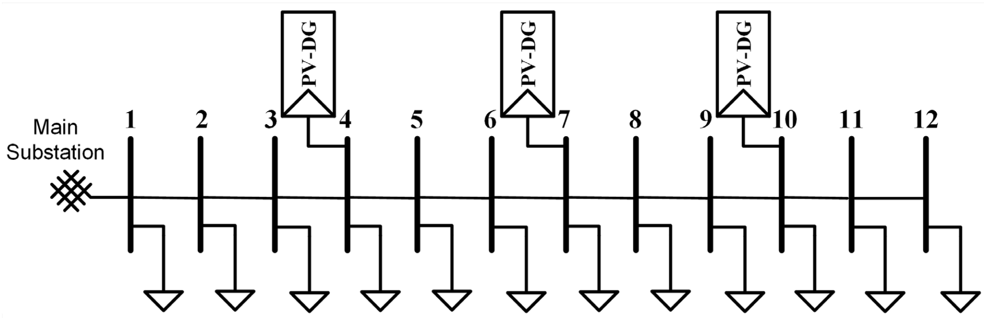

Figure 11. Single line diagram of 12-bus radial network.

5. Conclusion

In this paper, a new improved numerical approach has been proposed and tested on the radial distribution system 12 bus test system. Where the new algorithm is presented for DG allocation and sizing based on continual sensitivity analysis to determine the critical bus and minimize the total power losses. Using the proposed algorithm, the optimum DG allocation and sizing results minimize the burden of system losses. The results have been compared with GA Algorithm, ELF and LSF methods. ELF method has consumed more time than all other used algorithms. It can be noted that the triple PV-DG achieved the minimum loss compared with two and single PV-DG. Finally, this research can be considered a step towards employing dynamic model of PV-DG in power flow analysis based on time series power flow and variable solar radiation in order to present more accurate results than using a constant value to PV-DG size.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Gonzalez-Longatt, F. and Torres, J.L.R. (2018) Advanced Smart Grid Functionalities Based on Power-Factory. Springer. https://doi.org/10.1007/978-3-319-50532-9

[2] Massoud Amin, S. and Wollenberg, B.F. (2005) Toward a Smart Grid: Power Delivery for the 21st Century. *IEEE Power and Energy Magazine*, **3**, 34-41. https://doi.org/10.1109/mpae.2005.1507024

[3] Xu, X., Zhang, T., Qiu, Z. and Gao, H. (2023) A Co-Simulation Framework for Distribution Network Analysis: Case Study of Hosting Capacity Analysis. 2023 *China Automation Congress* (*CAC*), Chongqing, 17-19 November 2023, 6558-6562. https://doi.org/10.1109/cac59555.2023.10451457

[4] Bhusal, N., Abdelmalak, M., Kamruzzaman, M. and Benidris, M. (2020) Power System Resilience: Current Practices, Challenges, and Future Directions. *IEEE Access*, **8**, 18064-18086. https://doi.org/10.1109/access.2020.2968586

[5] Sridevi, J., Rani, V.U. and Rao, B.L. (2019) Integration of Renewable DGs to Radial Distribution System for Loss Reduction and Voltage Profile Improvement. 2019 1*st International Conference on Electrical*, *Control and Instrumentation Engineering* (*ICECIE*), Kuala Lumpur, 25 November 2019, 1-6. https://doi.org/10.1109/icecie47765.2019.8974670

[6] Arya, A., Verma, S.S., Mehroliya, S., Tomar, S. and Rajeshwari, C.S. (2021) Optimal Placement of Distributed Generators in Power System Using Sensitivity Analysis. In: *Lecture Notes in Electrical Engineering*, Springer, 749-759. https://doi.org/10.1007/978-981-16-1476-7_67

[7] Acharya, N., Mahat, P. and Mithulananthan, N. (2006) An Analytical Approach for DG Allocation in Primary Distribution Network. *International Journal of Electrical Power & Energy Systems*, **28**, 669-678. https://doi.org/10.1016/j.ijepes.2006.02.013

[8] Gözel, T. and Hocaoglu, M.H. (2009) An Analytical Method for the Sizing and Siting of Distributed Generators in Radial Systems. *Electric Power Systems Research*, **79**, 912-918. https://doi.org/10.1016/j.epsr.2008.12.007

[9] Ghosh, S., Ghoshal, S.P. and Ghosh, S. (2010) Optimal Sizing and Placement of Distributed Generation in a Network System. *International Journal of Electrical Power & Energy Systems*, **32**, 849-856. https://doi.org/10.1016/j.ijepes.2010.01.029

[10] Hung, D.Q. and Mithulananthan, N. (2013) Multiple Distributed Generator Placement in Primary Distribution Networks for Loss Reduction. *IEEE Transactions on Industrial Electronics*, **60**, 1700-1708. https://doi.org/10.1109/tie.2011.2112316

[11] Aman, M.M., Jasmon, G.B., Mokhlis, H. and Bakar, A.H.A. (2012) Optimal Placement and Sizing of a DG Based on a New Power Stability Index and Line Losses. *International Journal of Electrical Power & Energy Systems*, **43**, 1296-1304. https://doi.org/10.1016/j.ijepes.2012.05.053

[12] Murty, V.V.S.N. and Kumar, A. (2015) Optimal Placement of DG in Radial Distribution Systems Based on New Voltage Stability Index under Load Growth. *International Journal of Electrical Power & Energy Systems*, **69**, 246-256. https://doi.org/10.1016/j.ijepes.2014.12.080

[13] Sa'ed, J.A., Jubran, M.K., Favuzza, S. and Massaro, F. (2016) Reassessment of Voltage Stability for Distribution Networks in Presence of DG. 2016 *IEEE* 16*th International Conference on Environment and Electrical Engineering* (*EEEIC*), Florence, 7-10 June

2016, 1-5. https://doi.org/10.1109/eeeic.2016.7555686

[14] Marti, J.R., Ahmadi, H. and Bashualdo, L. (2013) Linear Power-Flow Formulation Based on a Voltage-Dependent Load Model. *IEEE Transactions on Power Delivery*, **28**, 1682-1690. https://doi.org/10.1109/tpwrd.2013.2247068

[15] Maduforo, P.C., Emechebe, J.N., Eronu, E.M. and Benson, S.A. (2020) Sensitivity Based Approach for the Optimal Sizing and Allocation of Distributed Generation in a Radial Network. *European Journal of Engineering Research and Science*, **5**, 751-756. https://doi.org/10.24018/ejers.2020.5.6.1978

[16] Sa'ed, J.A., Ismail, N., Favuzza, S., Ippolito, M.G. and Massaro, F. (2015) Effect of Voltage Deviations on Power Distribution Losses in Presence of DG Technology. 2015 *International Conference on Renewable Energy Research and Applications* (*ICRERA*), Palermo, 22-25 November 2015, 766-771. https://doi.org/10.1109/icrera.2015.7418515

[17] Hung, D.Q., Mithulananthan, N. and Bansal, R.C. (2010) Analytical Expressions for DG Allocation in Primary Distribution Networks. *IEEE Transactions on Energy Conversion*, **25**, 814-820. https://doi.org/10.1109/tec.2010.2044414

[18] Ayanlade, S.O., Ariyo, F.K., Jimoh, A., Akindeji, K.T., Adetunji, A.O., Ogunwole, E.I., *et al*. (2023) Optimal Allocation of Photovoltaic Distributed Generations in Radial Distribution Networks. *Sustainability*, **15**, Article 13933. https://doi.org/10.3390/su151813933

[19] Prabawa, P. and Choi, D. (2021) Hierarchical Volt-Var Optimization Framework Considering Voltage Control of Smart Electric Vehicle Charging Stations under Uncertainty. *IEEE Access*, **9**, 123398-123413. https://doi.org/10.1109/access.2021.3109621

[20] Vlachogiannis, J.G. and Østergaard, J. (2009) Reactive Power and Voltage Control Based on General Quantum Genetic Algorithms. *Expert Systems with Applications*, **36**, 6118-6126. https://doi.org/10.1016/j.eswa.2008.07.070

[21] Thangaraj, Y. and Kuppan, R. (2017) Multi-Objective Simultaneous Placement of DG and DSTATCOM Using Novel Lightning Search Algorithm. *Journal of Applied Research and Technology*, **15**, 477-491. https://doi.org/10.1016/j.jart.2017.05.008

[22] Kashem, M.A., Ganapathy, V., Jasmon, G.B. and Buhari, M.I. (2000) A Novel Method for Loss Minimization in Distribution Networks. *DRPT*2000. *International Conference on Electric Utility Deregulation and Restructuring and Power Technologies*. *Proceedings* (*Cat*. No.00EX382), London, 4-7 April 2000, 251-256. https://doi.org/10.1109/drpt.2000.855672

[23] Das, D. (1994) Novel Method for Solving Radial Distribution Networks. *IEE Proceedings—Generation*, *Transmission and Distribution*, **141**, 291-298. https://doi.org/10.1049/ip-gtd:19949966

[24] Sa'ed, J.A., Amer, M., Bodair, A., Baransi, A., Favuzza, S. and Zizzo, G. (2019) A Simplified Analytical Approach for Optimal Planning of Distributed Generation in Electrical Distribution Networks. *Applied Sciences*, **9**, Article 5446. https://doi.org/10.3390/app9245446

Appendix

Line & load data of IEEE 12-bus radial distribution system.

Branch no	Sending end	Receiving end	Line Data		Load data		
			R (ohms)	X (ohms)	Node no.	P_L (kW)	O_L (kVA)
					1	0	0
1	1	2	1.093	0.455	2	60	60
2	2	3	1.184	0.494	3	40	30
3	3	4	2.095	0.873	4	55	55
4	4	5	3.188	1.329	5	30	30
5	5	6	1.093	0.455	6	20	15
6	6	7	1.002	0.417	7	55	55
7	7	8	4.403	1.215	8	45	45
8	8	9	5.642	1.59	9	40	40
9	9	10	2.89	0.818	10	35	30
10	10	11	1.514	0.428	11	40	30
11	11	12	1.238	0.351	12	15	15