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Bose-Einstein Condensation $Yb_2Si_2O_7$ Retaining Si Applied in UV-Etching 1 nm Quantum Wires

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Abstract

A Bose-Einstein condensate (BEC) is a topic of significant interest within the scientific community. It is well understood that Rb-87 and $Yb_2Si_2O_7$ have been utilized in experiments to explore this phenomenon. These studies have demonstrated that these materials can achieve the BEC phase, a state that has been experimentally validated. In this paper, we further establish, from the perspective of theoretical physics, that silicon is also capable of exhibiting BEC properties. Our approach differs from prior studies in that it uses innovatively certain boundary conditions. Specifically, we employed Yb-70 as a gamma-ray radiation source and a 1 nm linewidth (as the half-width of a 2 nm line). Additionally, we utilized the concept of half-value thickness from nuclear physics absorption models to optimize the semiconductor process. This method effectively removes ytterbium (Yb) during the process, leaving only silicon, silicon-based materials, or silicon topological superconductors on the wafer. This technical procedure results in the creation of "BEC silicon" at absolute zero temperature (0 K), introducing a novel material for BEC realization.

Keywords

Rb-87, BEC Phase, Yb-70, BEC Silicon, Absolute Zero Temperature

1. Introduction

In Ref. [1], it is well established that the first complete realization of Bose-Einstein Condensation (BEC) in the history of science occurred on June 5, 1995, at MIT and the University of Colorado Boulder in the United States. In Ref. [2], scientists discovered that silicon alloys $Yb_2Si_2O_7$ can successfully achieve the BEC phase. This finding represents a significant milestone in materials physics, highlighting

the profound potential of silicon-based materials. It suggests that silicon, silicon alloys, and silicon topological superconductors possess either latent potential or demonstrated capability to achieve the BEC phase. The findings discussed in Refs. [3]-[25] demonstrate that the technology of Electromagnetically Induced Transparency (EIT) for light storage (quantum storage or mode locking) relies on laser light sources provided externally rather than generated internally by $Yb_2Si_2O_7$. By effectively applying the concept of half-value thickness from nuclear physics absorption models, it can be concluded that the ytterbium (Yb) atoms in a Bose-Einstein Condensate (BEC) may be considered absent. This also implies that single-crystal silicon ingots could be excluded at the initial stages of manufacturing. In other words, doping substrates with Yb atoms is unnecessary (as supported in Appendix A), and Yb can be eliminated entirely from the process.

This paper addresses a long-standing issue in the semiconductor industry: the challenges posed by 1-nanometer circuit nodes (quantum wires). Our work derives and verifies a solution to this critical problem. We have conducted extensive pioneering studies in this area and are now presenting them here. The focus of this paper is to explore whether silicon can achieve BEC and to conduct verification work related to this hypothesis.

Additionally, we have demonstrated that electron transport through 0.5-nanometer channels is fundamentally prohibited due to quantum mechanical scattering states. Specifically, electrons are scattered by the potential barriers within the channels. Detailed discussions and supporting evidence can be found in Appendices B and C.

We aim to ensure that these discussions align with experimental procedures and engineering realities. We confirm that this work is the same as the current BEC experiments. This is based on the properties of solid silicon crystals, which naturally play the role of a MOT, and the process of BEC laser cooling silicon is also the same as that of the current laser cooling Rb-87.

2. Method

2.1. Discourses on the Existence of Silicon in BEC Phases

Based on two fundamental assumptions outlined in Sect. 2.3, the energy gap of diamond-structured silicon at room temperature is denoted as:

$$E_g\left(\mathrm{Si}, 300\ \mathrm{K}\right) = \frac{\left(\hbar\kappa\right)^2}{2m_e} \tag{1}$$

As is widely known, this is an experimental value where the wave vector κ is used instead of k, since electrons are assumed to exist within the periodic potentials of the lattice structure in single-crystal silicon ingots. When employing Bose-Einstein Condensation (BEC) technology to form Fermi condensation, and in accordance with the principle of energy conservation, it follows that:

$$E_g\left(\mathrm{Si}, 0\ \mathrm{K}\right) = 1.12\ \mathrm{eV} - E_F = 1.12\ \mathrm{eV} - \frac{\left(\hbar k\right)^2}{2m_e} \tag{2}$$

where $C \equiv 1.12$ eV. Therefore,

$$E_g\left(\mathrm{Si},0\ \mathrm{K}\right) = C - \lim_{\substack{T_c \to 0+ \\ T_\gamma \to T_c}} \frac{\left(\hbar k\right)^2}{2m_e}\frac{T_\gamma}{T_c} \overset{(L.H.)}{=} C - \lim_{\substack{T_c \to 0+ \\ T_\gamma \to T_c}} \frac{\left(\hbar k\right)^2}{2m_e}\left(\frac{\mathrm{d}T_\gamma}{\mathrm{d}T_\gamma}\Big/\frac{\mathrm{d}T_c}{\mathrm{d}T_\gamma}\right) \tag{3}$$

(Here two fundamental assumptions would be provided in later sections.)

Hence

$$E_g\left(\mathrm{Si},0\ \mathrm{K}\right) = C - \lim_{\substack{T_c \to 0+ \\ T_\gamma \to T_c}} \frac{\left(\hbar k\right)^2}{2m_e}\left(\frac{\mathrm{d}T_c}{\mathrm{d}T_\gamma}\right)^{-1} \tag{4}$$

When the single-crystal silicon ingot is cooled using a laser, and based on the equation we derived theoretically (partially inspired by the seminal experiment (^{87}Rb, 170 nK) conducted in 1995), namely,

$$T_\gamma = \left(1+\frac{1}{10^\gamma}\right)T_C, \gamma \geq 3 \tag{5}$$

The minimum $\gamma = 3$ is specifically chosen. The "quantum temperature" (as we refer to it here) of the photons possess[1]—derived as a general result of black-body radiation—is given by

$$T_\gamma = \left(1+1/1000\right)T_C \tag{6}$$

Thus we have:

$$\mathrm{d}T_\gamma = \left(1+1/1000\right)\mathrm{d}T_C \tag{7}$$

By substituting the above equation back into Equation (4), we obtain:

$$\begin{aligned} E_g\left(\mathrm{Si},0\ \mathrm{K}\right) &= C - \lim_{\substack{T_c \to 0+ \\ T_\gamma \to T_c}} \frac{\left(\hbar k\right)^2}{2m_e}\left(\frac{\mathrm{d}T_c}{\left(1+1/1000\right)\mathrm{d}T_c}\right)^{-1} \\ &= C - \frac{\left(\hbar k\right)^2}{2m_e}\left(1+1/1000\right) \end{aligned} \tag{8}$$

With $C = \dfrac{\left(\hbar\kappa\right)^2}{2m_e}$ substituted back into Equation (8), hence

Similarly, with $C = \dfrac{\left(\hbar\kappa\right)^2}{2m_e}$ substituted back into Equation (8), we derive:

$$\begin{aligned} E_g\left(\mathrm{Si},0\ \mathrm{K}\right) &= \frac{\left(\hbar k\right)^2}{2m_e} - \frac{\left(\hbar k\right)^2}{2m_e}\left(1+\frac{1}{1000}\right) \\ &= \frac{\left(\hbar k\right)^2}{2m_e}\left[1-\left(1+\frac{1}{1000}\right)\right] \\ &= -\frac{1}{1000}\cdot\frac{\left(\hbar k\right)^2}{2m_e} \end{aligned} \tag{9}$$

[1]In thermal equilibrium, the energy that the photon gas has, which reflects to the energy of the absorption material of silicon is $E = nh\nu = \frac{1}{2}\left(\gamma kT + h\nu\right)$ where $n > 1$, $\gamma = 3$. Via an approximation by large number we confirm that $T_\gamma \equiv \left(1+\varepsilon\right)T_c$, $\varepsilon \equiv 1/10^\gamma$.

Since

$$k \xrightarrow{\hat{T}} \kappa \tag{10}$$

Therefore,

$$E_g(\mathrm{Si}, 0\ \mathrm{K}) = -\frac{1}{1000} \cdot \underbrace{\frac{(\hbar\kappa)^2}{2m_e}}_{=E_g(\mathrm{Si}, 300\ \mathrm{K})} < 0 \tag{11}$$

It is evident that as the temperature of the single-crystal silicon ingot approaches absolute 0 K, the energy gap of the ingot enters a bound state. This indicates that the electron becomes completely confined within the material.

When considering only the first Brillouin zone, any arbitrary wave vector in the reciprocal lattice space, based on the wave vector k, can be analyzed using Bloch's theorem. After the application of the translation operator $\hat{T}$, the resulting state becomes $k \xrightarrow{\hat{T}} \kappa$, as described above. In the next section, we introduce and discuss the concepts of coherent states.

The above discussion strongly supports the argument that 1 nm wire manufacturing necessitates the use of BEC-silicon, achieved through laser cooling. In practice, $Yb_2Si_2O_7$ can serve as a suitable BEC material, as supported by known experimental data. Furthermore, due to the influence of channel sizes (as detailed in Equations (21) - (25) below), silicon retains its prominence as a widely used material for 1 nm wires.

2.2. Examination: Consistency (I)

It is widely known that silicon atoms possess four covalent electrons in their outer shells. When silicon atoms bond with one another, they form shared electron pairs. These electron pairs exhibit physical behavior analogous to "Cooper pairs" as described in BCS theory, particularly at temperatures approaching absolute zero (0 K). For simplicity, we express the relevant quantum mechanical (QM) equations as follows:

$$\hat{E}\sum_{n=1}^{\infty}|\psi_n\rangle = E_n \tag{12}$$

$$\hat{E}\sum_{n'=0}^{\infty}|\psi_{n'}\rangle = E_{n'} \tag{13}$$

Here, $\{E_n, E_{n'}\}$ represents a mathematical "ensemble of energy eigenvalues" for quantum particles in physics. Photons produced through laser cooling are effectively described by Equation (12) (photon ensembles), while the pair-electrons in silicon atoms are accurately modeled by Equation (13) (electron ensembles).

When coherent states emerge in silicon systems, it indicates that $n = n'$, $n \neq 0$ Equation (12) specifically applies to laser photons. In this context, we attempt to compute the inner product of the states described by the above two equations. In other words,

$$\sum_{n=1}^{\infty}\sum_{n'=0}^{\infty}\left(\langle\psi_{n'}|^*\right)\cdot|\psi_n\rangle \tag{14}$$

Next, we apply the energy operator twice to Equation (14), yielding:

$$\hat{E}^2 \sum_{n=1}^{\infty} \sum_{n'=0}^{\infty} \left(\langle \psi_{n'} |^* \right) \cdot |\psi_n\rangle \tag{15}$$

Furthermore, leveraging the linearity of $\hat{E}$, we derive:

$$\hat{E} \sum_{n=1}^{\infty} \left[\hat{E} \sum_{n'=0}^{\infty} \langle \psi_{n'} |^* \right] |\psi_n\rangle \tag{16}$$

By applying Equation (12) and Equation (13), and considering the photon ensembles from the laser source (as demonstrated in the calculation of the equation's right-hand side below), we obtain[2]:

$$\hat{E} \sum_{n=1}^{\infty} \left[\hat{E} \sum_{n'=0}^{\infty} \langle \psi_{n'} |^* \right] |\psi_n\rangle = E_{n'}^* E_n = |E_{n'}||E_n| = |E_n|^2 = E_n^2 \tag{17}$$

Finally, taking the square root of both sides of Equation (17),

$$\hat{E} \left| \sum_{n'=0}^{\infty} \sum_{n=1}^{\infty} \langle \psi_{n'} | \psi_n \rangle \right|^{1/2} = |E_n| > 0, n = n' \tag{18}$$

And,

$$\hat{E} \left| \sum_{n'=0}^{\infty} \sum_{n=1}^{\infty} \langle \psi_{n'} | \psi_n \rangle \right|^{1/2} = 0, n \neq n' \tag{19}$$

Finally, taking the square root of both sides of Equation (17), Equation (18) reveals that coherent states emerge as $n = n'$. Consequently, we obtain the eigenvalues of $E_n > 0$ and $E_n < 0$, respectively, with the latter corresponding exactly to Equation (11).

It is important to note that the energy gap is one of the eigenvalue solutions of the Schrödinger equation in the context of solid-state physics. Through quantum mechanics (QM), we have provided strong evidence to support this paper, particularly the precise arguments presented in this section.

2.3. Examination of Consistency (II)

According to Equation (11) in Sect. 2.1, it is evident that

$$\begin{aligned} \left| E_g \left(\text{Si}, 0\ \text{K} \right) \right| &= \frac{1}{1000} \cdot E_g \left(\text{Si}, 300\ \text{K} \right) = \frac{1}{1000} \left(1.12\ \text{eV} \right) \\ &= 0.00112\ \text{eV} \approx 10^{-3}\ \text{eV} \end{aligned} \tag{20}$$

Based on the fundamental conditions of a Fermi condensate and the formation of "Cooper pairs," the system's energy for electron pairs (e.g., the energy gap) must be reduced below ~10^{-3} eV. This inference aligns well with widely recognized facts, reinforcing the consistency of our findings.

[2]Here, we focus on Equation (2) and Equation (3) and subsequently provide two fundamental assumptions for them, as outlined below.

Fundamental Assumption I

[2]To simplify the discussion, we have adopted the use of absolute value notation to exclude complex numbers when analyzing the projection of one quantum state onto another.

One of a silicon atom's covalent bonds shares two outer electrons, whose physical behavior during Fermi condensation at extremely low temperatures resembles ^{87}Rb - ^{87}Rb in solid-state systems at absolute zero.

Fundamental Assumption II

Considering the Fermi-Dirac distribution, the probability of 1/2 corresponds to the energy level at which an electron occupies the Fermi energy level. This resultsin $0 = \frac{\varepsilon - \mu}{kT}$, which is directly associated with the discussion above. Hence, we can sufficiently assume $k\varepsilon = k\mu$, and it is evident that $1 = k\mu/k\varepsilon \equiv T_\gamma / T_C$ at absolute 0 K. Notably, electrons are nearly fully populated at the Fermi energy ε during Bose-Einstein Condensation (BEC). As the system's temperature is further reduced toward absolute zero, the Fermi energy of the electron aligns precisely with the Fermi energy level. Consequently, the electron possesses $1 = T_\gamma / T_C$, as described in the assumption above. Furthermore, this relationship forms an "implicit equation," which can be mathematically expressed as $T_\gamma - T_C = 0$, using principles from calculus.

3. Results And Discussion

3.1. Results Analysis

Let us directly apply the technique of topological mapping in advanced physical mathematics (*i.e.*, conformal mapping) to the manufactured line widths of the material $Yb_2Si_2O_7$, such that

$$2\text{ nm} \mapsto 1\text{ nm} \tag{21}$$

This is because 1 nm is precisely half the width of 2 nm, and a quantum barrier occurred at solid angle of $\Omega = 4\pi$ (refer to Appendix B). It is widely known that the half-value thickness in an absorption model of nuclear physics refers to the thickness of a barrier material required to reduce the intensity of an incident ray to half its original value. Consequently, the value of the semi-absorbing thickness is

$$d_{1/2} = \frac{\ln 2}{\gamma} \approx \frac{0.693}{\gamma} \neq 0 \tag{22}$$

Here, γ denotes the absorption coefficient of the barrier material for incident rays. It is well known that when the circuit's short width is 2 nm, no significant quantum effects occur (as commonly understood). Consequently, there is no need to consider whether it could represent a Bose-Einstein Condensate (BEC) phase.

However, when the circuit component is exactly reduced to 1 nm, the situation changes significantly. According to the Fresnel diffraction formula and the Huygens-Fresnel principle in near-field optics, a pronounced quantum tunneling effect is observed. Notably, the probability of the electron's tunneling effect in a 2 nm circuit is precisely $1/e$ times that of a 1 nm circuit. Considering the physical significance of "skin depth" and referencing Equation (21), we have strictly

derived[3]:

$$0.5d_{1/2} = \frac{\ln 1}{\gamma'} = 0 \tag{23}$$

Here, $\gamma' \equiv 2\gamma$. This result arises from considering the inverse scenario of using Yb as a "gamma-ray source" at the 1 nm circuit component line width's node. Using this perspective, we have identified the following mathematical techniques to calculate the "half-value thickness of absorption" for gamma-rays in the circuit component line width (specifically, the numerator of ln2 in Equation (22)):

$$\left.\begin{aligned} &\frac{\mathrm{d}(x-1)!}{(x-1)!} = \mathrm{d}\left(\ln(x-1)!\right) \approx \mathrm{d}\left((x-1)\ln(x-1)-(x-1)\right), x \neq 1 \\ &\frac{\mathrm{d}x}{x} = \mathrm{d}(\ln x), \int_{x=1\,\mathrm{nm}}^{2\,\mathrm{nm}} \mathrm{d}(\ln x) \overset{(B.C.)}{=} \ln x\Big|_{x=1\,\mathrm{nm}}^{2\,\mathrm{nm}} = \ln 2 \approx 0.693 \end{aligned}\right\} \tag{24}$$

It is not difficult to find that the above is in agreement with what is revealed by the half-value thickness of an absorption model in nuclear physics.

Furthermore, by considering the continuous variations in the component line width during the manufacturing process (*i.e.*, a homeomorphism mapping, where all manufacturing processes can theoretically be represented along the time-axis, $2\,\mathrm{nm} \mapsto 1\,\mathrm{nm}$, $1\,\mathrm{nm} \mapsto 1^{-}\,\mathrm{nm}$), thus we obtain:

$$\left.\begin{aligned} &\frac{\mathrm{d}(x-1)!}{(x-1)!}\Big|_{x=2} \approx \mathrm{d}\left((x-1)\ln(x-1)-(x-1)\right)\Big|_{x=2} = \ln(x-1)\mathrm{d}x\Big|_{x=2} = 0 \\ &\frac{\mathrm{d}x}{x} = \mathrm{d}(\ln x), \int_{x=1^{-}\,\mathrm{nm}}^{1\,\mathrm{nm}} \mathrm{d}(\ln x) \overset{(B.C.)}{=} 0^{+} \end{aligned}\right\} \tag{25}$$

Evidently, the result of the formula in Equation (25) is physically equivalent to $\ln 1 = 0$ (*i.e.*, the extremely small error can be completely ignored in physics). Thus, it is identical to the numerator of Equation (23).

It is important to note that after topological mapping, the left and right boundary conditions (B.C.) of the 1 nm component line width become significantly narrower than those described in Equation (24), as indicated by $\left(1-1^{-}\right)\mathrm{nm} \ll 1\,\mathrm{nm}$.

3.2. Theoretical Analysis

According to Equation (23), the gamma-ray's penetrating ability is revealed to be zero, which physically signifies "no penetration." This paradox can be resolved in one of two ways:

(A) the absence of gamma-ray photons, or (B) the component has automatically undergone light storage (or quantum storage).

[3]In the Fresnel approximation, supposed that a nano tunnel in 2nm, the widely-known expression as practical material of silicon, $\rho = \pm\sqrt{(x_1 - x_2')^2 + (x_2 - x_2')^2}$, $r = \sum_{i=1}^{3} x_i$ for the wavelengths of gamma-rays: $\lambda_\gamma \ll \rho \ll x_3$ and then $r \approx x_3 + \frac{\rho^2}{2x_3}$, $\Omega = 4\pi$ such that $r \approx 1 + \frac{\rho^2}{2}$ and $r' \approx 2 + \frac{\rho^2}{4}$. Therefore $\frac{r'}{2} = 1 + 4 \times \frac{\rho^2}{8}$ with $\psi_n = 0$, this leads to corresponding: $\ln 2 \to \ln 1$.

It is clear that case (B) must be discarded. This is because light storage relies on photons [3]-[25] supplied by an external EIT light source rather than being generated internally by Yb atoms. Therefore, the correct solution is case (A): the source of gamma-rays does not exist or is not necessary initially. Consequently, it becomes evident that doping Yb is redundant and unnecessary in the semiconductor manufacturing process for a 1 nm line width. It is worth noting that the above represents a simple and elegant application of phenomenological techniques in theoretical physics.

It is important to note that the $U(1)$ gauge symmetry breaking discussed in Ref. [2] actually refers to the $SU(3)\times U(1)$ gauge symmetry group. Specifically, this indicates that the physical size of the electron remains constant at the turning point $\left(r_0 \approx 1.954 \text{ fm}\right)$ of the Yukawa potential [26]. (The detailed discussion of this topic is lengthy and highly complex; we will provide full proof in a forthcoming paper.)

In this context, electrons and photons involved in the EIT effect remain quantum entangled to sustain Bose-Einstein Condensation (BEC) at extremely low temperatures approaching absolute zero. This quantum entanglement manifests in the behavior of photoelectrons, wherein these two elementary particles $\left(\gamma, e^{-}\right)$ exchange their wave functions during the process [3]-[25]. Evidently, silicon is entirely suitable as a material for achieving BEC, reinforcing its critical role in the semiconductor industry. The reasons for this have already been successfully deduced using principles of solid-state physics, as discussed earlier.

Particular we indicated it that in Equation (C.2), since that in current semiconductor industrial, where one overcomes the problem of electron tunneling effects (*i.e.*, so-called "punch through") is by using solutions: the method named "high-κ dielectric". As far, we can see later on (Equation (C.2) in Appendix C), the terms of voltage: V (inversely proportional to above $\sqrt{\kappa}$ dielectric ($1/t \propto \kappa$)) such reflects to the shorter thickness t of the tunnels (notice that the structure is around 3 nm). This complete agrees with the deductions and physical pictures by Equations. (21) - (25). Since that at constant wavelengths, the higher κ dielectric as smaller thickness of the tunnels, but the difference is that we adopt the BEC laser and the EIT artificial, in contrast, instead of adopting the solution by "high-κ dielectric"[4]. This grant reduces its multiple-steps in semiconductor manufacturing, and simultaneously obtained the same results of extending Moore's law.

4. Conclusion

Based on the discussions in **sections 2 and 3 previously**, we have deduced that

[4]In other words, it is not essential that the structures required such as Gate-SiO_2-Si Substrate to be manufactured in a semiconductor manufactures by this paper. (BEC Si plus EIT can be feasible.) Here given reasons: $\varepsilon_r = \varepsilon(\omega)/\varepsilon_0$ and due to Equation (11), Equation (17), and Equation (18) ω in BEC is obviously coherent. Therefore, BEC resonant ω is regarded as contributing to imaginary parts of dielectric constants: $\varepsilon_r \approx \frac{3 \text{ nm}}{1 \text{ nm}} \varepsilon(\omega)' = 3 \times \underbrace{3.9}_{SiO_2} \approx \underbrace{11.68}_{Si}$ which is compared with 3 nm (Gate-SiO_2-Si substrate) and 1nm tunnels. It also can be computed by Kramers–Kronig relations that we confirm in Appendix G.

silicon can serve as one of the standard materials for a Fermi condensate. With appropriate transformations or technical manipulations—typically within the framework of BCS theory (e.g., effective applications of Cooper pairs)—a Fermi condensate can be sufficiently transformed into a Bose-Einstein condensate. In the manufacturing process, the intrinsic properties of silicon are preserved. Using quantum processing technologies, such as the so-called "light storage of EIT," silicon can be advanced into a novel material known as a "silicon topological superconductor." However, the fabrication of 0.5 nm channels is not feasible (it is explicitly prohibited). For further details on this specific case, refer to Appendices B and C.

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Conflicts of Interest

The authors declare no conflicts of interest.

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Appendix A: Numbers of Atoms Allowed

Fundamentally, $Yb_2Si_2O_7$ contains 11 atoms but does not allow its 11th atom to be accommodated within 1-nanometer wires, assuming an average atomic size of 0.1 nm, *i.e.*,

$$11\cdot\left(0.1\text{ nm}\right)=1.1\text{ nm}\geq 1\text{ nm} \tag{A.1}$$

Clearly, a matter of 0.1 nm cannot fit, indicating that one atom would be cut, leading to boundary effects. Such effects are not permissible in a 1-nanometer channel.

For silicon, with a lattice constant of $a=0.543\text{ Å}\approx\frac{1\text{ nm}}{18.4162}$ and a face-centered cubic (F.C.C.) structure, silicon atoms can be fully accommodated without cutting. This configuration allows for a maximum of 10 silicon atoms to be arranged within 1-nanometer wires.

Restriction of Boundary Effect: Nanowires cannot permit non-integer numbers of atoms to be accommodated within a channel. As the size of the nanowire decreases, the ratio of surface atoms to the total number of atoms increases, making the "boundary effect" more pronounced [27]. At this scale, if an atom is cut, it leads to a violation of the boundary effect, which also contradicts the quantization effect. As noted in Ref. [27], quantum size effects, quantum interference effects, and general quantization effects strongly support the arguments presented in this paper.

Remark.

Considering the atomic radius of silicon (111 pm), we next examine the constraints of a 1 nm channel. Specifically, this requires the exclusion of two silicon atoms to avoid boundary effects. Based on this, we initially obtain

$$\begin{aligned}&\frac{1\text{ nm}}{2\times 111\text{ pm}+a}-2=\frac{10^{-9}\text{ m}}{222\times 10^{-13}\text{ m}+0.543\text{ Å}}-2\approx 13.072-2=11.072,\\&\left[11.072\right]=11\end{aligned} \tag{A.2}$$

Returning to Equation (A.1), it becomes evident that one additional silicon atom must also be excluded artificially. Consequently, the final result is:

$$\frac{1\text{ nm}}{2\times 111\text{ pm}+a}-3=10.072,\left[10.072\right]=10 \tag{A.3}$$

This implies that only 10 silicon atoms can be accommodated within 1 nanowire. Such a reversal solution for Equation (A.1) indicates that $Yb_2Si_2O_7$ cannot be applied to 1 nanowire unless Yb atoms are not doped, leaving Si_2O_7.

However, even in this scenario, $Yb_2Si_2O_7$ still provides valuable insights (or contributes ideas) that can promote experimental advancements. These advancements may help transform silicon (Si) or Si_2O_7 into novel materials when subjected to BEC laser cooling, respectively.

Appendix B: 0.5 nm Channel as a Barrier Is Forbidden in Physics

Consider a special case: wave-numbers of electrons in 1D quantum wells of 0.5 nm-channels, namely 0.5 nm channel as a barrier is forbidden in physics. Consider a special case involving the wave numbers of electrons in one-dimensional (1D) quantum wells within 0.5 nm channels. Specifically:

$$\kappa = \frac{2\pi}{\lambda} \tag{B.1}$$

If the channel is contracted from 1 nm to 0.5 nm, the boundary conditions (B.C.) of the infinite potential wells change from $x = \pm L/2$ (for the 1 nm channel, used as the reference level) to $x = \pm L/4$ (for the 0.5 nm channel).With $\kappa = \pi/\lambda$ applied, this contraction produces the eigenvalues of potential energy as follows[5]:

$$E = \frac{\hbar^2\kappa^2}{2mL^2} = \frac{\hbar^2}{2m}\left(\frac{\pi/\lambda}{L/4}\right)^2 = 16\frac{(\pi\hbar)^2}{2m}\frac{1}{(\lambda L)^2} \quad \text{in channels (1D-wells)} \tag{B.2}$$

Moreover,

$$E = \frac{\hbar^2}{2m}\left(\frac{4\pi}{L}\right)^2\frac{1}{\lambda^2} \tag{B.3}$$

Derive the above equation by varying λ when an electron is input into the 0.5 nm channel from the tuning voltage source of integrated circuits

$$\frac{\partial E}{\partial \lambda} = -\frac{\hbar^2}{m}\left(\frac{4\pi}{L}\right)^2\frac{1}{\lambda^3} < 0 \quad \text{with fixed} \quad L = 0.5\text{ nm} \tag{B.4}$$

And it is in the bound state due to the negative sign mentioned above. However, when taking the second derivative of the above equation by varying λ as an electron is "quasi-standing" in the 0.5 nm channel space (regarding this part, it shall be seen in new sections (Appendix C)), so that

$$\frac{\partial^2 E}{\partial \lambda^2} = \frac{3\hbar^2}{m}\left(\frac{4\pi}{L}\right)^2\frac{1}{\lambda^4} \propto \frac{1}{\lambda^4}, \Omega \equiv 4\pi \tag{B.5}$$

With a constant $L = 0.5$ nm retained and solid angles $\Omega \equiv 4\pi$ considered in a spherical coordinate system (Refer to Ref. [28]), the actual result implies Rayleigh scattering in the case of a Fermi gas particle as the subject of interaction—provided we disregard the effects of the Pauli exclusion principle and spin (*i.e.*, treating it as a classical particle for faster solutions). Note that the barrier, acting as the scattering center, is exactly half the width of the 1 nm channels (*i.e.*, 0.5 nm channels). Since scattering states occur in 0.5 nm channels, we propose that it is unnecessary to produce these in physical engineering based on this section's deduction, thereby avoiding unnecessary waste. Notably, other solutions, such as destructive interference in quantum mechanics, can be explored to address this issue. Therefore with eigenfunctions:

[5]The electron Hamiltonian is given as $H = E + V(x)$, since it behaves as a free particle when affected, so that $H = E$.

$$\begin{aligned}
&\psi_n = \sqrt{\frac{2}{L/2}}\sin\left(\frac{n\pi x}{L/2}\right), x \neq 0, \\
&\psi_n'' = -\sqrt{\frac{2}{L/2}}\left(\frac{n\pi}{L/2}\right)^2 \sin\left(\frac{n\pi x}{L/2}\right), x \neq 0, \\
&n = 1, 2, 3, \cdots
\end{aligned} \tag{B.6}$$

The above equation implies that electrons disappear at the scattering center $(x = 0)$. In other words, in the case of $x = 0.05$ nm,

$$\begin{aligned}
&\psi_n = \sqrt{\frac{2}{L/2}}\sin\left(\frac{n\pi x}{L/2}\right)\Bigg|_{x=L/2=0.5\text{ nm}} = 0, \\
&n = 1, 2, 3, \cdots
\end{aligned} \tag{B.7}$$

Thus, electrons are completely absent within the 0.5 nm channels $(x \leq 0.5 \text{ nm})$ when affected by circuit nodes. In this case, the complete quantum mechanical solution with significant allowed values is

$$\left.\begin{aligned}
&\psi_n = \sqrt{\frac{2}{L/2}}\sin\left(\frac{n\pi x}{L/2}\right), x > 0.5 \text{ nm} \\
&\psi_n'' = -\sqrt{\frac{2}{L/2}}\left(\frac{n\pi}{L/2}\right)^2 \sin\left(\frac{n\pi x}{L/2}\right), x > 0.5 \text{ nm} \\
&n = 1, 2, 3, \cdots
\end{aligned}\right\} \tag{B.8}$$

For all space, whereas in industrial settings, the space in laboratories or fabrication facilities is considered complete.

Remark.

Note that if the scattering angle $\theta = \pi$ $\theta = \pi$ (with cylindrical coordinates chosen) possibly occurs in the 0.5 nm channel, this choice retains valid, since that

$$\Omega = 4\pi = 2\pi\left(1 - \cos\theta\right)\Big|_{\theta=\pi} \tag{B.9}$$

Appendix C: Mie's & Rayleigh's Scatterings

By applying the Heisenberg uncertainty principle, we obtain the lifetime of the quasi-standing state represented in terms of

$$\Delta\tau_{\min} = \frac{\hbar}{2\Delta E} = \frac{\hbar m}{16}\left(\frac{\lambda L}{\pi\hbar}\right)^2 \approx 3.4340\times10^{-36}\ \text{sec} \ll 10^{-9}\ \text{sec} \tag{C.1}$$

Under the given conditions $L \approx \lambda$, the photon-electronic sensitive time (10^{-9} sec) cannot immediately match the lifetime of the system. Therefore, it can be considered that the electron is initially well-scattered, resulting in outcomes similar to those observed in other solutions. We have primarily derived energy λ as a "twice," caused by electron quantum tunneling. This tunneling process occurs at the boundaries of 0.5 nm channels (quantum wells), which serve as scattering centers at $r = 0$.

It is important to note that the parameter L must approach the wavelengths of λ, making it necessary to achieve a Mie scattering pattern $(L \approx \lambda)$. Next, we examine f κ for the matter-wave properties of the electron within the 0.5 nm channel. Here, V_0 is determined by the applied voltages in the integrated circuits, which can be manually adjusted to fine-tune the required wavelengths.

$$\begin{aligned} \kappa &= \frac{2\pi}{\lambda}, \\ \lambda &= \frac{2\pi}{\kappa} = \frac{h}{p} = \frac{h}{\sqrt{2m_e V_0}} \equiv \frac{C}{\sqrt{V_0}} \end{aligned} \tag{C.2}$$

When the standing wave patterns formed by electron subjects are established within the 0.5 nm channel, and $\lambda = \frac{2\pi}{\kappa} = 0.5\ \text{nm}$ is tuned (*i.e.*, fixed at specific points a and b):

$$\left.\begin{aligned} |x_{ab}| = |x_b - x_a| = 0.5\ \text{nm} \equiv L/2 \\ L = 1\ \text{nm} \end{aligned}\right\} \tag{C.3}$$

When the wavelength is confined within the 0.5 nm channel, consider the relationship between the wavelengths and the particle size under Mie scattering conditions. Here, the scattering center (corresponding to the particle size, $L = 1\ \text{nm}$) and the wavelengths of electron matter-waves (analogous to light wavelengths) are key factors. Associated with $\lambda = 0.5\ \text{nm}$ and $L = 1\ \text{nm}$, the relationship is expressed as:

$$L \approx \lambda \tag{C.4}$$

This clearly corresponds to the Mie scattering conditions outlined in Equation (C.1). Furthermore, by manually decreasing the value of V_0 (tuned to smaller values), a greater λ would be applied. As a result, Mie scattering transitions into **Rayleigh scattering**, while maintaining a fixed L (*i.e.*, $L \gg \lambda$) to remain consistent with the prior discussions.

Appendix D: Examinations Through Mathematical Physics

Associated with Equations (B.4) - (B.8) (excluding Equation (B.7)), it is evident that the results align with the sinc function. Based on Equation (B.4) and Equation (B.5), observation leads to the corresponding wavelengths of 2λ versus the phase angle 4π, represented in figures of sinc functions. This observation is also consistent with Equation (B.1).

Rewriting Equation (B.4) such yields:

$$\frac{\partial E}{\partial \lambda} = -\frac{\hbar^2}{m}\left(\frac{4\pi}{\lambda L}\right)^2 \frac{1}{\lambda} < 0 \quad \text{with fixed} \quad L = 0.5 \text{ nm} \tag{D.1}$$

Make the same procedure, so

$$\frac{\partial^2 E}{\partial \lambda^2} = \frac{3\hbar^2}{m}\left(\frac{4\pi}{\lambda L}\right)^2 \frac{1}{\lambda^2} \propto \frac{1}{\lambda^4}, \Omega \equiv 4\pi \tag{D.2}$$

Referring to the second formulations of Equation (B.6) and Equation (B.8), we derive:

$$\frac{\psi_n''}{C} = -\sqrt{\frac{2}{L/2}} C^2 \frac{\sin(C)}{C}, x \neq 0 \tag{D.3}$$

And

$$\frac{\psi_n''}{C} = -\sqrt{\frac{2}{L/2}} C^2 \frac{\sin(C)}{C}, x > 0.5 \text{ nm} \tag{D.4}$$

where $C \equiv \frac{n\pi x}{L/2}$ is defined here.

It is evident that both Equation (D.3) and Equation (D.4) can be represented as sinc functions[6].

$$\frac{\psi_n''}{C} = -\sqrt{\frac{2}{L/2}} C^2 \sin c(C), x \neq 0 \tag{D.5}$$

And

$$\frac{\psi_n''}{C} = -\sqrt{\frac{2}{L/2}} C^2 \sin c(C), x > 0.5 \text{ nm} \tag{D.6}$$

This implies that they serve as strong evidence precisely supporting the previous discussions (*i.e.*, Equation (D.1) and Equation (D.2)). Specifically, through a step-by-step approach using mathematical physics, these results directly correspond to "Mie scattering."

[6] $\sin c^2(c)$, exhibits extremely small differences in area relative to $\sin c(c)$, making it an effective approximation. Note that, evidently the sinc functions effectively represent quantum barriers.

Appendix E: Approximation of Scattering States

In this section, we briefly demonstrate how the general interference patterns of an incident wave degenerate into particle scattering patterns.

In case of a 0.5 nm channel, when electron matter-waves encounter a slit with a width of a, the diffraction patterns observed on the screen degenerate into an optical spot governed by the properties of sinc functions. This occurs due to the limiting condition with $C \to 0$ in Equation. (D.5) and Equation (D.6) (*i.e.*, $a \approx \lambda$). In other words, when the wavelength $\lambda \cong 0.5\ \text{nm}$ of the incident wave approaches or equals the slit width a, the wave-like behavior transitions into a particle-like spot. However, in earlier sections of this paper, nano-silicon particles were arranged within a 0.5 nm channel (as described in Equation (A.1), where up to five silicon atoms can be partially accommodated within this space). Thus, the slit width can be approximately equated to the lattice spacing, estimated between 0.1 nm and 0.5 nm. These values align in terms of magnitude, allowing us to conceptually replace the slits (*i.e.*, lattice spacing) with the particles themselves. Through this reasoning, we conclude that wave-like behaviors degenerate into particle-like behaviors within the framework of quantum mechanics. This strongly supports the idea of specific scattering states, consistent with the results of the previous sections. These scattering states are well-established for a 0.5 nm channel. In summary, the work presented in this paper addresses related issues in a self-consistent and complete manner.

Appendix F: Mode Locking for Protection Tunneling of Electrons

Using Equation (C.1), within a 1 nm channel, if one aims to confine electron gas within the wires while imposing infinite pressure (or momentum), the tunneling probability becomes zero. Thus, we derive condition

$$\Delta\theta_{\min} = \lim_{\Delta P \to \infty} \frac{\hbar}{2\Delta P \Delta x}\bigg|_{\Delta x = 1\ \text{nm}} = 0 \quad \text{with mode locking} \tag{F.1}$$

This implies that the phase retains synchronized for all electrons in silicon atoms. The technological approach undoubtedly relies on Bose-Einstein Condensate (BEC) laser cooling and Electromagnetically Induced Transparency (EIT) to achieve mode locking. To ensure Moore's Law remains valid, the following must hold:

$$\text{BEC silicon} = \text{BEC laser cooling Si} + \text{EIT} \tag{F.2}$$

Appendix G: Calculation By Kramers-Kronig Relations

In Kramers-Kronig relations, the dielectric constant can be written as

$$\chi(\omega)=\chi_1(\omega)+i\chi_2(\omega) \tag{G.1}$$

where $\chi_1(\omega)$ is the real part, and $\chi_2(\omega)$ is the imaginary part (and $\omega'\in \mathrm{Im}$). Moreover,

$$\chi_2(\omega)=-\frac{2}{\pi}\Im\int_0^\infty \frac{\omega\chi_1(\omega')}{\omega'^2-\omega^2}\mathrm{d}\omega' \tag{G.2}$$

where $\Im$ means: taking imaginary parts. (The above integral must be convergence to a finite value.) In a BEC phase, obviously resonant ω keeps the same frequency, thus set $\omega\equiv 1$ [1/sec] fixed as a singular point in polar coordinates.

$$\chi_2(\omega)=\frac{2}{\pi}\Im\int_0^\infty \frac{\chi_1(\omega')}{1-\omega'^2}\mathrm{d}\omega' \tag{G.3}$$

Since that $\omega'\in \mathrm{Im}$, so that

$$\begin{aligned}\chi_2(\omega)&=\frac{2}{\pi}\Im\int_0^\infty \frac{\chi_1(i\omega)}{1-(i\omega)^2}\mathrm{d}\omega',\\ \chi_2(\omega)&=\frac{2}{\pi}\Im\int_0^\infty \frac{\chi_1(i\omega)}{\omega^2+1}\mathrm{d}\omega'\end{aligned} \tag{G.4}$$

Using the residue-method:

$$\begin{aligned}\frac{\chi_1(i\omega)}{\omega^2+1}&=\frac{\chi_1(i\omega)}{2i}\left(\frac{1}{\omega-i}-\frac{1}{\omega+i}\right),\\ Res_{\omega=i}f(\omega)&=\frac{\chi_1(i\omega)}{2i}\end{aligned} \tag{G.5}$$

Then due to the theorem of residues we have:

$$\Im\int_0^\infty \frac{\chi_1(i\omega)}{\omega^2+1}\mathrm{d}\omega'=\Im\left\{2\pi i Res_{\omega=i}f(\omega)\right\}=\Im\left\{2\pi i\frac{\chi_1(i\omega)}{2i}\right\}=\pi\chi_1(\omega) \tag{G.6}$$

Such that yields in Equation (G.4),

$$\chi_2(\omega)=\frac{2}{\pi}\Im\int_0^\infty \frac{\chi_1(i\omega)}{\omega^2+1}\mathrm{d}\omega'=2\chi_1(\omega) \tag{G.7}$$

As far, therefore:

$$\begin{aligned}&\chi(\omega)=\chi_1(\omega)+i\chi_2(\omega),\\ &\Im\chi(\omega)=\chi_2(\omega)=2\chi_1(\omega),\\ &\Re\chi(\omega)=\chi_1(\omega),\\ &\sum_{i=1}^{2}\chi_i(\omega)=\chi_1(\omega)+2\chi_1(\omega)=3\chi_1(\omega),\\ &\sum_{i=1}^{2}\varepsilon_{i,r}(\omega)=\varepsilon_1(\omega)+2\varepsilon_1(\omega)=3\varepsilon_1(\omega)\equiv 3\varepsilon(\omega)'\end{aligned} \tag{G.8}$$

Equation (G.8) complete fits the current solution way by using "*high-κ dielectric*" but the BEC laser plus EIT artificial promoted by this paper is much more advanced and affordable than it.

Notice that in Equation (G.6), the result of $\pi\chi_1(\omega)$ actually provides one that the Zak phase: $\varphi_{Zk}=\pi$. Associated with this quantity and its potential optical resonance, both ensure the symmetry-protected topological (SPT) order for Equation (F.2), namely this paper's highly-technology. In another respect, the axion-graphene system (Dr. 't Hooft condition 2D-graphenes with QFT-axions) reveals that the Berry curvature is zero (*i.e.*, a plane) as shown in Equations (36)-(37) of Ref. [29], *i.e.*, the electrons running in 1 nm wires which represent as the nature of topological invariance (*i.e.*, 3D topological insulators) undoubted. Moreover, the above deductions completely adhere *Chern-Simon theory*, since that due to Ref. [29] the *Chern-numbers* are 1,3,5 (gives Fractional Quantum Hall effect (FQHE) $\frac{1}{2},\frac{3}{2},\frac{5}{2}$) where $5/2$ denotes Dirac matters presented for Equation (F.2), and the technology provided by Equation (F.2) is a production better cheap than current graphene materials. —The big breakthrough.

Claims

***Point 1**: If a 0.5 nm channel is positioned at the center of scattering, it renders any manufacturing process impossible to directly implement. However, alternative approaches remain feasible. We emphasize that this paper does not endorse or support any commercial manufacturing without explicit authorization through proper purchasing agreements. Any commercial production of 1 nm chips based on the ideas presented in this paper must utilize materials developed through "silicon tomography" as outlined herein.*

***Point 2**: While AI calculations may independently derive results similar to those presented in this paper, the intellectual priority belongs to this work. This precedence is established by our earlier efforts and contributions in advancing these findings.*

***Point 3**: A possible issue and/or argument might be occurred at that the research done by this successful work whether if the first author's mother-school attended it or not? The first author claims that the all work done by himself independently (of course the second author has attended) and confirms that this work without any request(ing) from National Chung Cheng University (in Taiwan region) nor obtaining any help (e.g. its equipment or instruments) from it, although the first author had ever learned there. And this point of claims is only limited in this paper.*