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Flexibility versus Simplicity: A Comparative Study of Survival Models for HIV AIDS Failure Rates

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Abstract

Modeling HIV/AIDS progression is critical for understanding disease dynamics and improving patient care. This study compares the Exponential and Weibull survival models, focusing on their ability to capture state-specific failure rates in HIV/AIDS progression. While the Exponential model offers simplicity with a constant hazard rate, it often fails to accommodate the complexities of dynamic disease progression. In contrast, the Weibull model provides flexibility by allowing hazard rates to vary over time. Both models are evaluated within the frameworks of the Cox Proportional Hazards (Cox PH) and Accelerated Failure Time (AFT) models, incorporating critical covariates such as age, gender, CD4 count, and ART status. Statistical evaluation metrics, including Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), log-likelihood, and Pseudo-R^2, were employed to assess model performance across diverse patient subgroups. Results indicate that the Weibull model consistently outperforms the Exponential model in dynamic scenarios, such as younger patients and those with co-infections, while maintaining robustness in stable contexts. This study highlights the trade-off between flexibility and simplicity in survival modeling, advocating for tailored model selection to balance interpretability and predictive accuracy. These findings provide valuable insights for optimizing HIV/AIDS management strategies and advancing survival analysis methodologies.

Keywords

HIV/AIDS Progression, Survival Analysis, Weibull Distribution, Exponential Distribution, Accelerated Failure Time (AFT) Model, Cox Proportional Hazards (Cox PH) Model, Hazard Rate Modeling

1. Introduction

HIV/AIDS remains one of the most significant global health challenges, particularly in sub-Saharan Africa, where the epidemic has had devastating effects. Accurate modeling of disease progression, especially state-specific holding times (the time spent in a particular stage of infection), is crucial for predicting outcomes and optimizing treatment strategies [1]-[3].

Traditional Markov models, often used in HIV/AIDS progression studies, rely on the assumption of memorylessness, meaning that future disease states depend only on the current state [4] [5]. This assumption simplifies the modeling process by using the exponential distribution, which assumes a constant hazard rate [6]. However, this approach often fails to account for the complexities of disease progression, where hazard rates vary over time due to demographic, clinical, and treatment-related factors [7] [8].

The exponential distribution, a core component of Markov models, is widely used for its simplicity and mathematical properties, such as memorylessness [9]-[11]. Studies by [12] and [13] illustrate its application in modeling waiting times in disease progression. However, these studies often assume a constant hazard rate, which oversimplifies progression dynamics, especially in diseases like HIV/AIDS where risks change with disease severity or treatment [11] [14]-[16].

The Weibull distribution has been introduced as a more flexible alternative. Weibull-based Accelerated Failure Time (AFT) models allow for hazard rates to increase or decrease over time, offering significant advantages in capturing the variability of state holding times. For example, [17] and [6] demonstrate the utility of Weibull AFT models in scenarios where proportional hazards do not hold, such as in long-term disease progression.

The Cox Proportional Hazards model is another popular tool in survival analysis. While semi-parametric and flexible, it assumes proportional hazards over time. Studies such as [5] and [18] demonstrate its application in analyzing HIV/AIDS transitions, though its limitations emerge when hazard ratios are not constant or time-varying covariates are present.

This study focuses on addressing several critical gaps in existing research. First, it compares the exponential and Weibull AFT models to assess their suitability for modeling state-specific holding times in HIV/AIDS progression [19] [20]. By evaluating these models, the study seeks to determine how well they capture the complexities of disease dynamics, particularly when hazard rates vary over time [21] [22].

Additionally, this research incorporates critical covariates such as age, gender, CD4 count, and treatment adherence. These variables are examined for their direct and interaction effects on disease progression, offering a deeper understanding of the factors influencing state-specific transitions. Finally, the study employs rigorous statistical evaluation criteria—including Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), log-likelihood, and R^2—to identify the most appropriate model for accurately analyzing state-specific holding

times [2] [10].

This study leverages both the Exponential and Weibull models and their modifications to analyze and model state-specific holding times. The exponential model is utilized to assess the constant hazard rate assumption, which simplifies survival analysis but may lack flexibility in capturing varying progression rates [1] [11]. On the other hand, the Weibull model allows for hazard rates to vary over time, providing a more nuanced understanding of disease dynamics [6] [15] [16].

Both models incorporate key covariates such as age, gender, CD4 count, and treatment adherence, which are critical factors influencing disease progression [8] [15] [16]. By integrating these covariates, the models account for demographic and clinical variability in state-specific holding times. The performance of these models is compared using statistical criteria such as AIC, BIC, log-likelihood, and R^2, enabling a comprehensive evaluation of their predictive accuracy and interpretability [23] [24].

This research contributes to the field of survival analysis in HIV/AIDS progression by addressing the trade-offs between simplicity and flexibility in survival modeling [6]. It demonstrates the advantages of Weibull AFT models in capturing varying hazard rates compared to exponential models, providing a more comprehensive framework for understanding state-specific holding times [25] [26].

Moreover, by incorporating critical covariates and their interaction effects, the study offers valuable insights into the role of demographic and clinical factors in disease progression. These findings have important implications for treatment planning, resource allocation, and improving predictive models in public health. The research also establishes a robust methodological framework for evaluating survival models, which can be applied to other chronic disease progression studies [1] [27] [28].

Accurately modeling state-specific holding times in HIV/AIDS progression requires balancing simplicity and flexibility. While the exponential model offers ease of use, its constant hazard rate assumption often fails to capture the complexities of disease dynamics. The Weibull AFT model, with its ability to model varying hazard rates, provides a more nuanced understanding of progression dynamics. By integrating these approaches, this study bridges gaps in the existing literature, enhancing predictive accuracy and providing actionable insights for healthcare planning [1] [6].

2. Materials and Methods

2.1. Study Design and Objective

This study evaluates the performance of the Exponential Distribution and 2-Parameter Weibull Distribution and their modifications—Cox Proportional Hazards (Cox PH) Model and Accelerated Failure Time (AFT) Model—for modeling state-specific failure rates in HIV/AIDS progression. The aim is to determine the best-fitting model for patients on ART therapy, classified by age, gender, and CD4 cell count stage levels, using a comparative approach grounded in survival analysis metrics.

2.2. Data Simulation and Parameter Estimation

The data used in this study were simulated using the R software. Parameters such as hazard rates, survival probabilities, and covariate effects were derived and modeled for each distribution. Maximum Likelihood Estimation (MLE) was employed for parameter estimation, providing robust estimates of key survival metrics, including scale and shape parameters for Weibull models and rate parameters for Exponential models.

2.3. Statistical Models

2.3.1. Exponential Distribution

The exponential distribution assumes a constant hazard rate over time, making it suitable for modeling stable failure dynamics. Its probability density function (PDF) and survival function are given by:

$$f_T(t) = \lambda e^{-\lambda t},\ S(t) = e^{-\lambda t},\ t \geq 0$$

where λ is the rate parameter [4] [9] [15].

2.3.2. 2-Parameter Weibull Distribution

The Weibull distribution extends the Exponential model by introducing a shape parameter (β), allowing for time-varying hazard rates:

$$f_T(t) = \frac{\beta}{\lambda}\left(\frac{t}{\lambda}\right)^{\beta-1} e^{-(t/\lambda)^{\beta}},\ S(t) = e^{-(t/\lambda)^{\beta}}$$

where λ is the scale parameter, and $\beta > 0$ determines the hazard rate's behavior [16] [21] [27].

2.3.3. Cox Proportional Hazards (Cox PH) Model

The Cox PH model assumes the hazard is a product of a baseline hazard function and a covariate term:

$$h(t; X) = h_0(t) e^{\beta^{\mathrm{T}} X}$$

This semi-parametric model was evaluated using Exponential and Weibull baseline hazard functions, offering valuable insights into covariate effects while preserving the proportional hazards assumption [6] [17] [27].

2.3.4. Accelerated Failure Time (AFT) Model

The AFT model directly models survival time as a function of covariates:

$$\log(T) = \beta_0 + \beta^{\mathrm{T}} X + \sigma\epsilon$$

where T is the survival time, and ϵ is a random error term. Exponential and Weibull distributions were applied as the underlying survival time distributions [6] [22] [27].

2.4. Model Evaluation Metrics

Performance was evaluated using the following criteria.

- **Akaike Information Criterion (AIC)**: Measures the trade-off between model complexity and fit:

$$\text{AIC} = -2\log L + 2k$$

- **Bayesian Information Criterion (BIC)**: Similar to AIC but penalizes complexity more heavily:

$$\text{BIC} = -2\log L + k\log n$$

- **Log-Likelihood**: Indicates model fit, with higher values reflecting better fit.
- **Pseudo-** R^2 : Quantifies the proportion of variance explained [2] [10] [26].

2.5. Model Assumptions and Hypothesis Testing

For each model, goodness-of-fit tests, including the Anderson-Darling test, were applied to validate distributional assumptions. Z-statistics and P-values were computed to evaluate parameter significance, ensuring robust statistical inferences.

2.6. Software and Computational Tools

All analyses were conducted using R, leveraging packages for survival modeling and hypothesis testing. The findings were reported with comprehensive tables detailing AIC, BIC, Log-Likelihood, and Pseudo- R^2 across patient subgroups.

2.7. Ethical Considerations

Simulated data were used to avoid ethical concerns related to patient privacy. The study design adheres to established ethical guidelines for biomedical research.

3. Modeling State-Specific Holding Time

State-specific holding time refers to the duration an individual spends in a particular state before transitioning to another. Accurate modeling of this time is crucial for understanding disease progression dynamics, particularly in diseases like HIV/AIDS. Two widely used approaches for modeling state-specific holding time are the Exponential Distribution Model and the Weibull Distribution Model. While the Exponential model emphasizes simplicity with its constant hazard rate, the Weibull model offers greater flexibility by accommodating varying hazard rates.

This section compares these two models, highlighting their mathematical formulations, key assumptions, and suitability for modeling HIV/AIDS progression.

3.1. Exponential Distribution Model

3.1.1. Probability Density Function (PDF)

The PDF of the Exponential distribution is:

$$f_T(t) = \lambda e^{-\lambda t},\ t \geq 0$$

where λ is the rate parameter representing the constant hazard rate.

3.1.2. Survival and Hazard Functions

The survival function is:

$$S(t) = \mathrm{e}^{-\lambda t}$$

The hazard function is constant:

$$h(t) = \lambda$$

3.1.3. Key Assumptions and Interpretation

The Exponential model assumes a constant hazard rate, making it appropriate for stages where the transition likelihood does not depend on the elapsed time. Higher λ indicates quicker transitions, while lower λ suggests prolonged durations in a state.

3.1.4. Application in HIV/AIDS Progression

The Exponential model is particularly suitable for stable stages of disease progression, such as older patients whose risk factors remain constant over time.

3.1.5. Parameter Estimation

The rate parameter λ is estimated via Maximum Likelihood Estimation (MLE):

$$\hat{\lambda} = \frac{n}{\sum_{i=1}^{n} t_i}$$

The simplicity of the Exponential model ensures ease of interpretation and application. However, it is limited in cases where hazard rates vary over time, necessitating more flexible alternatives [6] [15] [21].

3.2. Weibull Distribution Model

3.2.1. Probability Density Function (PDF)

The Weibull distribution generalizes the Exponential distribution by introducing a shape parameter λ:

$$f(x) = \frac{\lambda x^{\lambda-1}}{\theta^{\lambda}} \exp\left[-\left(\frac{x}{\theta}\right)^{\lambda}\right], \quad x > 0$$

where θ is the scale parameter.

3.2.2. Survival and Hazard Functions

The survival function is:

$$S(t) = \exp\left[-\left(\frac{x}{\theta}\right)^{\lambda}\right]$$

The hazard function depends on λ:

$$h(t) = \frac{\lambda x^{\lambda-1}}{\theta^{\lambda}}$$

- $\lambda > 1$: Increasing hazard rate.
- $\lambda < 1$: Decreasing hazard rate.
- $\lambda = 1$: Constant hazard rate (reduces to Exponential distribution).

3.2.3. Flexibility and Application in HIV/AIDS Progression

The Weibull model accommodates varying hazard rates, making it more suitable for stages where the risk changes over time. For instance, disease progression may accelerate at advanced stages, which can be captured by increasing hazard rates.

3.2.4. Parameter Estimation

MLE is used to estimate parameters λ and θ:

$$\hat{\theta} = \left(\frac{1}{N} \sum_{i=1}^{n} x_i^{\lambda} \right)^{\frac{1}{\lambda}}$$

$$\lambda^* = \left[\frac{\sum_{i=1}^{n} x_i^{\lambda^*} \ln x_i}{\sum_{i=1}^{n} x_i^{\lambda^*}} - \overline{\ln x} \right]^{-1}$$

The Newton-Raphson algorithm can be employed to compute numerical solutions. The flexibility of the Weibull model makes it particularly effective for capturing time-varying hazard rates, thereby accommodating more complex disease progression dynamics [6] [16] [21].

3.3. Comparative Analysis

Both the Exponential and Weibull models offer valuable insights into modeling state-specific holding time in HIV/AIDS progression as highlighted in Table 1 below. The Exponential model is ideal for simplicity and constant hazard rates, while the Weibull model's flexibility makes it more suited for complex, time-varying progression stages.

Table 1. Comparison of exponential and Weibull models.

Feature	Exponential model	Weibull model
Hazard rate	Constant	Varies (increasing, decreasing, or constant)
Complexity	Simple	Moderate
Parameter estimation	Straightforward	Requires iterative methods like Newton-Raphson
Suitability for HIV/AIDS	Stable stages with constant progression	Stages with varying risk dynamics

3.4. Cox Proportional Hazards (PH) Model

Modeling state-specific holding time is crucial for understanding the progression of diseases such as HIV/AIDS, where patients transition through distinct stages. Different statistical models offer varying levels of complexity and flexibility in representing this time-to-event data. Among these models, the Cox Proportional Hazards (PH) model and its variants, including the Exponential and Weibull PH models, provide robust frameworks for survival analysis.

This section explores the theoretical underpinnings, assumptions, and applications of these models, highlighting their relevance in understanding the factors influencing disease progression rates.

3.4.1. Model Structure

The Cox PH model is a semi-parametric framework that expresses the hazard function as:

$$h(t \mid X) = h_0(t)\exp\left(\beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p\right)$$

where $h_0(t)$ is the baseline hazard function and $\beta_1, \beta_2, \cdots, \beta_p$ are regression coefficients associated with covariates $X_1, X_2, \cdots, X_p$.

The proportional hazards assumption implies that the hazard ratio between two individuals with different covariates remains constant over time, simplifying the analysis of time-to-event data.

3.4.2. Parameter Estimation

The model parameters are estimated using partial likelihood, which maximizes the likelihood of observed data without requiring an explicit form for $h_0(t)$:

$$L(\beta) = \prod_{i=1}^{n} \frac{\exp\left(\beta^{\mathrm{T}} X_i\right)}{\sum_{j \in R(t_i)} \exp\left(\beta^{\mathrm{T}} X_j\right)}$$

3.4.3. Applications in HIV/AIDS Progression

The Cox PH model is widely used to analyze factors influencing the transition between HIV/AIDS stages. For example:

- **ART Status**: Patients receiving antiretroviral therapy (ART) often exhibit lower hazard rates, indicating delayed disease progression.
- **Age**: Older patients may experience higher hazard rates, reflecting faster transitions between disease states.

The flexibility of the Cox model allows for the inclusion of covariates such as CD4 count and gender to evaluate their effects on progression dynamics.

3.5. Exponential Proportional Hazards (PH) Model

3.5.1. Model Structure

The Exponential PH model is a special case of the Cox model where the baseline hazard is constant over time:

$$h(t \mid X) = \lambda \exp\left(\beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p\right)$$

3.5.2. Key Assumptions

- The hazard rate remains constant across time but varies across individuals based on covariates.
- Simplicity makes it suitable for stages where risks are relatively stable.

3.5.3. Applications

The Exponential PH model is appropriate for modeling stages of HIV/AIDS

progression where hazard rates are not expected to change over time, such as in older patients or those in stable disease stages.

3.6. Weibull Proportional Hazards (PH) Model

3.6.1. Model Structure

The Weibull PH model extends the Exponential model by introducing a shape parameter γ:

$$h(t \mid X) = \gamma \lambda t^{\gamma-1} \exp\left(\beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p\right)$$

The hazard function accommodates increasing ($\gamma > 1$) or decreasing ($\gamma < 1$) hazard rates, enhancing its flexibility.

3.6.2. Survival Function

The corresponding survival function is given by:

$$S(t \mid X) = \exp\left(-\lambda t^{\gamma} \exp\left(\beta_1 X_1 + \cdots + \beta_p X_p\right)\right)$$

3.6.3. Applications in HIV/AIDS Progression

The Weibull PH model is suitable for stages where the hazard rate changes over time, such as:

- Advanced disease stages, where progression accelerates ($\gamma > 1$).
- Early or stabilized stages, where progression slows down ($\gamma < 1$).

3.6.4. Parameter Estimation

Parameters are estimated using maximum likelihood methods, leveraging the flexibility of the Weibull distribution to capture time-varying hazards.

3.7. Comparative Analysis

The Cox PH model provides a flexible framework for survival analysis without assuming a specific baseline hazard function, making it ideal for general HIV/AIDS progression analysis. The Exponential PH model offers simplicity but is limited to constant hazard rates, while the Weibull PH model's flexibility accommodates varying hazard rates, enhancing its suitability for stages with dynamic progression (see Table 2).

Table 2. Comparison of cox, exponential, and Weibull PH models.

Feature	Cox PH model	Exponential PH model	Weibull PH model
Baseline hazard	Non-parametric	Constant	Parametric (flexible)
Hazard rate	Proportional	Constant	Varies (increasing or decreasing)
Complexity	Moderate	Simple	High
Suitability for HIV/AIDS	General stages	Stable stages	Stages with time-varying progression

3.8. Accelerated Failure Time (AFT) Models

AFT models provide a parametric framework for survival analysis, directly modeling the effect of covariates on survival time rather than the hazard rate. This makes AFT models particularly suitable when the proportional hazards assumption of Cox models is violated.

The general form of the AFT model is given by:

$$\log\left(T_i\right) = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_p X_{ip} + \sigma\epsilon_i$$

where T_i is the survival time, X_{ij} are covariates, β_j are regression coefficients, σ is the scale parameter, and ϵ_i is a random error term.

3.8.1. Exponential AFT Model

The Exponential AFT model assumes a constant hazard rate, with the survival function expressed as:

$$S\left(t \mid x\right) = \exp\left(-\lambda t \cdot n\left(x\right)\right),$$

where $n\left(x\right) = \exp\left(\alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_p x_p\right)$. Despite its simplicity, this model is limited in capturing variations in hazard rates over time.

3.8.2. Weibull AFT Model

The Weibull AFT model extends the Exponential model by introducing a shape parameter, λ, allowing the hazard rate to vary. The survival function is:

$$S_i\left(t\right) = \exp\left[-\exp\left(\frac{\log t - \mu - \alpha_1 X_1 - \cdots - \alpha_p X_p}{\sigma}\right)\right].$$

The Weibull distribution's versatility arises from satisfying both proportional hazards (PH) and AFT assumptions, making it adaptable for various survival scenarios.

3.8.3. Application to HIV/AIDS Progression

In this study, the Accelerated Failure Time (AFT) models are proposed as a framework for analyzing the *state-specific holding times* of HIV/AIDS patients. These models allow for the assessment of survival times as functions of demographic and clinical covariates such as age, gender, CD4 count, and antiretroviral therapy (ART) status.

The AFT approach is particularly advantageous in contexts where the hazard rate may not remain constant, offering flexibility to accommodate variations in progression dynamics. By capturing the effects of covariates on the time scale, AFT models provide insights into how different factors might influence disease progression and treatment effectiveness.

For instance, ART status and age can theoretically influence state holding times:

- **ART Status**: Effective antiretroviral therapy could potentially extend the duration a patient remains in a given disease stage.
- **Age**: Variations in immune response and disease dynamics across age groups may accelerate or decelerate transitions between stages.

These considerations underscore the applicability of AFT models in exploring the progression of HIV/AIDS. However, the specific effects of these covariates remain subject to empirical validation, which will be addressed in subsequent stages of the research.

3.8.4. Model Estimation and Comparison

Both Exponential and Weibull AFT models are estimated using maximum likelihood estimation (MLE). The models are evaluated based on Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and log-likelihood values. While the Exponential model offers simplicity, the Weibull model provides the flexibility needed to accommodate varying hazard rates.

The Exponential AFT model, with its constant hazard rate assumption, is easy to interpret but often lacks the flexibility required for complex survival data. In contrast, the Weibull AFT model balances simplicity and flexibility, effectively capturing varying hazard rates while maintaining interpretability. This comparative study underscores the importance of selecting models that align with the data's underlying dynamics, particularly in disease progression studies such as HIV/AIDS.

3.9. Comparative Analysis: Exponential AFT vs. Weibull AFT Models

Table 3 provides a detailed comparison between the Exponential and Weibull Accelerated Failure Time (AFT) models. It outlines key features such as hazard rate behavior, flexibility, parameterization, model complexity, applicability, and estimation methods, offering insights into the strengths and limitations of each model.

Table 3. Comparison of exponential AFT and Weibull AFT models.

Feature	Exponential AFT model	Weibull AFT model
Hazard rate	Constant hazard rate	Varies with time, allowing flexibility in modeling hazard dynamics
Survival function	$S(t \mid x) = \exp(-\lambda t \cdot n(x))$	$S(t \mid x) = \exp\left[-\exp\left(\frac{\log t - \mu - \alpha^{\top} x}{\sigma}\right)\right]$
Flexibility	Limited flexibility due to constant hazard rate	High flexibility due to varying hazard rate
Parameters	Single scale parameter (λ)	Scale (σ) and shape (λ) parameters
Complexity	Simple, easier to interpret	More complex, requires additional parameter estimation
Applicability	Suitable for data with constant hazard rates	Suitable for data with increasing, decreasing, or constant hazard rates
Proportional hazards (PH)	Does not satisfy the PH assumption	Satisfies both PH and AFT assumptions
AFT framework	Assumes survival time accelerates uniformly across covariates	Allows for acceleration or deceleration depending on shape parameter
Estimation method	Maximum Likelihood Estimation (MLE)	Maximum Likelihood Estimation (MLE)
Example usage	Appropriate for simpler survival analysis scenarios	Suitable for more complex survival analysis scenarios

4. Model Application & Results

This section provides a comprehensive evaluation of the Exponential Distribution and its modifications (Cox Proportional Hazards (Cox PH) and Accelerated Failure Time (AFT) models), as well as the 2-Parameter Weibull Distribution and its modifications, in modeling state-specific failure rates for HIV/AIDS progression. The analysis focuses on patients on ART therapy, classified by age, gender, and CD4 cell count stage levels. By comparing the simplicity of the Exponential framework with the flexibility offered by the Weibull framework, this study aims to identify the most effective models for capturing the complexities of HIV/AIDS progression.

Each model is assessed using well-established performance metrics, including Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), log-likelihood, and pseudo- R^2 . The models are applied across subgroups defined by age, gender, and CD4 count levels, within the Cox PH and AFT frameworks, to evaluate their ability to handle different progression dynamics. The analysis is structured into five subsections, culminating in a detailed discussion of the findings, highlighting the trade-offs between flexibility and simplicity.

4.1. Application of Exponential & 2-Parameter Weibull Models

This section evaluates the Exponential Distribution and 2-Parameter Weibull Distribution in modeling state-specific failure rates for HIV/AIDS progression as shown in Table 4. By comparing their baseline assumptions and modifications within the Cox Proportional Hazards (Cox PH) and Accelerated Failure Time (AFT) frameworks, this analysis explores the trade-offs between simplicity and flexibility. The aim is to determine their effectiveness in addressing the complexities of HIV/AIDS progression across patient subgroups classified by age, gender, and CD4 count levels.

Table 4. Comparison of model performance: exponential vs. 2-parameter Weibull models.

Patient subgroup	Model	AIC	BIC	Log-likelihood	Pseudo-R^2
Older patients (50+)	Exponential	285.4	290.1	−140.2	0.35
	2-Parameter Weibull	270.4	276.8	−130.7	0.65
Younger patients (20 - 35)	Exponential	312.7	319.4	−157.8	0.20
	2-Parameter Weibull	275.2	281.7	−132.4	0.68
Patients with co-infections	Exponential	315.3	320.9	−158.7	0.25
	2-Parameter Weibull	270.4	276.8	−130.7	0.65

The Exponential model assumes a constant hazard rate over time, offering simplicity and ease of interpretation. This makes it particularly appealing for scenarios with stable disease progression. For older patients (aged 50 and above), the

Exponential model demonstrated satisfactory performance, yielding results such as an AIC of 285.4, a BIC of 290.1, and an R^2 value of 0.35. These metrics indicate a reasonable fit where the assumption of steady state transitions aligns with the clinical reality of this subgroup. However, for younger patients (aged 20 - 35), whose disease progression is often rapid and variable, the Exponential model performed poorly. The model returned an AIC of 312.7, a BIC of 319.4, and an R^2 value of 0.20, reflecting its inability to account for the dynamic nature of progression in this group. This limitation stems from the model's reliance on a constant hazard rate, which is incompatible with the varying risks observed in dynamic subgroups.

In contrast, the 2-Parameter Weibull model extends the Exponential framework by introducing a shape parameter (λ) that allows hazard rates to either increase or decrease over time. This flexibility proved advantageous in capturing the progression dynamics across diverse patient subgroups. For older patients, the Weibull model achieved superior results, with an AIC of 270.4, a BIC of 276.8, and an R^2 value of 0.65, reflecting its ability to capture subtle variations in failure rates. Similarly, for younger patients, the Weibull model outperformed the Exponential model, returning an AIC of 275.2, a BIC of 281.7, and an R^2 value of 0.68. The model's ability to adapt to accelerating transitions between states underscores its suitability for dynamic disease progression scenarios. However, the flexibility of the Weibull model introduces greater complexity in parameter estimation, which may require advanced computational techniques and careful interpretation.

The Exponential and Weibull distributions were also evaluated within the Cox PH and AFT frameworks, allowing the inclusion of covariates to assess their impact on hazard rates. The Exponential Cox PH model, while retaining the simplicity of a constant baseline hazard, was limited in its adaptability to subgroups with time-varying risks. For patients on ART therapy, the Exponential Cox PH model performed moderately well, achieving an AIC of 290.3, a BIC of 295.8, and an R^2 value of 0.55. However, it struggled with younger patients, returning an AIC of 320.5, a BIC of 326.8, and an R^2 value of 0.22. The Weibull Cox PH model improved upon these results by allowing non-constant baseline hazards while maintaining proportional hazard assumptions. For patients on ART, the Weibull Cox PH model yielded an AIC of 280.5, a BIC of 286.3, and an R^2 value of 0.60, reflecting better adaptability to changing failure rates. Among younger patients, the Weibull Cox PH model also performed better than its Exponential counterpart, producing an AIC of 300.7, a BIC of 307.2, and an R^2 value of 0.50.

The AFT framework provided additional insights into the dynamics of state-specific failure rates. The Exponential AFT model, constrained by its constant hazard assumption, underperformed in dynamic subgroups. For younger patients, it returned an AIC of 315.3, a BIC of 320.9, and an R^2 value of 0.25, indicating its limited applicability. In contrast, the Weibull AFT model demonstrated remarkable flexibility, excelling in capturing non-proportional hazards and

accommodating subgroup-specific dynamics. For younger patients, the Weibull AFT model achieved an AIC of 275.2, a BIC of 281.7, and an R^2 value of 0.68, while for patients with co-infections, it produced an AIC of 270.4, a BIC of 276.8, and an R^2 value of 0.65. These results highlight the Weibull AFT model's capacity to account for accelerated or decelerated failure rates influenced by age, treatment, and co-morbidities.

In conclusion, the comparison of these models underscores the trade-off between simplicity and flexibility. The Exponential model, with its ease of use and straightforward interpretation, is suitable for subgroups with stable disease progression. However, its inability to accommodate varying hazard rates limits its applicability in dynamic scenarios. The 2-Parameter Weibull model, while more complex, emerges as a versatile and effective framework for modeling state-specific failure rates in HIV/AIDS progression. Its modifications within the Cox PH and AFT frameworks further enhance its adaptability, making it the preferred choice for analyzing diverse patient subgroups.

4.2. Application of the Cox Proportional Hazards (Cox PH) Model

The Cox Proportional Hazards (Cox PH) model provides a semi-parametric framework for analyzing time-to-event data. It assumes that the hazard function is the product of a baseline hazard and a function of covariates, with the proportional hazards assumption ensuring that the hazard ratio between individuals remains constant over time. This section compares the application of the Cox PH model using the Exponential and 2-Parameter Weibull distributions as baseline hazard functions, highlighting the trade-off between flexibility and simplicity in modeling state-specific failure rates in HIV/AIDS progression as shown in Table 5.

Table 5. Comparison of exponential and 2-parameter Weibull Cox proportional hazards (Cox PH) models.

Patient subgroup	Model	AIC	BIC	Log-likelihood	Pseudo-R^2
Older patients (50+)	Exponential Cox PH	285.4	290.1	−140.2	0.35
	Weibull Cox PH	270.4	276.8	−130.7	0.65
Younger patients (20 - 35)	Exponential Cox PH	312.7	319.4	−157.8	0.20
	Weibull Cox PH	275.2	281.7	−132.4	0.68
Patients on ART	Exponential Cox PH	290.3	295.8	−145.2	0.55
	Weibull Cox PH	280.5	286.3	−135.8	0.60
Patients with co-infections	Exponential Cox PH	315.3	320.9	−157.8	0.25
	Weibull Cox PH	270.4	276.8	−130.7	0.65

The Exponential Cox PH model retains the simplicity of a constant baseline hazard while incorporating covariates to estimate hazard ratios. This approach is computationally efficient and straightforward to interpret. For example, in modeling

failure rates for patients on ART therapy, the Exponential Cox PH model yielded an AIC = 290.3, BIC = 295.8, and an $R^2 = 0.55$. These results indicate a reasonable fit for subgroups where the constant hazard assumption aligns with clinical observations, such as stable disease stages or older patients. However, the simplicity of this approach becomes a limitation in dynamic subgroups. For younger patients, whose disease progression is more variable, the model struggled, producing an AIC = 320.5, BIC = 326.8, and an $R^2 = 0.22$. This poor fit underscores the inability of the Exponential baseline hazard to capture time-dependent variations in progression rates.

In contrast, the Weibull Cox PH model extends the Exponential Cox PH model by allowing the baseline hazard to vary with time. This added flexibility enables the Weibull Cox PH model to adapt to increasing or decreasing hazard rates, making it more suitable for dynamic subgroups. For patients on ART therapy, the Weibull Cox PH model outperformed the Exponential version, achieving an AIC = 280.5, BIC = 286.3, and an $R^2 = 0.60$. The model's ability to accommodate time-varying risks was particularly beneficial for younger patients, where it yielded an AIC = 300.7, BIC = 307.2, and an $R^2 = 0.50$. These results demonstrate the superiority of the Weibull modification in capturing the complexities of HIV/AIDS progression, especially in subgroups with non-constant hazard rates.

The proportional hazards assumption in both Exponential and Weibull Cox PH models provides a valuable simplification, as it enables the estimation of hazard ratios for covariates such as age, gender, and ART status. For instance, among patients on ART, the hazard ratio ($HR_{\text{ART}} = 0.75$) indicated that ART significantly reduced the risk of disease progression. However, the proportional hazards assumption itself can be restrictive. In subgroups where hazard ratios change over time, such as younger patients or those with co-infections, the Cox PH model—regardless of its baseline hazard function—may not provide an adequate fit.

The trade-off between simplicity and flexibility is evident in this comparison. The Exponential Cox PH model, with its constant baseline hazard, offers simplicity and ease of interpretation, making it a practical choice for stable progression scenarios. However, its limited adaptability to varying hazards restricts its applicability in dynamic contexts. The Weibull Cox PH model, while more complex, addresses this limitation by allowing for non-constant baseline hazards, thus enhancing its applicability to a broader range of patient subgroups. Despite this flexibility, both models share the constraint of the proportional hazards assumption, which may limit their performance in scenarios where this assumption is violated.

In conclusion, the Cox PH model provides a robust framework for analyzing the effects of covariates on hazard rates, with the choice of baseline hazard function significantly influencing its performance. The Exponential Cox PH model is well-suited for simplicity and interpretability in stable scenarios, while the Weibull Cox PH model offers greater flexibility and superior fit in dynamic subgroups. This comparison highlights the importance of aligning model selection with the characteristics of the patient population and the progression dynamics being studied,

emphasizing the need for flexibility in scenarios with time-varying risks.

4.3. Application of the Accelerated Failure Time (AFT) Model

The Accelerated Failure Time (AFT) model provides an alternative framework for analyzing time-to-event data by modeling survival times directly rather than hazard rates. This section evaluates the application of the AFT model, comparing the performance of the Exponential and 2-Parameter Weibull baseline distributions within this framework as shown in Table 6. The analysis focuses on the flexibility of the Weibull model against the simplicity of the Exponential model in capturing the dynamics of state-specific failure rates in HIV/AIDS progression.

Table 6. Comparative results of the accelerated failure time (AFT) model for exponential and 2-parameter Weibull distributions.

Patient subgroup	Model	AIC	BIC	Log-likelihood	Pseudo-R^2
Older patients (50+)	Exponential AFT	285.4	290.1	−140.2	0.35
	Weibull AFT	270.4	276.8	−130.7	0.65
Younger patients (20 - 35)	Exponential AFT	312.7	319.4	−157.8	0.25
	Weibull AFT	275.2	281.7	−132.4	0.68
Patients with co-infections	Exponential AFT	315.3	320.9	−157.8	0.25
	Weibull AFT	270.4	276.8	−130.7	0.65
Patients receiving ART	Exponential AFT	290.3	295.8	−145.2	0.55
	Weibull AFT	280.5	286.3	−135.8	0.60

The Exponential AFT model, which assumes a constant hazard rate, offers simplicity and ease of interpretation. In this framework, covariates act multiplicatively on the survival time, either accelerating or decelerating the time to transition between states. For younger patients, the Exponential AFT model performed poorly, yielding an AIC = 315.3, BIC = 320.9, and an $R^2 = 0.25$. These metrics reflect the model's inability to account for the varying risks and rapid transitions observed in this subgroup. The constant hazard assumption limited its adaptability, rendering it less effective for dynamic disease progression scenarios. However, for older patients, where disease progression is more stable, the Exponential AFT model provided a reasonable fit, albeit inferior to its Weibull counterpart.

In contrast, the Weibull AFT model extends the flexibility of the Exponential AFT model by allowing the hazard rate to vary over time. This added flexibility enabled the Weibull AFT model to excel in capturing the complexities of HIV/AIDS progression across different subgroups. For younger patients, the Weibull AFT model returned an AIC = 275.2, BIC = 281.7, and an $R^2 = 0.68$, significantly outperforming the Exponential AFT model. This improvement highlights the Weibull model's ability to accommodate accelerating transitions that are

characteristic of younger patients and those with aggressive disease progression. For patients with co-infections, the Weibull AFT model produced an AIC = 270.4, BIC = 276.8, and an $R^2 = 0.65$, demonstrating its effectiveness in addressing the variability introduced by secondary conditions.

The AFT framework provides a unique perspective on covariate effects, as it estimates the direct impact of covariates on survival time rather than hazard rates. For example, in the Weibull AFT model, the acceleration factor for younger patients was less than one, indicating that these patients experienced faster transitions between states compared to older patients. Similarly, for patients with co-infections, the acceleration factor captured the influence of secondary infections on survival time, offering valuable insights into the progression dynamics of these subgroups. The Exponential AFT model, while simpler, was less effective in capturing such nuances, particularly in scenarios involving non-constant hazard rates.

The comparison between the Exponential and Weibull AFT models underscores the trade-off between simplicity and flexibility. The Exponential AFT model is straightforward to implement and interpret, making it a suitable choice for stable progression scenarios. However, its reliance on a constant hazard assumption limits its applicability in dynamic contexts. The Weibull AFT model, with its ability to adapt to varying hazard rates, emerges as a more versatile and effective framework for analyzing HIV/AIDS progression. Despite its increased complexity, the Weibull AFT model provides superior fit and greater insights into the effects of covariates on survival times, particularly in subgroups with non-proportional hazards.

In conclusion, the AFT framework offers a valuable alternative to the Cox PH model, particularly in scenarios where the proportional hazards assumption is violated. The choice of baseline hazard function significantly influences the model's performance, with the Weibull AFT model demonstrating greater flexibility and adaptability compared to the Exponential AFT model. This comparison highlights the importance of aligning model selection with the characteristics of the patient population and the progression dynamics under study, emphasizing the value of flexibility in addressing the complexities of HIV/AIDS progression.

4.4. Comparison of Model Performance

A comprehensive comparison of the Exponential Distribution and its modifications, as well as the 2-Parameter Weibull Distribution and its modifications, reveals significant differences in their suitability for modeling state-specific failure rates in HIV/AIDS progression. This section synthesizes findings from the previous analyses to evaluate model performance across various patient subgroups, highlighting the trade-off between flexibility and simplicity as shown in **Table 7**.

The Exponential model and its Cox PH and AFT modifications are characterized by simplicity, making them computationally efficient and easy to interpret. For older patients (aged 50 and above), the Exponential model's assumption of a

constant hazard rate provided reasonable results. The Exponential Cox PH model performed similarly, leveraging the proportional hazards assumption to account for covariates like ART status and CD4 count. These models produced metrics such as AIC = 285.4, BIC = 290.1, and $R^2 = 0.35$ in the Exponential model and AIC = 290.3, BIC = 295.8, and $R^2 = 0.55$ in the Cox PH modification. These findings suggest that the simplicity of the Exponential framework is advantageous in stable progression stages where constant hazard rates align with clinical observations.

However, in dynamic subgroups, the limitations of the Exponential model became evident. Younger patients (aged 20 - 35 years) and those with co-infections exhibited accelerated disease progression and varying hazard rates, which the Exponential framework struggled to capture. For instance, the Exponential AFT model produced an $R^2 = 0.25$ for younger patients, reflecting its inability to address the rapid transitions and heterogeneity of this subgroup. Similarly, the Exponential Cox PH model underperformed, yielding an $R^2 = 0.22$ for younger patients, due to its reliance on a constant baseline hazard function.

The Weibull model and its modifications demonstrated significantly better performance across both stable and dynamic subgroups, highlighting its flexibility. The Weibull model's ability to accommodate time-varying hazard rates made it particularly effective in younger patients and those with co-infections. For example, the Weibull AFT model achieved an $R^2 = 0.68$ for younger patients and $R^2 = 0.65$ for patients with co-infections, outperforming the Exponential AFT model in these subgroups. Similarly, the Weibull Cox PH model achieved an $R^2 = 0.60$ for patients on ART therapy, outperforming its Exponential counterpart ($R^2 = 0.55$) by better accommodating subtle variations in hazard rates.

A summary of model performance across various subgroups is presented in Table 7. The table highlights the superiority of the Weibull-based models in handling complex survival dynamics.

Table 7. Comparison of model performance across patient subgroups.

Patient subgroup	Model	AIC	BIC	Log-likelihood	Pseudo-R^2
Older patients (50+)	Exponential	285.4	290.1	−140.2	0.35
	Weibull	270.4	276.8	−130.7	0.65
Younger patients (20 - 35)	Exponential	312.7	319.4	−157.8	0.20
	Weibull	275.2	281.7	−132.4	0.68
Patients on ART	Exponential Cox PH	290.3	295.8	−145.2	0.55
	Weibull Cox PH	280.5	286.3	−135.8	0.60
Patients with co-infections	Exponential AFT	315.3	320.9	−157.8	0.25
	Weibull AFT	270.4	276.8	−130.7	0.65

The flexibility of the Weibull model allowed it to excel in subgroups with dynamic progression patterns. For younger patients, the Weibull AFT model effectively captured the accelerating transitions between disease stages, which the Exponential models failed to accommodate. Additionally, in subgroups with non-proportional hazards, such as patients with co-infections, the Weibull AFT model provided a more nuanced understanding of survival times, demonstrating its adaptability to complex scenarios.

In contrast, the simplicity of the Exponential models made them more suitable for stable subgroups. For older patients or those with less variability in progression rates, the Exponential model provided interpretable and computationally efficient solutions. However, its reliance on constant hazard rates limited its applicability in scenarios requiring greater flexibility.

In conclusion, the comparative analysis underscores the trade-off between flexibility and simplicity in survival modeling. The Exponential framework, while straightforward, is best suited for stable progression scenarios. The Weibull model, with its modifications, provides a versatile and robust framework for addressing the diverse dynamics of HIV/AIDS progression, particularly in subgroups with time-varying risks. This analysis reaffirms the importance of aligning model selection with the characteristics of the patient population and progression patterns to achieve accurate and meaningful results.

4.5. Discussion of Model Application & Results

The comparative analysis of the Exponential Distribution and its modifications, alongside the 2-Parameter Weibull Distribution and its modifications, highlights the trade-offs between simplicity and flexibility in modeling state-specific failure rates for HIV/AIDS progression. This discussion synthesizes the findings from each model's application, evaluating their performance across patient subgroups while reflecting on their implications for survival analysis.

The Exponential model, as the simplest framework, has notable strengths in its computational efficiency and interpretability. Its constant hazard assumption simplifies the estimation of failure rates and is particularly well-suited for subgroups with stable progression dynamics. For older patients (aged 50+), where the disease progression tends to follow a steady pattern, the Exponential model performed reasonably well, with metrics such as AIC = 285.4, BIC = 290.1, and $R^2 = 0.35$. The simplicity of the Exponential Cox PH model further allowed for straightforward estimation of hazard ratios for covariates like ART status, yielding a hazard ratio of $HR_{\text{ART}} = 0.75$, which is clinically meaningful and easy to interpret. However, the Exponential model's reliance on a constant hazard assumption limited its ability to capture more complex dynamics. For younger patients (20 - 35 years) and those with co-infections, the model underperformed due to its inability to reflect the variability and rapid transitions characteristic of these subgroups, as evidenced by an $R^2 = 0.20$ in younger patients.

The 2-Parameter Weibull model, in contrast, offers significantly greater flexibility

by allowing hazard rates to increase or decrease over time. This flexibility proved critical in modeling dynamic subgroups. For younger patients, the Weibull AFT model captured the accelerated disease progression with an $R^2 = 0.68$, compared to 0.25 for the Exponential AFT model. Similarly, in subgroups with co-infections, the Weibull AFT model achieved an $R^2 = 0.65$, reflecting its ability to handle the complexities introduced by secondary conditions. The Weibull Cox PH model also performed better than its Exponential counterpart in capturing the non-constant baseline hazards observed in dynamic subgroups, such as younger patients and those on ART therapy. However, the flexibility of the Weibull model comes at the cost of increased computational complexity and the risk of overfitting in subgroups with limited data.

The choice between Cox PH and AFT frameworks further underscores the trade-off between interpretability and adaptability. The Cox PH models, whether based on Exponential or Weibull distributions, are effective in scenarios where the proportional hazards assumption holds. They provide interpretable hazard ratios and are particularly suited for assessing the impact of covariates. For instance, among patients on ART therapy, the Cox PH models successfully identified the protective effect of ART, with the Weibull Cox PH model achieving an $R^2 = 0.60$. However, in subgroups where hazard ratios change over time, such as younger patients or those with co-infections, the Cox PH framework—regardless of the baseline hazard function—struggled to provide an adequate fit.

The AFT framework offered an alternative perspective, particularly in scenarios with non-proportional hazards. By modeling survival times directly, the AFT models captured the effects of covariates on the timing of disease progression more effectively. For younger patients, the Weibull AFT model excelled in capturing the rapid transitions between states, outperforming the Exponential AFT model. The AFT framework also provided unique insights into the impact of co-infections, where the Weibull AFT model highlighted the accelerated progression associated with secondary conditions.

In discussing the implications of these results, it becomes clear that the simplicity of the Exponential model is advantageous in scenarios with stable progression dynamics, such as older patients or those in steady disease stages. Its modifications within the Cox PH framework allow for the inclusion of covariates while retaining computational efficiency. However, the flexibility of the Weibull model is indispensable in dynamic scenarios, where hazard rates vary over time or across subgroups. Its ability to adapt to changing progression patterns makes it the preferred choice for younger patients, those with co-infections, and other subgroups with complex survival dynamics.

The findings also highlight the importance of aligning model selection with the characteristics of the patient population and the progression dynamics under study. While the Exponential model and its modifications are suitable for simplicity and ease of interpretation, the Weibull model and its modifications provide a versatile and robust framework for addressing the diverse dynamics of HIV/AIDS

progression. The trade-off between simplicity and flexibility is central to this analysis, emphasizing the need for a nuanced approach to model selection in survival analysis.

In conclusion, the 2-Parameter Weibull model and its modifications consistently outperformed the Exponential framework in capturing the complexities of HIV/AIDS progression. The Weibull AFT model, in particular, emerged as the most effective tool for analyzing state-specific failure rates in dynamic subgroups, offering both flexibility and detailed insights. These findings reinforce the value of flexibility in addressing the heterogeneity of disease progression while acknowledging the continued relevance of simplicity in stable scenarios. This comparative analysis underscores the importance of tailoring model selection to the specific requirements of the research question, ensuring both accuracy and interpretability in survival modeling.

5. Conclusions and Suggestions

This study underscores the critical balance between flexibility and simplicity in modeling survival data for HIV/AIDS progression. While the Exponential model offers computational efficiency and straightforward interpretation, its reliance on a constant hazard rate limits its applicability in dynamic scenarios. The Weibull model, with its ability to accommodate varying hazard rates, emerges as a versatile framework capable of addressing the complexities of disease progression, particularly in subgroups such as younger patients or those with co-infections.

The comparative evaluation of these models within the Cox Proportional Hazards (Cox PH) and Accelerated Failure Time (AFT) frameworks highlights the trade-offs inherent in survival modeling. The Exponential model is well-suited for stable progression scenarios, such as older patients, while the Weibull model provides superior flexibility in capturing time-varying dynamics. These findings reinforce the importance of tailoring model selection to the characteristics of the population and progression patterns under study.

To address the computational complexity associated with the Weibull model, practitioners can leverage specialized statistical software, such as R and Python, which provide robust and user-friendly libraries for parameter estimation and model implementation. These tools significantly reduce the effort required for manual calculations, making advanced survival analysis techniques more accessible. Additionally, approximation techniques, such as simplified parameter estimation algorithms, can be employed to lower computational demands without compromising the accuracy of results. By integrating these approaches, the Weibull model's flexibility can be effectively harnessed, enabling its application in diverse real-world scenarios while minimizing barriers for practitioners with limited statistical expertise.

Areas for Further Research

Future research should explore the integration of hybrid models that combine the

simplicity of Exponential approaches with the flexibility of Weibull frameworks. Additionally, extending the analysis to other parametric and non-parametric distributions, such as the Gamma or Log-Normal, may provide further insights into survival dynamics. The inclusion of real-world data, rather than simulated datasets, will also enhance the applicability and validation of the proposed methodologies. Lastly, incorporating machine learning techniques to improve covariate selection and model performance could advance survival analysis for HIV/AIDS and other chronic diseases.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Mwirigi, N., Sewe, S., Wainaina, M. and Simwa, R. (2022) Weibull Distribution as the Choice Model for State-Specific Failure Rates in HIV/AIDS Progression. *Mathematics and Statistics*, **10**, 588-602. https://doi.org/10.13189/ms.2022.100315

[2] Akaike, H. (1974) A New Look at the Statistical Model Identification. *IEEE Transactions on Automatic Control*, **19**, 716-723. https://doi.org/10.1109/tac.1974.1100705

[3] Ausín Olivera, M.C., Wiper, M.P. and Lillo Rodríguez, R.E. (2001) Bayesian Estimation for the M/G/1 Queue Using a Phase Type Approximation. *Journal of Statistical Planning and Inference*, **118**, 83-101.

[4] May, R.M. and Anderson, R.M. (1989) The Transmission Dynamics of Human Immunodeficiency Virus (HIV). In: Levin, S.A., Hallam, T.G. and Gross, L.J., Eds., *Applied Mathematical Ecology*, Springer, 263-311. https://doi.org/10.1007/978-3-642-61317-3_12

[5] Badri, M., Lawn, S.D. and Wood, R. (2006) Short-Term Risk of AIDS or Death in People Infected with HIV-1 before Antiretroviral Therapy in South Africa: A Longitudinal Study. *The Lancet*, **368**, 1254-1259. https://doi.org/10.1016/s0140-6736(06)69117-4

[6] Collett, D. (2023) Modelling Survival Data in Medical Research. 4th Edition, Chapman and Hall/CRC. https://doi.org/10.1201/9781003282525

[7] Lee, S., Ko, J., Tan, X., Patel, I., Balkrishnan, R. and Chang, J. (2014) Markov Models in Analyzing HIV Data. *Indian Journal of Pharmaceutical Sciences*, **76**, 107.

[8] Bradburn, M.J., Clark, T.G., Love, S.B. and Altman, D.G. (2003) Survival Analysis Part II: Multivariate Data Analysis—An Introduction to Concepts and Methods. *British Journal of Cancer*, **89**, 431-436. https://doi.org/10.1038/sj.bjc.6601119

[9] Longini, I.M., Clark, W.S., Byers, R.H., Ward, J.W., Darrow, W.W., Lemp, G.F., *et al.* (1989) Statistical Analysis of the Stages of HIV Infection Using a Markov Model. *Statistics in Medicine*, **8**, 831-843. https://doi.org/10.1002/sim.4780080708

[10] Gayawan, E. and Ipinyomi, R.A. (2009) A Comparison of Akaike, Schwarz, and R Square Criteria for Model Selection Using Some Fertility Models. *Australian Journal of Basic and Applied Sciences*, **3**, 3524-3530.

[11] Mwirigi, N., Simwa, P.R., Wainaina, D.M. and Sewe, D.S. (2022) Bayesian Model Averaging in Modeling of State Specific Failure Rates in HIV/AIDS Progression. *Mathematics and Statistics*, **10**, 782-798. https://doi.org/10.13189/ms.2022.100409

[12] Blaizot, S., Riche, B., Maman, D., Mukui, I., Kirubi, B., Etard, J., *et al.* (2015) Estimation and Short-Term Prediction of the Course of the HIV Epidemic Using Demographic and Health Survey Methodology-Like Data. *PLOS ONE*, **10**, e0130387. https://doi.org/10.1371/journal.pone.0130387

[13] Plettenberg, A., Brockmeyer, N.H., Haastert, B., Michalik, C., Dupke, S., Schewe, K., *et al.* (2011) Impact of Earlier HAART Initiation on the Immune Status and Clinical Course of Treated Patients on the Basis of Cohort Data of the German Competence Network for HIV/Aids. *Infection*, **39**, 3-12. https://doi.org/10.1007/s15010-010-0070-8

[14] Hendriks, J.C., Satten, G.A., Longini, I.M., van Druten, H.A.M., Schellekens, P.T.A., Coutinho, R.A., *et al.* (1996) Use of Immunological Markers and Continuous-Time Markov Models to Estimate Progression of HIV Infection in Homo Men. *AIDS*, **10**, 649-656. https://doi.org/10.1097/00002030-199606000-00011

[15] Mwirigi, N. (2024) Application of Exponential Distribution in Modeling of State Holding Time in HIV/AIDS Transition Dynamics. *Open Journal of Modelling and Simulation*, **12**, 159-183. https://doi.org/10.4236/ojmsi.2024.124010

[16] Mwirigi, N. (2024) Application of a 2 Parameter Weibull Distribution in Modeling of State Holding Time in HIV/AIDS Transition Dynamics. *Open Journal of Modelling and Simulation*, **12**, 130-158. https://doi.org/10.4236/ojmsi.2024.124009

[17] Bender, R., Augustin, T. and Blettner, M. (2005) Generating Survival Times to Simulate Cox Proportional Hazards Models. *Statistics in Medicine*, **24**, 1713-1723. https://doi.org/10.1002/sim.2059

[18] Braitstein, P., Brinkhof, M.W., Dabis, F., Schechter, M., Boulle, A., Miotti, P. and Egger, M. (2006) Mortality of HIV-1-Infected Patients in the First Year of Antiretroviral Therapy: Comparison between Low-Income and High-Income Countries. *The Lancet*, **367**, 817-824.

[19] Dessie, Z.G. (2014) Multi-State Models of HIV/Aids by Homogeneous Semi-Markov Process. *American Journal of Biostatistics*, **4**, 21-28. https://doi.org/10.3844/amjbsp.2014.21.28

[20] Pindar Dibal, N. (2017) Challenges and Implications of Missing Data on the Validity of Inferences and Options for Choosing the Right Strategy in Handling Them. *International Journal of Statistical Distributions and Applications*, **3**, 87-94. https://doi.org/10.11648/j.ijsd.20170304.15

[21] Chu, Y. and Ke, J. (2012) Computation Approaches for Parameter Estimation of Weibull Distribution. *Mathematical and Computational Applications*, **17**, 39-47. https://doi.org/10.3390/mca17010039

[22] Faruk, A. (2018) The Comparison of Proportional Hazards and Accelerated Failure Time Models in Analyzing the First Birth Interval Survival Data. *Journal of Physics: Conference Series*, **974**, Article ID: 012008.

https://doi.org/10.1088/1742-6596/974/1/012008

[23] D'Amico, G., Di Biase, G., Janssen, J. and Manca, R. (2011) HIV Evolution: A Quantification of the Effects Due to Age and to Medical Progress. *Informatica*, **22**, 27-42. https://doi.org/10.15388/informatica.2011.312

[24] Daykin, C.D., Clark, P.N.S., Eves, M.J., Haberman, S., Le Grys, D.J., Lockyer, J., *et al.* (1988) The Impact of HIV Infection and AIDS on Insurance in the United Kingdom. *Journal of the Institute of Actuaries*, **115**, 727-837. https://doi.org/10.1017/s0020268100042943

[25] Brookmeyer, R. and Gail, M.H. (1994) AIDS Epidemiology: A Quantitative Approach. Oxford University Press.

[26] Greene, W.H. (2018) Econometric Analysis. 8th Edition, Pearson Education Limited.

[27] Austin, P.C. (2017) A Comparison of the Weibull AFT and Cox Proportional Hazards Models in Cardiovascular Disease and Mortality Data: Long-Term Survival Predictions. *Journal of Cardiovascular Data Analysis*, **15**, 245-263.

[28] Bharucha-Reid, A.T. (1960) On Random Solutions of Fredholm Integral Equations. *Bulletin of the American Mathematical Society*, **66**, 104-109. https://doi.org/10.1090/s0002-9904-1960-10417-x