Journal of Modern Physics, 2024, 15, 2489-2514
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

The Matter in Newtonian Static Gravitational Field

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How to cite this paper: Xu, Z.L. (2024) The Matter in Newtonian Static Gravitational Field. *Journal of Modern Physics*, **15**, 2489-2514.
https://doi.org/10.4236/jmp.2024.1513102

Received: November 18, 2024
Accepted: December 28, 2024
Published: December 31, 2024

Abstract

In contemporary physics, there is an observed discrepancy in the mass calculations used to determine the strength of celestial gravitational fields. Therefore, physics is searching for dark matter candidate particles, such as weakly interacting massive particles (WIMPs) and axions, while attempting to modify Newtonian dynamics and the law of universal gravitation. Inspired by the classical theories of electric and magnetic field mass-energy calculations, the present work proposes a new theoretical attempt to explore the dark matter in the universe and challenge theories that modify Newtonian dynamics and the law of universal gravitation. Like the formulas for calculating the mass-energy density of electric and magnetic fields, Newtonian static gravitational fields also have a mass-energy density. The matter in the gravitational field will also generate a new gravitational field and thus derive the formula for calculating the mass-energy of matter in the gravitational field. In this way, the gravitational mass-energy of celestial bodies should consider the ordinary visible matter and invisible matter of the gravitational field. The strength of a gravitational field is a vector, and the mass-energy density of a gravitational field is proportional to the square of its strength. The greater the strength of the gravitational field, the greater the mass-energy density of the gravitational field at that location. Assuming that ordinary matter is distributed uniformly within a sphere, it deduces that the mass-energy of the celestial body is not only related to that of ordinary matter but also to its structure. The higher the celestial structure factor of that body, the greater the mass-energy density of matter in the gravitational field inside and outside the body.

Keywords

Dark Matter, Gravitational Field Strength, Gravitational Field Matter, Missing Mass, Celestial Structure Factor, Modified Newtonian Dynamics

1. Introduction

Since Newton discovered the law of universal gravitation in 1687, physics has never found cases where this law is not applicable in calculating fields such as the Earth and the solar system. In 1922, by studying the dynamic properties of celestial systems, Jacobus C. Kapteyn proposed that the possible existence of invisible matter and dark matter first entered the public eye. In the 1930s, California Institute of Technology astronomer Fritz Zwicky measured the motion of the galaxies in the Coma cluster. Based on simple gravitational calculations, he found that the trajectory speeds of galaxies around their center far exceed the results calculated using the law of universal gravitation based on observations of matter from luminosity [1]. So Fritz Zwicky believed there was a missing mass in galaxies or galaxy clusters and proposed a hypothesis that there was invisible matter inside and outside galaxies, galaxy groups, and galaxy clusters, called dark matter [2] [3]. In 1970, Vera C. Rubin and W. Kent Ford, when studying Andromeda, detected that the rotation speed of peripheral celestial bodies far from the center of the galaxy did not decrease with distance; instead, it remained constant, which means that even in the edge regions of the galaxy, there is still a significant amount of invisible matter. Based on Vera's research and subsequent measurements, astronomers established a theory that each spiral galaxy is closed by a ring-like mass that is transparent to any form of light. Later discoveries showed that the issue of missing mass in celestial bodies is common, although not all celestial objects exhibit this phenomenon, so astrophysicists employed various methods to search for this elusive dark matter.

Currently, there are several main categories of candidates considered as dark matter: the first type is the massive weakly interacting particle (WIMP); the second type is axions, which have a mass of approximately one billionth that of electrons; the third type is "inert" neutrinos, which rarely interact with ordinary matter and only undergo gravitational interactions; the fourth type is neutralino, which is a hypothetical particle predicted by supersymmetry theory; the fifth type is Kaluza Klein dark matter. While many other hypothetical particles have been proposed as potential dark matter candidates, decades of searching—ranging from astronomical observations to underground experiments—have provided no direct evidence of dark matter's existence. There has been some indirect evidence, mainly in high-energy particles believed to result from dark matter particle annihilation; however, conclusive direct evidence is still lacking.

Scholars have proposed attempts to modify Newtonian dynamics. For example, in 1983, Mordehai (Moti) Milgrom proposed "Modified Newtonian Dynamics (MOND)" [4]-[6]. Jacob Bekenstein, who proposed the theory of the origin of gravity [7], supported this theory. MOND is only an empirical modification of observational data formulas, which can explain some phenomena. Perhaps it is a significant step in modifying inertia, but it did not derive from physical principles, and its reasons seem insufficient [8] [9].

Eric Verlinde proposed a new theory of gravity, which suggests that universal gravity is an entropy force and that dark matter does not exist [10]. However, the view that gravity is an entropy force is considered a causal reversal, so physics did not accept it.

In 2009, Sudanese scholars proposed a new viewpoint called the generalized Newton's law of gravity. He believed that since moving charges generate a magnetic field, moving objects also produce a field corresponding to the magnetic field, called the gravitomagnetic field. This field generates a Lorentz-like force to explain the motion of celestial bodies [11] [12], which may seem reasonable, but upon closer examination of its theory, there are fatal flaws.

All these failures force scholars to ponder whether there are still flaws in matter theory in physics. Is there an error in the scientific understanding of matter? Should the gravitational field be treated as a matter with mass and energy? Can gravitational field matter further generate the gravitational field? Is the so-called dark matter gravitational field matter?

2. Inspiration from the Mass-Energy Formula for Electric and Magnetic Fields

The field is matter and also energy. The electric and magnetic fields are matter. For example, if the electric field strength in a vacuum electric field is E_e and the vacuum dielectric constant is ε_0, then the energy density of the electric field in a vacuum electric field is

$$\epsilon_e = \frac{1}{2}\varepsilon_0 E_e^2 \tag{1}$$

Similarly, if the magnetic induction strength of the magnetic field in the vacuum is B and the magnetic permeability of the vacuum is μ_0, then the energy density of the magnetic field in the vacuum is

$$\epsilon_m = \frac{1}{2\mu_0}B^2 \tag{2}$$

The charge of an electron is $-e$, and there is an electric field in the surrounding space. According to Coulomb's law, using the following formula can calculate the electric field strength at a distance of r from the center of the electron

$$E_e = -\frac{1}{4\pi\varepsilon_0}\frac{e}{r^2} \tag{3}$$

Substituting it into formula (1), the energy density in the space around the electron is

$$\epsilon_e = \frac{1}{2}\varepsilon_0 E_e^2 = \frac{1}{2}\varepsilon_0\left(-\frac{1}{4\pi\varepsilon_0}\frac{e}{r^2}\right)^2 = \frac{1}{32\pi^2\varepsilon_0}\frac{e^2}{r^4} \tag{4}$$

Assuming that an electron is a small ball and its classical radius is r_e, the total energy of the electric field in the entire space surrounding the electron is

$$\mathcal{E} = \int_{r_e}^{\infty}\frac{1}{32\pi^2\varepsilon_0}\frac{e^2}{r^4}4\pi r^2 \mathrm{d}r = \frac{1}{8\pi\varepsilon_0}\frac{e^2}{r_e} \tag{5}$$

This formula indicates that half of the mass-energy is distributed in the electric field outside the electron while the other half is in the electric field inside the electron and the magnetic field inside and outside the electron. Due to the complex internal structure of electrons, which possess electric and magnetic fields, the energy of the other half is hard to calculate using classical electrodynamics methods.

The above method for calculating the mass-energy of electric fields provides a good inspiration for Newton's theory of gravity, which suggests that gravitational fields should also have a mass-energy formula.

3. The Matter in the Gravitational Field of a Symmetrical and Uniform Spherical Celestial Body

3.1. The Matter of the Gravitational Field around Celestial Body

Any object will generate a gravitational field in the surrounding space. The expression for the universal gravitation between two objects is Newton's law of universal gravitation, which is similar in formula to Coulomb's law. The expression for the gravitational field is like (3), so there should also be a similar mass-energy calculation law in the gravitational field.

For celestial bodies with complex shapes, mass density distributions, and motion states, it is difficult to calculate their mass-energy. However, for celestial bodies with simple shapes, mass distributions, and motion states, such as the Earth and the Sun, it is easy to calculate their mass-energy. For example, to measure the mass of the sun, M_0, using the centripetal force of a planet orbiting to measure the gravitational force, expressed as

$$\frac{mv^2}{r} = G\frac{M_0 m}{r^2} \tag{6}$$

In Equation (6), m is the planet's mass, v is its orbital speed, and r represents its distance to the Sun.

Assuming a planet has an orbital radius of r, an orbital period of T, and an orbital speed $v = 2\pi r/T$, formula (6) can be used to derive the mass of the central celestial body, the Sun.

$$M_0 = \frac{\mathcal{E}_0}{c^2} = \frac{4\pi^2}{G}\frac{r^3}{T^2} \tag{7}$$

According to Equation (6), the speed of the planetary revolution around the Sun is

$$v = \sqrt{\frac{GM_0}{r}} \tag{8}$$

From the perspective of the materiality of the gravitational field, if using the radius R and period T of the Earth's orbit around the Sun to calculate the mass of the Sun, there is no doubt about the correctness of using this classical formula to calculate the mass of the Sun. However, the result value should not only include the mass of ordinary matter (baryons) inside the Sun but also the mass of the gravitational field between the Sun's interior and the Earth's orbit.

That is to say, this mass includes the mass of ordinary matter in the internal region of the Sun, as well as the mass of the gravitational field in a sphere with the radius of the Earth's orbit, but it does not distinguish between the mass of ordinary matter and the mass of gravitational field matter.

If the mass M_0 inside a sphere with a radius R is measured using the above method, then according to Newton's law of universal gravitation, the strength of the gravitational field at the outer radius r of the sphere is

$$g_0 = -G\frac{M_0}{r^2} = -G\frac{\mathcal{E}_0}{c^2 r^2} \tag{9}$$

According to formulas (1), (3), and (4) for calculating the mass-energy density around a spherical charge, corresponding to the gravitational field strength g, the mass-energy density formula for the gravitational field matter should be

$$\epsilon = \frac{1}{2}\frac{g^2}{4\pi G} \tag{10}$$

Substituting Equation (9) into Equation (10), the mass-energy density formula at radius r is

$$\epsilon_0 = \frac{g_0^2}{8\pi G} = \frac{1}{8\pi G c^4}\left(-\frac{G\mathcal{E}_0}{r^2}\right)^2 = \frac{G}{8\pi c^4}\frac{\mathcal{E}_0^2}{r^4} \tag{11}$$

Considering the mass-energy of the gravitational field, that inside a sphere with radius r must be modified. It should contain M_0 within the sphere with radius R and add the gravitational field matter within the sphere layer from radius R to radius r, as shown in **Figure 1**.

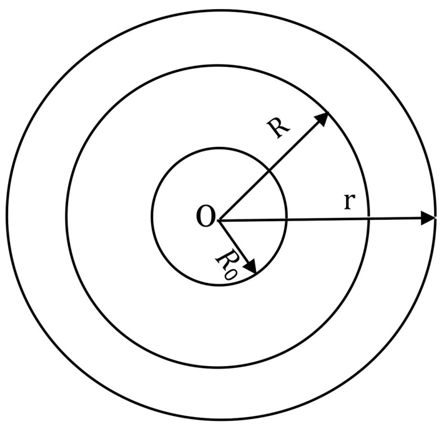

Figure 1. Calculation of the mass-energy of gravitational field matter in the sphere layer from radius R to r outside the sphere with R_0.

The first modification value of mass-energy is

$$\Delta\mathcal{E}_1 = \int_R^r \epsilon_0 \cdot 4\pi r^2 \mathrm{d}r = \int_R^r \frac{G}{8\pi c^4}\frac{\mathcal{E}_0^2}{r^4} \cdot 4\pi r^2 \mathrm{d}r \tag{12-1}$$

$$= \frac{G}{2c^4}\int_{r=R}^{r} \frac{\mathcal{E}_0^2}{r^2}\mathrm{d}r \tag{12-2}$$

So, after the first modification, the total mass-energy inside the sphere with radius r is

$$\mathcal{E}_1 = M_0c^2 + \Delta\mathcal{E}_1 \tag{13-1}$$

$$= \mathcal{E}_0 + \frac{G}{2c^4}\int_{r=R}^{r}\frac{\mathcal{E}_0^2}{r^2}\mathrm{d}r \tag{13-2}$$

A critical physical question arises: Will the added mass-energy in $\mathcal{E}_0 = M_0c^2$ generate new gravity? Scholar Charles T. Sebens believed [13] that gravitational mass-energy does not produce gravity, but this viewpoint lacks sufficient justification. Since it is mass-energy, there should be no essential difference between it and the mass-energy of ordinary matter. That is to say, $\Delta\mathcal{E}_1$ should also further generate a gravitational field.

After the first mass-energy modification, it is necessary to modify the gravitational field. So, the strength of the gravitational field is modified as follows

$$g_1 = -\frac{G\mathcal{E}_1}{c^2r^2} \tag{14}$$

After the first modification, the mass-energy density at radius r should be

$$\epsilon_1 = \frac{g_1^2}{8\pi G} = \frac{G\mathcal{E}_1^2}{8\pi c^4r^4} \tag{15}$$

The mass-energy within the sphere should be modified for the second time, with a modification value as follows

$$\Delta\mathcal{E}_2 = \int_{r=R}^{r}\epsilon_1\cdot 4\pi r^2\mathrm{d}r \tag{16-1}$$

$$= \int_{r=R}^{r}\frac{G\mathcal{E}_1^2}{8\pi c^4r^4}\cdot 4\pi r^2\mathrm{d}r \tag{16-2}$$

$$= \frac{G}{2c^4}\int_{r=R}^{r}\frac{\mathcal{E}_1^2}{r^2}\mathrm{d}r \tag{16-3}$$

So, after the second modification, the mass-energy in the entire sphere with radius r is

$$\mathcal{E}_2 = \mathcal{E}_0 + \Delta\mathcal{E}_2 \tag{17-1}$$

$$= \mathcal{E}_0 + \frac{G}{2c^4}\int_{r=R}^{r}\frac{\mathcal{E}_1^2}{r^2}\mathrm{d}r \tag{17-2}$$

After the second modification, the strength of the gravitational field generated by mass-energy $\mathcal{E}_2$ is

$$g_2 = -\frac{G\mathcal{E}_2}{c^2r^2} \tag{18}$$

The mass-energy density of the gravitational field is modified to be

$$\epsilon_2 = \frac{g_2^2}{8\pi G} = \frac{G\mathcal{E}_2^2}{8\pi c^4r^4} \tag{19}$$

The third modification of mass-energy from radius R to r within the sphere layer is

$$\Delta\mathcal{E}_3 = \int_{r=R}^{r} \epsilon_2 \cdot 4\pi r^2 \mathrm{d}r \tag{20-1}$$

$$= \int_{r=R}^{r} \frac{G\mathcal{E}_2^2}{8\pi c^4 r^4} 4\pi r^2 \mathrm{d}r \tag{20-2}$$

$$= \frac{G}{2c^4} \int_{r=R}^{r} \frac{\mathcal{E}_2^2}{r^2} \mathrm{d}r \tag{20-3}$$

In this way, after the third modification, the mass-energy of the entire sphere with radius r is

$$\mathcal{E}_3 = \mathcal{E}_0 + \Delta\mathcal{E}_3 \tag{21-1}$$

$$= \mathcal{E}_0 + \frac{G}{2c^4} \int_{r=R}^{r} \frac{\mathcal{E}_2^2}{r^2} \mathrm{d}r \tag{21-2}$$

By continuously modifying to obtain

$$\mathcal{E}_{n+1} = \mathcal{E}_0 + \frac{G}{2c^4} \int_{r=R}^{r} \frac{\mathcal{E}_n^2}{r^2} \mathrm{d}r \tag{22}$$

Modify infinitely multiple times to obtain the final mass-energy. It's obvious

$$\lim_{n\to\infty} \mathcal{E}_{n+1} = \lim_{n\to\infty} \mathcal{E}_n = \mathcal{E} \tag{23}$$

So, (22) ultimately becomes

$$\mathcal{E} = \mathcal{E}_0 + \frac{G}{2c^4} \int_{r=R}^{r} \frac{\mathcal{E}^2}{r^2} \mathrm{d}r \tag{24}$$

It is an integral equation. It is not easy to directly find $\mathcal{E}$.

For solving Equation (24), it is necessary to convert it into a differential equation, which requires taking the first-order derivative on both sides of the equation and considering that $\mathcal{E}_0 = M_0 c^2$ is a constant, the above equation becomes

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}r} = \frac{G}{2c^4} \frac{\mathcal{E}^2}{r^2} \tag{25}$$

Solving this differential equation yields

$$\mathcal{E} = \frac{1}{C + \frac{G}{2c^4} \frac{1}{r}} \tag{26}$$

In the Equation (26), C is the integral constant. $\mathcal{E} = \mathcal{E}_0 = M_0 c^2$ when $r = R$, obtained from Equation (26)

$$C = \frac{1}{\mathcal{E}_0} - \frac{G}{2c^4 R} \tag{27}$$

Substitute (27) into Equation (26) to obtain

$$\mathcal{E} = \frac{\mathcal{E}_0}{1 - \frac{G\mathcal{E}_0}{2c^4}\left(\frac{1}{R} - \frac{1}{r}\right)} \tag{28}$$

In other words, the mass-energy of the matter inside a sphere with the radius r is

$$\mathcal{E} = \frac{M_0 c^2}{1 - \frac{GM_0}{2c^2}\left(\frac{1}{R} - \frac{1}{r}\right)} \tag{26}$$

The mass inside a sphere with radius r is

$$M = \frac{M_0}{1 - \frac{GM_0}{2c^2}\left(\frac{1}{R} - \frac{1}{r}\right)} \tag{30}$$

Substitute it into Newton's law of universal gravitation to obtain

$$F = -G\frac{Mm}{r^2} = -\frac{GM_0 m}{r^2\left[1 - \frac{GM_0}{2c^2}\left(\frac{1}{R} - \frac{1}{r}\right)\right]} \tag{31}$$

According to the above equation, obtain the expression for the strength of the gravitational field.

$$g = \frac{F}{m} = -G\frac{M}{r^2} = -\frac{GM_0}{r^2\left[1 - \frac{GM_0}{2c^2}\left(\frac{1}{R} - \frac{1}{r}\right)\right]} \tag{32}$$

The mass-energy density of matter in the gravitational field at the radius is

$$\epsilon = \frac{g^2}{8\pi G} = \frac{G}{8\pi c^4}\frac{\mathcal{E}_0^2}{r^4\left[1 - \frac{G\mathcal{E}_0}{2c^4}\left(\frac{1}{R} - \frac{1}{r}\right)\right]^2} \tag{33}$$

According to Equation (32), the speed of the object's motion around the central celestial body at the orbital radius r is

$$v = \sqrt{\frac{GM_0}{r\left[1 - \frac{GM_0}{2c^2}\left(\frac{1}{R} - \frac{1}{r}\right)\right]}} = \sqrt{\frac{GM_0}{\left(1 - \frac{GM_0}{2c^2 R}\right)r + \frac{GM_0}{2c^2}}} \tag{34}$$

Formula (34) has an additional factor $1\Big/\sqrt{1 - \frac{GM_0}{2c^2}\left(\frac{1}{R} - \frac{1}{r}\right)}$ compared to formula (8). Obviously, under the condition of $r \geq R$, this factor is greater than or equal to 1, resulting in the speed of the surrounding celestial bodies

$$v \geq \sqrt{\frac{GM_0}{r}} \tag{35}$$

Moreover, as the factor $\frac{GM_0}{2c^2}\left(\frac{1}{R} - \frac{1}{r}\right) \to 1 - 0$, the deviation of the speed of surrounding celestial bodies from (8) becomes apparent. Therefore, the formula (8) is not applicable, but it is necessary to use (34).

3.2. The Matter inside Celestial Bodies

The shape and internal structure of the celestial bodies are often complex, and their mass distribution density and galaxy contours are not easy to express using simple functions. Even if expressed using simple functions, solving the equation may be difficult.

To understand the concept of gravitational field matter, assume that the mass distribution of ordinary matter inside celestial bodies is uniform, the mass density is ρ_0, the outline of the bodies is a sphere with a radius of R_0, the total mass of ordinary matter is M_R, and the total mass-energy is $\mathcal{E}_R$. Obviously, there is a relation as follows

$$\mathcal{E}_R = M_R c^2 = \frac{4\pi}{3}\rho_0 c^2 R_0^3 \tag{36}$$

The mass-energy of the ordinary matter within a sphere with a radius of r is

$$\mathcal{E}_0 = M_0 c^2 = \frac{4\pi}{3}\rho_0 c^2 r^3 \tag{37}$$

According to Newton's law of universal gravitation, the strength of the gravitational field on a sphere with a radius of r is

$$g_0 = -G\frac{\mathcal{E}_0}{c^2 r^2} \tag{38}$$

The corresponding mass-energy density is

$$\epsilon_0 = \frac{g_0^2}{8\pi G} = \frac{1}{8\pi G}\left(-G\frac{\mathcal{E}_0}{c^2 r^2}\right)^2 = \frac{G}{8\pi c^4}\left(\frac{\mathcal{E}_0}{r^2}\right)^2 \tag{39}$$

The first modification value of the mass-energy inside this sphere is

$$\Delta\mathcal{E}_1 = \int_0^r \epsilon_0 4\pi r^2 \mathrm{d}r = \frac{G}{2c^4}\int_0^r \frac{\mathcal{E}_0^2}{r^2}\mathrm{d}r \tag{40}$$

The mass-energy inside the sphere after the first modification is

$$\mathcal{E}_1 = M_0 c^2 + \Delta\mathcal{E}_1 \tag{41-1}$$

$$= M_0 c^2 + \frac{G}{2c^4}\int_0^r \frac{\mathcal{E}_0^2}{r^2}\mathrm{d}r \tag{41-2}$$

Similar to the modification process in 2.1, the mass-energy value after the ($n+1$) times modification is

$$\mathcal{E}_{n+1} = \mathcal{E}_0 + \frac{G}{2c^4}\int_0^r \frac{\mathcal{E}_n^2}{r^2}\mathrm{d}r \tag{42}$$

$\mathcal{E}_{n+1} \to \mathcal{E}_n \to \mathcal{E}$ when $n \to \infty$, so the above formula changes to

$$\mathcal{E} = \mathcal{E}_0 + \frac{G}{2c^4}\int_0^r \frac{\mathcal{E}^2}{r^2}\mathrm{d}r \tag{43}$$

Both $\mathcal{E}$ and $\mathcal{E}_0$ are functions on r. This equation is an integral one whose solution cannot be obtained directly through integration.

It is necessary to transform the equation into a differential one to solve $\mathcal{E}$. Differentiating both sides of the equation with respect to r yields

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}r} = \frac{\mathrm{d}\mathcal{E}_0}{\mathrm{d}r} + \frac{G}{2c^4}\frac{\mathcal{E}^2}{r^2} \tag{44}$$

Using Equation (37), Equation (44) changes to

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}r} = \frac{G}{2c^4}\frac{\mathcal{E}^2}{r^2} + 4\pi\rho_0 c^2 r^2 \tag{45}$$

It is a nonlinear ordinary differential equation, and it is impossible to directly find $\mathcal{E}$, which requires further changes to a new equation. It needs a Riccati substitution. Suppose there is a new function $u(r)$ whose first-order derivative to r is $u' = \mathrm{d}u/\mathrm{d}r$ and its second-order derivative is $u'' = \mathrm{d}^2u/\mathrm{d}r^2$, as a substitution.

$$\mathcal{E}(r) = -\frac{2c^4 r^2}{G}\frac{u'}{u} \tag{46}$$

Then Equation (45) becomes

$$ru'' + 2u' + \frac{2\pi G\rho_0 r}{c^2}u = 0 \tag{47}$$

It is a second-order homogeneous ordinary differential equation with variable coefficients. Set a constant

$$\alpha = \sqrt{\frac{2\pi G\rho_0}{c^2}} \tag{48}$$

Then Equation (47) becomes

$$ru'' + 2u' + \alpha^2 ru = 0 \tag{49}$$

Expanding the function into a Taylor series

$$u(r) = \sum_{n=0}^{\infty} a_n r^n \tag{50}$$

Substituting (50) to (49) gives

$$\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}r^{n+1} + 2\sum_{n=0}^{\infty}(n+1)a_{n+1}r^n + \alpha^2\sum_{n=0}^{\infty}a_n r^{n+1} = 0 \tag{51}$$

Derive the relationship among coefficients from this

$$a_1 = 0 \tag{52}$$

$$a_2 = -\frac{1}{3!}\alpha^2 a_0 \tag{53}$$

$$a_3 = 0 \tag{54}$$

$$a_4 = -\frac{1}{20}\alpha^2 a_2 = \frac{1}{5!}\alpha^4 a_0 \tag{55}$$

$$a_5 = 0 \tag{56}$$

$$a_6 = -\frac{1}{42}\alpha^2 a_4 = -\frac{1}{7!}\alpha^6 a_0 \tag{57}$$

$$a_7 = 0 \tag{58}$$

$$\vdots$$

$$a_{n=2k} = \frac{(-1)^k}{(2k+1)!}\alpha^{2k}a_0 \quad (k \ge 0) \tag{59}$$

$$a_{n=2k+1} = 0 \quad (k \ge 0) \tag{60}$$

Summarizing the above relationship can conclude that

$$u(r)=a_0\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}\alpha^{2k}r^{2k} \tag{61-1}$$

$$=\frac{a_0}{\alpha r}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}(\alpha r)^{2k+1} \tag{61-2}$$

$$=a_0\frac{\sin(\alpha r)}{\alpha r} \tag{61-3}$$

Substituting it into (46) obtains

$$\mathcal{E}(r)=-\frac{2c^4r^2}{G}\frac{u'}{u} \tag{62-1}$$

$$=\frac{2c^4r}{G}\left[1-(\alpha r)\cdot\operatorname{ctg}(\alpha r)\right] \tag{62-2}$$

Set

$$\chi=\alpha r \tag{63}$$

Then (62) becomes

$$\mathcal{E}(r)=\frac{2c^4}{G\alpha}\chi(1-\chi\cdot\operatorname{ctg}\chi) \tag{64}$$

When r reaches the radius of the sphere, R_0, *i.e.*, $r=R_0$, then $\chi=\alpha r$ becomes

$$\chi_0=\sqrt{\frac{3G\mathcal{E}_{0R}}{2c^4R_0}}=\sqrt{\frac{3GM_{0R}}{2c^2R_0}} \tag{65}$$

The mass-energy of ordinary matter inside the sphere is $\mathcal{E}_{0R}=M_{0R}c^2$, and the total mass-energy of matter in a gravitational field is

$$\mathcal{E}(R_0)=\frac{2c^4R_0}{G}(1-\chi_0\cdot\operatorname{ctg}\chi_0) \tag{66}$$

The total mass-energy $\mathcal{E}(R_0)$ inside the sphere is not equal to that of ordinary matter $\mathcal{E}_{0R}$.

3.3. The Mass-Energy Relationship inside and outside the Sphere

If $r=R=R_0$, the value $\mathcal{E}_0$ in Equation (28) equals $\mathcal{E}(R_0)$ in Equation (66), thus establishing a relationship

$$\mathcal{E}=\frac{M_0c^2}{1-\frac{GM_0}{2c^2R_0}\left(1-\frac{R_0}{r}\right)} \tag{67-1}$$

$$=\frac{2c^4R_0}{G}\frac{1-\chi_0\operatorname{ctg}\chi_0}{1-(1-\chi_0\operatorname{ctg}\chi_0)\left(1-\frac{R_0}{r}\right)} \tag{67-2}$$

This formula expresses the relationship between the gravitational matter and the ordinary matter of the sphere with radius r due to the presence of gravitational field matter inside and outside the sphere.

If $r \to \infty$, the above expression tends towards

$$\mathcal{E}_{\infty} = \frac{2c^4 R_0}{G}\left(\frac{\operatorname{tg}\chi_0}{\chi_0} - 1\right) \tag{68}$$

The graph of this function resembles the one shown in Figure 2.

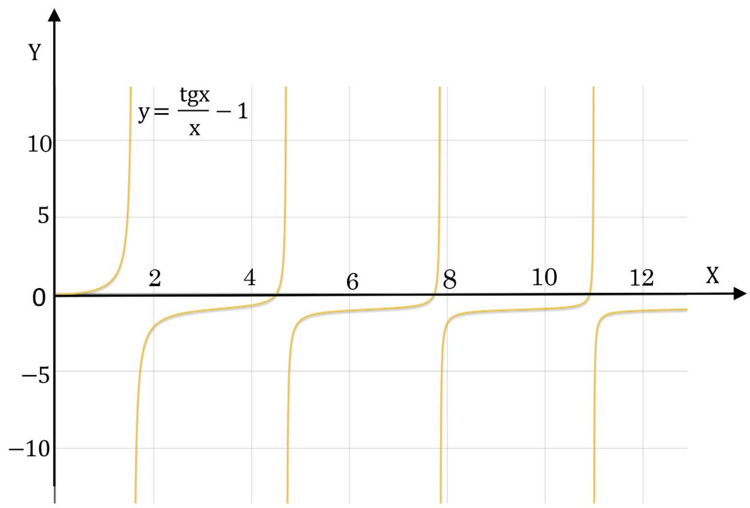

Figure 2. The graph of function $y = tgx/x - 1$.

$$y = \frac{\operatorname{tg} x}{x} - 1 \tag{69}$$

In this function, the independent variables $x = n\pi + \pi/2$ are singularities (where $n = 0,1,2,3,\cdots$). At these singularities, $\mathcal{E}_{\infty} = \pm\infty$. The graphs of functions (68) or (69) show that within some ranges, the gravitational matter of celestial bodies takes on negative values, indicating repulsive forces between the internal matter of the body, which is inconceivable.

The radius R_0 approaches its minimum values where $\chi_0 \to n\pi + \pi/2 - 0$.

$$R_{0n} = \frac{6}{(2n+1)^2 \pi^2} \frac{GM_{0R}}{c^2} + 0 \quad (n = 0,1,2,3,\cdots) \tag{70}$$

The curve with potential physical significance is the first one corresponding to $n = 0$. Due to $\chi_0 \to \pi/2 - 0$, obtained from Equation (65)

$$R_0 \geq \frac{6}{\pi^2} \frac{GM_{0R}}{c^2} \tag{71}$$

The minimum value of the celestial body's radius can only be

$$R_{0\min} = \frac{6}{\pi^2} \frac{GM_{0R}}{c^2} \tag{72}$$

However, this minimum radius is the largest value in formula (70).

3.4. The Gravitational Field of Newtonian Black Holes

Since gravitational field matter is related to ordinary matter and the structure factor $GM_0/2c^2R_0$ of celestial bodies, this is particularly significant for dense objects. Black holes are usually such dense objects with strong gravitational fields, making the mass of gravitational field matter non-negligible.

The measured mass of black holes using surrounding celestial bodies always yields only part of their gravitational mass rather than the entire gravitational mass of the objects or their actual mass.

The structure of black holes is discussed below from two different perspectives.

1) According to the theory of special relativity

To find the relationship between the mass and structural radius of ordinary matter, suppose there is a black hole with the mass of ordinary matter being M_{0R} and its radius being R_0. According to formula (67-2), when measured on the surface of the black hole, the mass of the black hole is

$$M_0 = \frac{2c^2 R_0}{G}\left(1 - \chi_0 \cdot \operatorname{ctg}\chi_0\right) \tag{73}$$

If an object with a rest mass of m_0 is launched radially from the surface of a black hole with an initial speed of v_0, and its speed at infinity distance is zero, with gravitational potential energy also being zero, an equation based on the law of conservation of mechanical energy is given by

$$\frac{m_0 c^2}{\sqrt{1-\frac{v_0^2}{c^2}}} - \frac{G M_0 m_0}{R_0\sqrt{1-\frac{v_0^2}{c^2}}} = m_0 c^2 \tag{74}$$

This equation can turn into

$$1 - \frac{G M_0}{c^2 R_0} = \sqrt{1-\frac{v_0^2}{c^2}} \tag{75}$$

Since celestial bodies are black holes, even if an object's initial speed reaches that of light, it cannot escape the gravitational pull of the black hole. Therefore, the condition must meet with $v_0 = c$. The equation can turn into

$$1 - \frac{G M_0}{c^2 R_0} = 0 \tag{76}$$

2) According to the photon redshift perspective

One can also imagine that a photon with energy $h\nu$ is emitted radially from the surface of a black hole. Due to gravitational redshift, its energy becomes zero at infinity distance, which sets up an equation as the following

$$h\nu - \frac{G M_0}{R_0}\frac{h\nu}{c^2} = 0 \tag{77}$$

The solution to this equation is also Equation (76). It yields the same solution from two different starting points.

$$M_0 = \frac{c^2 R_0}{G} \tag{78}$$

Substituting Equation (78) into Equation (73) obtains

$$\frac{\operatorname{tg}\chi_0}{\chi_0} = 2 \tag{79}$$

This equation has infinitely many solutions. To solve this equation, let the function

$$y = \frac{\operatorname{tg} x}{x} - 2 \tag{80}$$

The intersection points of this function with the X-axis are the solutions to Equation (79): $x_1, x_2, x_3, \cdots$. Its graph resembles the one shown in **Figure 3**.

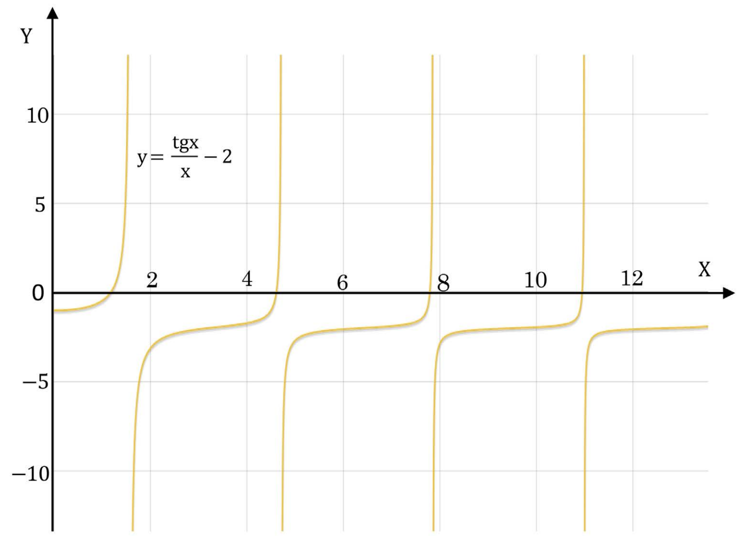

Figure 3. It is the graph of the function $y = tgx/x - 2$, from which the intersection points with the X-axis can be found, thus determining the solutions to the equation $tgx/x - 2 = 0$.

Use computer mathematical function graphing tools to obtain these solutions.

$$\chi_{01} = 1.165561185207 \tag{81-1}$$

$$\chi_{02} = 4.604216777200 \tag{81-2}$$

$$\chi_{03} = 7.789883751144 \tag{81-3}$$

$$\vdots$$

According to formula (63), each solution corresponds to a radius. However, since each solution is isolated by an infinitely deep potential well and an infinitely high potential barrier, the state of the black hole cannot transition between these radii. Therefore, solutions except for χ_{01} are not to be considered. Perhaps it poses a more profound physical significance hidden in this suspense that needs further exploration.

According to Equation (65), the critical radius of the black hole corresponding to χ_{01} is

$$R_C = \frac{3}{2\chi_{01}^2}\frac{GM_R}{c^2} = 1.104132278276 \times \frac{GM_R}{c^2} = \kappa\frac{GM_R}{c^2} \tag{82}$$

where

$$\kappa = \frac{3}{2\chi_{01}^2} = 1.104132278276 \tag{83}$$

Among all the solutions, χ_{01} is the smallest, corresponding to the maximum radius of the black hole.

If the distance R_0 in formula (67) exactly equals the critical structural radius of the black hole, R_C, then $M_0 = M_{0R}$, yields

$$M = \frac{M_0}{1 - \frac{GM_0}{2c^2}\left(\frac{1}{R_0} - \frac{1}{r}\right)} = \frac{M_0}{1 - \frac{GM_0}{2c^2 R_0} + \frac{GM_0}{2c^2 r}} \tag{84-1}$$

$$= \frac{2c^2}{G}\frac{R_0 r}{r + R_0} = \frac{2c^2}{G}\frac{R_C r}{r + R_C} \tag{84-2}$$

$$= \frac{2\kappa M_{0R} r}{r + \kappa \frac{GM_{0R}}{c^2}} \tag{84-3}$$

That is, the minimum mass of a black hole is

$$M_{\min} = \frac{2\kappa M_{0R} r}{r + \kappa \frac{GM_{0R}}{c^2}} \quad \left(r \ge R_{BH}\right) \tag{85}$$

At the surface of a critical black hole, the relationship between gravitational mass M and ordinary mass M_{0R} is such that the gravitational mass M is κ times the ordinary mass M_{0R}, that is κM_{0R}. If observing the critical black hole from an infinite distance, the gravitational mass is $2\kappa M_{0R}$. It proves that half of a critical black hole's gravitational mass is inside the black hole while the other half is outside.

This is somewhat similar to the mass-energy distribution characteristics of a single electron. So, a single electron is an electric field black hole.

On the other hand, according to Equation (65), setting $\chi_0 \to \pi/2 - 0$ yields

$$R_0 \to \frac{6}{\pi^2}\frac{GM_{0R}}{c^2} + 0 \tag{86}$$

Substituting it into (67-2) obtains

$$\lim_{\chi_0 \to \pi/2} M = \lim_{\chi_0 \to \pi/2} \frac{2c^2 R_0}{G} \frac{1 - \chi_0 \operatorname{ctg} \chi_0}{1 - \left(1 - \chi_0 \operatorname{ctg} \chi_0\right)\left(1 - \frac{R_0}{r}\right)} = \frac{2c^2 r}{G} \quad \left(r \ge R_C\right) \tag{87}$$

It is the limit value of black hole mass measured at a distancer from the black hole's center that is impossible to reach, of course, so there always exists a relationship as

$$M < \frac{2c^2 r}{G} \quad \left(r > R_C\right) \tag{88}$$

So, if a black hole's radius reaches the value $6GM_{0R}/\pi^2 c^2$, no object can escape its gravitational pull, even if it travels at the speed of light, regardless of how far away it is.

If the ordinary mass of a black hole is M_{0R}, then its gravitational mass, M, as measured from outside the black hole entity, would be

$$\frac{2\kappa M_{0R}r}{r+\kappa\frac{GM_{0R}}{c^2}} \le M < \frac{2c^2 r}{G} \quad (r > R_C) \tag{89}$$

As shown in formula (89), the gravitational mass of a black hole increases with the increase in radius r. In contrast, the gravitational mass of celestial bodies calculated according to Newton's conventional law of universal gravitation does not increase with an increase in measurement radius r.

Equation (89) indicates that at least half of the black hole's gravitational mass is distributed outside the black hole, or in other words, most of the gravitational mass spreads out in space outside the black hole.

Combining (82) and (86) obtain the range of structural radii for the black hole

$$\frac{6}{\pi^2}\frac{GM_{0R}}{c^2} < R_{BH} \le \kappa\frac{GM_{0R}}{c^2} \tag{90}$$

It is just a range for the radius of the black hole structure, but its specific expression remains unknown.

4. Explore Some Celestial Issues

The previous text presented a series of formulas based on the assumption that celestial bodies are spherical and have a uniform density distribution. This idealized model, characterized by its simple structure and shape, makes mathematical derivation easier and helps understand the underlying concepts. In reality, however, celestial structures are highly complex, featuring uneven mass density distributions and irregular shapes.

4.1. The Earth's Gravitational Field Matter

The Earth's ordinary mass is $M_e = 5.965\times10^{24}$ kg, with an Earth radius of $R_e = 6.37\times10^6$ m. This results in

$$\eta_e = \frac{GM_e}{2c^2 R_e} = 1.04\times10^{-9} \ll 1 \tag{91}$$

According to formula (65), this factor is negligible unless precise calculation is necessary. Therefore, the Earth's mass-energy can be expressed as $\mathcal{E} = M_e c^2$.

In the space near the Earth's surface, the mass-energy density of the gravitational field is given by

$$\epsilon = \frac{g^2}{4\pi G} = 1.146\times10^{11}\ \mathrm{J/m^3} \tag{92}$$

or mass density is

$$\rho_e = \frac{\epsilon}{c^2} = \frac{1}{4\pi G}\frac{g^2}{c^2} = 1.273\times10^{-6}\ \mathrm{kg/m^3} \tag{93}$$

The gravitational field matter is negligible compared to ordinary matter on Earth. However, there is indeed gravitational field matter inside and outside the Earth.

4.2. The Gravitational Field Matter of the Sun

The sun is a typical example of a star. The mass of the sun is $M_{sun} = 2.0 \times 10^{30}$ kg, and its radius is $R_{sun} = 6.963 \times 10^{8}$ m.

As calculated in Equation (91)

$$\eta_{sun} = \frac{GM_{sun}}{2c^2 R_{sun}} = 1.06 \times 10^{-6} \ll 1 \tag{94}$$

The mass of the sun, calculated using formula (67) with the condition $r = \infty$, is $M_{sun} = M_R = 2.0 \times 10^{30}$ kg.

When calculating the solar total mass, the mass of the gravitational field matter is negligible. Thus, the solar total mass equals the mass of ordinary matter unless precise calculation is necessary.

4.3. The Gravitational Field Matter of Neutron Stars

A typical neutron star forms from the collapse of a star with a mass of about 1.4 times that of the Sun, that is $M_n = 2.8 \times 10^{30}$ kg. The collapse process leads to a supernova explosion, during which the lost mass is roughly 5.0×10^{-4} times of the star's pre-collapse mass, which is negligible in rough calculations. The radius of a neutron star is approximately 1.1×10^{4} m. Thus

$$\eta_n = \frac{GM_n}{2c^2 R_n} = 0.094323 \tag{95}$$

Utilizing formula (65), determine

$$\chi_0 = \sqrt{\frac{3GM_n}{2c^4 R_n}} = 0.53195 \tag{96}$$

Substitute this result into formula (68) to calculate the desired value.

$$\mathcal{E}_{n\infty} = \frac{2c^4 R_0}{G}\left(\frac{\operatorname{tg}\chi_0}{\chi_0} - 1\right) = 2.842 \times 10^{47}\ \text{J} \tag{97}$$

The total mass of a neutron star measured from an infinite distance is

$$M_{n\infty} = \frac{\mathcal{E}_{\infty}}{c^2} = 3.158 \times 10^{30}\ \text{kg} = 1.1278 M_n \tag{98}$$

The gravitational mass of a neutron star is 1.1278 times that of the ordinary matter of the star before it exploded, M_n. It means that after a star undergoes a supernova explosion and becomes a neutron star, its total mass increases to 1.1278 times. What has increased is the mass of the gravitational field.

Does this violate the law of conservation of energy? Of course not. In fact, after the collapse of a celestial body, the total mass-energy increases while the gravitational potential energy decreases, and the total energy before and after changes in the system remains conserved.

4.4. The Gravitational Field Matter of Galaxies

Galaxies have shapes such as spherical, ellipsoidal, and spiral. Even if galaxies are spiral-shaped, there is also a bulge-dominated spiral (NGC7814), disk-dominated spiral (NGC6503), and gas-dominated dwarf (NGC3741). While it is theoretically possible to derive a series of formulas to calculate the gravitational field matter and other physical quantities based on observed data, the complexity of these calculations poses significant challenges. Nevertheless, the mentioned formulas can still help make approximate estimations.

Take the Milky Way as an example. Currently, there is no precise value for the total mass of visible matter in the Milky Way, but it is estimated to be on the order of magnitude represented by $M_G \approx 10^{42}$ kg, with a disk radius of $R_G = 3.9\times10^{20}$ m. At the center of the Milky Way, a supermassive black hole is the point with the highest mass density.

Preliminary estimation indicates

$$\eta_G = \frac{GM_G}{2c^2R_G} \approx 9.5014245\times10^{-7} \tag{99}$$

According to Equation (65) obtains

$$\chi_0 = \sqrt{\frac{3GM_G}{2c^2R_G}} = 1.68832\times10^{-3} \tag{100}$$

Use formula (68) to perform the calculation as follows

$$M_{G\infty} = \frac{2c^2R_G}{G}\left(\frac{\operatorname{tg}\chi_0}{\chi_0} - 1\right) \tag{101-1}$$

$$= 1.00000243\times10^{42}\ \text{kg} \approx 1.00000243 M_G \tag{101-2}$$

The calculation results show that the gravitational mass of the galaxy is slightly greater than its ordinary mass. So, the structure factor of the entire galaxy is tiny, the macroscopic gravitational field matter of the visible matter is negligible, and the conventional Newton's law of universal gravitation is applicable.

A spiral galaxy consists of three parts: the galactic nucleus, the disk, and the spiral arms. The majority of the mass in the Milky Way consists of stars, black holes, interstellar dust, gas, and other matter.

Based on the luminosity pattern [1] and assuming that the mass density of the disk is $\sigma = \sigma_0 e^{-\beta r}$, the mass within a radius r on the disk's surface can be expressed as follows

$$M = \int_0^R \sigma \mathrm{d}\left(\pi r^2\right) = 2\pi\sigma_0 \int_0^r e^{-\beta r} r\mathrm{d}r = \frac{2\pi\sigma_0}{\beta^2}\left[1-\left(1+\beta r\right)e^{-\beta r}\right] \tag{102}$$

If $r \to \infty$, the formula above yields $M \to M_G$, that is

$$M_G = \frac{2\pi\sigma_0}{\beta^2} \tag{103}$$

Using formulas (8) and (102), obtain

$$v = \sqrt{\frac{GM}{r}} = \left\{\frac{2\pi G\sigma_0\left[1-\left(1+\beta r\right)e^{-\beta r}\right]}{\beta^2 r}\right\}^{\frac{1}{2}} = v_0 y \tag{104}$$

Here, $x = \beta r$, $v_0 = \sqrt{2\pi G\sigma_0/\beta} = \sqrt{\beta GM_G}$, y can be expressed by the function as follows

$$y = \left[\frac{1-(1+x)\mathrm{e}^{-x}}{x}\right]^{\frac{1}{2}} \tag{105}$$

Its graph resembles as shown in **Figure 4**, where the unit for the Y-axis is $v_0 = \sqrt{\beta GM_0}$.

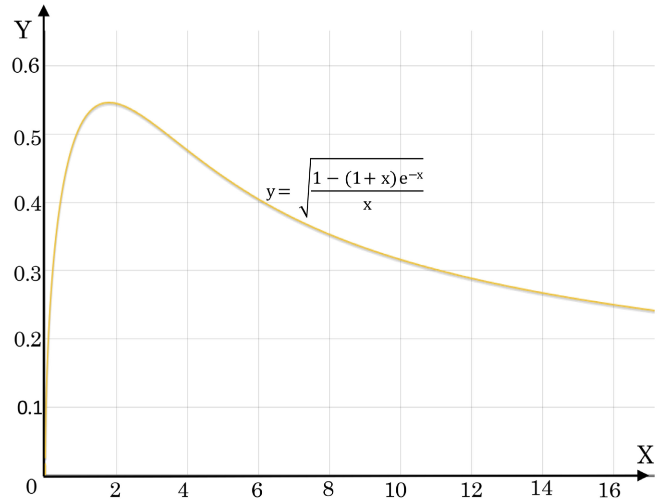

Figure 4. The relationship between the speed and radius of an object in a disk galaxy with a giant black hole at the center undergoing circular motion around the center.

The maximum speed of an object's circular motion within a galaxy depends on the mass M_G of its visible matter and the attenuation rate in mass density, β. Under certain matter conditions, the higher the value of β, the more mass is concentrated at the center, leading to a higher maximum speed.

Of course, the distribution of galaxy mass density does not strictly follow an exponential law pattern. However, interpreting β as the degree of mass concentration, it is unnecessary to question whether the above models align with actual celestial structures. The speed curve in the image is very similar to the observed situation. However, the estimated speed is numerically smaller than the observed value. Thus, the problem of a large amount of invisible matter still existing in the galaxy arises. So, what exactly are these invisible substances?

Apart from approximately 200 billion stars and interstellar dust, black holes' mass is a significant component. Although current estimates suggest about 100 million black holes in the Milky Way, their ordinary mass constitutes a small proportion of the total mass. Nevertheless, due to the celestial structure factor of black holes possibly being close to 1, with a very narrow range of variation from critical radius $R_C = \kappa GM_{0R}/c^2$ to limitation radius $R_{lim} = 6GM_{0R}/\pi^2c^2$, so black holes significantly affect the gravitational mass of celestial bodies.

For example, the mass of the supermassive black hole at the center of the Milky Way, Sagittarius A*, is about 4.3 million times that of the Sun, *i.e.*, $M_R = 8.6\times10^{36}$ kg, with an observed radius of approximately $R_{BH} = 2.2\times10^{10}$ m. This data is controversial; even if calculated using the Schwarzschild radius formula $R_S = 2GM/c^2$, it should be $R_S = 1.186\times10^{10}$ m, and according to the critical radius formula for black holes (82), it should be $R_C = \kappa GM_{0R}/c^2 = 6.374\times10^9$ m. The value $R_{BH} = 2.2\times10^{10}$ m cannot represent a black hole. Of course, black holes don't have to be spherical or have uniformly distributed mass, nor are they necessarily static. However, based on R_{BH}, the structure factor of celestial body is

$$\eta_s = \frac{GM_S}{2c^2R_S} = \frac{6.67\times10^{-11}\times8.6\times10^{36}}{2\times\left(3\times10^8\right)^2\times2.2\times10^{10}} = 0.1448535 \tag{106}$$

Thus

$$\chi_0 = \sqrt{\frac{3GM_S}{2c^2c^2R_S}} = 0.659212 \tag{107}$$

Substitute into formula (68) to obtain

$$M_\infty = \frac{2c^2R_S}{G}\left(\frac{\operatorname{tg}\chi_0}{\chi_0}-1\right) = 10.414\times10^{36}\ \text{kg} = 1.211M_S \tag{108}$$

It is evident that this gravitational field mass is 1.211 times greater than ordinary mass.

Since a celestial body becomes a black hole, its radius must be less than or equal to the critical radius. Even if the radius equals the critical radius, according to formula (85), the gravitational mass of the black hole is at least $M_C = \kappa M_{0R}$. As the radius approaches $6GM_{0R}/\pi^2c^2$, the gravitational mass of the black hole increases, with no upper limit.

Therefore, the missing mass in galaxies is likely to be the mass of the gravitational field matter of black holes.

4.5. The Gravitational Field Matter of Galaxy Cluster

Take the Coma Cluster as an example. Its total visible mass is 10^{15} times the mass of the Sun, that is, $M_{Coma} = 2\times10^{45}$ kg, and its radius is approximately 4 times that of the Milky Way [14], which is $R_{Coma} = 1.56\times10^{21}$ m.

The structure factor of celestial body is

$$\eta_C = \frac{GM_{Coma}}{2c^2R_{Coma}} \approx 4.75071225\times10^{-4} \tag{109}$$

Use formula (65) to perform the calculation as follows

$$\chi_0 = \sqrt{\frac{3GM_{Coma}}{2c^2R_{Coma}}} = 3.7752\times10^{-2} \tag{110}$$

Use formula (68) to perform the calculation as follows

$$M_{C\infty} = \frac{2c^2R_{Coma}}{G}\left(\frac{\operatorname{tg}\chi_0}{\chi_0}-1\right) = 2.00114\times10^{45}\ \text{kg} = 1.00057M_C \tag{111}$$

Therefore, the mass of the galaxy's gravitational field matter has increased slightly, so the speed formula (8) is still valid.

Suppose the mass density distribution function of clusters is

$$\rho = \rho_0 e^{-\beta r} \tag{112}$$

The ordinary mass within a sphere of radius r is

$$M = \int_0^r \rho_0 e^{-\beta r} 4\pi r^2 dr = \frac{8\pi\rho_0}{\beta^3}\left\{1-\left[1+\beta r+\frac{1}{2}(\beta r)^2\right]e^{-\beta r}\right\} \tag{113}$$

If $r \to \infty$, the formula above yields $M \to M_C$, that is

$$M_{Coma} = \frac{8\pi\rho_0}{\beta^3} \tag{114}$$

The speed of an object moving in circular motion around the center of the galaxy cluster is

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM_{Coma}\left\{1-\left[1+\beta r+\frac{1}{2}(\beta r)^2\right]e^{-\beta r}\right\}}{r}} \tag{115-1}$$

$$= \sqrt{\frac{\beta GM_{Coma}\left[1-\left(1+x+\frac{1}{2}x^2\right)e^{-x}\right]}{x}} \tag{115-2}$$

Here $x = \beta r$, $v_0 = \sqrt{\beta GM_{Coma}}$, and $v = v_0 y$, y can be expressed by the function as follows

$$y = \sqrt{\frac{1-\left(1+x+\frac{1}{2}x^2\right)e^{-x}}{x}} \tag{116}$$

The unit on the Y-axis is $v_0 = \sqrt{\beta GM_{coma}}$. Its graph resembles as shown in Figure 5.

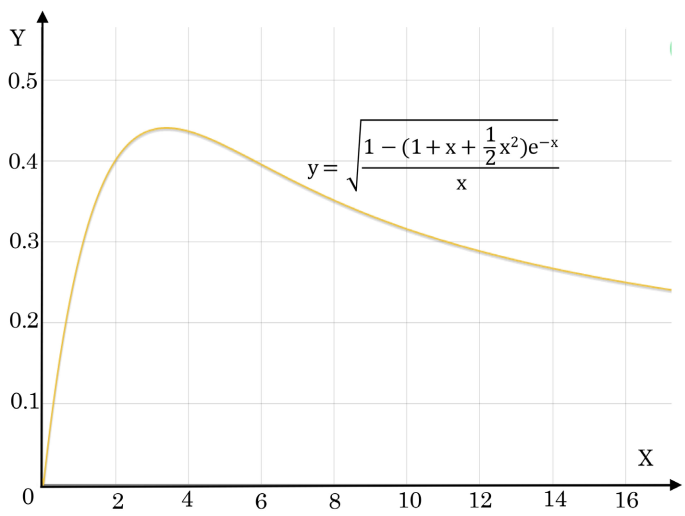

Figure 5. The relationship between the speed and radius of an object in a spherical galaxy with a giant black hole at the center undergoing circular motion around the center.

Similar to disk galaxies, the maximum speed of an object's circular motion within a galaxy depends on the mass M_{Coma} of its visible matter and the attenuation rate β in mass density. Under certain matter conditions, the higher the value of β, the more mass is concentrated at the center, leading to a higher maximum speed.

Equation (67) illustrates that the magnitude of gravitational matter depends on the mass of ordinary matter and the structure factor $GM_R/2c^2R_0$. It indicates that the higher the structure factor of a celestial body, the greater its effect on gravitational matter, which is associated with dark matter. In smaller structure factor objects such as the Earth and the Sun, the factor $GM_R/2c^2R_0$ is negligible, so it seems that there is no dark matter. However, for celestial bodies such as neutron stars and black holes, the factors $GM_R/2c^2R_0$ are so large that they cannot be ignored.

Using the speed of outer galaxy objects orbiting the center to calculate the galaxy mass and comparing it with luminous mass, there is inevitably a problem which gravitational mass exceeds luminous mass. A black hole not only has massive ordinary matter of its own but also generates gravitational field matter much more than its ordinary matter. Galaxies and galaxy clusters contain a large number of massive black holes, contributing enormous gravitational field matter.

5. Discussion

5.1. What Are the Characteristics of Gravitational Field Matter?

1) The gravitational field matter is generated by ordinary matter and exists inside and outside the ordinary matter. There is almost no gravitational field matter in spaces far from galaxies, galaxy groups, and galaxy clusters, especially in voids.

It can explain why dark matter always accompanies ordinary matter.

2) Because the gravitational field is a vector, it results from the vector superposition of gravitational fields generated by different parts of ordinary matter. Therefore, in space where the gravitational field cancels out to zero, the density of gravitational field matter is also zero. In space where the gravitational field weakens, the density of gravitational field matter decreases. In space where the gravitational field strengthens, the gravitational field matter increases.

It can explain the presence of dense dark matter halos around galaxies or star groups.

3) The strength of the gravitational field is related to the structure factor of the celestial body, $GM_R/2c^2R_0$.

It can explain why dark matter always exists inside and outside celestial bodies with high structure factors. In contrast, dark matter appears relatively scarce around loosely structured celestial bodies. Within and surrounding tightly structured celestial bodies, however, dark matter is found to be denser. It is particularly evident around neutron stars and black holes, where the concentration of dark matter is significantly higher.

4) The gravitational field matter only interacts through gravity. It does not

participate in electromagnetic, strong, and weak interactions. Consequently, non-gravitational methods cannot detect gravitational field matter. In other words, gravitational field matter is transparent to these non-gravitational forces.

It can explain why dark matter is uneasy to detect using non-gravitational methods.

5) The gravitational field matter does not consist of particles.

This view can explain why experimental methods such as particle colliders cannot detect this substance.

6) The gravitational field matter, like electromagnetic field matter, can excite waves and propagate through space. Static ordinary matter generates static gravitational fields, while dynamic ordinary matter produces dynamic gravitational fields that can propagate over long distances.

It can explain gravitational wave phenomena.

7) The mass-energy of the gravitational field matter can exist in a vacuum.

It can explain why dark matter is cold.

5.2. Is the Mass-Energy of the Gravitational Field Matter Negative?

There is literature [15] that states that the mass-energy of matter in a gravitational field is negative, with its calculation formula being

$$\mathcal{E}_p = \frac{1}{2}\int_0^V \rho\phi \mathrm{d}V \tag{117}$$

where, ϕ represents the gravitational potential, ρ is the mass density of ordinary matter, and V is the volume of the space in which ordinary matter exists.

Since the gravitational potential ϕ is always negative, the gravitational potential energy $\mathcal{E}_p$ obtained from the above integral is also always negative. Does this mean that the mass-energy of the gravitational field is negative? It is not. The above expression represents the gravitational potential energy of ordinary matter rather than the mass-energy of the gravitational field matter. While gravitational potential energy is negative, it does not imply that the mass-energy of the gravitational field is negative. The calculation of the mass-energy density of the gravitational field must utilize the formula (10) instead of this formula, which results in a positive mass-energy for the gravitational field matter.

5.3. Is It Necessary to Continue Searching for Dark Matter?

Some scientists believe that dark matter must be some unknown fundamental particles, proposing various hypotheses about it, with two main ones: one called Weakly Interacting Massive Particles (WIMPs), which are relatively heavy; the other called axions, which are very light. However, beyond this, they know nothing. Physicists have debated its properties for decades. But so far, attempts to find these particles have been unsuccessful. It has led some researchers to speculate that particle-based dark matter might not exist; instead, it could be a gravitational field matter generated by celestial bodies with high structure factors, such as neutron stars and black holes [16].

Due to the inability to find dark matter in particle form and the seemingly unnecessary modification of Newtonian dynamics, gravitational field matter provides a possible alternative to dark matter.

5.4. How to Understand the Radius of a Black Hole?

According to general relativity, if an object has mass M_R, its radius $2GM_R/c^2$ becomes the singularity known as the Schwarzschild radius or event horizon, which is the twice orbital radius of a photon in circular motion around a celestial body.

In contrast, the structure of a celestial body within the Newtonian static gravitational field has critical radius $R_C = \kappa GM_{0R}/c^2$, and a minimum radius $R_{\min} = 6GM_{0R}/\pi^2 c^2$. However, these radii do not numerically equal the Schwarzschild radius.

The critical radius $R_C = \kappa GM_{0R}/c^2$ of a black hole means that it can become a black hole only if the structural radius of the celestial body is less than or equal to this value. The minimum radius $R_{\min} = 6GM_{0R}/\pi^2 c^2$ of a celestial body refers to the limit radius that can only infinitely approach but can never reach when a celestial body collapses.

6. Conclusion

Astrophysicists have discovered a problem of mass shortage, which has led to the dark matter hypothesis, possibly due to the neglect of the gravitational field matter. Regarded the gravitational field as a form of matter might solve the problem of missing matter in galaxies, which means that the so-called dark matter in cosmology is gravitational field matter. Every celestial body has an internal and external gravitational field, meaning every celestial body generates gravitational field matter whose mass-energy density is proportional to the square of the gravitational field strength. However, for the systems of celestial bodies like planets, stars, galaxies, galaxy groups, and galaxy clusters, their gravitational field's matter can be ignored, making their mass equal to ordinary matter. For the celestial bodies of high structure factors such as neutron stars and black holes, their gravitational field's matter significantly increases; in these cases, gravitational field matter could be the dominant component. Although galaxies, galaxy groups, and galaxy clusters are not high structure factor celestial bodies, they often contain massive black holes that generate gravitational field matter, leading to mass anomalies where gravitational matter deviates from luminous mass. Dark matter might be gravitational field matter, and its mass density depends on the vector superposition of gravitational field strength. Gravitational field matter interacts only through gravity and not through other forces, giving it transparency. Gravitational field matter may play the role of dark matter, eliminating the need for physics to search for hypothetical particles like WIMPs and axions or to modify Newtonian dynamics and Newton's law of gravitation. Introducing the concept of gravitational field matter modifies the theory of inertia, thus adjusting Newtonian dynamics and universal gravitation law. The dark matter hypothesis arises from the geometric interpretation of gravity; correcting this theory involves treating gravity as a form of matter. The missing mass in the universe is due to the gravitational field mass of black holes. No celestial body can collapse infinitely; the final radius of a spherical body cannot reach the limit value $6GM_{0R}/\pi^2c^2$.

Acknowledgements

Thank my family for supporting my scientific research!

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] He, X.T. (2007) Observational Cosmology, Postgraduate Textbook of New Century Colleges and Universities. Beijing Normal University Press.

[2] Zwicky, F. (1933) The Redshift of Extragalactic Nebulae [Die Rotverschiebung von extragalaktischen Nebeln]. *Helvetica Physica Acta*, **6**, 110-127.

[3] Zwicky, B., Meyling, J. and Appell, D. (2021) Fritz Zwicky and the Earliest Prediction of Dark Matter. *Physics World*, **34**, 24-25. https://doi.org/10.1088/2058-7058/34/05/28

[4] Scarpa, R. (2005) Modified Newtonian Dynamics, an Introductory Review. arXiv: astro-ph/0601478

[5] Sanders, R.H. (2009) Modified Newtonian Dynamics: A Falsification of Cold Dark Matter. *Advances in Astronomy*, **2009**, Article ID: 752439. https://doi.org/10.1155/2009/752439

[6] Sanders, R.H. (2015) A Historical Perspective on Modified Newtonian Dynamics. *Canadian Journal of Physics*, **93**, 126-138. https://doi.org/10.1139/cjp-2014-0206

[7] Bekenstein, J.D. (2009) Relativistic MOND as an Alternative to the Dark Matter Paradigm. *Nuclear Physics A*, **827**, 555c-560c. https://doi.org/10.1016/j.nuclphysa.2009.05.122

[8] Trippe, S. (2013) A Derivation of Modified Newtonian Dynamics. *Journal of The Korean Astronomical Society*, **46**, 93-96. https://doi.org/10.5303/jkas.2013.46.2.93

[9] Milgrom, M. (1983) A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis. *The Astrophysical Journal*, **270**, 365-370. https://doi.org/10.1086/161130

[10] Verlinde, E. (2011) On the Origin of Gravity and the Laws of Newton. *Journal of High Energy Physics*, **2011**, Article No. 29. https://doi.org/10.1007/jhep04(2011)029

[11] Arbab, A.I. (2009) The Generalized Newton's Law of Gravitation. *Astrophysics and Space Science*, **325**, 37-40. https://doi.org/10.1007/s10509-009-0145-0

[12] Arbab, A.I. (2015) Flat Rotation Curve without Dark Matter: The Generalized Newton's Law of Gravitation. *Astrophysics and Space Science*, **355**, 343-346. https://doi.org/10.1007/s10509-014-2152-z

[13] Sebens, C.T. (2022) The Mass of the Gravitational Field. *The British Journal for the Philosophy of Science*, **73**, 211-248. https://doi.org/10.1093/bjps/axz002

[14] Ho, M., Ntampaka, M., Rau, M.M., Chen, M., Lansberry, A., Ruehle, F., *et al.* (2022)

The Dynamical Mass of the Coma Cluster from Deep Learning. *Nature Astronomy*, **6**, 936-941. https://doi.org/10.1038/s41550-022-01711-1

[15] Peters, P.C. (1981) Where Is the Energy Stored in a Gravitational Field? *American Journal of Physics*, **49**, 564-569. https://doi.org/10.1119/1.12460

[16] Vera, R. (1998) Dark Matter in the Universe. *Scientific American*, **9**, 106-110.