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Further Exploration of the Gauge Transformation across Fundamental Interactions

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Abstract

This paper constructs a gauge transformation where two photons convert into a graviton, revealing that the graviton can be viewed as a result generated by the interaction of two photons through gauge transformations that traverse different fundamental interactions, and vice versa. So it can be speculated that at any spacetime point in our universe, there exists a corresponding the generalized gauge transformation for each fundamental interaction (such as gravitation, electromagnetism, strong or weak interaction). This generalized gauge transformation corresponds to a specific physical process, allowing the conversion from one fundamental interaction to another. This significant conclusion indicates that the theory of principal fiber bundles and its geometric picture of the universe exhibits remarkable adaptability, capable of encompassing both quantum systems and general relativity systems. This characteristic allows the theory to bypass many difficulties faced in the construction of quantum gravity, providing a seamless unification of quantum effects and classical gravity, and presenting a new research pathway.

Keywords

Photons and Gravitons, Principal Fiber Bundles, General Gauge Transformations, Unification of Electromagnetism and Gravity

1. Introduction

One of the core tasks of physics is to construct a theoretical framework that can uniformly describe the four fundamental interactions in nature. These four types of interactions: gravity, electromagnetic force, strong interaction, and weak interaction, each control physical phenomena at different spatiotemporal scales and domains. Although researchers have made significant progress in the Grand Unified Theory (GUT) over the past few decades, the complete unification of the four interactions has not yet been achieved.

The Standard Model has successfully unified electromagnetic forces with weak interactions, known as electroweak interactions (Weinberg, Salam, Glashow, 1979), and combined with quantum chromodynamics (QCD) to provide a robust description of strong interactions. However, the quantization of gravity remains one of the biggest challenges facing physics today. Traditional general relativity, as a classical theory of gravity, cannot be described within the framework of quantum field theory. Therefore, finding a new theory that is compatible with quantum mechanics and general relativity has become the key to the grand unified theory [1].

In recent years, researchers have proposed a series of theoretical attempts to unify the four fundamental interactions. For example, string theory and M-theory provide a high-dimensional theoretical framework, in which gravity and other interactions can be viewed as different vibration modes of strings [2]. Loop Quantum Gravity attempts to quantize gravity from the perspective of geometric quantization [3]. However, these theories still face many challenges in terms of mathematical complexity and experimental verification.

After 2020, some new developments have provided new perspectives for achieving great unification. For example, A. Ashtekar *et al.* used the loop quantum gravity framework to study the microscopic properties of quantum gravity and explored how to integrate quantum effects with spatiotemporal geometry at extremely small scales [4]. In addition, E. Witten recently re-examined the low energy limit of string theory within the framework of gauge field theory and proposed a new unified path [5]. These studies indicate that gauge transformations and geometric structures may play a central role in the unification of interactions.

Aside from Loop Quantum Gravity and string theory, some new methods have begun to attract attention. In 2023, Jonathan Oppenheim proposed an alternative theory exploring the possibility of combining classical gravity with quantum mechanics, suggesting that gravity might not need to be fully quantized but could interact with quantum systems through probabilistic mechanisms [6]. Furthermore, research by Anupama B. and P. K. Suresh *et al.* introduced quantum effects into early cosmological inflation models to explain discrepancies in measurements of the Hubble constant, indicating the potential role of quantum gravity in cosmology [7].

Simultaneously, a new experimental research project has commenced in Antarctica to test the existence of quantum gravity by observing the interactions between high-energy neutrinos and gravitational effects [8]. This direct observational experiment will provide new experimental grounds for validating or excluding existing theories.

This study is based on the theory of principal fiber bundles in differential geometry, combined with the foundational principles of gauge transformations, proposing a new method to preliminarily achieve the unification of electromagnetism and gravity [9]-[12]. Through rigorous mathematical derivations, we derive the gauge transformation process where two photons form a graviton, interpreting gravity as a quantum effect. The quantum entanglement or strong correlation between photons is seen as one of possible mechanisms for the generation of gravitons, providing new possibilities for unifying electromagnetism and gravity from the perspective of gauge transformations [13].

The purpose of this paper is to further elucidate our proposed "generalized gauge transformation theory hypothesis" [9] [10]. This theory views the potential unification of the four fundamental interactions as a natural outcome of gauge transformations in higher-dimensional fiber bundle structures. Through this theory, we not only preliminarily achieve the unification of electromagnetism and gravity but also provide a theoretical basis for the further incorporation of strong and weak interactions.

2. Physical Significance of the Cosmic Structural Picture of the Principal Associated Bundle

The principal fiber bundle consists of a bundle manifold *P*, a base manifold *M*, and a structure group (Lie group) *G*, formed through some complex mappings [13]. These mappings require:

- G has a free right action on $R: P \times G \to P$;
- $\exists C^{\infty}$ projection mapping $\pi: P \to M$ satisfies $\pi^{-1}\left[\pi(P)\right] = \left\{pg \middle| g \in G\right\}$, $\forall p \in P$;
- There is an open neighborhood *U*, $\forall x \in U \subset M$ and the diffeomorphic map $T_U: \pi^{-1}[U] \to U \times G$, where T_U takes the form

$$T_U(p) = \left(\pi(p), S_U(p)\right), \forall p \in \pi^{-1}[U]$$

And the mapping $S_U: \pi^{-1}[U] \to G$ must satisfy $S_U(pg) = S_U(p)g$, $\forall g \in G$.

Associated bundle is a fiber bundle that associated the principal fiber bundle. Let $P(M, G)$ be the principal bundle, F be the manifold, and G have a left effect on *F*, $\chi: G \times F \to F$, *i.e.*, for $g \in G$, $f \in F$, $\chi_g(f) = gf$. The combined induction of G's free right action R on P and G's left action χ on F can lead to the free right action of G on $P \times F$ as

$$\xi: (P \times F) \times G \to P \times F$$

Namely, defined as

$$\xi_g(P, f) := \left(pg, g^{-1}f\right) \in P \times F, \forall g \in G, p \in P, f \in F$$

Redefine $\tau: P \times F \to P$ represents natural projection mapping, *i.e.*

$$\tau(p, f) := p, \forall p \in P, f \in F$$

We consider each orbit formed by ξ on $P \times F$ due to the variation of $g \in G$ as an element, and obtain the set $Q \equiv (P \times F)/\sim$, where ~ represents the equivalence relationship, that is, the two points of $P \times F$ are called equivalent if and only if they belong to the same orbit. So any element $q \in Q$ must be an orbital on $P \times F$. It can be proven that Q is a manifold, and this manifold is called an associated fiber bundle [9]-[13]!

Of course, there are also two natural projection mappings that are related to the Q manifold and the principal bundle: 1) the mapping corresponding to τ above is defined as $\hat{\tau}: P \times F \to Q$ (Note: $\tau(p, f) := p \in P$); 2) The mapping $\hat{\pi}: Q \to M$ corresponding to the principal bundle π is defined as $\hat{\pi}: Q \to M$, namely defining $\hat{\pi}(q) := \pi(p) \in M$, $\forall q = p \cdot f \in Q$, and further defined as $\hat{T}_U$ corresponding to T_U, etc. See **Figure 1**.

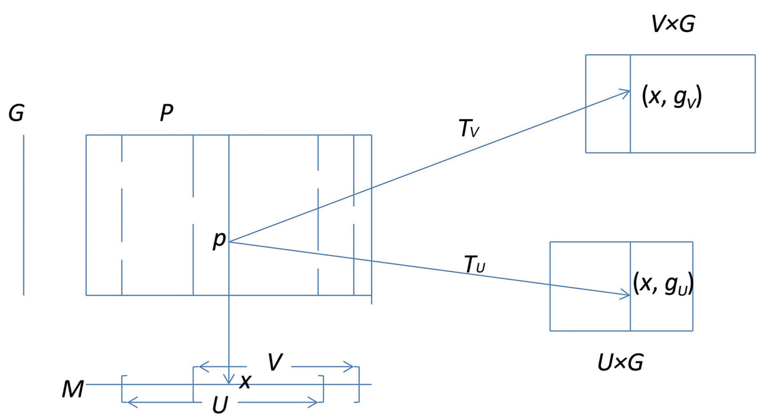

Figure 1. Illustrates the image of $p \in P$ under the mapping of T_U and T_V, where g_U and g_V belong to subgroups in G corresponding to different fundamental interactions, such as electromagnetic interaction: $g_U \in U(1), SU(2)$. gravitational interaction: $g_V \in O(4), SO(1,3)$ so that $g_{UV}(x) = g_U g_V^{-1} = S_U(p) S_V(p)^{-1}$ can form a gauge transformation across electromagnetic and gravitational interactions.

Now, two questions deserve exploration: 1) Can the bottom manifold accurately express the physical world of the universe? 2) What does the principal associated bundle structure signify for the universe?

Regarding the first question, gauge transformations based on the $U(1)$ and $SU(2)$ groups can keep the Lagrangian constant, successfully addressing physical issues such as electromagnetic fields and the zero rest mass of photons. Additionally, the invariance of gauge transformations based on $U(1) \times SU(2) \times SU(3)$ and the theory of the grand unification of weak, electromagnetic, and strong interactions suggest that the bottom manifold of the principal bundle structure can accurately describe the quantum physical world. Furthermore, from the perspective of general relativity, the bottom manifold equipped with a Riemannian metric is naturally suited for the relativistic universe, indicating that the base manifold of the principal bundle can also accurately represent the macroscopic and cosmological worlds.

Regarding the second question, the structural Lie group G can be regarded as a certain set of rules or laws governing the universe. Therefore, the principal fiber bundle manifold $P \equiv M \times G$ beyond the bottom manifold can be seen as the universe with inherent rules or laws. That's all. Driven by internal rules or laws, the universe is projected onto the bottom manifold, which is our observable universe. The connection between the principal fiber bundle manifold reflects the gauge potential, the curvature of the principal fiber bundle manifold reflects the strength of the gauge field, the cross-section of the associated fiber bundle is the physical gauge field, and the cross-section of the principal fiber bundle presents the selection of the gauge. All these projections on the bottom manifold are only projection components, and the gauge transformation reflects the changes between these components. The quantities on the principal bundle are invariant to the gauge transformation. This is the true meaning of the great unity of the interactions of universe [9]-[13]!

The reason why choosing the frame bundle as the principal bundle can better describe our physical world is that among all manifolds that can be constructed as a principal bundle, the unique feature is that the projection of the connections on the frame bundle onto the bottom manifold manifests as derivative operators, which are linked to the connections, curvature, and other geometric properties of Riemannian spacetime or special and general relativistic spacetime. Other manifolds constructed with Lie groups lack this property. Meanwhile, the connections in the physical world are precisely those connected to derivative operators in Riemannian spacetime or relativistic spacetime. This suggests that using the frame bundle as the principal bundle aligns with the observed facts of the physical world. Below, we present another theorem from a mathematical perspective, with a detailed proof provided in [13].

Theorem: A derivative operator ∇ on the bottom maniform M of the frame bundle FM gives a connection on FM, *i.e.* $(M,\nabla)\Leftrightarrow(FM,\tilde{\boldsymbol{\omega}})$, whose inverse is also true.

The proof approach involves first establishing $(M,\nabla)\Rightarrow(FM,\tilde{\boldsymbol{\omega}})$. This involves defining a horizontal subspace H_p on the tangent space T_pFM through the given (M,∇), and then demonstrating that this H_p satisfies all the properties of the horizontal space, thereby establishing its equivalence to $\tilde{\boldsymbol{\omega}}$. Subsequently, it is shown that $(FM,\tilde{\boldsymbol{\omega}})\Rightarrow(M,\nabla)$, where the derivative operator ∇ on the bottom manifold M is determined through the connection $\tilde{\boldsymbol{\omega}}$ of the frame bundle.

The aforementioned theorem unequivocally demonstrates that the frame bundle FM is compatible with the physical world, and the same applies to the tangent bundle. The cross-sectional expression of the tangent bundle corresponds to the physical gauge field. Furthermore, it can be proven that any connection on the principal bundle P naturally induces a connection on its associated bundle Q [13].

Therefore, there exists a solid physical rationale for selecting the frame bundle coupled with the associated vector bundle (or tensor bundle) as the principal fiber bundle for describing our universe. This framework serves as the backbone for cross-fundamental interaction gauge transformations and models of cosmic evolution [9]-[13]. Next, let's delve into the mathematical and physical implications of cross-fundamental interactions.

3. Definition of Gauge Transformation across Fundamental Interactions

Assuming $P(M,G)$ is the principal bundle, and $T_U:\pi^{-1}[U]\to U\times G$ and $T_V:\pi^{-1}[V]\to V\times G$ are two local trivialities with $U\cap V\neq\varnothing$, then each $p\in\pi^{-1}[U\cap V]$ in G has two image points mapped by S_U and S_V, which are $g_U=S_U(p)$ and $g_V=S_V(p)$, respectively. Here, they belong to two subgroups in G that can express different basic interactions. For example, for electromagnetic fields, $g_U=S_U(p)\in SU(2)$, for the gravitational field, $g_V=S_V(p)\in SO(1,3)$, so there are $T_U(p)=(x,g_U)$, and $T_V(p)=(x,g_V)$. In this way, both g_U and g_V are elements of the general linear transformation group G. Therefore, we have:

$$g_U=g_U g_V^{-1}g_V=S_U(p)S_V(p)^{-1}g_V\equiv g_{UV}(x)g_V \tag{1}$$

Note here: If $\{U,V,\cdots\}$ is an open cover of M and Σ represents the non union of $U\times G,V\times G,\cdots$, then Σ is larger than P because every $p\in P$ corresponds to more than one image point in Σ. For example, in the above expression, p corresponds to at least two points (x,g_U) and (x,g_V). However, as long as all the image points of $p\in P$ in Σ are identified as one point, Σ can represent P. A more accurate equivalence relationship can be defined, that is, assuming $(x,g)\in U\times G$, $(x',g')\in V\times G$, if and only if

$$x=x',\ g=g_{UV}(x)g' \tag{2}$$

Just say that (x,g) is equivalent to (x',g'), denoted as $(x,g)\sim(x',g')$. Identify all equivalent points in Σ, and the result is P, denoted as $P=\Sigma/\sim$.

From this, the following definition can be derived: Let $T_U:\pi^{-1}[U]\to U\times G$ and $T_V:\pi^{-1}[V]\to V\times G$ be the two local ordinariness of the principal bundle $P(M,G)$, where $U\cap V\neq\varnothing$. The mapping $g_{UV}:U\cap V\to G$ is called the transition function from T_U to T_V. If there are

$$g_{UV}(x)=S_U(p)S_V(p)^{-1},\forall x\in U\cap V,\pi(p)=x \tag{3}$$

where, the images of $p\in P$ under the mappings of T_U and T_V are $(x,g_U)\epsilon U\times G_U$ and $(x,g_V)\in V\times G_V$, respectively. Here, $g_U\in G_U$, $g_V\in G_V$, G_U and G_V are the subgroups corresponding to the basic interactions described in $G=GL(m,C)$: for example, $G_U=U(1)$ or $SU(2)$ for electromagnetic interactions, $G_V=SO(1,3)$ for gravitational interactions, then g_{UV} defines a gauge transformation from electromagnetic interactions to gravitational interactions. If G_U and G_V only belong to a subgroup that describes the same fundamental interaction, then g_{UV} only defines the gauge transformation within the same fundamental interaction. This can be referred to as two types of gauge transformations, namely the gauge transformation across fundamental interactions and the traditional gauge transformation. We can refer to them as the cross basic gauge transformation and the basic gauge transformation [9]-[12], as shown in **Figure 1**.

The conversion function $g_{UV}(x)$ defined above can prove that the following theorem (1) is satisfied:

1) $g_{UU}(x) = g_U g_U^{-1} = e, \forall x \in U$;

2) $g_{VU}(x) = g_V g_U^{-1} = \left(g_U g_V^{-1}\right)^{-1} = \left(g_{UV}(x)\right)^{-1}, \forall x \in U \cap V$;

3) $g_{UV}(x) g_{VW}(x) g_{WU}(x) = g_U g_V^{-1} g_V g_W^{-1} g_W g_U^{-1} = e, \forall x \in U \cap V \cap W$.

Moreover, in order to make the non intersecting product set Σ equal to *P*, the different image points in *P* that are mapped from each point $p \in P$ by T_U and T_V, *i.e.* the terms $U \times G, V \times G, \cdots$ belonging to Σ, should be identified. More accurately, define an equivalence relationship, namely

Let $(x, g) \in U \times G$, $(x', g') \in V \times G$, if and only if the present equation holds

$$x = x',\ g = g_{UV}(x) g' \tag{4}$$

Then we say that (x, g) is equivalent to (x', g'), denoted as $(x, g) \sim (x', g')$. If we identify all the equivalent points in Σ, the result is *P*, denoted mathematically as $P = \Sigma / \sim$. It can be proven that this mathematical equivalence relationship has three properties, which are specifically expressed as:

1) Reflexivity $(x, g) \sim (x, g)$;

2) Symmetry $(x, g) \sim (x', g') \Leftrightarrow (x', g') \sim (x, g)$; (5)

3) Transitivity $(x, g) \sim (x', g'), (x', g') \sim (x'', g'') \Rightarrow (x, g) \sim (x'', g'')$.

Below, we use Equation (4) to prove that the above equations indeed define an equivalence relationship:

1) Proof of reflexivity: since $x = x$, then $g = g_{UU}(x) g = eg = g$, $\forall x \in U$, hence $(x, g) \sim (x, g)$;

2) Proof of symmetry: since $x = x'$, $g = g_{UV}(x) g'$
$\Rightarrow g' = g_{UV}(x)^{-1} g = g_{VU}(x) g$, then $(x, g) \sim (x', g') \Leftrightarrow (x', g') \sim (x, g)$;

3) Proof of transitivity: since $g = g_{UV}(x) g'$ and $g' = g_{VW}(x) g''$ $\Rightarrow$
$g = g_{UV}(x) g_{VW}(x) g'' = g_U g_V^{-1} g_V g_W^{-1} g'' = g_U g_W^{-1} g'' = g_{UW}(x) g''$, so the transitivity is established.

q.e.d.

Here is a proof of a theorem that will be used later: Let g_{UV} be the conversion function from the local triviality T_U to T_V, $x \in U \cap V$. If $\breve{p}_U$ and $\breve{p}_V$ respectively represent the special points determined on $\pi^{-1}[x]$, *i.e.*
$S_U(\breve{p}_U) = S_V(\breve{p}_V) = e \in G$, then there is

$$\breve{p}_V = \breve{p}_U g_{UV}(x) \tag{6}$$

Proof: Suppose $\breve{p}_U, \breve{p}_V \in \pi^{-1}[x]$ to keep $\exists g \in G$ s.t. $\breve{p}_V = \breve{p}_U g$; considering again $S_U(\breve{p}_U g) = S_U(\breve{p}_U) g$, then we have:

$$g_{UV}(x) = S_U(\breve{p}_V) S_V(\breve{p}_V)^{-1} = S_U(\breve{p}_V) e = S_U(\breve{p}_U g) = S_U(\breve{p}_U) g = eg = g \tag{7}$$

q.e.d.

Now define a local cross section: let $P(M, G)$ be the principal bundle and U be an open subset of *M*. C^∞ mapping $\sigma : U \to P$ is defined as a local cross-section, if there is

$$\pi(\sigma(x)) = x, \forall x \in U \tag{6}$$

That is, the equation $\forall x \in U$ guarantees that $\sigma(x) \in \pi^{-1}[x]$. Furthermore, there is a natural correspondence between local cross-sections and local triviality:

In fact, given $T_U : \pi^{-1}[U] \to U \times G$, there exists a fiber $\pi^{-1}[x]$ with $\forall x \in U$, which has a special point $\breve{p}_U$, satisfying the condition that $S_U(\breve{p}_U) = e\epsilon G$. If $\breve{p}_U \equiv \sigma(x)$, a local cross-section σ is naturally obtained: $\sigma : U \to P$; On the contrary, given $\sigma : U \to P$, then $\forall p \in U$ makes $x = \pi(p)$, and there can be p and $\sigma(x)$ belonging to the same fiber $\pi^{-1}[x]$. Therefore, there exists a unique right acting group element $g \in G$ such that $p = \sigma(x) g$, and thus defining $T_U(p)$ as (x, g). This theorem can also be extended to show that in the region of $x \in U \cap V$, there is a natural correspondence between the local cross-section $\sigma(x)$ and the local trivial $T_U(p)$ and $T_V(p)$, and its smoothness is evident, namely

If $T_U : \pi^{-1}[U] \to U \times G$ and $T_V : \pi^{-1}[V] \to V \times G$ are local triviality, $U \cap V \neq \varnothing$, g_{UV} is the conversion function from T_U to T_V, and $\sigma_U : U \to P$ and $\sigma_V : V \to P$ are the local cross-sections corresponding to T_U and T_V, respectively, then according to Equation (4), we have

$$\sigma_V(x) = \sigma_U(x) g_{UV}(x), \forall x \in U \cap V \tag{7}$$

where, if $U = M$, $\sigma : U \to P$ is called as the global section.

The above Equation (7) indicates that different regions U or V correspond to different local cross-sections $\sigma_U(x)$ or $\sigma_V(x)$, $\forall x \in U \cap V$, which can be transformed through the conversion function $g_{UV}(x)$. Then we have:

$$\boldsymbol{\omega}_U = \sigma_U^* \boldsymbol{\omega}$$

$$\boldsymbol{\omega}_V = \sigma_V^* \boldsymbol{\omega}$$

where σ_U^* or σ_V^* are the pull back mapping of σ_U or σ_V, respectively. $\boldsymbol{\omega}$ is the connection of principal bundle, $\boldsymbol{\omega}_U$ or $\boldsymbol{\omega}_V$ is the connection of bottom manifold M in region U or V, respectively.

In this sense, it can be said that the local cross-section selects a gauge, and the conversion function determines a gauge transformation. Furthermore, for the cross basic gauge transformation, the image points corresponding to local trivial T_U and T_U, denoted as $\sigma_U(x)$ and $\sigma_V(x)$ 1-1 respectively, belong to two groups corresponding to different basic interactions (such as electromagnetic or gravitational interactions corresponding to $U(1)$, $SU(2)$ or $SO(1,3)$). These groups are subgroups of $GL(m,C)$. Due to the fact that this type of conversion function across basic subgroups does not affect the differential isomorphism of the conversion function, which is crucial, the relevant formulas for the conversion function in the principal associated bundle theory now still hold, such as the generalized gauge equation.

4. The Generalized Gauge Equation

There are three definitions of the connection between the principal bundle, namely the horizontal subspace H_p of the tangent vector at each point $p \in P$

of the principal bundle that satisfies a certain condition, or the 1-form field $\tilde{\boldsymbol{\omega}}$ of a C^{∞} Lie algebra value on the principal bundle P that satisfies a certain condition, or in the bottom manifold, that is, the 1-form field $\boldsymbol{\omega}_U$ of a C^{∞} Lie algebra $\mathcal{G}$ value on U specified by each local trivial T_U in our universe. If T_V is another local trivial, $U \cap V \neq \varnothing$, and the conversion function from T_U to T_V is g_{UV}, then the generalized gauge equation holds for both the cross basic gauge transformation and the basic gauge transformation:

$$\boldsymbol{\omega}_V\left(Y\right) = \mathcal{A}d_{g_{UV}(x)^{-1}}\boldsymbol{\omega}_U\left(Y\right) + L^{-1}_{g_{UV}(x)*}g_{UV*}\left(Y\right), \forall x \in U \cap V, Y \in T_x M \tag{8}$$

where $L^{-1}_{g_{UV}(x)}$ is the inverse mapping of left translation $L_{g_{UV}(x)}$ generated by $g_{UV}\left(x\right) \in G$, $L^{-1}_{g_{UV}(x)*} \equiv \left(L^{-1}_{g_{UV}(x)}\right)_*$.

The author needs to emphasize that after careful analysis and scrutiny of the formula proof process in the references [9]-[13], the author found that the conversion function g_{UV} defined above for the cross basic interaction subgroup does not affect the diffeomorphism of the originally defined conversion function, *i.e.* the relevant T_U or T_V is still diffeomorphic. Therefore, the conversion function for the subgroup of the cross basic interaction satisfies all the equations of the original conversion function equally.

Furthermore, it can be proven that if the structural group G is a matrix group, then the generalized gauge Equation (8) can be simplified as follows:

$$\boldsymbol{\omega}_V = g^{-1}_{UV}\boldsymbol{\omega}_U g_{UV} + g^{-1}_{UV}\boldsymbol{d}g_{UV} \tag{9}$$

where $\boldsymbol{d}$ is defined as the exterior differential of g_{UV}. A very interesting phenomenon is that the graviton spin 2 has a rest mass of 0, while the photon spin 1 has a rest mass of 0. Macroscopically, within the scope of Newtonian mechanics, both are proportional to the "charge" (such as charge, mass) of the field and inversely proportional to the square of the distance. From the perspective of standard quantum field theory, the gravitational field is composed of gravitons, while the electromagnetic field is composed of photons. The both are too similar [14]-[18]! At the level of the above generalized gauge Equation (9), we seem to glimpse a natural mystery, which is that in a sense, graviton is synthesized by photons under certain conditions.

We use the generalized gauge equation here to propose a model for transforming 2 photons into 1 graviton through gauge transformation, suggesting that if gravitons exist, they can be formed by combining 2 photons with positive and negative helices of 1 to form 1 graviton with positive and negative helices of 2. Can it be said that the final graviton is composed of two photons? The physical essence and significance of this process still need to be analyzed. However, no matter what, gravitons are not ancillary products of high-order infinitesimal electromagnetic interactions. There is no electromagnetic interaction between gravitons; it is another independent fundamental interaction. They are all projection components of the principal fiber bundle manifold's connections or curvatures within a certain scale range, and are (non) linearly independent of each other.

We first make certain theoretical preparations [14] [15]:

1) The plane electromagnetic wave equation in vacuum needs to satisfy the wave equation and Lorentz condition, respectively

$$\partial^a \partial_a A_b = \Box A_\mu = 0 \tag{10}$$

$$\partial^a A_a = A^\mu_{,\mu} = 0 \tag{11}$$

Then we can observe that the electromagnetic 4 potential (electromagnetic gauge potential) A_μ is in the same direction as the polarization vector e_μ, and its magnitude is proportional to that of e_μ. So for a photon we have

$$\begin{pmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{pmatrix} \sim \begin{pmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{pmatrix} \tag{12}$$

where electromagnetic wave propagation is a transverse field, that is, when "one photon" propagates we get

$$\begin{pmatrix} 0 \\ A_1 \\ A_2 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 0 \\ e_1 \\ e_2 \\ 0 \end{pmatrix} \tag{13}$$

where the helicity of its two polarization directions e_1 or e_2 is ±1, and the rest mass is 0; and the helicity of its longitudinal field $(A_3 \sim e_3)$ and time like field $(A_0 \sim e_0)$ are both 0. In order to coordinate with the second-order tensor of the subsequent gravitons, we consider that when two photons propagate (such as certain two-photon entangled states have a spiral shape), their polarization vector matrix expression is

$$\begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \tag{14}$$

2) On the other hand, plane gravitational waves are equivalent to being far enough from the source and with a sufficiently small receiving surface. With the Riemann metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, the vacuum Einstein equation can be formulated as

$$\Box h_{\mu\nu} = 0 \tag{15}$$

Therefore, it can be proved that the propagation of gravitational waves is also a transverse field, and the direction and magnitude of the propagated polarization vector $e_{\mu\nu}$ are also proportional to $h_{\mu\nu}$, and its helicity is ± 2, that is,

$$\begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \sim \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix} = \begin{pmatrix} 2 & \pm 2 \\ \mp 2 & -2 \end{pmatrix} \tag{16}$$

The helicity of the longitudinal field $(e_{33} \sim h_{33})$ and the time-like field $(e_{00} \sim h_{00})$ are both 0. Therefore, we can assume that $h_{\mu\nu}$ here is the gravitational gauge potential! So applying the general gauge Equation (9), we can get

$$\begin{pmatrix} 2 & \pm 2 \\ \mp 2 & -2 \end{pmatrix} = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}^{-1} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} + \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}^{-1} \boldsymbol{d} \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}$$

$$\Rightarrow\begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}\begin{pmatrix} 2 & \pm 2 \\ \mp 2 & -2 \end{pmatrix}=\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}+\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} \tag{17}$$

where $u_{\mu\nu}$ is a function of a vector space [13]; $\boldsymbol{d}$ is the exterior differential. From the proof of formula (9), we can see that $\boldsymbol{d}$ here is equivalent to taking the derivative of $\{u_{\mu\nu}\}$ with respect to the parameter *t*.

Now expanding the Equation (17), we have

$$\begin{pmatrix} 2u_{11}\mp 2u_{12} & \pm 2u_{11}-2u_{12} \\ 2u_{21}\mp 2u_{22} & \pm 2u_{21}-2u_{22} \end{pmatrix}=\begin{pmatrix} u_{11}-u_{21} & u_{12}-u_{22} \\ -u_{11}-u_{21} & -u_{12}+u_{22} \end{pmatrix}+\begin{pmatrix} \partial_t u_{11} & \partial_t u_{12} \\ \partial_t u_{21} & \partial_t u_{22} \end{pmatrix} \tag{18}$$

So the above formula is expanded into a system of differential equations

$$\begin{cases} u_{11}\mp 2u_{12}+u_{21}=\partial_t u_{11} \\ \pm 2u_{11}-3u_{12}+u_{22}=\partial_t u_{12} \\ 3u_{21}\mp 2u_{22}+u_{11}=\partial_t u_{21} \\ \pm 2u_{21}-3u_{22}+u_{12}=\partial_t u_{22} \end{cases} \tag{19}$$

Here, $\begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}$ is equivalent to a scalar field with *t* as parameter. Fortunately, the above differential equations can be solved to determine $\begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}$.

3) Solve the differential equation system (19). First, solve the characteristic equation of the above differential equation system:

$$\begin{vmatrix} 1-\lambda & \mp 2 & 1 & 0 \\ \pm 2 & -3-\lambda & 0 & 1 \\ 1 & 0 & 3-\lambda & \mp 2 \\ 0 & 1 & 0 & -3-\lambda \end{vmatrix} \tag{20}$$

According to the characteristic equation, we have the following determinant equal to 0, that is

$$\begin{vmatrix} 1-\lambda & \mp 2 & 1 & 0 \\ \pm 2 & -3-\lambda & 0 & 1 \\ 1 & 0 & 3-\lambda & \mp 2 \\ 0 & 1 & 0 & -3-\lambda \end{vmatrix}=0 \tag{21}$$

Solve a 4-degree equation as

$$\lambda^4+2\lambda^3-13\lambda^2+18\lambda+13=0 \tag{22}$$

Although the solution of this univariate 4-order equation is relatively complicated, its characteristic roots, $\lambda_1,\lambda_2,\lambda_3,\lambda_4$ can be expressed as the four roots of the following two equations, respectively:

$$\lambda^2+\left(2+\sqrt{2y+14}\right)\lambda+\left(y+\frac{y-9}{\sqrt{2y+14}}\right)=0 \tag{23}$$

$$\lambda^2+\left(2-\sqrt{2y+14}\right)\lambda+\left(y-\frac{y-9}{\sqrt{2y+14}}\right)=0 \tag{24}$$

where y is any real root of the following cubic equation,

$$8y^3 - 4y^2 + (72 - 104)y + 13(-52 - 4) - 13^2 = 0 \tag{25}$$

We choose

$$y = \frac{49}{72\sqrt[3]{\frac{1}{144}\sqrt{3}\sqrt{21969727} + \frac{24365}{432}}} - \frac{1}{3} \approx 0.47419 \tag{26}$$

So we can get:

$$\begin{cases} \lambda_1 = 0.28156 \\ \lambda_2 = -6.1478 \\ \lambda_3 = 0.93315 + 1.3449i \\ \lambda_4 = 0.93315 - 1.3449i \end{cases} \tag{27}$$

So the four linearly independent solutions of the above differential equations are:

$$\begin{cases} \beta_1 \mathrm{e}^{\lambda_1 t} = \beta_1 \mathrm{e}^{0.28156t} \\ \beta_2 \mathrm{e}^{\lambda_2 t} = \beta_2 \mathrm{e}^{-6.1478t} \\ \beta_3 \mathrm{e}^{0.93315t} \cos(1.3449t) \\ \beta_4 \mathrm{e}^{0.93315t} \sin(1.3449t) \end{cases} \tag{28}$$

Among them, β_1, β_2, β_3, and β_4 are all unknown coefficients; the general solution of the original differential equation system is the combination of these four linearly independent solutions, for example:

$$\begin{cases} u_{11} = \beta_1 \mathrm{e}^{0.28156t} + \beta_2 \mathrm{e}^{-6.1478t} + \beta_3 \mathrm{e}^{0.93315t} \cos(1.3449t) + \beta_4 \mathrm{e}^{0.93315t} \sin(1.3449t) \\ u_{12} = \beta_1 \mathrm{e}^{0.28156t} - \beta_2 \mathrm{e}^{-6.1478t} + \beta_3 \mathrm{e}^{0.93315t} \cos(1.3449t) + \beta_4 \mathrm{e}^{0.93315t} \sin(1.3449t) \\ u_{22} = \beta_1 \mathrm{e}^{0.28156t} - \beta_2 \mathrm{e}^{-6.1478t} - \beta_3 \mathrm{e}^{0.93315t} \cos(1.3449t) - \beta_4 \mathrm{e}^{0.93315t} \sin(1.3449t) \\ u_{21} = \beta_1 \mathrm{e}^{0.28156t} - \beta_2 \mathrm{e}^{-6.1478t} - \beta_3 \mathrm{e}^{0.93315t} \cos(1.3449t) + \beta_4 \mathrm{e}^{0.93315t} \sin(1.3449t) \end{cases} \tag{29}$$

In this way, we can determine the matrix represented by the gauge transformation g_{UV}. From this solution, we can see that the physical process it represents may be complicated, but it clearly shows that two photons with a helicity (equivalent to spin) of positive or negative 1 can be transformed into a graviton with a helicity (spin) of positive or negative 2 through gauge transformation, and vice versa. Obviously, the secret of its physical process is hidden in the conversion function or gauge transformation. Understanding this process is an important task of physics and its experiments, but the direction is clear. This general gauge equation is the basic formula for the mutual transformation between electromagnetic force and gravity. Let's write it again:

$$\boldsymbol{\omega}_V = g_{UV}^{-1} \boldsymbol{\omega}_U g_{UV} + g_{UV}^{-1} \mathbf{d} g_{UV} \tag{30}$$

And prove it [13] to see if we can find any clues about the physical process.

Proof:

1) First, let's analyze: Note that G is a matrix group, which means that its elements are $N \times N$ matrices, so the elements in its Lie algebra $\mathcal{G}$ are also $N \times N$ matrices. If $\mathcal{V} \equiv \{N \times N \text{ matrices}\}$, that is, the set of $N \times N$ matrices, then $\mathcal{V}$ is a vector space, and $G \subset \mathcal{V}$, $\mathcal{G} \subset \mathcal{V}$. And both $\boldsymbol{\omega}_U$ and $\boldsymbol{\omega}_V$ are $\mathcal{G}$

-valued 1-form fields on $U \cap V$, $\boldsymbol{\omega}_U, \boldsymbol{\omega}_V \in \Lambda_{U\cap V}(1,\mathcal{V})$, that is, they all belong to the set of 1-form fields in the vector space. So the above equation is the $\mathcal{V}$-valued 1-form field equation. In addition, because

$$g_{UV} : U \cap V \to G \subset \mathcal{V} \tag{31}$$

Hence $g_{UV}, g_{UV}^{-1} \in \Lambda_{U\cap V}(0,\mathcal{V})$, so $g_{UV}^{-1}\boldsymbol{\omega}_U g_{UV} \equiv \boldsymbol{\Phi}$ is a $\mathcal{V}$-valued 1-form field on $U \cap V$, which gives an $N \times N$ matrix after specifying $x \in U \cap V$ and $Y \in T_x \in T_x M$ (*i.e.*, the tangent space at point *x* in *M*), that is,

$$\boldsymbol{\Phi}_x(Y) \equiv g_{UV}^{-1}(\boldsymbol{x})\boldsymbol{\omega}_U(Y)g_{UV}(x) \tag{32}$$

Looking at the second term on the right side of the Equation (30), $g_{UV} \in \Lambda_{U\cap V}(0,\mathcal{V})$ will lead to $dg_{UV} \in \Lambda_{U\cap V}(1,\mathcal{V})$, so we can further get

$$\boldsymbol{\psi}_x(Y) \equiv g_{UV}^{-1}(x)dg_{UV}(Y), x \in U \cap V, Y \in T_x M \tag{33}$$

2) Proof 1: According to the first term on the right-hand side of the generalized gauge equation and the definition of adjoint isomorphism, it can be concluded that

$$\mathcal{A}d_{g_{UV}(x)^{-1}}\boldsymbol{\omega}_U(Y) = I_{g_{UV}(x)*}\boldsymbol{\omega}_U(Y) = \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0} I_{g_{UV}(x)}\mathrm{Exp}\left[t\boldsymbol{\omega}_U(Y)\right] \tag{34}$$

However, we have

$$\begin{aligned}
&I_{g_{UV}(x)}\mathrm{Exp}\left[t\boldsymbol{\omega}_U(Y)\right]\\
&= I_{g_{UV}(x)}\left[I + t\boldsymbol{\omega}_U(Y) + \frac{1}{2}\left(t\boldsymbol{\omega}_U(Y)\right)^2 + \cdots\right]\\
&= g_{UV}(x)\left[I + t\boldsymbol{\omega}_U(Y) + \frac{1}{2}\left(t\boldsymbol{\omega}_U(Y)\right)^2 + \cdots\right]g_{UV}(x)^{-1}\\
&= I + tg_{UV}(x)\boldsymbol{\omega}_U(Y)g_{UV}(x)^{-1} + \frac{t^2}{2}\left(g_{UV}(x)\boldsymbol{\omega}_U(Y)g_{UV}(x)^{-1}\right)^2 + \cdots\\
&= \mathrm{Exp}\left[tg_{UV}(x)\boldsymbol{\omega}_U(Y)g_{UV}(x)^{-1}\right]
\end{aligned} \tag{35}$$

Therefore we obtain

$$\begin{aligned}
\mathcal{A}d_{g_{UV}(x)^{-1}}\boldsymbol{\omega}_U(Y) &= \boldsymbol{\Phi}_x(Y)\\
&= \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0}\mathrm{Exp}\left[tg_{UV}(x)\boldsymbol{\omega}_U(Y)g_{UV}(x)^{-1}\right]\\
&= g_{UV}(x)\boldsymbol{\omega}_U(Y)g_{UV}(x)^{-1}
\end{aligned} \tag{36}$$

3) Proof 2: If the curve $\eta : I \to U \cap V$ satisfies $\eta(0) = x$, $\left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0}\eta(t) = Y$, then the second term on the right side of the generalized gauge equation is expressed as:

$$\begin{aligned}
L_{g_{UV}(x)^{-1}*}g_{UV*}(Y) &= L_{g_{UV}(x)^{-1}*}g_{UV*}\left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0}\eta(t)\\
&= \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0}\left[L_{g_{UV}(x)^{-1}}g_{UV}(\eta(t))\right]\\
&= \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0}\left[g_{UV}(x)^{-1}g_{UV}(\eta(t))\right]\\
&= g_{UV}(x)^{-1}\left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0}g_{UV}(\eta(t))
\end{aligned} \tag{37}$$

Then, we represent a set of basis vectors of the vector space $\mathcal{V}$ with $\{e_r\}$, then g_{UV} can be expanded as $g_{UV} = f^r e_r$ by using the basis vectors, where f^r is a real (complex) function on U $U \cap V$. So the second factor on the right above can be expressed as:

$$\left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0} g_{UV}\left(\eta(t)\right) = e_r \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0} f^r\left(\eta(t)\right) \tag{38}$$

Here, $\forall t \in I$, $\eta(t)$ is a point of $U \cap V$, and $f^r\left(\eta(t)\right)$ is the value of f^r at $\eta(t)$. Therefore, by definitions of the tangent and the external differentiation, we get

$$\left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0} f^r\left(\eta(t)\right) = Y\left(f^r\right) = \left(df^r\right)(Y) \tag{39}$$

Therefore, the second item on the right is

$$g_{UV}(x)^{-1} \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0} g_{UV}\left(\eta(t)\right) = g_{UV}(x)^{-1} e_r \left(df^r\right)(Y) = g_{UV}(x)^{-1}\left(dg_{UV}\right)(Y) \tag{40}$$

Finally, the proofs of (b) and (c) demonstrate that the generalized gauge Equation (30) is correct.

q.e.d.

5. Gauge Transformation across Fundamental Interactions

Unfortunately, we cannot find any clues to the physical process from the above mathematical proof. However, we can be sure that from the perspective of principal bundle theory [9]-[13] [16]-[19], there is no problem with the existence of this generalized gauge transformation. This shows that at the microscopic and macroscopic scales, electromagnetic interaction and gravitational interaction can be transformed into each other through generalized gauge transformation. The physical process corresponding to our calculation results above may be the process of two photons transforming into a graviton [20]. Understanding the details of this physical process is obviously very important for the future use of the mutual transformation between gravity and electromagnetic force to achieve long-distance interstellar travel.

It should be pointed out that this gauge transformation does not represent the view that gravitational interaction is only high-order residual field of electromagnetic interaction, but essentially belongs to high-order residual product of electromagnetic interaction. It does not necessarily mean that electromagnetic interaction is just a manifestation of gravitational interaction or a certain decomposition of the gravitational field. The two interactions are essentially relatively independent basic interactions, which are only the projection components and expressions of the connection or curvature of the principal fiber manifold of the universe in the bottom manifold (our universe). They cannot be considered mutually inclusive, and the relationship between weak, strong, and electromagnetic interactions is the same. The description of quantum mechanics is only because the scale structure of space-time becomes a "representation" that must be adapted to the microscopic and subtle worlds. All the paradoxes and misunderstandings faced by quantum mechanics may be caused by the influence effect of the size of spacetime scale.

Here we generalize the above general gauge equations and relevant results into a more general physical law as an important corollary of this article:

Law 1: At any point in spacetime, there exists a generalized gauge transformation for any fundamental interaction, corresponding to a physical process that can transform one fundamental interaction into another, and the transformation process satisfies a generalized gauge equation.

Or more generally, Law 2: At any point in spacetime, for any gauge interaction (force), there exists a gauge transformation, corresponding to a physical process that can transform one gauge interaction (force) into another, and the transformation process satisfies a generalized gauge equation.

In conclusion, from the above results, and considering the results and discussions of [9]-[13], we can basically say that the universe picture based on the principal bundle is suitable for describing both quantum and classical systems. This can also be proved by the unification of weak, strong and electromagnetic interactions by $U(1)\times SU(2)\times SU(3)$ in the quantum field on the basis of gauge transformation invariance. So quantization is, in a sense, just a gauge expression of a certain structure group selected by the principal bundle section, and the choice of this gauge is related to the scale of the bottom manifold. It is ultimately reflected in the choice of structure group. Generally speaking, the $U(1)\times SU(2)\times SU(3)$ group is useful for weak, strong and electromagnetic interactions, while $O(n)$ or $SO(1,3)$ is useful for gravitational interactions. In the framework of the principal bundle, they satisfy the gauge transformation invariance form and are unified in the connection or curvature of the principal fiber bundle. They can all be Lie subgroups of the linear transformation group.

Moreover, the physical gauge invariance is reflected in the Lagrangian invariance, which is equivalent to satisfying the generalized gauge equations in a sense. However, the construction of the Lagrangian of a system usually does not involve the transformation across the fundamental interactions, which is precisely where the generalized gauge equations make outstanding contributions.

In essence, the four basic interactions are all geometric expressions of the connection or curvature of the principal fiber bundle manifold. The quantization of the microscopic and cosmic fields, the exchange of virtual particles, and the gravitational force in the macroscopic and cosmic fields are the manifestations of the curvature of space-time, etc., are all caused by choosing different standard representations. All the basic interactions of the universe are unified in the connection or curvature of the principal fiber bundle manifold. These interactions are the components of the connection or curvature projected in the universe at different scales, such as cosmic, macroscopic, microscopic, and microcosm. And these components can be transformed into each other through the generalized gauge transformation at any same space-time point. The basic interactions are ultimately unified in "geometry".

On the other hand, from the experimental facts discovered by existing quantum field theory, many basic interactions are unified in the "exchange of virtual particles" [21]-[23]. Gravity may also originate from the exchange of "virtual gravitons", just as electromagnetic interaction originates from the exchange of virtual photons. The model of two photons constructed above that is transformed into a graviton through the generalized gauge transformation may provide clues to the origin of the graviton of gravity.

Therefore, it can be inferred that the basic interactions may be unified in the "duality" of "geometry" and "virtual particle exchange", and the two are in opposition and unity. Of course, this is very philosophical.

6. Conclusions and Outlook

1) This article reveals that a graviton can be viewed as the result of two photons undergoing gauge transformations across different fundamental interactions, and vice versa, by constructing a gauge transformation that converts two photons into one graviton. This process indicates that the transformation between different fundamental interactions does not occur in isolation, but depends on deep mathematical and physical laws. Specifically, all these transformation processes strictly follow Generalized Gauge Equations, ensuring theoretical consistency and completeness.

2) At any point in spacetime in our universe, every fundamental interaction (such as electromagnetic force, gravity, strong interaction, or weak interaction) has a corresponding gauge transformation. This gauge transformation corresponds to a specific physical process, through which the transition from one fundamental interaction to another can be achieved. Moreover, these transformations are not limited to the interaction of specific forces, more generally, at any point in spacetime, for any gauge interaction (force), a suitable gauge transformation can always be found to transform it into another gauge interaction, and the entire transformation process must satisfy the constraints of the generalized gauge equation.

3) This important conclusion indicates that the principal fiber bundle theory and its cosmic geometric picture exhibit excellent adaptability, capable of simultaneously covering quantum systems and general relativity systems. This characteristic enables the theory to bypass many difficulties faced in constructing quantum gravity [23]-[27], seamlessly connecting quantum effects with the description of classical gravity and providing a new research path. Therefore, the principal fiber bundle theory can serve as a strong theoretical foundation, providing a solid framework for the great unity of the four fundamental interactions.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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