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Solving the Conundrum of Dark Matter and Dark Energy in Galaxy Clusters

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Abstract

This paper develops the Dark Matter by Quantum Gravitation theory, DMbQG theory hereafter, in clusters of galaxies in the cosmologic model ΛCDM of the Universe. Originally this theory was developed by the author for galaxies, especially using MW and M31 rotation curves. An important result got by the DMbQG theory is that the total mass associated to a galactic halo depend on the square root of radius, being its dominion unbounded. Apparently, this result would be absurd because of divergence of the total mass. As the DE is negligible at galactic scale, it is needed to extend the theory to clusters in order to study the capacity of DE to counterbalance to DM. Thanks this property, the DMbQG theory finds unexpected theoretical results. In this work, it is defined, the total mass as baryonic matter plus DM and the gravitating mass as the addition of the total mass plus the negative mass associated to dark energy. In clusters it is defined the zero gravity radius (R_{ZG} hereafter) as the radius needed by the dark energy to counterbalance the total mass. It has been found that the ratio $R_{ZG}/R_{VIRIAL} \approx 7.3$ and its Total mass associated at R_{ZG} is $\approx 2.7\ M_{VIRIAL}$. In addition, it has been calculated that the sphere with the extended halo radius $R_E = 1.85\ R_{ZG}$ has a ratio DM density versus DE density equal to 3/7 and its total mass associated at R_E is $\approx 3.6\ M_{VIRIAL}$. This works postulates that the factor 3.6 may equilibrate perfectly the strong imbalance between the Local mater density parameter (0.08) versus the current Global matter density one (0.3). Currently, this fact is a big conundrum in cosmology, see chapter 7. Also it has been found that the zero velocity radius, R_{ZV} hereafter, *i.e.* the cluster border because of the Hubble flow, is $\approx 0.6\ R_{ZG}$ and its gravitating mass is $\approx 1.5\ M_{VIR}$. By derivation of gravitating mass function, it is calculated that at $0.49\ R_{ZG}$, this function reaches its maximum whose value is $\approx 1.57\ M_{VIR}$. Throughout the paper, some of these results have been validated with recent data published for the Virgo cluster. As Virgo is the nearest big cluster, it is the perfect benchmark to validate any new theory about DM and DE. These new theoretical findings offer to scientific community a wide number of tests to validate or reject the theory. The validation of DMbQG theory would mean to know the nature of DM that at the present, it is an important challenge for the astrophysics science.

Keywords

Dark Matter, Dark Energy, Galaxy Clusters, Quantum Gravitation

1. Introduction

The bases of this paper are developed in [1] Abarca, M. 2023, so it is highly recommended to read it to understand the meaning of this paper. The dark matter by quantum gravitation theory, DMbQG theory hereafter, is an original theory developed since 2013 through more than 20 papers, although in [1] Abarca, M. 2023 is published the best version as physical as mathematically. The theory has been stated studying the galactic rotation curves, specially the ones associated to MW and M31, see [2] Sofue, Y. 2015 and [3] Sofue, Y. 2020. Therefore, it is not possible to understand this paper if readers have not at least a general knowledge about the DMbQG theory.

The hypothesis of DMbQG theory is that the DM is generated by the own gravitational field. In order to study purely the DM phenomenon, it is needed to consider a radius dominion where it is supposed that baryonic matter is negligible. For example, for MW galaxy the radius must be bigger than 30 kpc and bigger than 40 kpc for M31 galaxy. For galactic clusters, the radius must be bigger than its virial radius.

This hypothesis has two main consequences: the first one is that the law of dark matter generation, in the halo region, has to be the same for all the galaxies and clusters; the second one is that the DM haloes are unlimited so the total dark matter goes up without limit.

The DMbQG theory has been developed assuming the hypothesis that DM is a quantum gravitational effect. See [4] Corda, C. 2012. However, it is possible to remain into the Newtonian framework to develop the theory. In my opinion, there are two factors to manage the DM phenomenon with a quite simple theory.

The first one, that it is developed into the halo region, where baryonic matter is negligible. The second one, that the mechanics movements of celestial bodies are very slow regarding velocity of light, which is supposed to be the speed of gravitational bosons. It is known that community of physics is researching a quantum gravitation theory since many years ago, but does not exist yet; however, my works in this area support strongly that DM is a quantum gravitation phenomenon.

Use a more simple theory instead the general theory is a typical procedure in physics.

For example, the Kirchhoff's laws are the consequence of Maxwell theory for direct current and remain valid for alternating current, introducing complex impedances, on condition that signals must have low frequency.

In [1] Abarca, M. 2023, in the framework of DMbQG, is demonstrated mathematically that the total mass (baryonic plus DM) enclosed by a sphere with a specific radius is given by the Direct mass into the galactic halo and that the total mass goes up proportionally to the root square of radius, formula (4.1).

It is well known that DE may be modelled as a constant density of negative mass in the whole space, see [5] Chernin, A.D. *et al.* 2013, therefore the total amount of DE grows up with the cubic power of the sphere radius, so it is clear that DE is able to counterbalance the total mass of the clusters, which grows up more slowly. Precisely, the main goal of this paper is to study the relation between both phenomenons in clusters. Namely in cluster haloes.

This paper explores the mutual counterbalance between DM and DE in the framework of DMbQG theory and the result got have been fructiferous, with a dozen of new formulas never published before.

The following paragraphs will be introduced the paper structure:

The newness of the important results got in this paper are due to the possibility to approximate the virial radius to R_{200} and the virial mass to M_{200}, the chapter 3 is dedicated to validate this approximation using recent data published for some important clusters such as Virgo or some others.

The chapter 4 is dedicated to extend the direct mass formula to clusters. The direct mass formula (4.1) has only one parameter "a^2" whose units are $m^{5/2}/s^2$. Using the approximation R_{200} as virial radius and M_{200} as virial mass into the direct mass formula it is got the formula $a^2 = \left(G \cdot M_{VIR}\right)^{5/6} \cdot \left(10 \cdot H\right)^{1/3}$ which is on the basis for some important results got in the following chapters.

The chapter 5 contains three main concepts:

Firstly, it is defined the zero gravity radius, R_{ZG}, as the sphere radius where the total mass is counterbalanced by the DE. It is found that the ratio $R_{ZG}/R_{VIRIAL} \approx 7.3$ is universal.

Secondly, it is defined the gravitating mass as the addition of total mass plus the dark energy and the gravitating mass function using a dimensionless parameter f = Radius/R_{ZG}. It is found that for any cluster at ≈0.5 R_{ZG} is reached the maximum of gravitating mass and its value is $M_G\,(<R_M) \approx 1.57\ M_{VIR}$.

Finally, it is defined the concept of extended halo (R_E) as the spherical region where the ratio $\dfrac{Density_{TM}^{SPHERE}\left(<R_E\right)}{\rho_{DE}} = 3/7$ *i.e.* the local ratio of such densities is equal to the current global ratio one, and it is found that $R_E \approx 1.85\ R_{ZG}$.

In the sixth chapter it is defined the zero velocity radius, R_{ZV}, as the sphere radius where the escape velocity is zero because of the Hubble flow. It is demonstrated that the ratio $R_{ZV}/R_{ZG} \approx 0.602$ is universal.

In the seventh chapter, it is validated the gravitating mass formula into the Virgo cluster for a couple of radius. Namely at 7.3 Mpc and at 3.4 Mpc. The calculus made with the formula of gravitating mass is compared successfully with recent results published in 2020. Also it is validated the theoretical result of $R_{ZV}/R_{ZG} \approx$

0.602 with result of measures published.

Two of the most important results got in this work are the formula $M_{TOTAL}\left(<R_{ZG}\right)=U\cdot M_{VIR}$ being $U\approx 2.7$ and the formula $M_{\text{TOTAL}}\left(<R_{E}\right)\approx 3.67\cdot M_{VIR}$ associated to zero gravity radius and the extended radius respectively.

Thanks these formulas this work suggest the possibility to solve the current discrepancy between the local parameter of matter density, $\Omega_{m}^{\text{local}}=0.08$, see [6] Karachentsev *et al.* 2014, and the current global one $\Omega_{m}^{\text{global}}=0.3$. This discrepancy is an open problem for the current cosmology.

Finally the chapter 8 is devoted to the concluding remarks.

The reader can consult the paper [7] Abarca, M, 2024, which is an extension of this paper, 24 pages, where the theory is validated through the Local Group and the Coma cluster data published as well.

2. Virial Mass and Virial Radius in Cluster of Galaxies

In cluster, it is a good estimation about virial radius and virial mass to consider $R_{vir} = R_{200}$ and $M_{vir} = M_{200}$. Where R_{200} is the radius of a sphere whose mean density is 200 times bigger than the critic density of Universe

$$\rho_{C}=\frac{3H^{2}}{8\pi G} \tag{2.1}$$

and M_{200} is the total mass enclosed by the radius R_{200}.

Considering the spherical volume formula, it is right to get the following relation between both concepts

$$R_{VIR}^{3}\approx R_{200}^{3}=\frac{G\cdot M_{200}}{100\cdot H^{2}} \tag{2.2}$$

or

$$M_{VIR}\approx M_{200}=\frac{100H^{2}R_{200}^{3}}{G} \tag{2.3}$$

Checking the Virial Mass Aproximation on a Sample of Clusters and Group of Galaxies

See **Table 1**. The data of second and third column have been taken from [8] R. Ragusa *et al.* 2022 and using the formula $M_{200}=\frac{100H^{2}R_{200}^{3}}{G}$ it is calculated its mass associated for each radius. In the last column shows the relative difference for masses, which is always under 10%.

Table 1. Data [8] R. Ragusa *et al.* 2022.

Group of galaxies G. or Clusters C.	Virial Radius	Virial Mass	Mass calculated	Relative diff for M
Name	Mpc	$\times 10^{13}\ M_{\odot}$	$\times 10^{13}\ M_{\odot}$	%
Antlia C.	1.28	26.3	2.39E+01	−9.21E+00

Continued

NGC596/584 G.	0.5	1.55	1.42E+00	−8.18E+00
NGC 3268 G.	0.9	8.99	8.30E+00	−7.67E+00
NGC 4365Virgo SubG.	0.32	0.4	3.73E-01	−6.73E+00
NGC 4636 Virgo SubG.	0.63	3.02	2.85E+00	−5.73E+00
NGC 4697Virgo Sub G.	1.29	26.9	2.44E+01	−9.14E+00
NGC 5846 G.	1.1	16.6	1.52E+01	−8.71E+00
NGC 6868 G.	0.6	2.69	2.46E+00	−8.57E+00

As the Virgo cluster is the nearest between the big clusters it is crucial to check the approximation for virial mass and radius with its data.

See in Table 2 the data of Virgo cluster, according [9] Olga Kashibadze, I. Karachentsev. 2020.

Using formula (2.3) with R_{200} =1.7 Mpc it is got $M_{200} = 5.59 \times 10^{14}\ M_{\Theta}$ which matches with the mass published if it is considered the range of errors.

Table 2. Virgo cluster.

	Virial Radius	Virial mass	Calculated M_{200}	Relative diff. of mass
Cluster	Mpc	$\times 10^{14}\ M_{\Theta}$	$\times 10^{14}\ M_{\Theta}$	%
Virgo	1.7	6.3 ± 0.9	5.59	11

In conclusion: R_{200} and M_{200} are a very good estimation for the virial radius and the virial mass for a group of galaxies and a cluster of galaxies, when they are in dynamical equilibrium.

3. Virial Theorem as a Method to Get the Direct Mass Formula in Galaxy Clusters

In chapter 9, of paper [1] Abarca, M. 2023 was demonstrated that the direct formula

$$M_{TOTAL}\left(<r\right) = \frac{a^2 \cdot \sqrt{r}}{G} \tag{3.1}$$

is the most suitable formula to calculate the total mass (baryonic and DM) depending on radius in the galactic halo.

3.1. Parameter a^2 Formula Depending on Virial Radius and Virial Mass

Due to the fact that the Direct mass formula has one parameter only, is enough to know the mass associated to a specific radius to be able to calculate parameter a^2. That is the situation when it is known the virial mass and the virial radius for a cluster of galaxies.

If it is considered that the virial radius is the border of halo cluster where galaxies are in dynamical equilibrium and at the same time is negligible the amount of Baryonic matter outside the sphere with such radius, then according DMbQG theory is possible to do an equation between M_{VIRIAL} ($<R_{VIRIAL}$) = M_{DIRECT} ($<R_{VIRIAL}$) (3.2.1) *i.e.*

$$M_{VIRIAL} = M_{TOTAL}\left(< R_{VIRIAL}\right) = \frac{a^2 \cdot \sqrt{R_{VIRIAL}}}{G} \tag{3.2}$$

and clearing up

$$a^2 = \frac{G \cdot M_{VIRIAL}}{\sqrt{R_{VIRIAL}}} \tag{3.3}$$

this formula is called parameter a^2 (M_{VIR}, R_{VIR}) because depend on both measures.

3.2. Parameter a^2 Formula Depending on Virial Mass Only

In chapter 2 was got this formula $R_{VIR}^3 = \frac{G \cdot M_{VIR}}{100 \cdot H^2}$ (2.2) as a good approximation between virial mass and virial radius. So using that formula and by substitution of virial radius in $a^2 = \frac{G \cdot M_{VIRIAL}}{\sqrt{R_{VIRIAL}}}$ (3.3) it is right to get parameter a^2 depending on M_{VIR} only

$$a^2 = \left(G \cdot M_{VIR}\right)^{5/6} \cdot \left(10 \cdot H\right)^{1/3} \tag{3.4}$$

This formula will be called parameter a^2 (M_{VIR}) as depend on M_{VIR} only.

With the virial data for Virgo cluster, see Table 2, will be calculated its parameter $\boldsymbol{a^2}$ with the formula (3.3) *i.e.* a^2 (M_{VIR}, R_{VIR}) and with the formula (3.4) *i.e.* a^2 ($\mathrm{M_{VIR}}$).

The last formula is an approximation of the previous formula as it is supposed that $R_{VIR} \approx R_{200}$. In Table 3 are calculated both formulas and fortunately its relative difference is negligible.

Table 3. Calculated data.

	Parameter a^2 (M_{VIR}, R_{VIR})	Parameter a^2 (M_{VIR})	Relative diff.
Clusters	I.S. units $m^{5/2}/s^2$	I.S. units $m^{5/2}/s^2$	%
Virgo	3.6527E23	3.581E23	2

4. Dark Matter Is Counter Balanced by Dark Energy at Zero Gravity Radius

The basic concepts about DE on the current cosmology can be studied in [5] Chernin, A.D.

As currently there is a tension regarding the experimental value of Hubble constant, in this paper will be used H = 70 Km/s/Mpc and $\Omega_{DE} = 0.7$ as the fraction of Universal density of DE.

4.1. Zero Gravity Radius Depending on Parameter a^2 Formula

According [5] Chernin, A. D. in the current cosmologic model ΛCDM, dark energy has an effect equivalent to antigravity *i.e.* the mass associated to dark energy is negative and the dark energy have a constant density for all the Universe equal to $\rho_{DE} = \rho_C \cdot \Omega_{DE} = 6.444 \times 10^{-27}\ \mathrm{kg/m^3}$ being $\Omega_{DE} = 0.7$ and $\rho_C = \frac{3H^2}{8\pi G} = 9.205 \times 10^{-27}\ \mathrm{kg/m^3}$ the critic density of the Universe.

As DE density is constant, the total DE mass is proportional to Radius with power 3, whereas DM mass grows with radius power 0.5 so it is right to get a radius where DM is counter balanced by DE.

According to [5] Chernin, A. D.

$$M_{DE}(<R) = -\frac{\rho_{DE} 8\pi R^3}{3}, \tag{4.1}$$

Is the mass associated to DE or equivalently

$$M_{DE}(<R) = -\rho_{DE}\frac{8\pi R^3}{3} = \frac{-H^2 \cdot \Omega_{DE}}{G} \cdot R^3 \tag{4.2}$$

Notice that the author multiplies by two the volume of a sphere by reasons explained in his work.

[5] Chernin defines gravitating mass

$$M_G(<R) = M_{DE}(<R) + M_{TOTAL}(<R) \tag{4.3}$$

where M_{TOTAL} is baryonic plus dark matter mass, and defines R_{ZG}, Radius at zero Gravity as the radius where $M_{DE}(<R_{ZG}) + M_{TOTAL}(<R_{ZG}) = 0$. *i.e.* where the gravitating mass is zero.

According to the previous equation it is got

$$M_{TOTAL}(<R_{ZG}) = \rho_{DE}\frac{8\pi R_{ZG}^3}{3} \tag{4.4}$$

Using (3.1) formula $M_{TOTAL}(<R_{ZG}) = \frac{a^2 \cdot \sqrt{R_{ZG}}}{G}$ the Equation (4.4) leads to

$$\frac{a^2 \cdot \sqrt{R_{ZG}}}{G} = \rho_{DE} \cdot \frac{8\pi \cdot R_{ZG}^3}{3}, \tag{4.5}$$

where it is possible to clear up

$$R_{ZG} = \left[\frac{3a^2}{8\pi G \rho_{DE}}\right]^{2/5} \tag{4.6}$$

and as

$$\rho_{DE} = \frac{3 \cdot H^2}{8\pi G}\Omega_{DE} \tag{4.7}$$

then by substitution

$$R_{ZG} = \left[\frac{a^2}{H^2 \cdot \Omega_{DE}}\right]^{2/5} \tag{4.8}$$

This formula will be called R_{ZG} (parameter a^2).

As the radius R_{ZG} is the distance to cluster centre where is zero the gravitating mass, it is right to consider R_{ZG} as the halo radius and its sphere defined as the halo cluster.

4.2. Zero Gravity Radius Formula Depending on Virial Mass

In previous chapter was got the value for $a^2 = \left(G \cdot M_{VIR}\right)^{5/6} \cdot \left(10 \cdot H\right)^{1/3}$ (3.4), depending on M_{VIR}, so by substitution in (4.8) it is right to get

$$R_{ZG} = \frac{\left(G \cdot M_{VIR}\right)^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \Omega_{DE}^{2/5}} \tag{4.9}$$

In Table 4 are calculated R_{ZG} by two ways: Formulas (4.8) and (4.9). Both calculi are mathematically equivalents. See in Table 4 how the both values match perfectly. See in Table 2 the values for the Virial mass of Virgo.

Table 4. Calculi for Virgo Rzg.

	Parameter a^2 (M_{VIR})	R_{ZG} (parameter a^2)	R_{ZG} (M_{VIR})	Relative diff.
Clusters	I.S. units $m^{5/2}/s^2$	Mpc	Mpc	%
Virgo	3.581E23	12.871	12.871	0

With this important cluster of galaxies, it has been illustrated how the total mass, calculated by $M_{TOTAL}\left(<r\right) = \frac{a^2 \cdot \sqrt{r}}{G}$, is counter balanced by dark energy at mega parsecs scale, and precisely this Radius at zero gravity determines the region size where the cluster has gravitational influence.

4.3. Zero Gravity Radius versus Virial Radius

From (2.2) $R_{VIR}^3 \approx R_{200}^3 = \frac{G \cdot M_{200}}{100 \cdot H^2} \approx \frac{G \cdot M_{VIR}}{100 \cdot H^2}$ it is right to get

$R_{VIR} = \left(\frac{G \cdot M_{VIR}}{100 \cdot H^2}\right)^{1/3}$. In previous epigraph was got $R_{ZG} = \frac{\left(G \cdot M_{VIR}\right)^{1/3} \cdot \sqrt[15]{100}}{H^{2/3} \Omega_{DE}^{2/5}}$ (4.9).

So it is right to get the ratio

$$\frac{R_{ZG}}{R_{VIR}} = \left(\frac{100}{\Omega_{DE}}\right)^{2/5} \approx 7.277 \tag{4.10}$$

which is universal as it does not depend on virial mass associated to a specific cluster.

Using the data from Table 1, first, second and third columns, the fourth column of Table 5 is calculated the zero gravity radius formula (4.9). In the last column is calculated the ratio.

It is clear that the ratio R_{ZG}/R_{VIR} got in this sample of celestial bodies match very well with the theoretical ratio formula (4.10)

Table 5. Celestial bodies.

	Virial Radius	Virial Mass	Zero Gravity R.	Ratio
Celestial Body	Mpc	1E13 $M_{\odot}$	Mpc	R_{ZG}/R_{VIR}
Antlia cluster	1.28	26.3	9.62E+00	7.52E+00
NGC596/584	0.5	1.55	3.74E+00	7.49E+00
NGC 3268	0.9	8.99	6.73E+00	7.47E+00
NGC 4365	0.32	0.4	2.38E+00	7.45E+00
NGC 4636	0.63	3.02	4.68E+00	7.42E+00
NGC 4697	1.29	26.9	9.69E+00	7.51E+00
NGC 5846	1.1	16.6	8.25E+00	7.50E+00
NGC 6868	0.6	2.69	4.50E+00	7.50E+00

Similarly in Table 6, it is done the same ratio for the Virgo cluster with an optimal result, for the Virial radius see Table 3, for the Zero gravity radius see Table 4.

Table 6. Virgo cluster.

	VirialRadius	Zero Grav R	Ratio
Cluster	Mpc	Mpc	R_{ZG}/R_{VIR}
Virgo	1.7	12.871	7.57

4.4. Total Mass Associated to a Cluster of Galaxies

4.4.1. Total Mass Associated to the Sphere with Zero Gravity Radius

In epigraph 4.1 was shown how this equation $M_{TOTAL}(<R_{ZG}) = \rho_{DE} \cdot \frac{8\pi \cdot R_{ZG}^3}{3}$ see (4.4) is used to define R_{ZG}. By simplification it is got

$$M_{TOTAL}\left(<R_{ZG}\right) = \frac{H^2 \cdot \Omega_{DE}}{G} \cdot R_{ZG}^3 = M \cdot R_{ZG}^3 \tag{4.11}$$

being $M = 5.3984 \cdot 10^{-26}$ (I.S.)

So that is the total mass formula, using the equation between total mass and dark energy at zero gravity radius.

In Table 7 is shown calculus of total mass for Virgo.

Table 7. Calculus of total mass for Virgo.

	Radius ZG	$M_{TOTAL}(<R_{ZG}) = M \cdot R^3{}_{ZG}$
Clusters	Mpc	$M_{\odot}$
Virgo	12.871	1.699945E15

4.4.2. Total Mass at Zero Gravity Radius Using the Virial Mass

Using rightly the direct mass formula see (3.1) at R_{ZG} it is got

$$M_{TOTAL}\left(<R_{ZG}\right)=\frac{a^{2}\cdot\sqrt{R_{ZG}}}{G}\tag{4.12}$$

By substitution of (3.4) $a^{2}=\left(G\cdot M_{VIR}\right)^{5/6}\cdot\left(10\cdot H\right)^{1/3}$ and (4.9)

$R_{ZG}=\frac{\left(G\cdot M_{VIR}\right)^{1/3}\cdot\sqrt[15]{100}}{H^{2/3}\Omega_{DE}^{2/5}}$ in (4.12) it is got

$$M_{TOTAL}\left(<R_{ZG}\right)=\frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}}\cdot M_{VIR}\tag{4.13}$$

and calling

$$U=\frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}}\tag{4.14}$$

$U\approx 2.7$ then

$$M_{TOTAL}\left(<R_{ZG}\right)=U\cdot M_{VIR}\tag{4.15}$$

So may be stated that according Dark matter by gravitation theory, the total mass (baryonic plus DM) enclosed by the sphere with radius R_{ZG} is equivalent to 2.7 times the Virial Mass.

In Table 8 are compared the masses for Virgo using the formula (4.15) and the previous one (4.11). It is clear that both are mathematically equivalents.

Table 8. Compared data.

	Virial mass	$M_{TOTAL}\,(<R_{ZG})=U\cdot M_{VIR}$	$M_{TOTAL}\,(<R_{ZG})=M\cdot R^{3}_{ZG}$
Clusters	$\cdot 10^{14}\ M_{\odot}$	$M_{\odot}$	$M_{\odot}$
Virgo	6.3 ± 0.9	1.6994E15	1.6994E15

By the direct formula for total mass, may be calculated the total mass associated to a radius, which is a fraction of R_{ZG}, with the following property: if $\left(R=f\cdot R_{ZG}\right)$ then

$$M_{TOTAL}\left(<R\right)=M_{TOTAL}\left(<f\cdot R_{ZG}\right)=\sqrt{f\cdot R_{ZG}}\cdot\frac{a^{2}}{G}=\sqrt{f}\cdot U\cdot M_{VIR}\tag{4.16}$$

because $M_{TOTAL}\left(<R_{ZG}\right)=U\cdot M_{VIR}$. See (4.15)

e.g., using the data for Virgo cluster $R_{ZG}=12.87$ and $R_{VIR}=R_{200}=1.7687$ Mpc, then $R_{VIR}/R_{ZG}=f=0.137428$ and by the formula (4.16)

$$M_{TOTAL}\left(<R_{VIR}\right)=M_{TOTAL}\left(<f\cdot R_{ZG}\right)=\sqrt{f}\,U\cdot M_{VIR}\approx 0.99999M_{VIR}.$$

4.5. Total Dark Energy at Zero Gravity Radius

The formula (4.2) at R_{ZG} becomes

$$M_{DE}\left(<R_{ZG}\right)=\frac{-H^{2}\cdot\Omega_{DE}}{G}\cdot R_{ZG}^{3}\tag{4.17}$$

that it is just the opposite value to $M_{TOTAL}\left(<R_{ZG}\right)=\frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}}\cdot M_{VIR}$ see (4.13) because

the total gravitating mass enclosed into the sphere of zero gravity radius is zero by definition.

$$M_G\left(<R_{ZG}\right)=M_{TOTAL}\left(<R_{ZG}\right)+M_{DE}\left(<R_{ZG}\right)=0 \quad (4.18)$$

See epigraph 4.1

Therefore $M_{DE}\left(<R_{ZG}\right)=-\frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}}\cdot M_{VIR}=-U\cdot M_{VIR}$ see (4.15), being $U\approx 2.7$, so joining (4.17) and (4.18) it is got

$$M_{DE}\left(<R_{ZG}\right)=\frac{-H^2\cdot\Omega_{DE}}{G}\cdot R_{ZG}^3=-U\cdot M_{VIR} \quad (4.19)$$

The formula (4.19) may be used to calculate the mass associated to *DE* at a specific radius *R*, writing such radius as a fraction of R_{ZG} *i.e.* $\left(R=f\cdot R_{ZG}\right)$ then (4.2) is written as

$$M_{DE}\left(<R\right)=M_{DE}\left(<f\cdot R_{ZG}\right)=\frac{-f^3\cdot H^2\cdot\Omega_{DE}}{G}\cdot R_{ZG}^3=-f^3\cdot U\cdot M_{VIR} \quad (4.20)$$

For example using the Virgo data R_{ZG} =12.871 Mpc and $R_{VIR}=R_{200}=1.7687$ Mpc, the ratio $R_{VIR}/R_{ZG}=f=0.137428$ and
$M_{DE}\left(<R_{VIR}\right)=M_{DE}\left(<f\cdot R_{ZG}\right)=-f^3\cdot U\cdot M_{VIR}=-0.007\cdot M_{VIR}$ *i.e.* $M_{DE}\left(<R_{VIR}\right)$ is negligible, whereas $M_{DE}\left(<R_{ZG}\right)=-U\cdot M_{VIR}$.

4.6. Gravitating Mass Function

In the epigraph 4.1 was defined the gravitating mass $M_G=M_{DE}+M_{TOTAL}$, where M_{TOTAL} is baryonic plus dark matter mass and M_{DE} is the negative mass associated to DE.

The best way to calculate the gravitating mass is using the formulas got in epigraph 4.4 and 4.5 where are calculated the both types of masses associated to a radius, which is a fraction of R_{ZG}, *i.e.* $R=f\cdot R_{ZG}$, these formulas are (4.16) and (4.20).

Joining both ones it is got

$$M_G\left(<R\right)=M_G\left(<f\cdot R_{ZG}\right)=\left[\sqrt{f}-f^3\right]\cdot U\cdot M_{VIR} \quad (4.21)$$

where $U\approx 2.7$. This way the gravitating mass depends on the dimensionless factor *f*.

Dominion of the Gravitating Mass Function

As the gravitating mass is defined into the halo cluster, its dominion begins at R_{VIR} so the gravitating mass depending on f begins at R_{VIR}/R_{ZG}. By (4.10) formula R_{ZG}/R_{VIR} =7.277 then R_{VIR}/R_{ZG} = 1/7.277 therefore the dominion of the gravitating mass function depending on $f=R/R_{ZG}$ begins at 0.13732.

In this epigraph this function will be studied up to $f=1$ *i.e.* when the radius reaches the zero gravity radius.

In **Figure 1** is represented the ratio *gravitating mass/virial mass* versus the ratio $f=R/R_{ZG}$ in its dominion and close to $f=0.5$ the function reach the maximum.

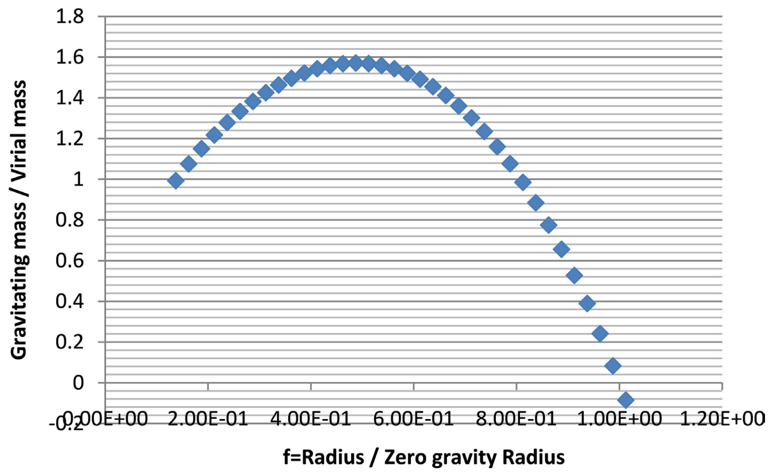

Figure 1. Gravitating mass function/Virial mass vs *f*.

4.7. Calculus for the Maximum of the Gravitating Mass

It is clear that such function will have a maximum, that it is found easily by derivation, $f_M = \frac{1}{\sqrt[5]{36}} \approx 0.488$, so the radius at maximum is

$$R_M = \frac{1}{\sqrt[5]{36}} \cdot R_{ZG} \approx 0.488 \cdot R_{ZG}. \tag{4.22}$$

By substitution of this value f_M into the mass gravitating formula (5.21) it is got

$$M_G^{MAXIMUM}\left(< R_M\right) = 1.57 \cdot M_{VIR} \tag{4.23}$$

So may be stated the following important result:

For any cluster of galaxies at a half of R_{ZG}, it is reached the maximum of gravitating mass which is $1.57 \cdot M_{VIR}$.

4.8. Density of the Total Mass into the Halo Cluster Theorem

Assuming the hypothesis that $R_{vir} = R_{200}$ and $M_{vir} = M_{200}$, *i.e.* the virial sphere $Density_{VIR} = \frac{M_{vir}}{Vol_{VIR}} = 200\,\rho_C$ being $\rho_C = \frac{3H^2}{8\pi G}$ the critical density of Universe.

Then into the halo cluster sphere the

$$Density_{TOTAL\ MASS}\left(< R_{ZG}\right) = 1.4 \cdot \rho_C \tag{4.24}$$

Being R_{ZG} the halo radius.

Proof:

$$Density_{TOTAL\ MASS}^{CLUSTER}\left(< R_{ZG}\right) = \frac{M_{TOTAL}\left(< R_{ZG}\right)}{Vol_{CL}\left(< R_{ZG}\right)} = \frac{M_{VIR} \cdot U}{Vol_{vir} \cdot \left(\frac{R_{ZG}}{R_{VIR}}\right)^3} = \frac{M_{VIR} \cdot U}{Vol_{vir} \cdot U^6}$$

$$= \frac{200 \cdot \rho_C}{U^5} = \frac{200 \cdot \rho_C \cdot \Omega_{DE}}{100} = 1.4 \cdot \rho_C$$

In that chained equalities has been used two properties before got: (4.15) and (4.10).

Dark Energy Density into the Halo Cluster

As into the current ΛCDM model the DE density has a constant value, $\rho_{DE} = 0.7\rho_C$ in every place, in particular

$$Density_{DARK\ ENERGY}^{CLUSTER}\left(< R_{ZG}\right) = 0.7 \cdot \rho_c \quad (4.25)$$

Corolarius

The ratio density of total mass versus density of DE into the halo cluster is 2.

4.9. Extended Halo Where the Ratio Total Mass versus Dark Energy Is 3/7

At the present, it is accepted that $\frac{\Omega_M}{\Omega_{DE}} = 3/7$ as a global average in the current Universe, so it is worth to calculate an extension of halo cluster, R_E, where it is reached such ratio. Now it is calculated the radius of a sphere that verify

$$\frac{Density_{TM}^{SPHERE}\left(< R_E\right)}{\rho_{DE}} = 3/7 \quad (4.26)$$

$As\, M_{TM}\left(< R\right) = Density_{TM} \cdot Volume$ and by (4.2) $M_{DE}\left(< R\right) = 2 \cdot \rho_{DE} \cdot Volume$ then by substitution into (4.26) it is got

$$\frac{Density_{TM}^{SPHERE}}{\rho_{DE}} = \frac{2 \cdot M_{TM}\left(< R\right)}{M_{DE}\left(< R\right)} = 3/7 \quad (4.27)$$

Using (4.16) and (4.20) it is right to get the second one equality below.

$$\frac{Density_{TM}^{SPHERE}\left(< R\right)}{\rho_{DE}} = 2 \cdot \frac{M_T\left(< R\right)}{M_{DE}\left(< R\right)} = \frac{2 \cdot \sqrt{f}}{f^3} \quad (4.28)$$

By equation (4.27) and (4.28) it is got $\frac{2 \cdot \sqrt{f}}{f^3} = \frac{3}{7}$ whose solution is

$$f = \left(\frac{14}{3}\right)^{2/5} \approx 1.85 \quad (4.29)$$

Therefore the radius of extended halo searched is

$$R_E \approx 1.85 R_{ZG} \quad (4.30)$$

and by (4.16)

$$M_{TOTAL}\left(R_E\right) \approx 3.67 M_{VIR} \quad (4.31)$$

In order to validate such calculus is enough to check that the Density of total mass at R_E radius is $0.3 \cdot \rho_C$

$$D_{TM}\left(< R_E\right) = \frac{M_{VIR} \cdot U \cdot \sqrt{1.85}}{Vol_{vir} \cdot \left(\frac{R_{ZG}}{R_{VIR}}\right)^3 \cdot 1.85^3} = \frac{200 \cdot \rho_C \cdot \Omega_{DE}}{100 \cdot 1.85^{5/2}} = \frac{1.4}{1.85^{5/2}} \cdot \rho_C = 0.3 \cdot \rho_C \quad (4.32)$$

as it was expected.

For example for the Virgo cluster $R_E = 1.85 \cdot 12.9$ Mpc = 23.9 Mpc.

5. Zero Velocity Radius Because of the Hubble Flow

It is defined the zero velocity radius as the distance to the cluster centre, where the escape velocity from gravitation field is equal to Hubble flow velocity. *i.e.*

$$V_E = V_{HF} \tag{5.1}$$

From classical dynamic it is taken the formula

$$\frac{V_E^2}{2} = -V(R). \tag{5.2}$$

i.e. the kinetic energy associated to escape velocity compensates the potential energy getting zero as total energy ad infinitum.

It is not possible to use the classical escape velocity formula

$$V_E^2 = \frac{2GM}{R}. \tag{5.3}$$

because of two reasons:

1) The gravitational potential in DMbG they is different to $V = -\frac{GM}{R}$.

2) The border of the halo cluster is R_{ZG} where the gravitating mass is zero.

5.1. Gravitational Potential into the Halo Cluster

The gravitating mass is zero at R_{ZG} the line integral for potential goes up to R_{ZG}, so the formula is

$$V = \int_R^{R_{ZG}} \left[\frac{-GM_G(<r)}{r^2} \right] \cdot \mathrm{d}r \tag{5.4}$$

The gravitating mass $M_G(<r) = M_G(<f \cdot R_{ZG}) = \left[\sqrt{f} - f^3\right] \cdot U \cdot M_{VIR}$ (4.21) will be used to do the integral, doing some little changes.

As $f = r/R_{ZG}$ and calling $K = U \cdot M_{VIR}$, by substitution in (4.21) it is got

$$M_G(<r) = K \cdot \left[\frac{\sqrt{r}}{\sqrt{R_{ZG}}} - \frac{r^3}{R_{ZG}^3} \right] \tag{5.5}$$

that by substitution in (5.4) results

$$V(R) = -GK \int_R^{R_{ZG}} \left[\frac{r^{-3/2}}{\sqrt{R_{ZG}}} - \frac{r}{R_{ZG}^3} \right] \cdot \mathrm{d}r \tag{5.6}$$

whose result is

$$V(r) = \frac{5GK}{2 \cdot R_{ZG}} - GK \cdot \left[\frac{2}{\sqrt{r} \cdot \sqrt{R_{ZG}}} + \frac{r^2}{2 \cdot R_{ZG}^3} \right] \tag{5.7}$$

where its dominion ranges from R_{VIR} up to R_{ZG}.

Notice that $V(R_{ZG}) = 0$ and $V(r)$ is negative inside its dominion.

5.2. Equation for Zero Velocity Radius

In this epigraph will be developed the equation $V_E(r) = V_{HF}(r)$ (5.1) to calculate the R_{ZV}

From (5.2) $V_E^2 = -2 \cdot V(R)$ So

$$-2 \cdot V(R) = V_{HF}^2 = H^2 \cdot R^2 \tag{5.8}$$

and by substitution of potential formula (5.7) it is got

$$\frac{-5GK}{R_{ZG}} + 2GK \cdot \left[\frac{2}{\sqrt{r} \cdot \sqrt{R_{ZG}}} + \frac{r^2}{2 \cdot R_{ZG}^3} \right] = H^2 \cdot r^2 \tag{5.9}$$

reorganising that equation it is got:

$$\frac{4GK}{\sqrt{R_{ZG}}} \cdot \frac{1}{\sqrt{r}} + \left[\frac{GK}{R_{ZG}^3} - H^2 \right] \cdot r^2 = \frac{5GK}{R_{ZG}} \tag{5.10}$$

and multiplying that equation by the factor $\frac{R_{ZG}}{GK}$ it is got a better expression

$$4\sqrt{R_{ZG}} \cdot \frac{1}{\sqrt{r}} + \left[\frac{1}{R_{ZG}^2} - \frac{H^2 \cdot R_{ZG}}{GK} \right] \cdot r^2 = 5 \tag{5.11}$$

Which is the equation for zero velocity radius.

That equation is not possible to solve with algebraic methods, but is quite easy to solve numerically for specific data.

5.3. Zero Velocity Radius for Virgo Cluster

In Table 9 there are the Virgo cluster data. Virial radius and mass data come from [9] O. Kashi-badze.

Table 9. Virgo cluster data.

	Virial Radius	Virial mass	R_{ZG}
Cluster	Mpc	$\cdot 10^{14}$ M_Θ	
Virgo	1.7	6.3 ± 0.9	12.87 Mpc

Using such data it is possible to calculate the coefficient for the equation (5.11)

$4\sqrt{R_{ZG}} \cdot \frac{1}{\sqrt{r}} + \left[\frac{1}{R_{ZG}^2} - \frac{H^2 \cdot R_{ZG}}{GK} \right] \cdot r^2 = 5$ As $U \approx 2.7$ and $K = U \cdot M_{VIR}$ then

$GK = 2.259\text{E}+35$ (I.S. units), $\frac{H^2 \cdot R_{ZG}}{GK} = 9.047\text{E}-48\ \text{m}^{-2}$,

$\frac{1}{R_{ZG}^2} = 6.3397\text{E}-48\ m^{-2}$ and $4\sqrt{R_{ZG}} = 2.5208\text{E}+12 m^{1/2}$

The equation (5.11) is easy to be solved numerically.

Using f = Radius/R_{ZG} it is got that $f = 0.602$ is a very good approximation for the solution. Therefore the $R_{ZV} = f \cdot R_{ZG} = 7.75$ Mpc.

5.4. Zero Velocity Radius Theorem

Considering (5.11) *as the equation for the* R_{ZV} *then according DMbQG theory the ratio* R_{ZV}/R_{ZG} *is universal and its value is* $R_{ZV}/R_{ZG} \approx 0.602016$.

Proof

The equation (5.11) is $4\sqrt{R_{ZG}}\cdot\frac{1}{\sqrt{r}}+\left[\frac{1}{R_{ZG}^{2}}-\frac{H^{2}\cdot R_{ZG}}{GK}\right]\cdot r^{2}=5$ where

$K=U\cdot M_{VIR}$ being $U=\frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}}$ (4.14) and $R_{ZG}=\frac{\left(G\cdot M_{VIR}\right)^{1/3}\cdot\sqrt[15]{100}}{H^{2/3}\Omega_{DE}^{2/5}}$ (4.9)

then

using the values for K, U and R_{ZG}, doing some algebraic substitutions and transformations it is not difficult to get the following equation.

$$4\sqrt{R_{ZG}}\cdot\frac{1}{\sqrt{r}}+\frac{1}{R_{ZG}^{2}}?\left[\frac{\Omega_{DE}-1}{\Omega_{DE}}\right]\cdot r^{2}=5 \tag{5.12}$$

and defining $f=r/R_{ZG}$ the equation (5.12) becomes

$$4\cdot\frac{1}{\sqrt{f}}+\left[\frac{\Omega_{DE}-1}{\Omega_{DE}}\right]\cdot f^{2}=5 \tag{5.13}$$

that considering $\Omega_{DE}=0.7$ becomes:

$$4\cdot\frac{1}{\sqrt{f}}+\left[\frac{-3}{7}\right]\cdot f^{2}=5 \tag{5.14}$$

By elementary algebraic operations it is got this equivalent equation:

$$9\cdot f^{5}+210\cdot f^{3}+1225\cdot f-784=0$$

Thanks Wolfram alpha software, this is its only real solution

$$f\approx 0.602016 \tag{5.15}$$

By this new formulation of the equation for R_{ZV} calculus, it has been demonstrated that the ratio $f=R_{ZV}/R_{ZG}$ is universal and depends on Ω_{DE} solely.

Notice how close are the value (5.15) with the one got for Virgo in epigraph 5.3.

5.5. Gravitating Mass at Zero Velocity Radius

Using the gravitating mass formula (4.21) being $R_{ZV}=f\cdot R_{ZG}$ and by substitution of $f\approx 0.602$ at formula (4.21) where $U=\frac{100^{1/5}}{\Omega_{DE}^{1/5}}\approx 2.6976$ it is right to get that

$$M_{G}\left(<R_{ZV}\right)\approx 1.5\cdot M_{VIR} \tag{5.16}$$

6. Validation of the Theory with Results Published about Virgo Cluster

In this chapter, some theoretical results got in this paper will be validated with three results published about the Virgo cluster.

The first test is relative to the R_{ZV} and its associated mass. These results were studied in chapter 5.

The second test is relative to the gravitating mass associated to the twice of virial radius.

In the third test, the most important, it is postulated that DMbQG theory is able to multiply by the factor $U \cdot \sqrt{1.85} = 2.7 \times 1.36 = 3.67$ the current parameter of local matter density $\Omega_m^{local} = 0.08$. reaching 0.294 which match with the value $\Omega_m^{Global} = 0.3$. accepted currently by the scientific community.

6.1. Gravitating Mass Associated to the Zero Velocity Radius at 7.3 Mpc

In the clipped text below, from [9] O. Kashibadze. 2020, the authors gives the interval for [7, 7.3] Mpc for the R_{ZV}.

rding DMbQG theory, the zero velocity radius R_{ZV} =7.75 Mpc, see epigraph 5.3, so the relative difference is only 6%.

This is a very good match between experimental results and theoretical results by DMbQG theory.

Also at the concluding remarks from [9] O. Kashibadze.2020, the authors give for the total mass $M_T(<R_0) = (7.4 \pm 0.9) \cdot 10^{14}\ M_\Theta$. As this value is got by dynamical measures in fact this value must be considered as gravitational mass.

The theoretical value (5.12) calculated in epigraph 5.5 is $M_G(<R_{ZV}) \approx 1.5 \cdot M_{VIR} =$ 9.48E14 M_Θ which is 14 % bigger regarding the value given by the authors, if it is considered the upper value of the interval. So both results may be considered compatibles.

Clipped text of [9] O. Kashibadze. 2020

7. Concluding remarks
The analysis of galaxy motions in the outskirts of the Virgo cluster makes it possible to measure the radius of the zero velocity surface, Ro = 7 - 7.3 Mpc (Karachentsev *et al.* 2014, Shaya *et al.* 2017, Kashibadze *et al.* 2018) corresponding to the total mass of the Virgo cluster $M_T = (7.4 \pm 0.9) \times 10^{14}\ M_\Theta$ inside the Ro. The numerical simulated trajectories of nearby galaxies with accurate distance estimates performed by Shaya *et al.* (2017) confirmed the obtained estimate of the total mass of the cluster. The virial mass of the cluster, being determined independently at the scale Rg = 1.7 Mpc from the internal motions, is nearly the same M_{VIR} $(6.3 \pm 0.9) \times 10^{14}\ M_\Theta$.

Table 10 summarized and compared the observational results and theoretical results.

Table 10. Virgo cluster.

	[9] O. Kashibadze	DMbG theory	Relative difference
R_{ZV}	[7, 7.3] Mpc	7.75 Mpc	6%—Very good
M_{VIR}	$(6.3 \pm 0.9) \cdot 10^{14}\ M_\Theta$		
$M_G(<R_{ZV})$	$(7.4 \pm 0.9) \cdot 10^{14}\ M_\Theta$	$1.5 \cdot M_{VIR} = 9.45 \cdot 10^{14}\ M_\Theta$	14% Compatibles

6.2. Gravitating Mass Associated Up to the Twice of Virial Radius

As R_{VIR} = 1.7 Mpc its twice value is 3.4 Mpc. As R_{ZG} = 12.9 Mpc then f= 3.4/12.9

= 0.2635 and by (4.21) $M_G\left(<2\cdot R_{VIR}\right)=1.335\cdot M_{VIR}=8.4\times10^{14}M_\Theta$.

This value match perfectly with the interval of masses given below in the clipped text.

Clipped text from page 9 of [9] O. Kashibadze.2020

The Plank collaboration (2016) performed a detailed study of the Virgo cluster through Sunyaev-Zeldovich effect and found the total mass of warm/hot gas to be (1.4 - 1.6) × 10^{14} M_Θ. Assuming the cosmic value for the baryon fraction, $f_b=\Omega_b/\Omega_m=0.1834$, they found that the total mass of the cluster would be (7.4 - 8.7) × 10^{14} M_Θ on a scale up to 2 times larger than the virial radius.

The total mass mentioned in Kashibadze paper it is considered in this paper as gravitating mass because in the whole paper of Kashibadze *et al.* they use always the concept of total mass as a result of dynamical measures so it is more suitable to interpret his total mass as gravitating mass.

Anyway, considering the total mass given by the formula (4.16) then $M_{TOTAL}\left(<2R_{VIR}\right)=1.386\cdot M_{VIR}=8.73\times10^{14}M_\Theta$ that match with the upper value of mass range given by the authors.

6.3. Solving the Conundrum: The Local Density Matter versus the Global Density Matter

Below is the clipped text of a paper published for a team of well known astrophysicist.

Clipped text from introduction of paper [6] Karachentsev I. D., R. Brent Tully. 2014

As it has been noted by different authors (Vennik, 1984, Tully, 1987, Crook *et al.* 2007, Makarov & Karachentsev, 2011, Karachentsev, 2012), the total virial masses of nearby groups and clusters leads to a mean local density of matter $\Omega_m\approx0.8$, that is 1/3 the mean global density $\Omega_m\approx0.24\pm0.03$ (Spergel *et al.*, 2007). One possible explanation of the disparity between the local and global density estimates may be that the outskirts of groups and clusters contain significant amount of dark matter beyond their virial radii, beyond what is anticipated from the integrated light of galaxies within the infall domain. If so, to get agreement between local and global values of Ω_m, the total mass of the Virgo cluster (and other clusters) must be 3 times their virial masses.

In page 3 of that paper, they state that at the nearby clusters the mean local density of matter is $\Omega_m^{local}=0.08$, whereas the global mass density in the Universe is $\Omega_m^{Global}=0.24$ (data year 2007).

Currently that data updated for scientific community is $\Omega_m^{Global}=0.3$.

The authors suggest that a possible solution for this tension would be that the total mass for cluster haloes must be three times the virial mass. That is justly what is found in this paper studying the DM at cluster scale as universal law in the framework of DMbQG theory.

In chapter 4, the formula (4.13) $M_{TOTAL}\left(<R_{ZG}\right)=\frac{\sqrt[5]{100}}{\Omega_{DE}^{1/5}}\cdot M_{VIR}=U\cdot M_{VIR}$,

shows that the total mass (baryonic and DM) enclosed into the halo cluster is $U\approx$ 2.7 times the virial mass equal to $M_{TOTAL}\left(<R_{VIR}\right)\approx M_{200}$.

However in the epigraph 4.9 has been calculated an extension of the halo cluster up to radius $R_E = 1.85 \cdot R_{ZG}$ to obtain a ratio $\frac{\Omega_{TM}^{LOCAL}}{\Omega_{DE}^{LOCAL}}=\frac{\Omega_{TM}^{GLOBAL}}{\Omega_{DE}^{GLOBAL}}=3/7$. Now using the formula (4.16) it is right to calculate the total mass enclosed by such sphere $M_{TOTAL}\left(<R_E\right)=M_{TOTAL}\left(<f\cdot R_{ZG}\right)=\sqrt{f}\cdot U\cdot M_{VIR}$ being $f = 1.85$ and the factor $\sqrt{f}\cdot U=3.67$

then

$$M_{TOTAL}\left(<R_E\right)=3.67\cdot M_{VIR} \tag{6.1}$$

Therefore with such factor the parameter $\Omega_m^{local}=0.08$ is increased up to a

$$\Omega_m^{local}=0.08\times 3.67=0.2936 \tag{6.2}$$

because according [6] Karachentsev *et al.* 2014, the coefficient $\Omega_m^{local}=0.08$ is calculated considering the virial masses of clusters into the Local Universe, and as according DMbQG such total mass is increased by the factor 3.67 if it is considered an extended halo with radius $R_E = 1.85 \cdot R_{ZG}$ then the coefficient $\Omega_m^{local}=0.08\times 3.67=0.293$ match perfectly with $\Omega_m^{global}=0.3$.

This result enables an experimental test to validate these theoretical findings: If at the present Universe, the average distance between clusters is about its R_E associated to each one, then the DMbQG theory would explain the current $\Omega_m^{GLOBAL}=0.3$.

In other words, considering that almost the total baryonic matter at the current Universe is enclosed inside the virial radius of the clusters, as it is confirmed by multiples measures, the DMbQG is able to justify that $\Omega_m^{LOCAL}=\Omega_m^{GLOBAL}$ on condition that the average distance between clusters is the extended radius R_E.

7. Concluding Remarks

Thanks direct mass $M_{DIRECT}\left(<r\right)=\frac{a^2\cdot\sqrt{r}}{G}$ (3.1) and using the approximation of virial mass for M_{200} and the virial radius for R_{200} it is possible to get the formula $a^2=\left(G\cdot M_{VIR}\right)^{5/6}\cdot\left(10\cdot H\right)^{1/3}$ (3.4).

In chapter 3, the direct mass has been extended to clusters, and it is possible to state that all the new theoretical results obtained in this paper are based on this formula, co working with the well known properties of DE.

The main results got in this paper are summarized below.

A) The universality of ratio $R_{ZG}/R_{VIR} \approx 7.3$ (4.10) and its total mass associated, being M_{TOTAL} $(<R_{ZG}) \approx 2.7\ M_{VIR}$ (4.15).

B) For any cluster at 0.488 R_{ZG} (4.22) is reached the maximum of gravitating mass and M_G $(<R_M) \approx 1.57 \cdot M_{VIR}$ (4.23).

C) The universality of the ratio $R_{ZV}/R_{ZG} \approx 0.602$ (5.15) and its gravitating mass M_G ($<R_{ZV}$) ≈ 1.5 M_{VIR} (5.16).

D) The universality of the ratio $R_E/R_{ZG} \approx 1.85$, (4.29) where the ratio total mass density versus DE density is 3/7 and its total mass $M_{TOTAL}\left(<R_E\right) = 3.67 \cdot M_{VIR}$ (4.31).

Finally in chapter 6, some results published about Virgo cluster are introduced that back fully the previous theoretical findings:

1) Regarding the property D) may be consulted [6] Karachentsev I. D., R. Brent Tully 2014 to understand more in deep the current tension between the low value for Local mass density parameter $\Omega_m = 0.08$ and the current global matter density parameter $\Omega_m = 0.3$.

2) Calculus of the zero velocity radius and its associated gravitating mass of Virgo are compatibles with the result published by [9] Olga Kashibadze *et al.* 2020, see epigraph 6.1.

3) Calculus of the mass gravitating at two times the virial radius match fully with the result published by [9] Olga Kashibadze *et al.* 2020, see epigraph 6.2.

These new theoretical findings offer to scientific community a number of tests to validate the theory. The validation of DMbQG theory would suggest that DM is a quantum gravitation effect, see [4] Corda 2012, see [10] Corda 2018 and see [11] Abarca 2014, giving to scientific community new elements to continue searching a quantum gravitation theory.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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