Journal of Modern Physics, 2024, 15, 1708-1716
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Predicting High Precision Hubble Constant Determinations Based on a New Theoretical Relationship between CMB Temperature and H_0

Eugene Terry Tatum[1], **Espen Gaarder Haug**[2], **Stéphane Wojnow**[3]

[1]Bowling Green, KY, USA
[2]Norwegian University of Life Sciences, Ås, Norway
[3]Limoges, France
Email: ett@twc.com, espenhaug@mac.com, wojnow.stephane@gmail.com

How to cite this paper: Tatum, E.T., Haug, E.G. and Wojnow, S. (2024) Predicting High Precision Hubble Constant Determinations Based on a New Theoretical Relationship between CMB Temperature and H_0. *Journal of Modern Physics*, **15**, 1708-1716.
https://doi.org/10.4236/jmp.2024.1511075

Received: September 6, 2024
Accepted: October 15, 2024
Published: October 18, 2024

Abstract

Based on considerable progress made in understanding the Cosmic Microwave Background (CMB) temperature from a deep theoretical perspective, this paper demonstrates a useful and simple relationship between the CMB temperature and the Hubble constant. This allows us to predict the Hubble constant with much higher precision than before by using the CMB temperature. This is of great importance, since it will lead to much higher precision in various global parameters of the cosmos, such as the Hubble radius and the age of the universe. We have improved uncertainty in the Hubble constant all the way down to 66.8712 ± 0.0019 km/s/Mpc based on data from one of the most recent CMB studies. Previous studies based on other methods have rarely reported an uncertainty much less than approximately ±1 km/s/Mpc for the Hubble constant. Our deeper understanding of the CMB and its relation to H_0 seems to be opening a new era of high-precision cosmology, which may well be the key to solving the Hubble tension, as alluded to herein. Naturally, our results should also be scrutinized by other researchers over time, but we believe that, even at this stage, this deeper understanding of the CMB deserves attention from the research community.

Keywords

Hubble Constant, CMB, Planck Temperature, Upsilon Constant

1. Introduction: Hubble Constant from CMB Temperature

This paper is motivated by very recent developments in theoretical cosmology, specifically where it concerns new cosmological models. For example, in 2015, Tatum *et al.* [1] offered their own model, including its formula for the cosmic temperature in the following form:

$$T_H = \frac{\hbar c^3}{k_b 8\pi G\sqrt{M_c m_p}} \tag{1}$$

where T_H is the Hubble temperature, k_b is the Boltzmann constant, $\hbar$ is the reduced Planck constant, c is the speed of light, G is the gravitational constant, m_p is the Planck mass and M_c is the critical mass in the Friedmann [2] equation $M_c = \frac{c^3}{2GH_0}$ that also is part of Einstein's [3] general relativity and the Λ-CDM cosmological model, as well as other lesser-known cosmological models. Equation (1) was initially investigated heuristically as many good ideas often start out; however, a more solid foundation must be established over time. A derivation of Equation (1) based on the Stefan-Boltzmann law has just been published [4] [5] and clearly shows that the formula is valid within thermodynamics and general relativity theory. Whether our way to predict the Hubble constant can be incorporated into the Λ-CDM model should be the subject of future investigation.

However, it is also important to investigate its predictive capacity for different so-called $R_h = ct$ cosmological models [6]-[11] that are consistent with general relativity theory, including growing black hole models. Equation (1) was introduced in Tatum *et al.*'s growing black hole $R_h = ct$ cosmological model. Such cosmological models are actively discussed to this day; see, for example [12]-[16]. We are pointing this out because it is too early to say which cosmological models will ultimately be found consistent with this way to predict the Hubble constant and which ones will not. The most important point in this article is that our proposed mathematical relationship between the Hubble constant and the CMB temperature should lead to much lower uncertainty in Hubble constant determinations based on measurement of the CMB temperature. This approach might also improve the prediction of how the CMB temperature evolved in the past, such as the exact time of the decoupling of the CMB. However, correlation with past cosmic epochs is outside the scope of this paper.

The formula above can also be expressed as:

$$T_{CMB} = \frac{T_p}{8\pi}\sqrt{\frac{2l_p}{R_{H_0}}} = \frac{T_p}{8\pi}\sqrt{\frac{2l_p H_0}{c}} \tag{2}$$

where T_{CMB} is the CMB temperature, R_{H_0} is the Hubble radius, H_0 is the Hubble constant, c is the speed of light, T_p is the Planck [17] temperature and l_p is the Planck length. Equations (1) and (2) are just two ways to write the same formula, so we can start with either of these and solve for H_0. Solving for H_0 gives:

$$H_0 = \frac{T_{CMB}^2}{T_p^2}\frac{64\pi^2 c}{2l_p} \tag{3}$$

And since the Planck length $l_p = \sqrt{\frac{G\hbar}{c^3}}$ and $T_p = \frac{1}{k_b}\sqrt{\frac{\hbar c^5}{G}} = \frac{m_p c^2}{k_b}$, if we insert that into Equation (3), we get:

$$H_0 = \frac{T_{CMB}^2}{\left(\frac{1}{k_b}\sqrt{\frac{\hbar c^5}{G}}\right)^2} \frac{64\pi^2 c}{2\sqrt{\frac{G\hbar}{c^3}}}$$

$$H_0 = \frac{T_{CMB}^2}{\left(\frac{1}{k_b} c\hbar\sqrt{\frac{c^3}{\hbar G}}\right)^2} \frac{32\pi^2 c}{\sqrt{\frac{G\hbar}{c^3}}}$$

$$H_0 = \frac{T_{CMB}^2 k_b^2 32\pi^2}{c\hbar\sqrt{\frac{c^3\hbar}{G}}} \tag{4}$$

In the equation above, we can even separate out the part only containing constants:

$$\frac{k_b^2 32\pi^2}{c\hbar\sqrt{\frac{c^3\hbar}{G}}} = \frac{k_b^2 32\pi^2 G^{1/2}}{c^{5/2}\hbar^{3/2}} = 2.91845601\times10^{-19} \pm 0.00003279\times10^{-19}\ \mathrm{s}^{-1}\cdot\mathrm{K}^{-2} \tag{5}$$

And we could call this composite constant Upsilon: Ʊ (Latin version of Upsilon). The uncertainty in Upsilon only comes from the uncertainty in $G = 6.67430\times10^{-11} \pm 0.00015\times10^{-15}\ \mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$, as all other constants in the composite constant are defined exactly in NIST CODATA 2018. The relation between the Hubble constant and the CMB temperature is, therefore, just a composite constant multiplied by the CMB temperature squared:

$$H_0 = \mho T_{CMB}^2 \tag{6}$$

Still, naturally, part of this Upsilon composite constant contains G, and we would still naturally need to take into account uncertainty in G, as well as the uncertainty in the CMB temperature when finding the uncertainty in the Hubble constant from this method, so the uncertainty will be the same as we will get from Equation (4), as will be explored in the next section.

To summarize this section, all of the above formulae are effectively produced by different substitutions and rearrangements of Equation (1). The results are the same with respect to calculating the value and precision of the Hubble constant for a given CMB temperature value [18]. In the next section, we will demonstrate that this formula is not only of theoretical interest to describe the relationship between the Hubble constant and the CMB temperature, but that it surprisingly leads to much higher precision in Hubble constant predictions after properly accounting for the full uncertainty in all input parameters.

2. High Precision Hubble Constant

Since the discoveries by Lemaître [19] and Hubble [20], extensive observational studies have been ongoing for many decades in order to increase the precision in the Hubble constant, something that is of great importance for a more precise understanding of the cosmos. See, for example [21]-[30]. Even the more precise of these studies have not much less than 1 standard deviation uncertainty in their measured or estimated Hubble constant values in units of 1 km/s/Mpc.

In our formulae, we are using the NIST CODATA (2018) value for G, which is $6.67430 \times 10^{-11} \pm 0.00015 \times 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$. Therefore, we are fully accounting for the uncertainty in G. Additionally, we consider the uncertainty in CMB temperature as provided in the respective studies we represent in **Table 1**. The speed of light $c = 299792458\ \mathrm{m \cdot s^{-1}}$, the reduced Planck constant (also known as the Dirac constant) $\hbar = \frac{h}{2\pi} = 1.054571817 \times 10^{-34}\ \mathrm{J \cdot s}$ and the Boltzmann constant $k_b = 1.380649 \times 10^{-23}\ \mathrm{J \cdot K^{-1}}$ that we need as inputs have no uncertainty, as they are exactly defined according to NIST 2018 CODATA. This approach allows us to incorporate the complete input uncertainty into predicting H_0.

To convert our value into units km/s/Mpc, we use the resolution B2 adopted at the 2015 General Assembly of the International Astronomical Union (IAU), where the parsec is defined as exactly 648,000/π astronomical units, and for AU, we use 149,597,870,700 m (IAU 2012 Resolution B1). So, the conversion factor we need to multiply the results from our formula is the product of 1000 × 648,000/π × 149,597,870,700 km/Mpc. There is no uncertainty in these conversion numbers, since they are merely conversion factors that are exactly defined.

For example, from the recent Dhal *et al.*'s [31] CMB study, we obtain a value of $H_0 = 66.8712 \pm 0.0019$ km/s/Mpc. This uncertainty of ±0.0019 km/s/Mpc represents one standard deviation. Compared to other published methods and studies, our Equations (4) and (6) provide for dramatically improved precision. We do not know of a previous study with much less than about 1 standard deviation below 1 km/s/Mpc. This breakthrough lies in a much deeper understanding of the relationship between the CMB temperature and the Hubble constant. **Table 1** displays Hubble constant values (H_0) estimated from a series of different CMB studies, but using our new high-precision method to determine H_0 while accounting for the full uncertainty in the input parameters.

Table 1. This table shows Hubble constant estimates using our new calculation method from several different CMB studies.

CMB Study	Temperature Measurement	High-Precision Method for H_0
Dhal *et al.* [31]	2.725007 ± 0.000024K	$H_0 = 66.8712 \pm 0.0019$ km/s/Mpc
Noterdaeme *et al.* [32]	2.725 ± 0.002K	$H_0 = 66.8708 \pm 0.0989$ km/s/Mpc
Fixsen *et al.* [33]	2.72548 ± 0.00057K	$H_0 = 66.8944 \pm 0.0287$ km/s/Mpc
Fixsen *et al.* [34]	2.721 ± 0.010K	$H_0 = 66.68 \pm 0.49$ km/s/Mpc

Figure 1 graphically illustrates the estimates provided in **Table 1**, along with error bars of 1 Standard Deviation (STD), using our new theoretical understanding

of the precise relationship between the Cosmic Microwave Background (CMB) temperature and H_0. The error bars in the most recent study by Dhal *et al.* [31] are so small that they are barely discernible on the graph, without significantly reducing the visibility of the observation points themselves. This is why we are confident enough to claim that this appears to be leading us into a new realm of high-precision cosmology. The improvement in precision is so dramatic that it is easy to think that it is too good to be true. We were initially skeptical as well, but have carefully retraced our steps, and it is clear that it is the newly established direct theoretical relationships between CMB temperature and the Hubble constant that make this possible. We naturally do not ask any researcher to take this for granted, but hope that more researchers will scrutinize this method carefully as well as determine its relevance with respect to the different cosmological models in the literature.

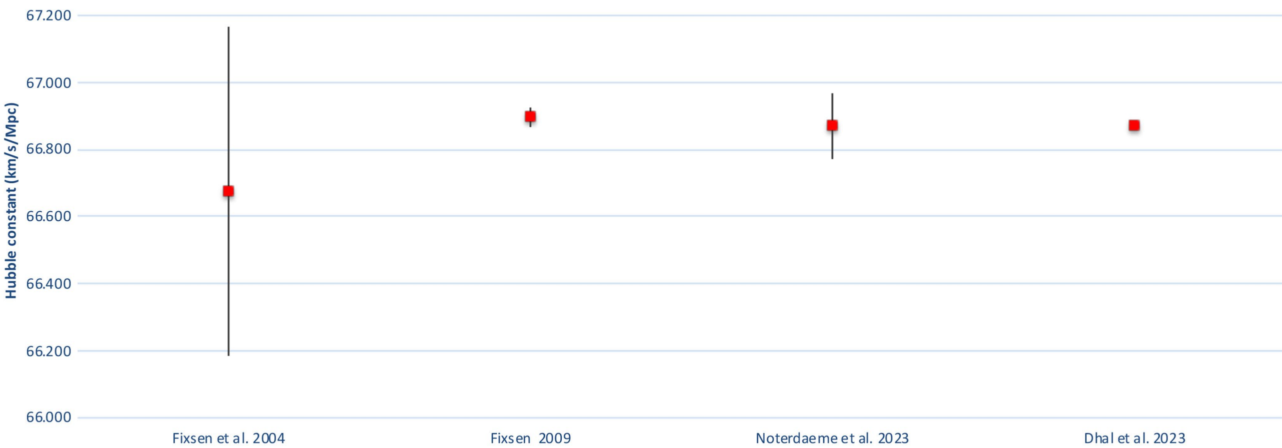

Figure 1. Hubble constant estimates from different CMB studies using new method.

An outstanding issue in relation to the Hubble constant is the Hubble tension, as discussed in, for example [27] [35] [36]. This tension results from markedly different Hubble constant measurements based on apparently conflicting early universe [28] and local universe [26] research studies. However, on the basis of the new theoretical relationship between the CMB temperature and H_0 introduced herein, we have recently published preprints [37] [38], presently in the journal submission stage, in which we claim to have solved the Hubble tension in favor of the Planck Collaboration Hubble constant value. We believe that the basis for this longstanding tension hinges upon using the correct distance-vs-redshift formula appropriate for the original Tatum *et al.*'s growing black hole $R_h = ct$ cosmological model. We now refer to the model that uses such a distance-vs-redshift formula as the Haug-Tatum cosmological model, which can be explored by our readers in our highly detailed preprints referenced above. In addition, because the Haug-Tatum model naturally implies a cosmic age significantly greater than 13.8 billion years, we also refer readers to preprints that give a cosmic age value of approximately 14.622 billion years [39] [40]. This significantly longer cosmic time frame may well have important implications with respect to better understanding the surprisingly "early" galaxy structure formation recently observed in the early universe.

3. High Precision Hubble Cosmology

Due to a significantly higher precision in the determination of the Hubble constant, we can now predict various cosmological parameters that employ the Hubble constant, such as the Hubble time and the Hubble radius, with much greater accuracy than before. The Hubble radius, denoted as R_{H_0}, is typically calculated using the formula $R_{H_0} = \frac{c}{H_0}$. Since there is no uncertainty in the speed of light c, the uncertainty in R_H is essentially the same as that in H_0. The Hubble time, defined as $t_h = \frac{1}{H_0}$, similarly benefits from the reduced uncertainty in H_0. In addition, because of the linear nature of $R_h = ct$ models, the concept of an accelerating dark energy is not relevant to the derivations of the key equations presented herein.

In the context of the Λ-CDM model, the critical mass, denoted as M_c, is calculated as $M_c = \frac{c^3}{2GH_0}$. Here, the uncertainty is slightly higher due to the additional factor of the gravitational constant G. Nonetheless, this method still provides significantly higher precision in such a model than any other approaches, thanks to the considerably reduced uncertainty in the Hubble constant value.

4. Conclusion

Any of our quantum cosmology formulae displayed in Section 1 can predict H_0 with much higher precision than before due to a breakthrough in understanding the CMB temperature in relation to H_0. Based on recent high-precision CMB temperature observations in combination with our new and deeper understanding of the relationship between CMB temperature and H_0, we obtain a 1 standard deviation uncertainty of no greater than ±0.49 km/s/Mpc, when using the 2004 data by Fixen *et al.* [34], to as low as one standard deviation of ±0.0019 km/s/Mpc from the 2023 data provided by Dhal *et al.* [31]. We claim that our formulaic method to find H_0 from precise CMB temperature observations is quite revolutionary and deserves attention from the research community. As a prime example of its potential value, we refer the reader to the above-mentioned Haug and Tatum's references, which provide a highly detailed analysis and apparent solution of the Hubble tension problem. Over time, the research community can either confirm our findings or point out possible weaknesses in our reasoning. So far, we have not identified any such weaknesses, despite searching for them. It indeed appears that the recent breakthrough in understanding the theoretical relationship between CMB temperature and H_0 offers significantly improved precision regarding the large-scale global parameters of the universe. However, a theory must undergo scrutiny by multiple researchers over time to demonstrate its robustness. Therefore, the first step must be to make our discoveries accessible. We sincerely hope that this publication will encourage more researchers to look into this fascinating relationship between CMB temperature and H_0.

Data Availability Statements

All data used in this article are properly referenced and incorporated into our table and figure, so that anyone can easily check our calculations ("predictions") in comparison to observations.

Acknowledgements

We sincerely thank Dr. Ratan Kumar Paul for suggesting to present the table information also as a whisker chart.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Tatum, E.T., Seshavatharam, U.V.S. and Lakshminarayana, S. (2015) The Basics of Flat Space Cosmology. *International Journal of Astronomy and Astrophysics*, **5**, 116-124. https://doi.org/10.4236/ijaa.2015.52015

[2] Friedman, A. (1922) Über die Krümmung des Raumes. *Zeitschrift für Physik*, **10**, 377-386. https://doi.org/10.1007/bf01332580

[3] Einstein, A. (1916) Die Grundlage der allgemeinen Relativitätstheorie. *Annalen der Physik*, **354**, 769-822. https://doi.org/10.1002/andp.19163540702

[4] Haug, E.G. and Wojnow, S. (2023) How to Predict the Temperature of the CMB Directly Using the Hubble Parameter and the Planck Scale Using the Stefan-Boltzman Law. Research Square. https://doi.org/10.21203/rs.3.rs-3576675/v1

[5] Haug, E.G. (2024) CMB, Hawking, Planck, and Hubble Scale Relations Consistent with Recent Quantization of General Relativity Theory. *International Journal of Theoretical Physics*, **63**, Article No. 57. https://doi.org/10.1007/s10773-024-05570-6

[6] Melia, F. and Shevchuk, A.S.H. (2011) The $r_h = ct$ Universe. *Monthly Notices of the Royal Astronomical Society*, **419**, 2579-2586. https://doi.org/10.1111/j.1365-2966.2011.19906.x

[7] Melia, F. (2013) The $r_h = ct$ Universe without Inflation. *Astronomy & Astrophysics*, **553**, A76. https://doi.org/10.1051/0004-6361/201220447

[8] Melia, F. (2016) The Linear Growth of Structure in the $r_h = ct$ Universe. *Monthly Notices of the Royal Astronomical Society*, **464**, 1966-1976. https://doi.org/10.1093/mnras/stw2493

[9] Tatum, E.T. and Seshavatharam, U.V.S. (2018) How a Realistic Linear $r_h = ct$ Model of Cosmology Could Present the Illusion of Late Cosmic Acceleration. *Journal of Modern Physics*, **9**, 1397-1403. https://doi.org/10.4236/jmp.2018.97084

[10] John, M.V. (2019) $r_h = ct$ and the Eternal Coasting Cosmological Model. *Monthly Notices of the Royal Astronomical Society*: *Letters*, **484**, L35-L37.

https://doi.org/10.1093/mnrasl/sly243

[11] John, M.V. and Joseph, K.B. (2000) Generalized Chen-Wu Type Cosmological Model. *Physical Review D*, **61**, Article ID: 087304. https://doi.org/10.1103/physrevd.61.087304

[12] Stuckey, W.M. (1994) The Observable Universe Inside a Black Hole. *American Journal of Physics*, **62**, 788-795. https://doi.org/10.1119/1.17460

[13] Popławski, N. (2016) Universe in a Black Hole in Einstein-Cartan Gravity. *The Astrophysical Journal*, **832**, Article No. 96. https://doi.org/10.3847/0004-637x/832/2/96

[14] Tatum, E.T. (2020) A Heuristic Model of the Evolving Universe Inspired by Hawking and Penrose. In: Tatum, E., Eds., *New Ideas Concerning Black Holes and the Universe*, IntechOpen, 5-21. https://doi.org/10.5772/intechopen.87019

[15] Akhavan, O. (2022) The Universe Creation by Electron Quantum Black Holes. *Acta Scientific Applied Physics*, **2**, 34-45. https://actascientific.com/ASAP/pdf/ASAP-02-0046.pdf

[16] Lineweaver, C.H. and Patel, V.M. (2023) All Objects and Some Questions. *American Journal of Physics*, **91**, 819-825. https://doi.org/10.1119/5.0150209

[17] Planck, M. (1899) Natuerliche Masseinheiten. Der Königlich Preussischen Akademie der Wissenschaften. https://www.biodiversitylibrary.org/item/93034#page/7/mode/1up

[18] Tatum, E.T. (2024) Upsilon Constants and Their Usefulness in Planck Scale Quantum Cosmology. *Journal of Modern Physics*, **15**, 167-173. https://doi.org/10.4236/jmp.2024.152007

[19] Lemaître, G. (1927) Un Univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques. *Annales de la Société Scientifique de Bruxelles*, **A47**, 49-59. https://adsabs.harvard.edu/full/1927ASSB...47...49L

[20] Hubble, E.P. (1926) Extragalactic Nebulae. *The Astrophysical Journal*, **64**, 321-369. https://doi.org/10.1086/143018

[21] Abbott, B.P., Abbott, R., Abbott, T.D., Acernese, F., Ackley, K., Adams, C., *et al.* (2017) A Gravitational-Wave Standard Siren Measurement of the Hubble Constant. *Nature*, **551**, 85-88. https://doi.org/10.1038/nature24471

[22] Hotokezaka, K., Nakar, E., Gottlieb, O., Nissanke, S., Masuda, K., Hallinan, G., *et al.* (2019) A Hubble Constant Measurement from Superluminal Motion of the Jet in Gw170817. *Nature Astronomy*, **3**, 940-944. https://doi.org/10.1038/s41550-019-0820-1

[23] Freedman, W.L., Madore, B.F., Hatt, D., Hoyt, T.J., Jang, I.S., Beaton, R.L., *et al.* (2019) The Carnegie-Chicago Hubble Program. VIII. An Independent Determination of the Hubble Constant Based on the Tip of the Red Giant Branch. *The Astrophysical Journal*, **882**, Article No. 34. https://doi.org/10.3847/1538-4357/ab2f73

[24] Gayathri, V., Healy, J., Lange, J., O'Brien, B., Szczepanczyk, M., Bartos, I., *et al.* (2021) Measuring the Hubble Constant with GW190521 as an Eccentric Black Hole Merger and Its Potential Electromagnetic Counterpart. *The Astrophysical Journal Letters*, **908**, L34. https://doi.org/10.3847/2041-8213/abe388

[25] Sedgwick, T.M., Collins, C.A., Baldry, I.K. and James, P.A. (2020) The Effects of Peculiar Velocities in SN Ia Environments on the Local H_0 Measurement. *Monthly Notices of the Royal Astronomical Society*, **500**, 3728-3742. https://doi.org/10.1093/mnras/staa3456

[26] Riess, A.G., Yuan, W., Macri, L.M., Scolnic, D., Brout, D., Casertano, S., *et al.* (2022)

A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 Km S⁻¹ Mpc⁻¹ Uncertainty from the Hubble Space Telescope and the SH0ES Team. *The Astrophysical Journal Letters*, **934**, L7. https://doi.org/10.3847/2041-8213/ac5c5b

[27] Di Valentino, E., Mena, O., Pan, S., Visinelli, L., Yang, W., Melchiorri, A., *et al.* (2021) In the Realm of the Hubble Tension—A Review of Solutions. *Classical and Quantum Gravity*, **38**, Article ID: 153001. https://doi.org/10.1088/1361-6382/ac086d

[28] Aghanim, N., Akrami, Y., Ashdown, M., Aumont, J., Baccigalupi, C., Ballardini, M., *et al.* (2021) *Planck* 2018 Results. *Astronomy & Astrophysics*, **652**, C4. https://doi.org/10.1051/0004-6361/201833910e

[29] Kelly, P.L., Rodney, S., Treu, T., Oguri, M., Chen, W., Zitrin, A., *et al.* (2023) Constraints on the Hubble Constant from Supernova Refsdal's Reappearance. *Science*, **380**, eabh1322. https://doi.org/10.1126/science.abh1322

[30] Balkenhol, L., Dutcher, D., Spurio Mancini, A., Doussot, A., Benabed, K., Galli, S., *et al.* (2023) Measurement of the CMB Temperature Power Spectrum and Constraints on Cosmology from the SPT-3G 2018 *TT*, *TE*, and *EE* Dataset. *Physical Review D*, **108**, Article ID: 023510. https://doi.org/10.1103/physrevd.108.023510

[31] Dhal, S., Singh, S., Konar, K. and Paul, R.K. (2023) Calculation of Cosmic Microwave Background Radiation Parameters Using COBE/FIRAS Dataset. *Experimental Astronomy*, **56**, 715-726. https://doi.org/10.1007/s10686-023-09904-w

[32] Noterdaeme, P., Petitjean, P., Srianand, R., Ledoux, C. and López, S. (2011) The Evolution of the Cosmic Microwave Background Temperature. *Astronomy & Astrophysics*, **526**, L7. https://doi.org/10.1051/0004-6361/201016140

[33] Fixsen, D.J. (2009) The Temperature of the Cosmic Microwave Background. *The Astrophysical Journal*, **707**, 916-920. https://doi.org/10.1088/0004-637x/707/2/916

[34] Fixsen, D.J., Kogut, A., Levin, S., Limon, M., Lubin, P., Mirel, P., *et al.* (2004) The Temperature of the Cosmic Microwave Background at 10 GHZ. *The Astrophysical Journal*, **612**, 86-95. https://doi.org/10.1086/421993

[35] Krishnan, C., Mohayaee, R., Colgáin, E.Ó., Sheikh-Jabbari, M.M. and Yin, L. (2021) Does Hubble Tension Signal a Breakdown in FLRW Cosmology? *Classical and Quantum Gravity*, **38**, Article ID: 184001. https://doi.org/10.1088/1361-6382/ac1a81

[36] Capozziello, S., Benetti, M. and Spallicci, A.D.A.M. (2020) Addressing the Cosmological H_0 Tension by the Heisenberg Uncertainty. *Foundations of Physics*, **50**, 893-899. https://doi.org//10.1007/s10701-020-00356-2

[37] Haug, E.G. and Tatum, E.T. (2024) Solving the Hubble Tension Using the Union2 Supernova Database. https://doi.org/10.20944/preprints202404.0421.v1

[38] Haug, E.G. and Tatum, E.T. (2024) Planck Length from Cosmological Redshifts Solves the Hubble Tension. https://hal.science/hal-04520966/document

[39] Tatum, E.T. and Haug, E.G. (2024) Extracting a Cosmic Age of 14.6 Billion Years from all 580 Type IA Supernova Redshifts in the Union2 Database. https://doi.org/10.13140/RG.2.2.28246.87360

[40] Haug, E.G. and Tatum, E.T. (2024) How a Thermodynamic Version of the Friedmann Equation Appears to Solve the Early Galaxy Formation Problem. https://doi.org/10.20944/preprints202404.0159.v1