Viscoelastic Property of Honeycomb Paperboard and Dynamic Response Analysis

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Abstract: The creep experiment and the quasi-static are carried out to learn the properties of honeycomb paperboard roughly, the creep and hysteresis phenomena are important manifestations of honeycomb paperboard viscoelastic properties. In addition to the static and quasi-static effects, the viscoelastic property also influences the dynamic properties of honeycomb paperboard. A linear differential equation is adopted to account for the complexity of honeycomb paperboard dynamic properties. Base on the Laplace transform, the model for the honeycomb paperboard properties is formulated. The relaxation kernel is used to account for the viscoelasticity, and it is expressed as the sum of complex exponentials. The free response of the mass loaded honeycomb paperboard is analyzed based on the Laplace transform and substitution method, the free response of the viscoelastic system have multiple vibration modals. The forced response of the mass loaded honeycomb paperboard to general force function is analyzed based on the state-space approach. The simulation results indicates the viscoelasticity can provide more stiffness and damping forces than the equivalent single degree of freedom system.

Keywords: honeycomb paperboard; viscoelasticity; free response; forced response

1. Introduction

Honeycomb paperboard is a kind of sandwich panel, in recent years, because of the environment protection concerns, it has been widely used in packaging as the cushion material. However, much of the packaging design is based on experience from previous designs without the benefit of a clear understanding of the properties of the honeycomb paperboard used in packaging. The packaging designers have a strong need to learn and characterize the quasi-static and dynamic properties of the honeycomb paperboard. As reported in [1-5], the researchers studied the honeycomb paperboard properties by experiments, the experiment results can be used in packaging designs, but the designers still need accurate models that can predict the behavior of honeycomb paperboard.

During the transportation, the motion of the packages varies in very short duration if excited by shock. It is necessary to learn the dynamic response of packages excited by the shock. In present work, a model is formulated to account for the viscoelastic property of honeycomb paperboard, the relaxation kernel is expressed as the sum of complex exponentials. The mass loaded honeycomb paperboard can be seen as a viscoelastic system. Methods are developed to analyze the free and forced response of mass loaded honeycomb paperboard system excited by shock respectively. The simulations indicate the free response of the viscoelastic system have multiple vibration modals, while in the forced response, the viscoelasticity can provide more stiffness and damping forces comparing with the equivalent spring-dashpot system.

2. Viscoelastic Model

Viscoelastic property is an important aspect of the dynamic behavior of honeycomb paperboard. Because of the specific structure, honeycomb paperboard has a special mechanical behavior. In contrast with many engineering materials whose mechanical behavior depends mainly on the value of the applied stress or strain. The response of honeycomb paperboard is significantly influenced by the rate of loading. Roughly speaking, honeycomb paperboard exhibit a combination of elastic and viscous behavior. This property is usually called viscoelastic property. The creep and hysteresis phenomena under quasi-static load are important manifestations of the viscoelasticity.

If a honeycomb paperboard sample is loaded with a given mass, the compressive strain increases over time from the initial value. A typical static creep measurement for the honeycomb paperboard used in this study is shown in Fig. 1. A constant loading 50kg was applied to the 25cm×25cm×4cm honeycomb paperboard specimen, and the resulting compression displacement was measured over a period of 96 hours. Although this creep behavior is similar to plasticity, the honeycomb paperboard sample recover to its undeformed state once the loading

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is recovered, excluding possible extreme loading conditions.

On the other hand, in the quasi-static experiment, the honeycomb paperboard sample is compressed very slowly(1mm/min), and then unloaded slowly, the schematic of the experiment set-up and the imposed deformation are shown in Fig. 2 and force-deformation curve is shown in Fig. 3, it is easy to see that in the deformation cycle, the response force in the unloading phase is less than that in the loading phase. This behavior has been investigated in detail in previous research and is considered as manifestation of the viscoelastic memory-like behavior[6,7].

In addition to these static and quasi-static effects, viscoelasticity also influences the behavior of honeycomb paperboard under dynamic conditions. When subjected to a dynamic excitation, a mass loaded honeycomb paperboard system is seen to display additional dynamic creep beyond its static equilibrium. It has also been shown that the stiffness and damping characteristics of honeycomb paperboard undergoing forced vibration depend on the length of time that the material has been exercised.

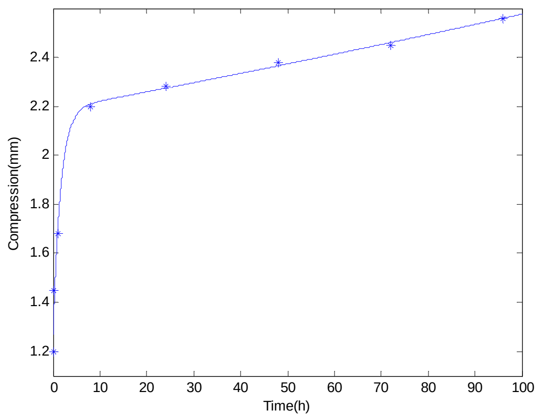

Fig. 1 Typical static creep under constant loading

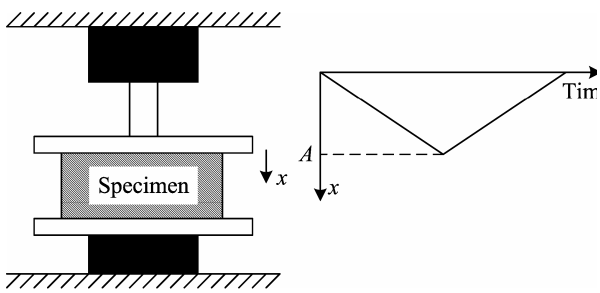

Figure 2. Schematic of the experiment set-up and imposed deofrmation

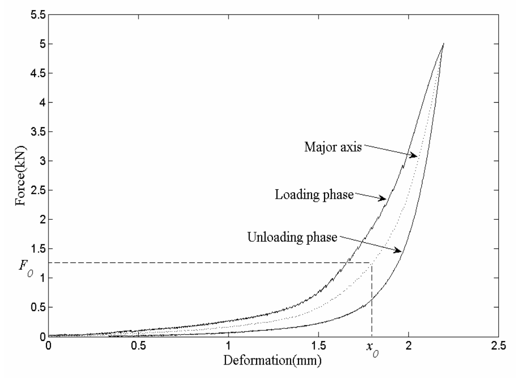

Fig. 3 The Force-Deformation curve of the quasi-static experiment

There are several approaches to modeling the viscoelastic nature of honeycomb paperboard. The early attempts of describing the viscoelastic effects consisted in the use of simple mechanical models which exhibit superposed elastic and viscous behavior. Such mechanical models form the basis of the superposition principle and lead to exponential laws for the description of relaxation and creep. These models include Maxwell model, Voigt model, Kelvin model etc, which are shown in Fig. 4. Different models of viscoelastic material are discussed in [8]. These models are rather restrictive and they are incapable of describing the mechanical behavior of real viscoelastic materials. The constitutive relations of the models shown in Fig. 4 can be expressed as differential equation of the stress and strain, a further generalization of these models is the following linear differential equation[9]

$$a_0\sigma + a_1\frac{d\sigma}{dt} + \cdots + a_M\frac{d^M\sigma}{dt^M} = b_0\varepsilon + b_1\frac{d\varepsilon}{dt} + \cdots + b_N\frac{d^N\varepsilon}{dt^N} \tag{1}$$

Assuming that at t=0, the material was unstressed and undeformed, we take the Laplace transformation of (1), the following equation can be obtained:

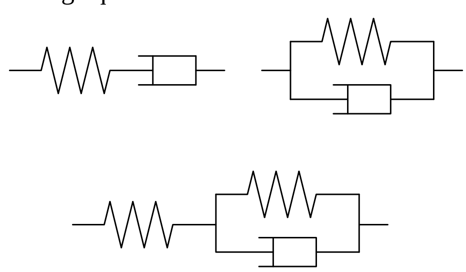

Fig. 4 Elementary models for viscoelasticity

$$\bar{\sigma} = \frac{b_0 + b_1 s + \cdots + b_N s^N}{a_0 + a_1 s + \cdots + a_M s^M}\bar{\varepsilon} \tag{2}$$

The overbar denotes the counterparts in Laplace domain.

Assuming that M=N, (2) can be represented as a sum of fractions, i.e.,

$$\bar{\sigma} = E(1 - \sum_{i=1}^{N} \frac{a_i}{s + \alpha_i})\bar{\varepsilon} \tag{3}$$

Where E is the instantaneous Young's modulus of the material. Taking the inverse Laplace transform of (3), one can obtain:

$$\sigma = E[\varepsilon - \int_0^t \Gamma(t-\tau)\varepsilon(\tau)d\tau] \tag{4}$$

The weight function $\Gamma(t\text{-}\tau)$ is called the relaxation kernel, and it is the sum of exponentials

$$\Gamma(t-\tau) = \sum_{i=1}^{N} a_i \exp[-\alpha_i(t-\tau)]$$

Where a_i and α_i are real or complex conjugates.

In the packages, the product and honeycomb paperboard forms a mass loaded viscoelastic system, a linear viscous damping term is included in the model to account for the viscous losses in the material, according to (4), the restoring force in the honeycomb paperboard is assumed to possess the following form:

$$F = k[x - \int_0^t \Gamma(t-\tau)x(\tau)d\tau]$$

And the equation of motion of the mass loaded viscoelastic system can be expressed as:

$$m\ddot{x} + c\dot{x} + kx - k\int_0^t \sum_{i=1}^{N} a_i e^{-\alpha_i(t-\tau)} x(\tau)d\tau = f(t) \tag{5}$$

Where m is the mass of the system, k is the stiffness coefficient, c is the viscous damping coefficient, a_i and α_i, i=1,2,…N are the N viscoelastic parameters, $f(t)$ is the force exerted to the mass.

3. Free Response of Viscoelastic System

At free response condition, in (5), $f(t)$=0, the motion equation of the system is

$$m\ddot{x} + c\dot{x} + kx - k\int_0^t \sum_{i=1}^{N} a_i e^{-\alpha_i(t-\tau)} x(\tau)d\tau = 0 \tag{6}$$

the initial conditions are assumed to be:

$$x(0)=x_0 \quad \dot{x}(0) = \dot{x}_0$$

applying the Laplace transform to (5), one obtains:

$$\bar{x} = \frac{m(\dot{x}(0) + sx(0)) + cx(0)}{s^2 m + sc + k(1 - \sum_{i=1}^{N} \frac{a_i}{s + \alpha_i})} = \frac{R(s)}{T(s)}$$

Knowing the roots p_i of the denominator, and assuming they are distinct, one can obtain the solution to (6) in the form[10]

$$x(t) = \sum_{i=1}^{N+2} \frac{R(p_i)}{T'(p_i)} e^{p_i t} = \sum_{i=1}^{N+2} C_i e^{p_i t} \tag{7}$$

Equation (7) is substituted into (6), a parameters identification procedure is formulated in [5,11] based on the Prony method, where the identification results indicated N=2 can account for the viscoelastic effects well, the stiffness coefficient k, the damping coefficients c, the viscoelastic parameters a_i and α_i(i=1,2) are function of the weight of mass.

Substituting (7) into (6), one can obtain:

$$mp_j^2 + cp_j + k - k\sum_{i=1}^{N} \frac{a_i}{p_j + \alpha_i} = 0 \qquad j = 1,2,\cdots,N+2 \tag{8}$$

The initial conditions

$$\sum_{i=1}^{4} C_i = x(0) \;, \quad \sum_{i=1}^{4} C_i p_i = \dot{x}(0) \tag{9}$$

Assuming $a_{1,2}$=a_r±ja_i, $\alpha_{1,2}$=α_r±jα_i, one can rewrite (8) as

$$A_4 p^4 + A_3 p^3 + A_2 p^2 + A_1 p + A_0 = 0 \tag{10}$$

Where

$A_4 = m$, $A_3 = 2m\alpha_r + c$, $A_2 = m|\alpha|^2 + 2c\alpha_r + k$,

$A_1 = c|\alpha|^2 + 2k\alpha_r - 2ka_r$, $A_0 = k(|\alpha|^2 - 2a_r\alpha_r + 2a_i\alpha_i)$

Solve (10), p_i(i=1,2,3,4) can be found, C_i can be obtained based on the relations in (9) and identified p_i. Substituting p_i and C_i into (7), we can obtain the free response of the viscoelastic system.

Once knowing the stiffness coefficient k, the damping coefficient c and the viscoelastic parameters a_i and α_i in (5), one can find the free response of the mass loaded honeycomb paperboard system by implementing the method outlined above. To verify this approach, a free response experiment is carried out. A honeycomb paperboard sample is loaded and compressed to a set value x_0, then the outer load is removed suddenly, the free response of the mass loaded honeycomb paperboard system is recorded, this free response is also simulated by the approach outlined above, the parameters in (9) and (10) have been identified in [5,11]. The experiment data and the simulation results in Fig 5 have a good agreement.

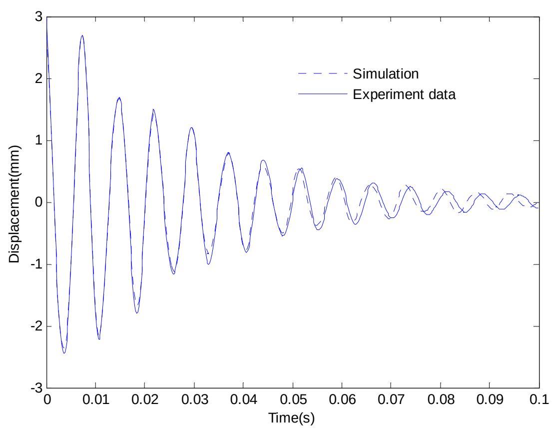

Fig. 5 Free response of the viscoelastic system

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4. Forced Response of Viscoelastic System

The forced response of the viscoelastic system should satisfy the equation of motion with forcing function (5) but with zero initial conditions, taking the Laplace transform, we obtain:

$$(P_4 s^4 + P_3 s^3 + P_2 s^2 + P_1 s + P_0)\bar{x} = (s^2 + d_1 s + d_0)\bar{f} \qquad (11)$$

where

$P_4 = m$, $P_3 = 2m\alpha_r + c$, $P_2 = m|\alpha|^2 + 2c\alpha_r + k$,

$P_1 = c|\alpha|^2 + 2k\alpha_r - 2ka_r$, $P_0 = k(|\alpha|^2 - 2a_r\alpha_r + 2a_i\alpha_i)$,

$d_1 = 2\alpha_r$, $d_0 = |\alpha|^2$.

The basic idea of the approach for solving the viscoelastic system is to find a first-order state-space model that is equivalent to its Laplace domain equation (11). One can then use the well-developed state-space algorithms, such as the modal analysis, modal expansion method or direct integration method to solve for the forced response of the viscoelastic system expressed by (11).

Let's consider the following system,

$$P_4 \frac{d^4 y}{dt^4} + P_3 s \frac{d^3 y}{dt^3} + P_2 \frac{d^2 y}{dt^2} + P_1 \frac{dy}{dt} + P_0 y = f \qquad (12)$$

with zero initial conditions, i.e.,

$y(0)=y'(0)=y''(0)=y^{(3)}(0)=0$

we may conclude that the forced response of the original viscoelastic system with zero initial conditions can be given as

$$x = d_0 y + d_1 \dot{y} + \ddot{y} \qquad (13)$$

This can be verified by taking Laplace transform of (12) and (13),

$$\bar{x} = \frac{s^2 + d_1 s + d_0}{P_4 s^4 + P_3 s^3 + P_2 s^2 + P_1 s + P_0}\bar{f} \qquad (14)$$

Equation (14) is exactly the same as (11). This indicates the forced response of the viscoelastic system can be expressed as linear combinations of the state variables of an auxiliary dynamic system described by (12). Equations (12) and (13) can then be cast into the state-space format,

$$\dot{\mathbf{Y}} = \mathbf{AY} + \mathbf{B}f$$
$$\dot{\mathbf{Y}}(0) = \mathbf{0}$$
$$x = \mathbf{CY}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -P_0/P_4 & -P_1/P_4 & -P_2/P_4 & -P_3/P_4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/P_4 \end{bmatrix},$$

$$\mathbf{Y} = \begin{bmatrix} y \\ y' \\ y'' \\ y^{(3)} \end{bmatrix}, \mathbf{C}=[d_0, d_1, 1, 0]$$

By the state-space approach, the forced response of the mass loaded honeycomb paperboard system with viscoelastic property can be simulated. Assuming the mass loaded honeycomb paperboard system is excited by a rectangle impulse and a half-sine impulse. The duration of the impulse is 0.02 seconds, the amplitude of the impulse F is 500N. Based on the earlier experimental results[11], the parameters in (5) are chosen to be:

N=2, m=12kg, k=9.32×10^{6}N/m, c=712Ns/m, $a_{1,2}$=-217±105i,$\alpha_{1,2}$=204±1532i.

Applying the state-space approach outlined above to the forced response problem, one can simulate the response to the rectangle impulse in Fig. 6(a), where x denotes the displacement of honeycomb paperboard specimen, $x_{st}=mg/k$. The forced response of the mass loaded honeycomb paperboard system to the half-sine impulse is also shown in Fig. 6(b). The forced response of equivalent single degree of freedom system is also displayed in Fig. 6 for comparison.

From Fig. 6, the response of the viscoelastic system to a rectangle and half-sine impulse is quite different from

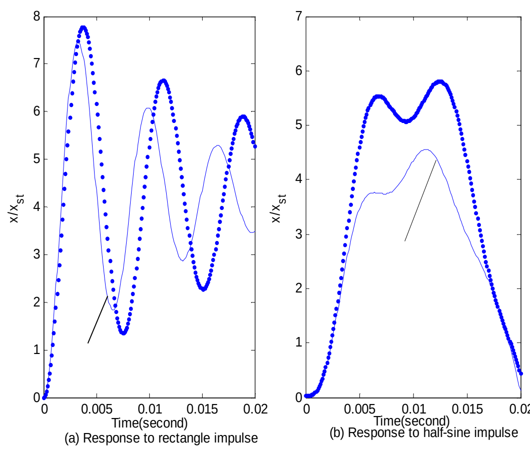

Fig. 6. response to the impulse

that of the equivalent single degree of freedom system. If we don't take the viscoelastic property into account, both the stiffness coefficient and the damping coefficient will be underestimated. In other words, the viscoelastic property can provide extra stiffness and damping force in honeycomb paperboard.

5. Summary

In this paper, to learn the property of honeycomb paperboard roughly, the creep experiment and the quasi-static experiment are carried out. The creep and hysteresis phenomena are manifestation of viscoelastic property. The viscoelastic model is performed based on the linear differential equation, the relaxation kernel is used in the motion equation to account for the viscoelastic property of honeycomb paperboard. The relaxation kernel can be expressed as the sum of complex exponentials. The free response of the mass loaded honeycomb paperboard is analyzed based on the Laplace transform and the substitution method, while the forced response of the system is analyzed by the use of state-space approach. From the analysis, because of the viscoelastic property, in the free response, there are multiple modals, while in the forced response, comparing with the equivalent single degree of freedom system, the viscoelastic property in the mass loaded honeycomb paperboard system can provide more stiffness and damping force.

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