Honeycomb Paperboard Vibration Transmissibility Analysis and Modeling

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Abstract: The realization of the potential of the paper honeycomb sandwich panel as an important packaging material has inspired a close scrutiny of its properties. It is well known the mechanical properties of honeycomb paperboard exhibit significant frequency dependence and nonlinear dynamic behavior. Nonlinear characteristics in deformation are reflected in mechanical properties such as stiffness and damping. In present work, the treatment involves developing phenomenological models of stiffness and damping properties as a function of excitation amplitude. It is found that as the excitation amplitude increases, the response amplitude decreases and the resonance frequency shifts to the left. The simulation results indicates the model in this paper can predict the vibration transmissibility of honeycomb paperboard well.

Keywords: honeycomb paperboard; transmissibility; phenomenological model

1. Introduction

Honeycomb paperboard is a kind of paper sandwiched panel. It's made up of three parts: the upper and lower face sheets between which is the honeycomb core, which are all made of reusable paper. Because of its specific structure, honeycomb paperboard has many advantages over other cushion material, such as the light weight, high strength-to-weight ratio, the ease to process and recycle. It has widely been used in agriculture, architecture and manufacture. In resent years, because of the environmental protection concerns and the command for eliminating white pollution, honeycomb paperboard has been used in packaging as the cushion material to substitute the foam, especially in the packages for the electric appliance such as the washing machine and refrigerator. The realization of the potential of honeycomb paperboard as an important cushion material has inspired a close scrutiny of its properties.

In [1], the shock absorbing characteristics and vibration transmissibility of honeycomb paperboard have been studied through the experiments, dynamic cushion curves and vibration transmissibility data are obtained to provide the design basis for the protective packaging. But in the shock test, the honeycomb core is crushed by the shock of the dropping head, which is not permitted in the transportation packaging. Hidetoshi Kobayashi et al. [2] studied the effect of loading rate on the strength and absorbed energy of paper honeycomb cores through the quasi-static and dynamic compression experiments. The critical buckling load of honeycomb paperboard under out-of-plane pressure is investigated by analyzing the structure and the collapse mechanism [3], the models and the calculation method in the paper can be used to predict the static critic buckling load. Dongmei Wang and Zhiwei Wang [4] investigated the cushioning properties by means of the experiment, the effects of the relative density, the thickness of honeycomb paperboard and liner on the cushioning properties have been discussed in detail.

In the packaging design, the honeycomb paperboard dynamic properties excited by vibration is important for packaging designs. The vibration transmissibility curve is useful for the design. In this paper, a series of vibration experiments are carried out to learn the vibration transmissibility of honeycomb paperboard under different excitation amplitudes. According to the experiment results, a phenomenological model is formulated, the nonlinearity of honeycomb paperboard is taken into account in this model, this model is analyzed based on the harmonic balance method. This model can be used to predict the vibration transmissibility of the honeycomb paperboard.

2. Experimental Investigation

Vibration transmissibility curve of honeycomb paperboard is useful for packaging design. It has been observed in [5] that the restoring force is nonlinear function of displacement under quasi-static load condition. Because of the small compression under the shock, the nonlinearity of honeycomb paperboard is not taken into account, the honeycomb paperboard is modeled as a linear material. While in vibration condition, the nonlinearity can not be ignored, because the displacement of honeycomb paperboard is considerable large if the packaging system is excited at the resonance frequency.

In this paper, the vibration transmissibility is meas-

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ured by the forced resonance tests. The experiment set-up is shown schematically in Fig. 1. The honeycomb paperboard specimen is sandwiched between the mass and the vibration exciter, the mass and the honeycomb paperboard are used to simulate the packaging system, the upper and lower surfaces of the specimens are glued to the mass and the vibration exciter respectively in case they may lose contact with each other. Two accelerometers are used to measure the acceleration of the vibration excitation $\ddot{y}$ and the system response $\ddot{x}$ respectively. A compressible displacement transducer is sandwiched between the mass and the vibration exciter to measure the honeycomb paperboard specimen compression directly. The data acquisition system involves three parts: the low-pass anti-aliasing filter, charge amplifier and the dynamic data acquisition equipment. The low-pass filter is used to reduce the noise in the signals, the charge amplifier is used to transform the charge signal from the piezoelectric accelerometers into voltage signal. A dynamic data acquisition equipment is used to transform the voltage signal from charge amplifier into digital data, and transmit the data to the computer. All the equipments above are provided by Sinocera Piezotronics company. All the data collection process in this work was under the control of the YE7600 software package.

As with any nonlinear system, the control of input amplitude and frequency is of critical importance to the measured results. The commonly used techniques in linear vibration analysis of using random input or continuous frequency sweep are not used in present work. Instead, the input signal to the system is stepped through a set of discrete frequencies. First, the amplitude of the excitation $|\ddot{x}|$ and the response $|\ddot{y}|$ are recorded, the ratio $T_r=|\ddot{x}|/|\ddot{y}|$ is defined as the transmissibility under this frequency. The input frequency is then slowly increased to the next value and the transmissibility of the frequency is recorded when the system come the steady state. Repeat this process, one can obtain the vibration transmissibility versus excitation frequency curve. In regions sufficiently far away from resonance, the step in

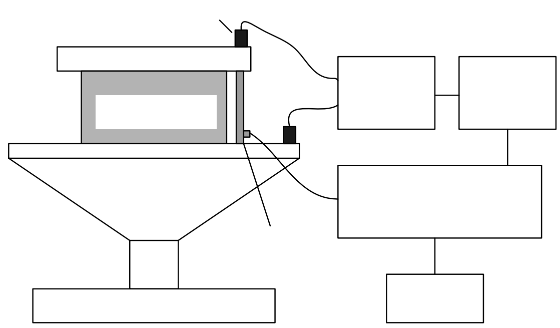

Figure 1. Forced response vibration experiment set-up

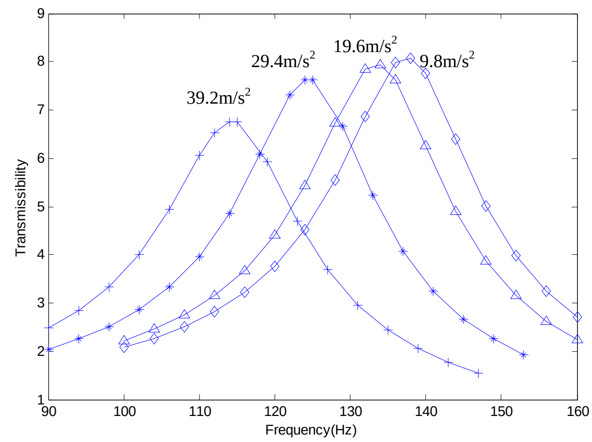

Figure 2. Transmissibility under different excitation amplitude

frequency is 4Hz. However, in regions where the response magnitude is seen to change rapidly with excitation frequency, the frequency step is reduced to 2 Hz or 1Hz in order to accurately capture the response. This measurement is repeated under different excitation altitude varied from 9.8m/s^2 to 39.2m/s^2. The inertia of the mass in the experiment is 15kg.
From Fig. 2, the transmissibility curves under different excitation altitude level are quite different, the transmissibility measurements show that as the excitation magnitude increases, the stiffness decreases, while the damping increases. The resonance peaks shift to the left and the peak magnitudes are reduced. The compression of the honeycomb paperboard specimen at the resonance frequency is also measured directly by the displacement transducer, the resonance frequency, transmissibility and the honeycomb paperboard compression at resonance frequency under different excitation condition is shown in table 1.

The stiffness coefficient k of the mass loaded honeycomb paperboard system can be obtained by $k=m\omega_n^2$, where ω_n is the resonance frequency. The stiffness force F_k, can be estimated by $F_k=kd$, where d is the compression of the honeycomb paperboard specimen. The stiffness force F_k versus the compression d is shown in Fig. 3(a). From the figure, the stiffness force is obviously a nonlinear

Table 1. results measured at resonance frequency

Excitation amplitude	***Resonance frequency***	***transmissibility***	***compression***
9.8m/s^2	138Hz	8.08	0.082mm
19.6m/s^2	134 Hz	7.94	0.192mm
29.4m/s^2	124 Hz	7.62	0.308mm
39.2m/s^2	114 Hz	6.76	0.391mm

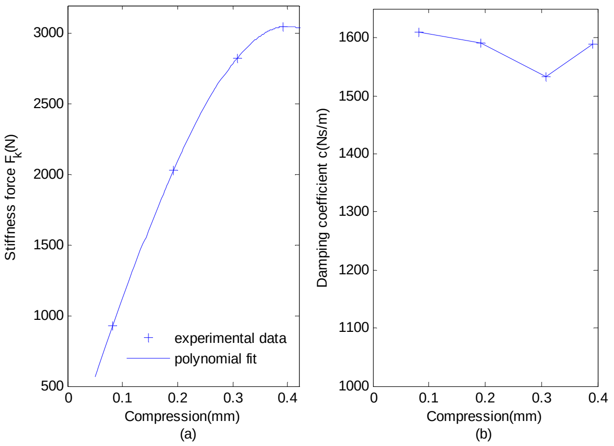

Figure 3. stiffness and damping coefficient versus the compression

function of the compression, the nonlinear stiffness force can be expressed by a polynomial of compression. Because F_k is usually odd function of compression d, in the polynomial, only odd terms is presented. In this paper, the coefficients in the polynomial is identified by the least-square principle, using the measured data in table 1. This polynomial is

$$F_k=1.14\times10^7 d-2.38\times10^{13} d^3$$

where d is the compression of the specimen in m.

At the resonance frequency, the damping ratio of the mass loaded honeycomb paperboard system can be estimated approximately by

$$\xi \approx \frac{1}{2T_r}$$

The damping coefficient can be estimated by

$$c=2m\xi\omega_n$$

the damping coefficient versus the compression is shown in Fig. 3(b). From the figure, the damping coefficient can be considered as a constant. $c\approx1580.9$Ns/m.

From the results discussed above, the vibration transmissibility of honeycomb paperboard-mass system can be modeled as

$$m\ddot{z}+c\dot{z}+kz+k_3z^3=f(t) \tag{1}$$

Where $f(t)$ is the external force exerted onto the system, z is the compression of the specimen in mm. In (1), $c\approx$ 1580.9Ns/m, $k=1.14\times10^7$N/m, $k_3=-2.38\times10^{13}$N/m^3.

From (1), we can learn that the resonance frequency and resonance peak under different excitation amplitude is quite different, and this is caused by the stiffness nonlinearity, the damping coefficient can be considered approximately as a constant. The vibration transmissibility model in (1) can be used to predict the vibration properties of honeycomb paperboard-mass system.

3. Model analysis

If the system represented by (1) is excited by a harmonic input, the approximate periodic solutions can be constructed by using the method of harmonic balance[6,7]. In this method, it is assumed that the system has a periodic solution of appropriate period. The Fourier series coefficients for the approximate periodic solution are obtained by the usual method of substituting the solution into (1) and equating coefficients of like terms.

In (1), we assume $f(t)=G\cos(\omega t)$. Since the motion equation only contains cubic order nonlinearity, the periodic response is expected to contain only odd harmonics of the driving frequency. It is assumed that, at the levels of excitation considered, the first harmonics are more dominant than the higher harmonics. Hence, the harmonic balance solution to an excitation $f(t)=G\cos(\omega t)$ is

$$z(t)=Ae^{i\omega t}+\bar{A}e^{-i\omega t} \tag{2}$$

where $A=A_r+iA_i$ is the complex Fourier coefficient, and $\bar{A}$ is complex conjugate of A.

When (2) is substituted into (1), and the coefficients of $e^{i\omega t}$ are equated, it gives

$$-m\omega^2A+ic\omega A+kA+3k_3A^2\bar{A}-G/2=0 \tag{3}$$

By multiplying (3) by $\bar{A}$ and splitting the resulting equation into its real and imaginary components, the following two equation are generated:

$$-GA_r/2+(k-m\omega^2)|A|^2+3k_3|A|^4=0 \tag{4a}$$

$$-GA_i/2-c\omega|A|^2=0 \tag{4b}$$

Where $|A|$ denotes the absolute value of A. The equation for the amplitude of A can be obtained by squaring and adding (4a) and (4b), which yields

$$\begin{aligned}&9k_3^2|A|^6+6k_3(k-m\omega^2)|A|^4\\&+[(k-m\omega^2)^2+c^2\omega^2]|A|^2=m^2G^2/4\end{aligned} \tag{5}$$

This is a 3rd-order polynomial equation in the square of amplitude of response, $|A|^2$. Because negative roots for $|A|$ are meaningless and zero is not a root, there are at most 3 real roots for the amplitude of A at any given excitation frequency.

Once the $|A|$ is obtained, the phase $\angle A=\tan^{-1}(A_i/A_r)$ at the input frequency can be obtained from:

$$\angle A=\tan^{-1}\frac{-c\omega}{-m\omega+k+3k_3|A|^2} \tag{6}$$

From (5) and (6), if the acceleration altitude of the harmonic input G is known, the response of the mass loaded honeycomb paperboard system can be expressed as:

$$x=z+y=2|A|\cos(\omega t+\angle A)-\frac{G}{\omega^2}\cos\omega t$$

The transmissibility of the system can be estimated by

$$T_r=\frac{|\ddot{x}|}{|\ddot{y}|}=\frac{|x|}{|y|} \tag{7}$$

From (7), the transmissibility curves of the mass loaded honeycomb paperboard system under different excitation altitudes can be simulated in Fig. 4, the experimental data are also shown in this for comparison.

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From Fig. 4, the simulated transmissibility curves have good agreement with the experimental data at the low excitation altitude. If the excitation altitude is high(39.2m/s^2 or 29.4m/s^2), the simulated transmissibility curve peak is about 10% higher than the experimental transmissibility peak. There are several possible explanations for the discrepancies between the measured and predicted altitudes.

1). The viscoelastic property of honeycomb paperboard is an important aspect of dynamic properties, especially on the high load condition. The viscoelasticity influences the behavior of material under dynamic conditions. When subjected to a dynamic excitation, a mass loaded material is seen to display additional dynamic creep beyond its static equilibrium. From the comparison of the experimental data with the simulated curve, the viscoelasticity can provide additional damping in the system. The model for viscoelasticity should be added to the model, and this is a topic for the further research.

2). The nonlinearity not only exists in the stiffness property, it may also affect the damping property. The nonlinearity of the damping can be detected and identified by the Hilbert transform-based technology[8,9].

3). The response of the system is calculated by the method based on the harmonic balance approach, in this method, we assume only the first harmonic is more dominant, while the higher harmonics of the response are ignore, which make the results inaccurate. The third harmonic and the higher harmonics should be taken into account to make the simulation more accurate.

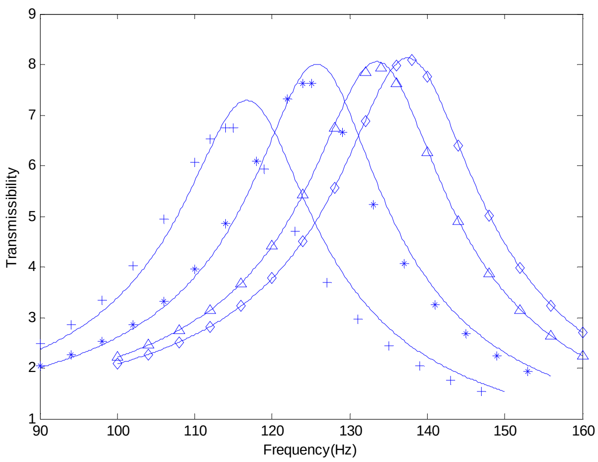

Figure 4. Simulated transmissibility and experimental data

4. Summary

Honeycomb paperboard has become a kind of important cushion material in recent years. Its potential as packaging material has inspired many research on its property testing and modeling. In present work, the vibration transmissibility experiments are carried out under different excitation altitude. The experimental data indicates the transmissibility curves are quite different under different excitation altitudes. With the increase of the excitation altitude, the resonance peak shift to the left and the resonance peaks decrease. The stiffness coefficient is model as nonlinear function of the compression, while the damping coefficient is considered as a constant. The motion equation is founded and the dynamic property of honeycomb paperboard is modeled. The model is analyzed by the harmonic balance approach, the simulation and the experimental data show good agreement under low excitation amplitude condition, but at the high excitation amplitude condition, the simulated transmissibility curve is about 10% higher than the experimental data. The possible reason is discussed. The model obtained in this paper can be used to predict the vibration transmissibility curve of honeycomb paperboard, and provide some data for the packaging design.

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