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The Solution of the Einstein's Equations in the Vacuum Region Surrounding a Spherically Symmetric Mass Distribution

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Abstract

In this article, we address the solution of the Einstein's equations in the vacuum region surrounding a spherically symmetric mass distribution. There are two different types of mathematical solutions, depending on the value of a constant of integration. These two types of solutions are analysed from a physical point of view. The comparison with the linear theory limit is also considered. This leads to a new solution, different from the well known one. If one considers the observational data in the weak field limit this new solution is in agreement with the available data. While the traditional Schwarzschild solution is characterized by a horizon at $r = 2GM/c^2$, no horizon exists in this new solution.

Keywords

General Theory of Relativity, Schwarzschild Solution, Event Horizon, Black Hole

1. The First Part That Is Equivalent to the Commonly Accepted Treatment

We will first recall the treatment leading to the traditional Schwarzschild solution [1]-[3] and will use the formulas with $c \equiv 1$.

We solve the Einstein's equations in the vacuum region surrounding a spherically symmetric mass distribution.

Now, the corresponding gravitational field must also have spherical symmetry [3]. Since it is of interest to deal with a collapsing spherical mass, that is, a sphere that shrinks in size, we begin by assuming a space-time metric that includes a time dependence for the components of the metric tensor [3]. At the end of our calculation one finds that in the above vacuum region this time dependence disappears: as in the case of the Newtonian gravitational potential, the exterior solution for the metric tensor of a spherical mass does not depend on the size of the mass and remains static even when the mass collapses, due to Birkhoff's theorem [3].

The spherical symmetry of the gravitational field imposes severe restrictions on the form of the space-time interval. The rectangular displacements $\mathrm{d}x, \mathrm{d}y$, $\mathrm{d}z$ must occur in the combination $\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$ characteristic of spherical symmetry; this combination takes the familiar form $\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2$ when is expressed in spherical polar coordinates [3]. Furthermore, although the product $\mathrm{d}r\mathrm{d}t$ is consistent with spherical symmetry, the products $\mathrm{d}\theta\mathrm{d}t$ and $\mathrm{d}\varphi\mathrm{d}t$ are not consistent with spherical symmetry, because they are different when moving in the direction of increasing or decreasing θ and φ [3]. Accordingly, the space-time interval must be of the form [3]:

$$\mathrm{d}s^2 = A(r,t)\mathrm{d}t^2 - B(r,t)\left[\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2\right] - 2F(r,t)\mathrm{d}r\mathrm{d}t \tag{1}$$

where $A(r,t)$, $B(r,t)$ and $F(r,t)$ are some functions of the radial coordinate r and the time coordinate t.

As H. C. Ohanian and R. Ruffini note [3], the angular coordinates θ and φ are unambiguous; the measurement of these coordinates depends only on our ability to divide a circumference concentric with the mass into equal parts, which we can do even when the functions $A(r,t)$, $B(r,t)$ and $F(r,t)$ are not specified. The radial coordinate r is ambiguous because we do not yet know its precise relation to the measurement of distance. For a start, as [3], we will treat r simply as a parameter that identifies different spherical surfaces concentric with the mass. Obviously r is always greater than zero since we are looking for a solution for vacuum region surrounding a spherically symmetric mass distribution. The time coordinate t suffers from similar ambiguities. However, we, as [3], will insist that when $r \to +\infty$ and the space-time becomes flat, the increments in r and t should equal the increments in the true distance and time, respectively. This requires that for $r \to +\infty$, $A(r,t)$ and $B(r,t) \to 1$.

The term involving $\mathrm{d}r\mathrm{d}t$ in the Equation (1) can be eliminated by a change in the time coordinate. We, as [3], introduce a new time coordinate $\tilde{t}$ such that

$$\mathrm{d}\tilde{t} = (A\mathrm{d}t - F\mathrm{d}r)Q(r,t) \tag{2}$$

where $Q(r,t)$ is a function of r and t that is to be chosen so as to make the right side of the Equation (2) into a perfect differential [this requires that Q satisfy the following differential equation: $(\partial/\partial t)(FQ) = -(\partial/\partial r)(AQ)$].

From the Equation (2) we obtain [3]:

$$A\mathrm{d}t^2 - 2F\mathrm{d}r\mathrm{d}t = \frac{1}{Q^2A}\mathrm{d}\tilde{t}^2 - \frac{F^2}{A}\mathrm{d}r^2 \tag{3}$$

which gives:

$$\mathrm{d}s^2 = \frac{1}{Q^2 A}\mathrm{d}\tilde{t}^2 - \left(B + \frac{F^2}{A}\right)\mathrm{d}r^2 - B\left[r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2\right] \tag{4}$$

We can note that we have obtained a formula equivalent to the traditional one for a spherically symmetric space-time, as we can see in H. Takeno [4].

We, as [3], can simplify the Equation (4) further by introducing a new radial coordinate $\tilde{r} = r\sqrt{B(r)}$. This has the advantage that the terms involving angular displacements reduce to $\tilde{r}^2\mathrm{d}\theta^2 + \tilde{r}^2\sin^2\theta\mathrm{d}\varphi^2$, so we obtain, as [3]:

$$\mathrm{d}s^2 = \frac{1}{Q^2 A}\mathrm{d}\tilde{t}^2 - \left(B + \frac{F^2}{A}\right)\left(\frac{\partial r}{\partial \tilde{r}}\right)^2\mathrm{d}\tilde{r}^2 - \tilde{r}^2\mathrm{d}\theta^2 - \tilde{r}^2\sin^2\theta\mathrm{d}\varphi^2 \tag{5}$$

For the solution of the Einstein's equations, it is convenient, following [3], to omit the tildes in the Equation (5) and to write the unknown functions multiplying $\mathrm{d}t^2$ and $\mathrm{d}r^2$ as exponentials:

$$\mathrm{d}s^2 = \mathrm{e}^{N(r,t)}\mathrm{d}t^2 - \mathrm{e}^{L(r,t)}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{6}$$

With $x^0 = t$, $x^1 = r$, $x^2 = \theta$, $x^3 = \varphi$, the metric tensor corresponding to the Equation (6), as H. C. Ohanian and R. Ruffini note [3], is:

$$g_{\mu\nu} = \begin{pmatrix} \mathrm{e}^N & 0 & 0 & 0 \\ 0 & -e^L & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix} \tag{7}$$

The unknown functions are now $N(r,t)$ and $L(r,t)$. We will use the Einstein's equations to find them.

As H. C. Ohanian and R. Ruffini note [3], the Christoffel symbols for a metric tensor of the form (7) are (the primes indicate space derivatives: $N' \equiv \frac{\partial N}{\partial r}$ and $L' \equiv \frac{\partial L}{\partial r}$; the dots indicate time derivatives: $\dot{N} \equiv \frac{\partial N}{\partial t}$ and $\dot{L} \equiv \frac{\partial L}{\partial t}$):

$$\begin{aligned} &\Gamma^0_{00} = \frac{1}{2}\dot{N},\ \Gamma^0_{01} = \Gamma^0_{10} = \frac{1}{2}N', \\ &\Gamma^0_{11} = \frac{1}{2}\dot{L}\mathrm{e}^{L-N},\ \Gamma^1_{00} = \frac{1}{2}N'\mathrm{e}^{N-L}, \\ &\Gamma^1_{10} = \Gamma^1_{01} = \frac{1}{2}\dot{L},\ \Gamma^1_{11} = \frac{1}{2}L', \\ &\Gamma^1_{22} = -r\mathrm{e}^{-L},\ \Gamma^1_{33} = -r\sin^2\theta\mathrm{e}^{-L}, \\ &\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r},\ \Gamma^2_{33} = -\sin\theta\cos\theta, \\ &\Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r},\ \Gamma^3_{23} = \Gamma^3_{32} = \cot\theta \end{aligned} \tag{8}$$

All other Christoffel symbols are zero.

As H. C. Ohanian and R. Ruffini note [3], the Ricci tensor $R_{\mu\nu}$ can be calculated from the Christoffel symbols; only the 00, 01, 10, 11, 22, 33 components of $R_{\mu\nu}$ are nonzero.

The Einstein's field equation in vacuum is [3]:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \tag{9}$$

By taking the trace of this equation, we find, as [3], that $R = 0$. Hence the Equation (9) reduces to:

$$R_{\mu\nu} = 0 \tag{10}$$

The 00, 01, 10, 11, 22 and 33 components of this equation, as H. C. Ohanian and R. Ruffini note [3], are as follows:

$$R_{00} = \mathrm{e}^{N-L}\left(-\frac{N''}{2} + \frac{L'N'}{4} - \frac{N'^2}{4} - \frac{N'}{r}\right) + \frac{\ddot{L}}{2} + \frac{\dot{L}\left(\dot{L} - \dot{N}\right)}{4} = 0 \tag{11}$$

$$R_{01} = -R_{10} = -\frac{\dot{L}}{r} = 0 \tag{12}$$

$$R_{11} = \frac{N''}{2} - \frac{L'N'}{4} + \frac{N'^2}{4} - \frac{L'}{r} + \mathrm{e}^{L-N}\left[\frac{\ddot{L}}{2} + \frac{\dot{L}\left(\dot{L} - \dot{N}\right)}{4}\right] = 0 \tag{13}$$

$$R_{22} = \mathrm{e}^{-L}\left[1 + \frac{1}{2} r\left(N' - L'\right)\right] - 1 = 0 \tag{14}$$

$$R_{33} = \sin^2\theta \mathrm{e}^{-L}\left[1 + \frac{1}{2} r\left(N' - L'\right)\right] - \sin^2\theta = 0 \tag{15}$$

Although these equations look complicated, they can be integrated quite easily. The Equation (12) says that L is time independent, and this implies that *all* the terms involving time derivatives in the Equations (11) and (13) drop out. The sum of the Equation (13) and e^{L-N} times the Equation (11) then gives:

$$-\frac{L'}{r} - \frac{N'}{r} = 0 \tag{16}$$

from which we have [3]:

$$N = -L + h(t) \tag{17}$$

where $h(t)$ is an arbitrary function of time [in regard to the space derivatives in the Equation (16), this function $h(t)$ behaves like a constant of integration].

When we substitute the Equation (17) into the Equation (14), we find, as [3]:

$$\mathrm{e}^{-L}\left(-rL' + 1\right) = 1 \tag{18}$$

This equation can be rewritten as:

$$\mathrm{e}^{-L}\left(-r\frac{\mathrm{d}L}{\mathrm{d}r} + 1\right) = 1 \tag{19}$$

From which we have:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r\mathrm{e}^{-L}\right) = 1 \tag{20}$$

from which we have:

$$r\mathrm{e}^{-L} = r + C \tag{21}$$

where C is a constant of integration that can have any real value (positive, negative or null) and r is always greater than zero.

If there had been a generic function of r on the first member of the (21) then we would not have any constraints connected to the values of C and r. But the particular shape of this first member entails, as we will see, further constraints on the admissible values of C and r.

From the Equation (19), because of the fact that e^{-L}, r and $\mathrm{d}r$ are all real, we can say that also $\mathrm{d}L$ must be real. For which, since $L = 0$ for $r = +\infty$, L is equal to a real number for all values of $r > 0$.

Thus in the first member of the Equation (21) there is an exponential with a real exponent, which exponential is always positive, multiplied by r which is also positive for all values of r greater than zero. Therefore also the second member of the (21) must be greater than zero for all positive values of r.

2. The Two Cases of $C \geq 0$ and $C < 0$

2.1. The Case of $C \geq 0$

If C is chosen greater than or equal to zero then the second member of the (21) is always greater than zero for all values of $r > 0$.

2.2. The Case of $C < 0$

If C is a negative number the second member of the (21) is greater than zero only for the values of r greater than $-C$.

This constraint does not come from the derivatives with respect to which a positive or negative or zero value of the constant C makes no difference, but from the particular form of the Equation (21) in which the constant C was introduced.

In the commonly accepted treatment C is assumed to be $-2GM$ [3] and this solution is considered valid even for r equal to or less than $-C = 2GM$, the so-called Schwarzschild radius.

This is not correct, since in the case of negative C the values of r cannot be less than or equal to $-C$, *i.e.* in this case $2GM$, otherwise the (21) is not satisfied.

The values of r equal to or less than $2GM$ are precisely those corresponding to the points on or inside the event horizon.

In conclusion there are two types of mathematical solutions, one for the values of $C \geq 0$ for which the solutions are valid for all positive values of r. A second one for the negative values of C for which the solutions are valid only for the values of r greater than $-C$ (that is, commonly for $r > 2GM$).

3. Physical Analysis of the Case with $C \geq 0$

We first consider the case with $C \geq 0$ which, as we have seen, has solutions for all values of $r > 0$.

The fact that these solutions hold for all values of $r > 0$ satisfies a physical requirement, since we are looking for a solution for the vacuum region surrounding a spherically symmetric mass distribution of radius R, which mass distribution can be concentrated into an arbitrarily small radius R.

Now, from the (21) we have:

$$e^{-L} = 1 + \frac{C}{r} \tag{22}$$

From which we have:

$$e^{L} = \frac{1}{1 + \frac{C}{r}} \tag{23}$$

where C is a constant ≥0.

According to the Equation (17), the solution for $N(r)$ is then:

$$\mathrm{e}^{N(r)} = \mathrm{e}^{-L(r)+h(t)} = \left(1 + \frac{C}{r}\right)\mathrm{e}^{h(t)} \tag{24}$$

With the solutions (23) and (24), the space-time interval for the spherically symmetric field takes the form:

$$\mathrm{d}s^2 = \left(1 + \frac{C}{r}\right)\mathrm{e}^{h(t)}\mathrm{d}t^2 - \frac{1}{1 + \frac{C}{r}}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{25}$$

where C is a constant ≥0.

Here, the time-dependent function $h(t)$ remains undetermined and unknown. The field equations do not determine this function, but we can eliminate it by means of a transformation of the time coordinate. If we, as H. C. Ohanian and R. Ruffini [3], adopt a new time coordinate $\tilde{t}$ such that $\mathrm{d}\tilde{t} \equiv \mathrm{e}^{\frac{h(t)}{2}}\mathrm{d}t$, then the function $h(t)$ disappears from view, and the first term of the right side of the Equation (25) becomes $\left(1 + \frac{C}{r}\right)\mathrm{d}\tilde{t}^2$. We can then omit the tilde, so we obtain, as H. C. Ohanian and R. Ruffini [3]:

$$\mathrm{d}s^2 = \left(1 + \frac{C}{r}\right)\mathrm{d}t^2 - \frac{1}{1 + \frac{C}{r}}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{26}$$

where C is a constant ≥0. Obviously, $C = 0$ corresponds to the absence of the gravitational field.

We can see that in the (26) there is no singularity in the coefficients of $\mathrm{d}t^2$ and $\mathrm{d}r^2$ for any value of $r > 0$. In particular, for $r = C$ we have that this metric is perfectly defined and regular.

Moreover, according to the (26), for any value of $r > 0$ the coefficient of $\mathrm{d}t^2$ always remains positive and the coefficient of $\mathrm{d}r^2$ always remains negative. Therefore, the time t never becomes as a spatial coordinate and the spatial coordinate r never becomes as a temporal coordinate. There is no an event horizon.

On the other hand, this is in agreement with the fact that the Einstein's field equation [1] [3] [5]-[7] is symmetric with respect to time, *i.e.* is invariant under time reversal T [8]-[13], as can also be easily obtained directly from the CPT symmetry, since in this case we have invariance under the parity transformation since we have a spherical symmetry, and invariance under the charge conjugation since the antiparticles behave in the same way as the particles under the action of the gravitation. Therefore we can always reverse the motion, that is due to the gravitational field, of any particle in such a way as if it went back in time simply by means of changing the sign of the velocity of that particle, *i.e.* by means of reversing the direction of the motion of that particle. This implies in general the non-existence of any event horizon, since the existence of an event horizon would imply the impossibility of going back for any particle once this particle has gone from the outside of the event horizon in the inside of the same event horizon and the Einstein's field equation implies always for any particle the possibility of going back by means of reversing the direction of the motion of the same particle.

Moreover, we know that A. Einstein in his article establishing the General Theory of Relativity [1] expressed the formally flat metric in the commonly used coordinates $\mathrm{d}s^2$ as a function of curved coordinates (that is, measured relatively to a reference frame that is integral with a curved space-time) for expressing the gravitational field as the curvature of the space-time [1]. Therefore, we can note that the fact that the coefficient of $\mathrm{d}t^2$ in the Equation (26) is >1 in the presence of a gravitational field entails that in the presence of a gravitational field the clocks go more slowly (that is, that the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a gravitational field flow more slowly): and we know that this is in agreement with the experimental results [3].

In fact, by using t_0 for the time in the commonly used coordinates, we have:

$$\mathrm{d}t_0^2 = \left(1 + \frac{C}{r}\right)\mathrm{d}t^2 \tag{27}$$

For which the time t measured by a clock positioned in the gravitational field (that is, measured relatively to a reference frame that is integral with a space-time curved for expressing the presence of a gravitational field) is less than the time t_0 measured by a clock positioned where there is not any gravitational field (that is, measured relatively to a reference frame that is integral with a flat space-time).

Finally, as we will see later, from the comparison with the linear theory carried out correctly we find that $C = 2GM$ and therefore $C \geq 0$ ($C = 0$ when $M = 0$). This value of $C = 2GM$, as we will see later, leads to a new metric that is different from the analogous one commonly used, but in the usual case of $\frac{2GM}{r} \ll 1$ we have that the difference between the previsions of the new metric and the previsions of the commonly used metric is only at the second order in $\frac{2GM}{r}$: in fact, in this case the two metrics are equal at the first order in $\frac{2GM}{r}$. And all the experiments conducted so far have not had errors so small as to test differences at the second order in $\frac{2GM}{r}$.

4. Physical Analysis of the Case with $C < 0$

Now, we consider the case with $C < 0$ which, as we have seen, has solutions only for the values of $r > -C$.

Since we are looking for a solution for the vacuum region surrounding a spherically symmetric mass distribution of radius *R*, which mass distribution can be concentrated into an arbitrarily small radius *R*, in this case there are values of $r \le -C$ which belong to that region for which values we have no solution. Clearly, this is physically unacceptable.

Using a procedure similar to that used in the case of $C \ge 0$ we arrive at a formula equal to the (26):

$$\mathrm{d}s^2 = \left(1+\frac{C}{r}\right)\mathrm{d}t^2 - \frac{1}{1+\frac{C}{r}}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \qquad (28)$$

where, in this case C is a constant <0 and, obviously, $r > -C$.

We can see that in the (28) there is not any singularity in the coefficients of $\mathrm{d}t^2$ and $\mathrm{d}r^2$ for any value of $r > -C$.

On the other hand, we can note that in the (28) there would have been a singularity for $r = -C$ (since for $r \to -C$ the coefficient of $\mathrm{d}r^2$ tends to infinity, while the coefficient of $\mathrm{d}t^2$ tends to zero), but, as we have seen, this value of r is excluded for mathematical reasons. We have already noted that this value of r corresponds in the commonly accepted treatment [3] to the event horizon at the so-called Schwarzschild radius.

Moreover, according to the (28), for any value of $r > -C$ the coefficient of $\mathrm{d}t^2$ always remains positive and the coefficient of $\mathrm{d}r^2$ always remains negative. Therefore, since the coefficient of $\mathrm{d}t^2$ is always not negative the time (that is, the temporal coordinate) does not become in any case as a spatial coordinate. On the other hand, since the coefficient of $\mathrm{d}r^2$ is always not positive the spatial coordinate r does not become in any case as a temporal coordinate. This implies that, in this case, for these values of r there is not any event horizon.

On the other hand, for the values of $r < -C$ we have that in the (28) the coefficient of $\mathrm{d}t^2$ would be negative and the coefficient of $\mathrm{d}r^2$ would be positive, so the time (that is, the temporal coordinate) would be as a spatial coordinate and the spatial coordinate r would be as a temporal coordinate, but, as we have seen, these values of r are excluded for mathematical reasons. We have already noted that these values of r correspond, in the commonly accepted treatment [3] [5], to the inside an event horizon.

Now, as we have already observed, we know that the Einstein field equation [1] [3] [5]-[7] is symmetric with respect to time, *i.e.* is invariant under time reversal T [8]-[13], therefore we can always reverse the motion, that is due to the gravitational field, of any particle in such a way as if it went back in time simply by means of changing the sign of the velocity of that particle, *i.e.* by means of reversing the direction of the motion of that particle. This implies in general the non-existence of any event horizon, since the existence of an event horizon would imply the impossibility of going back for any particle once this particle has gone from the outside of the event horizon in the inside of the same event horizon and the Einstein's field equation implies always for any particle the possibility of going back by means of reversing the direction of the motion of the same particle.

Therefore, we can say that the fact that the values of $r \le -C$ are excluded for mathematical reasons is strictly linked, in this case, to the fact that the Einstein's equations are mathematically symmetric with respect to time: in fact we have seen that the values of $r \le -C$, in this case, correspond to a black hole (that is, to an event horizon and its inside), which, as we have seen, is excluded by the symmetry with respect to time of the Einstein's equations.

Even A. Einstein wrote an article in 1939 [14] in which, using a metric equivalent to the (28), he argued that the values of r less than or equal to the Schwarzschild radius $2GM$ are excluded for physical reasons.

Furthermore, as we have already observed, we know that A. Einstein in his article establishing the General Theory of Relativity [1] expressed the formally flat metric in the commonly used coordinates $\mathrm{d}s^2$ as a function of curved coordinates (that is, measured relatively to a reference frame that is integral with a curved space-time) for expressing the gravitational field as the curvature of the space-time [1]. Therefore, we can note that the fact that the coefficient of $\mathrm{d}t^2$ in the Equation (28) is <1 in the presence of a gravitational field entails that in the presence of a gravitational field the clocks go more quickly (that is, that the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a gravitational field flow more quickly): and we know that this is contrary to the experimental results [3].

In fact, by using t_0 for the time in the commonly used coordinates, we have:

$$\mathrm{d}t_0^2 = \left(1 + \frac{C}{r}\right)\mathrm{d}t^2 \tag{29}$$

For which, in this case, the time t measured by a clock positioned in the gravitational field (that is, measured relatively to a reference frame that is integral with a space-time curved for expressing the presence of a gravitational field) is greater than the time t_0 measured by a clock positioned where there is not any gravitational field (that is, measured relatively to a reference frame that is integral with a flat space-time), contrary to the experimental results [3].

Finally, as we have already said and will see later, from the comparison with the linear theory carried out correctly we find that $C = 2GM$, therefore the case with $C < 0$ is also excluded for this reason.

In conclusion, we have that the solutions with $C < 0$, for various reasons, are unacceptable from a physical point of view.

5. The Comparison with the Linear Theory

For finding the value of C in the Equation (26) or (28), we must compare this expression (26) or (28) with the result obtained from the linear theory. Now, the formula (obtained from the linear theory) commonly used for the comparison, as reported for example by H. C. Ohanian and R. Ruffini [3], is:

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{r}\right)\mathrm{d}t^2 - \left(1 + \frac{2GM}{r}\right)\left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right) \tag{30}$$

But, as we said before, A. Einstein in his article establishing the General Theory of Relativity [1] expressed the formally flat metric in the commonly used coordinates $\mathrm{d}s^2$ as a function of curved coordinates (that is, measured relatively to a reference frame that is integral with a curved space-time) for expressing the gravitational field as the curvature of the space-time [1]. While the (30) is commonly interpreted, as we can see for example in the book of H. C. Ohanian and R. Ruffini [3] {for example in the page 131 in the formulas (4.13) and (4.14) of their book [3]}, as a formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates. Therefore the (30) is not analogue to the (26) or (28), in fact the (30) is not equal to a formally flat metric in the commonly used coordinates as the (26) or (28), but the (30) is equal to a formally flat metric in the curved coordinates for expressing the presence of the gravitational field as a curvature of space-time.

Consequently, we can make this fact explicit by rewriting the (30) as:

$$\mathrm{d}s_g^2 = \left(1 - \frac{2GM}{r}\right)\mathrm{d}t^2 - \left(1 + \frac{2GM}{r}\right)\left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right) \tag{31}$$

where $\mathrm{d}s_g^2$ is a metric formally flat in the curved coordinates.

Therefore, for making a comparison with the (26) or (28), we, instead of the expression (31), need to have the expression (obtained from the linear theory) of the formally flat metric in the commonly used coordinates $\mathrm{d}s^2$, which metric be expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time.

Now, from the formula (31) we have:

$$\mathrm{d}t_g^2 = \left(1 - \frac{2GM}{r}\right)\mathrm{d}t^2 \tag{32}$$

where $\mathrm{d}t_g$ is the time measured by clocks positioned in the gravitational field (that is, measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time). And therefore for $2GM \ll r$ we have:

$$\mathrm{d}t^2 \cong \left(1 + \frac{2GM}{r}\right)\mathrm{d}t_g^2 \cong \left(1 + \frac{2GM}{r_g}\right)\mathrm{d}t_g^2 \tag{33}$$

And analogously, from the formula (31) we have relatively to the spatial coordinates:

$$\left(1+\frac{2GM}{r}\right)\left(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2\right)$$
$$=\mathrm{d}x_g^2+\mathrm{d}y_g^2+\mathrm{d}z_g^2=\mathrm{d}r_g^2+r_g^2\mathrm{d}\theta_g^2+r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2 \tag{34}$$

where x_g, y_g, z_g or r_g, θ_g, φ_g are the spatial coordinates measured relatively to a reference frame that is integral with a space-time curved for expressing the gravitational field as the curvature of the space-time.

And therefore for $2GM \ll r$ we have:

$$\begin{aligned}\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2 &\cong \left(1-\frac{2GM}{r}\right)\left(\mathrm{d}x_g^2+\mathrm{d}y_g^2+\mathrm{d}z_g^2\right)\\ &\cong \left(1-\frac{2GM}{r_g}\right)\left(\mathrm{d}x_g^2+\mathrm{d}y_g^2+\mathrm{d}z_g^2\right)\\ &= \left(1-\frac{2GM}{r_g}\right)\left(\mathrm{d}r_g^2+r_g^2\mathrm{d}\theta_g^2+r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2\right)\end{aligned} \tag{35}$$

Therefore the expression (26) or (28) in the limit for $r \to +\infty$ must be compared not with the expression (30) but with this other expression:

$$\mathrm{d}s^2=\left(1+\frac{2GM}{r_g}\right)\mathrm{d}t_g^2-\left(1-\frac{2GM}{r_g}\right)\left(\mathrm{d}r_g^2+r_g^2\mathrm{d}\theta_g^2+r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2\right) \tag{36}$$

In fact the formula (36), in the case of $\frac{2GM}{r} \ll 1$, represents the formally flat metric in the commonly used coordinates $\mathrm{d}s^2$, which metric is expressed as a function of the coordinates curved for expressing the gravitational field as the curvature of the space-time.

Let us now rewrite the formula (26) or (28) for greater clarity, making explicit the fact that the coordinates on the second member of (26) or (28) are those curved for expressing the gravitational field as the curvature of the space-time:

$$\mathrm{d}s^2=\left(1+\frac{C}{r_g}\right)\mathrm{d}t_g^2-\frac{1}{1+\frac{C}{r_g}}\mathrm{d}r_g^2-r_g^2\mathrm{d}\theta_g^2-r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2 \tag{37}$$

Since we cannot compare the expression (37) with the expression (36) directly, because the (36) has a form which is different from that of the expression (37), let us change, as H. C. Ohanian and R. Ruffini [3], the Equation (37) by introducing a new coordinate $\tilde{r}_g$:

$$\tilde{r}_g \equiv \frac{1}{2}\sqrt{r_g^2+Cr_g}+\frac{1}{2}r_g+\frac{1}{4}C \tag{38}$$

or, equivalently,

$$r_g \equiv \tilde{r}_g\left(1-\frac{C}{4\tilde{r}_g}\right)^2 \tag{39}$$

This transformation gives:

$$\mathrm{d}s^2 = \left(\frac{1 + \frac{C}{4\tilde{r}_g}}{1 - \frac{C}{4\tilde{r}_g}} \right)^2 \mathrm{d}t_g^2 - \left(1 - \frac{C}{4\tilde{r}_g} \right)^4 \left(\mathrm{d}\tilde{r}_g^2 + \tilde{r}_g^2 \mathrm{d}\theta_g^2 + \tilde{r}_g^2 \sin^2\theta_g \mathrm{d}\varphi_g^2 \right) \tag{40}$$

The coordinates used in the expression (40) are called *isotropic* [3].

In the weak field limit ($\tilde{r}_g \to \infty$), the Equation (40) reduces to:

$$\mathrm{d}s^2 = \left(1 + \frac{C}{\tilde{r}_g} \right) \mathrm{d}t_g^2 - \left(1 - \frac{C}{\tilde{r}_g} \right) \left(\mathrm{d}\tilde{r}_g^2 + \tilde{r}_g^2 \mathrm{d}\theta_g^2 + \tilde{r}_g^2 \sin^2\theta_g \mathrm{d}\varphi_g^2 \right) \tag{41}$$

Obviously, this equation has the same form of the Equation (36). By comparing the Equation (41) with the Equation (36), we have:

$$C = 2GM \tag{42}$$

We can note that this is precisely the value of C that we announced in advance.

By means of the substitution of C with $2GM$, according to (42), the Equation (37) becomes:

$$\mathrm{d}s^2 = \left(1 + \frac{2GM}{r_g} \right) \mathrm{d}t_g^2 - \frac{1}{1 + \frac{2GM}{r_g}} \mathrm{d}r_g^2 - r_g^2 \mathrm{d}\theta_g^2 - r_g^2 \sin^2\theta_g \mathrm{d}\varphi_g^2 \tag{43}$$

where M is (a constant, which is equal to) the total mass of the system.

Now the (43) is equal to a formally flat metric in the commonly used coordinates and therefore it is not expressed in a form analogue to that in which the linear theory is commonly expressed. For returning to an expression analogue to that in which the linear theory is commonly expressed we must simply use a method which is the exact inverse to that we have used for transforming the coordinates of the linear limit, except for approximations.

In particular, by using t for the time in the commonly used coordinates, we have:

$$\mathrm{d}t^2 = \left(1 + \frac{2GM}{r_g} \right) \mathrm{d}t_g^2 \tag{44}$$

from which we have:

$$\frac{1}{1 + \frac{2GM}{r_g}} \mathrm{d}t^2 = \mathrm{d}t_g^2 \tag{45}$$

that is, the coefficient of the temporal part becomes the inverse with this transformation. And analogously the coefficient of $\mathrm{d}r_g^2$ becomes the inverse returning to the coordinates commonly used.

Therefore we have that the correspondent of the (43), *i.e.* in this case the formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, is:

$$ds_g^2 = \frac{1}{1+\frac{2GM}{r_g}}dt^2 - \left(1+\frac{2GM}{r_g}\right)dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2 \quad (46)$$

where ds_g^2 is a metric formally flat in the curved coordinates.

As for the relation between r_g and r, we have:

$$dr^2 = \frac{1}{1+\frac{2GM}{r_g}}dr_g^2 \quad (47)$$

From which, we have:

$$\int_0^{r^2} dr^2 = \int_0^{r_g^2} \frac{1}{1+\frac{2GM}{r_g}}dr_g^2 \quad (48)$$

From which, we have:

$$r^2 = \int_0^{r_g} \frac{2r_g^2 dr_g}{r_g + 2GM} \quad (49)$$

From which, we have:

$$r^2 = \left[r_g^2 - 4GMr_g + 8G^2M^2 \ln\left(2GM + r_g\right)\right]_0^{r_g} \quad (50)$$

From which, we have:

$$r^2 = r_g^2 - 4GMr_g + 8G^2M^2 \ln\left(1+\frac{r_g}{2GM}\right) \quad (51)$$

According to the (51) for $r_g \to 0$ also $r \to 0$ and for $r_g \to +\infty$ also $r \to +\infty$. Moreover, as r_g increases, r also increases as can be seen by differentiating the right side of the (51) with respect to r_g or directly from the (49). On the other side, for any value of $r_g > 0$ we have that $r < r_g$, as can be seen directly from the (48).

Furthermore, for $r_g \gg 2GM$ we have that $r \cong r_g\left(1-\frac{2GM}{r_g}\right)$ and also $r_g \cong r\left(1+\frac{2GM}{r}\right)$. This implies that for $\frac{2GM}{r} \ll 1$ the presence of r_g instead of r in the (46) implies only corrections to the second order in $\frac{2GM}{r}$.

On the other hand for $r_g \ll 2GM$ we have $r^2 \cong \frac{r_g^3}{3GM}$.

In particular for $r_g = 2GM$, $r^2 = 4G^2M^2\left(2\ln 2 - 1\right)$, and so in this case we have $r = 2GM\sqrt{2\ln 2 - 1} \cong 1.24GM$.

Moreover, for $r_g = 4GM$, $r^2 = 8G^2M^2 \ln 3$, and so in this case we have $r = 2GM\sqrt{2\ln 3} \cong 2.96GM$.

Instead for $r_g = 8GM$, $r^2 = 8G^2M^2\left(4+\ln 5\right)$, and so in this case we have $r = 2GM\sqrt{2\left(4+\ln 5\right)} \cong 6.70GM$.

For simplicity we have used the formulas with $c \equiv 1$ for calculating the correct Schwarzschild solution. Obviously, in the case more general (in which c is not defined equal to 1) we have that the (43) and the (46) become respectively:

$$\mathrm{d}s^2 = \left(1+\frac{2GM}{r_g c^2}\right)c^2\mathrm{d}t_g^2 - \frac{1}{1+\frac{2GM}{r_g c^2}}\mathrm{d}r_g^2 - r_g^2\mathrm{d}\theta_g^2 - r_g^2\sin^2\theta_g\mathrm{d}\varphi_g^2 \tag{52}$$

$$ds_g^2 = \frac{1}{1+\frac{2GM}{r_g c^2}}c^2\mathrm{d}t^2 - \left(1+\frac{2GM}{r_g c^2}\right)\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{53}$$

The (53) is the correct final form of the Schwarzschild solution, *i.e.* the correct solution for the case which Schwarzschild has analysed, when we use an expression analogue to that in which the linear theory is commonly expressed.

Instead, the common erroneous expression for the Schwarzschild solution is usually written as [3]:

$$\mathrm{d}s^2 = \left(1-\frac{2GM}{rc^2}\right)c^2\mathrm{d}t^2 - \frac{1}{1-\frac{2GM}{rc^2}}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{54}$$

This formula (54) is usually considered to be equal to a formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, in fact, for example, H. C. Ohanian and R. Ruffini obtained this formula through a direct comparison with the (30) [3]. Therefore, the (54) with the symbolism adopted by us becomes:

$$\mathrm{d}s_g^2 = \left(1-\frac{2GM}{rc^2}\right)c^2\mathrm{d}t^2 - \frac{1}{1-\frac{2GM}{rc^2}}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{55}$$

Therefore, we can note that the erroneous expression for the Schwarzschild solution was due to two errors: one error was the choice of a solution of the Einstein's equations that is incorrect for mathematical and physical reasons, and another error was the interpretation of that erroneous solution as analogous of the (30) or (31).

As we will see, the difference between the correct formula (53) and the incorrect formula (55) entails enormous physical consequences, even though, as we announced in advance, in the usual case of $\frac{2GM}{rc^2} \ll 1$ we have that the difference between the previsions of the erroneous Schwarzschild solution and the previsions of the correct Schwarzschild solution is only at the second order in $\frac{2GM}{rc^2}$: in fact, in this case the two formulas (53) and (55) are equal at the first order in $\frac{2GM}{rc^2}$.

6. The Behaviour of the Clocks Positioned in Gravitational Fields According to the Correct Schwarzschild Solution

Now, as we have already seen, the fact that the coefficient of $c^2\mathrm{d}t_g^2$ in the Equation (52), which is the correct Schwarzschild metric when we express the formally flat metric in the commonly used coordinates $\mathrm{d}s^2$ as a function of the curved coordinates, is >1 in the presence of a gravitational field entails that in the presence of a gravitational field the clocks go more slowly (that is, that the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of the gravitational field flow more slowly): and we know that this is in agreement with the experimental results [3]. In fact, we have:

$$\left(1+\frac{2GM}{r_g c^2}\right)\mathrm{d}t_g^2 = \mathrm{d}t^2 \tag{56}$$

For which the time t_g measured by a clock positioned in the gravitational field (that is, measured relatively to a reference frame that is integral with a space-time curved for expressing the presence of the gravitational field) is less than the time t measured by a clock positioned where there is not any gravitational field (that is, measured relatively to a reference frame that is integral with a flat space-time).

Analogously, the fact that the coefficient of $c^2\mathrm{d}t_g^2$ in the Equation (52) has the form $\left(1+\frac{2GM}{r_g c^2}\right)$ entails that the clocks positioned in a more intense gravitational field go more slowly than the clocks positioned in a less intense gravitational field (that is, that the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a more intense gravitational field flow more slowly than the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a less intense gravitational field), and we also know that this fact is in agreement with the experimental data. In fact we have:

$$\left(1+\frac{2GM}{r_{g1} c^2}\right)\mathrm{d}t_{g1}^2 = \left(1+\frac{2GM}{r_{g2} c^2}\right)\mathrm{d}t_{g2}^2 \tag{57}$$

Now, when the gravitational field is more intense (that is, when r_g is more small) the coefficient of $\mathrm{d}t_g^2$ is greater, for which the corresponding time flows more slowly (that is, the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a more intense gravitational field flow more slowly than the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a less intense gravitational field). In particular, we have:

$$\frac{\mathrm{d}t_{g2}}{\mathrm{d}t_{g1}} = \sqrt{\frac{1+\frac{2GM}{r_{g1}c^2}}{1+\frac{2GM}{r_{g2}c^2}}} \tag{58}$$

For which when $r_{g2} > r_{g1}$, *i.e.* when the second gravitational field is less intense than the first gravitational field, then $\mathrm{d}t_{g2} > \mathrm{d}t_{g1}$: we have the well-known result (which has been confirmed by numerous experiments [3]) that the more intense is the gravitational field the more slowly the clocks go (that is, that the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a more intense gravitational field flow more slowly than the measurements of time relatively to a reference frame that is integral with a space-time curved for expressing the presence of a less intense gravitational field).

Analogously, the ratio of the relative frequencies $\nu_1 = 1/\mathrm{d}t_{g1}$ and $\nu_2 = 1/\mathrm{d}t_{g2}$ is:

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1+\dfrac{2GM}{r_{g1}c^2}}{1+\dfrac{2GM}{r_{g2}c^2}}} \tag{59}$$

This is the correct formula for the gravitational redshift of light in the correct Schwarzschild metric. We can note that the two formulas (58) and (59) are different from those obtained usually from the incorrect expression of the Schwarzschild solution by means of an incorrect procedure [3], but are equal at the first order in $\dfrac{2GM}{rc^2}$ (to the incorrect formulas). In fact, the formulas commonly used, instead of the two correct formulas (58) and (59), are respectively [3]:

$$\frac{\mathrm{d}t_2}{\mathrm{d}t_1} = \sqrt{\frac{1-\dfrac{2GM}{r_2c^2}}{1-\dfrac{2GM}{r_1c^2}}} \tag{60}$$

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{1-\dfrac{2GM}{r_2c^2}}{1-\dfrac{2GM}{r_1c^2}}} \tag{61}$$

7. The Radial Velocity of Light According to the Correct Schwarzschild Solution

The light formally moves in a straight line relative to the reference system curved for expressing the presence of the gravitational field. Therefore we can impose the condition $\mathrm{d}s_g^2 = 0$ for the propagation of light. Therefore from the (53) if we take a radial motion of the light in the commonly used coordinates ($\mathrm{d}\theta = 0$ and $\mathrm{d}\varphi = 0$) we have:

$$\frac{1}{1+\dfrac{2GM}{r_gc^2}}c^2\mathrm{d}t^2 - \left(1+\frac{2GM}{r_gc^2}\right)\mathrm{d}r^2 = 0 \tag{62}$$

From which we have that the radial velocity of light in the commonly used coordinates is equal to [3]:

$$v_l = \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{1+\frac{2GM}{r_g c^2}} c \tag{63}$$

We can note that the radial velocity of light in the commonly used coordinates is always $\le c$, and is equal to c only when there is not a gravitational field. Moreover this formula (63) is in agreement with the available experimental data [3].

8. Other Consequences of the Correct Schwarzschild Solution

On the other hand, we have seen that the correct Schwarzschild solution, analogue to that in which the linear theory is commonly expressed, is a formally flat metric in the curved coordinates, which metric is expressed as a function of the commonly used coordinates, and is equal to:

$$\mathrm{d}s_g^2 = \frac{1}{1+\frac{2GM}{r_g c^2}} c^2 \mathrm{d}t^2 - \left(1+\frac{2GM}{r_g c^2}\right)\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{64}$$

We can easily see that this metric does not imply any event horizon and, consequently, this metric does not imply any black hole, contrary to the common treatment of the Schwarzschild solution [3] [5].

In fact, according to the (64), taking into account the (51), there is not any singularity at the Schwarzschild radius $r_s \equiv \frac{2GM}{c^2}$ (both for r and for r_g) and in general at any value of $r > 0$ (or of $r_g > 0$).

On the other hand, the gravitational redshift according to the (59) is always not infinite for any value of $r > 0$ (or of $r_g > 0$), in particular this is true both for $r = r_s \equiv \frac{2GM}{c^2}$ and for $r_g = r_s \equiv \frac{2GM}{c^2}$, contrary to the common treatment of the Schwarzschild metric [3] [5].

Moreover, according to the (64), for any value of $r > 0$ or of $r_g > 0$ (and in particular for any value of r or of r_g less than the Schwarzschild radius $r_s \equiv \frac{2GM}{c^2}$) the coefficient of $c^2\mathrm{d}t^2$ always remains positive and the coefficient of $\mathrm{d}r^2$ always remains negative. Therefore, since the coefficient of $c^2\mathrm{d}t^2$ is always not negative the time (that is, the temporal coordinate) does not become in any case as a spatial coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [5]. On the other hand, since the coefficient of $\mathrm{d}r^2$ is always not positive the spatial coordinate r does not become in any case as a temporal coordinate, contrary to the common treatment of the space-time inside the event horizon [3] [5].

Consequently, the light cones are always orientated in the usual way, in particular there is not any horizontal inclination of the light cones, contrary to the common treatment of the space-time inside the event horizon [3] [5].

9. Experimental Prospects

9.1. The Available Experimental Data

As for the experimental data obtained with the help of x-ray astronomy the proof that we have found black holes, and therefore event horizons, is based only on the fact that we have found invisible objects which have masses that are too great, according to the commonly accepted theory, for not being black holes [3] [15] [16]. But according to the correct theory of this article, whatever the masses and the dimensions of these invisible objects are, we never have black holes, and therefore we never have event horizons. Therefore such experimental data cannot discriminate between the commonly accepted theory and the same theory corrected according to this article.

On the other hand, with regard to the experimental data of the so-called gravitational waves (obtained by the LIGO collaboration) of a collision between two black holes, such gravitational waves were detected only below measurement errors, *i.e.* the signals detected were lower than the background noise (cf. chapter 6 of [15]). Furthermore the models expected from the theory were used for selecting the signals from the background noise (cf. chapter 6 of [15]) with the help of supercomputers: obviously, this is an incorrect practice which cannot produce any significant data. The awareness of the non-significance of the LIGO collaboration data is now widespread [17]-[20]. Obviously, also such data cannot discriminate between the commonly accepted theory and the same theory corrected according to this article.

As for the alleged photos of black holes, they were formed with the help of special algorithms from something compatible with the white noise. In other words, these photos were extracted from something compatible with the white noise only on the basis of the images that were expected by the researchers, with the help of appropriate algorithms loaded onto supercomputers (cf. the Section "Imaging a Black Hole" of [21]. See also [22]). Therefore, also in this case the researchers wanted to measure something that is below measurement errors, and so these photos are completely unreliable. On the other hand, serious doubts have now spread about the reliability of these photos [23] [24]. Consequently such photos cannot prove anything and in particular cannot discriminate in any way between the commonly accepted theory and the same theory corrected according to this article.

Moreover, the corrections, that we have proposed in this article, to the commonly accepted theory are very small in the normal experimental situations (for example in the solar system), so the fact that, in these situations, so far no difference has been noted between the commonly accepted theory and the experimental results is not strange. In fact, as we have seen, in the usual case of $\frac{2GM}{rc^2} \ll 1$ we have that the difference between the previsions of the erroneous Schwarzschild solution together with the erroneous classical limit and the previsions of the correct Schwarzschild solution together with the correct classical limit is only at the second order in $\frac{2GM}{rc^2}$. And all the experiments conducted so far in the solar system have not had errors so small as to test differences at the second order in $\frac{2GM}{rc^2}$ [3].

Therefore, in conclusion, there is no available experimental data that can discriminate between the commonly accepted theory and the same theory corrected according to this article.

9.2. A Proposal for a Crucial Experiment

On the other hand, a crucial experiment could be done, which discriminates between the commonly accepted theory and the same theory corrected according to this article, by taking advantage of the high precision and sensitivity of the latest atomic clocks.

In fact from the correct formula (58) and from the analogous one for the commonly accepted theory (60) we have that the ratio of the passage of time in the gravitational field according to the correct theory to that according to the commonly accepted theory is:

$$\frac{\mathrm{d}t_g}{\mathrm{d}t} = \sqrt{\frac{1}{\left(1+\frac{2GM}{r_g c^2}\right)\left(1-\frac{2GM}{rc^2}\right)}} \tag{65}$$

which in the case of $\frac{2GM}{rc^2} \ll 1$, since (as we have seen) in this case $r_g \cong r$, becomes:

$$\begin{aligned} \frac{\mathrm{d}t_g}{\mathrm{d}t} &= \sqrt{\frac{1}{\left(1+\frac{2GM}{r_g c^2}\right)\left(1-\frac{2GM}{rc^2}\right)}} \cong \sqrt{\frac{1}{1-\left(\frac{2GM}{rc^2}\right)^2}} \\ &\cong \sqrt{1+\left(\frac{2GM}{rc^2}\right)^2} \cong 1+\frac{1}{2}\left(\frac{2GM}{rc^2}\right)^2 \end{aligned} \tag{66}$$

Now throughout the solar system we have effectively $\frac{2GM}{rc^2} \ll 1$.

The term $\frac{1}{2}\left(\frac{2GM}{rc^2}\right)^2$ due to the solar mass on the surface of the Sun is approximately equal to 8.99×10^{-12}, while at the average distance of the Earth from the Sun this value becomes approximately equal to 1.95×10^{-16}. Obviously, externally to the Sun, such term decreases with the square of the distance from the centre of the Sun according to the formula.

On the other hand the same term due to the mass of the Earth on the surface of the Earth is approximately equal to 9.69×10^{-19} and obviously also here, externally to the Earth, decreases with the square of the distance from the centre of the Earth according to the formula. Of course, if we were to opt to use only the term due to the mass of the Earth we would have to do so between two points that have an approximatively equal contribution due to the mass of the Sun, so that the difference between these contributions due to the mass of the Sun be negligible compared to the difference between the correspondent contributions due to the mass of the Earth.

On the other hand, now we have atomic clocks that have an error of 7.6×10^{-21} [25] [26] and therefore we can measure such differences between the predictions of the commonly accepted theory and those of the same theory corrected according to this article with appropriate temporal measurements made in the solar system.

Now, in theory we could make a single measurement of time with one such atomic clock for being able to detect such differences, but in practice there can easily be errors due to the low precision in predicting the measurement results according to the commonly accepted theory (for example due to the low precision in knowing the mass of the Sun, the mass of the Earth and the universal gravitational constant G), so it is more convenient to make differential measurements, *i.e.* to measure the differences between the time measurements of two atomic clocks of such type placed respectively in two appropriate positions in the solar system. Indeed, it would be even better if the comparison between the time measurements of two atomic clocks of such type were made between several pairs of points of the solar system, thus revealing the difference between the two theories also on the basis of the trend of these differences as a function of the positions of at least one of these two clocks in the solar system.

Therefore we could do a crucial experiment, which discriminates between the commonly accepted theory and the same theory corrected according to this article, by taking one such atomic clock to diverse convenient locations in the solar system for comparing its time measurements made at those various locations with the corresponding time measurements made by another similar clock here on Earth.

10. General Conclusions

As we have seen, the corrections of this article imply that the correct solution does not entail any event horizon and, consequently, any black hole, since there is not any black hole without an event horizon. Therefore, this article confutes all the physics that on the basis of the Schwarzschild solution foresees the possibility of the existence of event horizons and black holes [3] [5] [8] [27].

Moreover, we have observed that the symmetry with respect to time, *i.e.* the invariance for time reversal T, of Einstein's field equation [8]-[13] excludes the possibility of event horizons, and therefore of black holes, in general.

On the other hand, we have seen that the alleged proofs in favour of the existence of black holes and event horizons based respectively on x-ray astronomy, on alleged gravitational waves and on alleged photos of black holes are not conclusive and therefore are not sufficient to discriminate between the commonly accepted theory and the same theory corrected according to this article.

Furthermore, we have observed that the corrections, that we have proposed here, to the commonly accepted theory are very small in the normal experimental situations, so the fact that so far no difference has been noted between the commonly accepted theory and the experimental results is not strange. In fact, as we have seen, in the usual case of $\frac{2GM}{rc^2} \ll 1$ we have that the difference between the previsions of the erroneous Schwarzschild solution and the previsions of the correct Schwarzschild solution is only in the second order in $\frac{2GM}{rc^2}$. And, as we announced in advance, all the experiments conducted so far have not had errors so small as to test such differences at the second order in $\frac{2GM}{rc^2}$ [3].

However, as we have already noted, recently atomic clocks have been constructed with a sensitivity such as to test these small differences in experiments that are feasible in the solar system. Therefore, it would be appropriate to try to make a crucial experiment that discriminates between the commonly accepted theory and the same theory corrected according to this article.

Finally, according to this article, all the physics that is based on the incorrect Schwarzschild solution should be modified on the basis of the correct formula that we have calculated in this same article.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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