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Winless Lottery Steak and Generalized Geometric Distribution

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Abstract

In the U.S., the two most popular lotteries are Mega Millions® and Powerball. In Mega Millions®, players pick six numbers from two separate pools of numbers, five different numbers from 1 to 70, and one number from 1 to 25, a total of six numbers. In Powerball, players choose five numbers from 1 to 69 and one number from 1 to 26. If there is no winning ticket this time, the amount of jackpot (matching all six numbers) increases for the next draw until a winning ticket is found. We estimate the average number of draws it takes for a winning lottery ticket to be found.

Keywords

Lottery, Geometric Distribution, Expectation, Telescopic Series, Geometric Series

1. Introduction

Lotteries have long captivated the imagination of hopeful participants, offering the possibility of instant wealth. However, for a select few, the experience becomes a perplexing journey through a winless lottery streak. In this work, we delve into the mathematics underlying such streaks, exploring the concepts of probability, randomness, and the elusive nature of winning. Specifically, we give an upper bound estimate of the streaks and apply it to two real-world examples.

2. Main Results

If p is the probability of winning the jackpot, then $q = 1 - p$ is the probability of not winning. Let X be the number of draws when a winning ticket is found. We denote the number of tickets sold at the k^{th} draw by n_k.

Theorem 1 *On average, the number of draws when the winning ticket is found is*

$$E[X]=1+q^{n_1}+q^{n_1+n_2}+q^{n_1+n_2+n_3}+\cdots.$$

Proof The probability mass function of X is

$$X=\begin{cases}1, & \text{with probability } 1-q^{n_1}\\ 2, & \text{with probability } q^{n_1}\left(1-q^{n_2}\right)\\ 3, & \text{with probability } q^{n_1}q^{n_2}\left(1-q^{n_3}\right)\\ & \vdots\\ k, & \text{with probability } q^{n_1}q^{n_2}\cdots q^{n_{k-1}}\left(1-q^{n_k}\right)\\ & \vdots\end{cases}$$

This is a generalized geometric distribution and the expectation of X is given by [1]

$$E[X]=1-q^{n_1}+2q^{n_1}\left(1-q^{n_2}\right)+3q^{n_1}q^{n_2}\left(1-q^{n_3}\right)+\cdots.$$

We rewrite this sum and add each row in **Table 1** and observe that the sum in each row is a telescopic series.

Table 1. Calculation of $E[X]$.

$E[X]$	$=$	$1-q^{n_1}$	$+2q^{n_1}\left(1-q^{n_2}\right)$	$+3q^{n_1}q^{n_2}\left(1-q^{n_3}\right)$	$+4q^{n_1}q^{n_2}q^{n_3}\left(1-q^{n_4}\right)$	$+\cdots$
	$=$	$1-q^{n_1}$	$+q^{n_1}\left(1-q^{n_2}\right)$	$+q^{n_1}q^{n_2}\left(1-q^{n_3}\right)$	$+q^{n_1}q^{n_2}q^{n_3}\left(1-q^{n_4}\right)$	$+\cdots$
			$+q^{n_1}\left(1-q^{n_2}\right)$	$+q^{n_1}q^{n_2}\left(1-q^{n_3}\right)$	$+q^{n_1}q^{n_2}q^{n_3}\left(1-q^{n_4}\right)$	$+\cdots$
				$+q^{n_1}q^{n_2}\left(1-q^{n_3}\right)$	$+q^{n_1}q^{n_2}q^{n_3}\left(1-q^{n_4}\right)$	$+\cdots$
					$+q^{n_1}q^{n_2}q^{n_3}\left(1-q^{n_4}\right)$	$+\cdots$
						$\ddots$

Therefore,

$$E[X]=1+q^{n_1}+q^{n_1+n_2}+q^{n_1+n_2+n_3}+\cdots \quad \blacksquare$$

If there is no winning ticket that matches all the lottery numbers in the current draw, the lottery authority raises the jackpot amount. As a result, more tickets will be sold in the next draw. It is reasonable to assume the number of tickets sold is increasing: $n_1<n_2<\cdots$.

Corollary 2 $E[X]$ *is at most* $\dfrac{1}{1-q^{n_1}}$.

Proof

$$\begin{aligned}E[X]&=1+q^{n_1}+q^{n_1+n_2}+q^{n_1+n_2+n_3}+\cdots\\ &\le 1+q^{n_1}+q^{2n_1}+q^{3n_1}+\cdots \quad \blacksquare\\ &=\frac{1}{1-q^{n_1}}\end{aligned}$$

Example The longest streak of non-winning draws in Mega Millions® [2] was 36 consecutive drawings that started on September 18, 2020 and ended on January 22, 2021. During this period, no ticket matched all the numbers drawn to win the jackpot. The number of the tickets sold on September 18, 2020 was 8,517,586. For each jackpot, between seven and nine million Mega Millions® tickets are sold initially.

The probability of winning the jackpot in Mega Millions® is $p = \frac{1}{\binom{70}{5} \times 25}$ and the probability of not winning the jackpot is $q = 1 - p = 0.99999999669504$.

From **Table 2** [3], the number of tickets sold is steadily increasing since the first draw: $n_1 < n_2 < \cdots$. By Corollary 2,

Table 2. Data of the longest Mega Millions® winless streak.

Date	# of Tickets Sold	Jackpot (Millions)
09/18/20	8,517,586	$20
09/22/20	8,400,426	$22
09/25/20	8,562,146	$24
09/29/20	8,515,402	$32
10/02/20	9,129,793	$41
10/06/20	9,094,373	$50
10/09/20	9,358,502	$60
10/13/20	9,172,316	$69
10/16/20	9,602,462	$77
10/20/20	9,702,164	$86
10/23/20	10,192,498	$97
10/27/20	10,555,378	$109
10/30/20	10,954,959	$118
11/03/20	11,459,802	$129
11/06/20	11,271,257	$142
11/10/20	11,436,317	$152
11/13/20	12,204,690	$165
11/17/20	12,069,849	$176
11/20/20	12,562,599	$188
11/24/20	14,299,681	$200
11/27/20	13,066,892	$214
12/01/20	14,521,615	$229
12/04/20	15,353,620	$244

Continued

12/08/20	15,913,041	$264
12/11/20	16,202,376	$276
12/15/20	17,089,898	$291
12/18/20	18,286,623	$310
12/22/20	20,975,658	$330
12/25/20	20,319,996	$352
12/29/20	25,118,295	$376
01/01/21	30,448,574	$401
01/05/21	45,438,292	$447
01/08/21	58,436,919	$520
01/12/21	85,860,269	$625
01/15/21	113,672,857	$750
01/19/21	130,054,138	$865
01/22/21	183,642,272	$1 B

$$E[X] \le \begin{cases} \dfrac{1}{1-0.99999999669504^{7000000}}, & \text{if } n_1 = 7000000 \\ \dfrac{1}{1-0.99999999669504^{9000000}}, & \text{if } n_1 = 9000000 \end{cases}$$
$$= \begin{cases} 43.7, & \text{if } n_1 = 7000000 \\ 34.1, & \text{if } n_1 = 9000000 \end{cases}$$

Example According to Powerball [4], when the jackpot is low, initially, around six to ten million tickets are sold. The longest streak of non-winning draws in Powerball was 41 consecutive drawings that started on August 6, 2022 and ended on November 7, 2022. The probability of winning the jackpot in Powerball is $p = \dfrac{1}{\binom{69}{5} \cdot 26}$ and the probability of not winning the jackpot is $q = 1 - p = 0.9999999965777$. By Corollary 2,

$$E[X] \le \begin{cases} \dfrac{1}{1-0.9999999965777^{6000000}}, & \text{if } n_1 = 6000000 \\ \dfrac{1}{1-0.9999999965777^{10000000}}, & \text{if } n_1 = 10000000 \end{cases}$$
$$= \begin{cases} 49.2, & \text{if } n_1 = 6000000 \\ 29.7, & \text{if } n_1 = 10000000 \end{cases}$$

Corollary 3 *If* $\{n_k\}$ *is an geometric sequence* $n_k = \alpha^{k-1} n_1$, *for* $k \ge 1$ *and* $\alpha > 1$, *then* $E[X] = q^{-\frac{1}{\alpha-1} n_1} f\left(q^{\frac{n_1}{\alpha-1}}, \alpha\right)$, *where* $f(x,y) = x + x^y + x^{y^2} + x^{y^3} + \cdots$.

Proof

$$\begin{aligned} E[X] &= 1 + q^{n_1} + q^{n_1+n_2} + q^{n_1+n_2+n_3} + \cdots \\ &= 1 + q^{n_1} + q^{(1+\alpha)n_1} + q^{\left(1+\alpha+\alpha^2\right)n_1} + \cdots \\ &= 1 + q^{\frac{\alpha-1}{\alpha-1}n_1} + q^{\frac{\alpha^2-1}{\alpha-1}n_1} + q^{\frac{\alpha^3-1}{\alpha-1}n_1} + q^{\frac{\alpha^4-1}{\alpha-1}n_1} + \cdots \\ &= q^{-\frac{1}{\alpha-1}n_1}\left(q^{\frac{n_1}{\alpha-1}} + \left(q^{\frac{n_1}{\alpha-1}}\right)^{\alpha} + \left(q^{\frac{n_1}{\alpha-1}}\right)^{\alpha^2} + \left(q^{\frac{n_1}{\alpha-1}}\right)^{\alpha^3} + \cdots \right) \end{aligned} \quad \blacksquare$$

Example Here are some values of $E[X]$ for various n_1 and α in Corollary 3: (Table 3)

Table 3. $E[X]$ for different values of n_1 and α.

n_1	$\{n_1, n_2, n_3, \cdots\}$	$E[X]$
6,000,000	$n_k = 1.05^{k-1} n_1$	22.0797
8,000,000	$n_k = 1.075^{k-1} n_1$	16.0259
10,000,000	$n_k = 1.1^{k-1} n_1$	12.6951
10,000,000	$n_k = 1.2^{k-1} n_1$	9.4422
7,000,000	$n_k = 1.5^{k-1} n_1$	7.0673
9,000,000	$n_k = 1.3^{k-1} n_1$	8.2070

3. Conclusion

The winless lottery streak, while disheartening for those experiencing it, is a mathematical phenomenon rooted in the principles of probability, randomness, and statistical patterns. Understanding the mathematics behind lotteries helps us appreciate the rarity of winning and the statistical inevitability of winless streaks. While participants may yearn for a breakthrough, it is crucial to remember that lotteries are primarily games of chance, where each drawing offers an independent opportunity for success. The mathematics of the winless lottery streak reminds us of the complexities of probability and the unpredictable nature of random processes, inviting us to approach lotteries with both hope and a realistic understanding of the odds.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Ross, S. (2023) A First Course in Probability. 10th Edition, Pearson.

[2] https://www.megamillions.com/

[3] https://lottoreport.com/

[4] https://www.powerball.com/