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Space-Time Chaos Filtering for the Incoherent Paradigm for 6G Wireless System Design from Theoretic Perspective

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Abstract

The following material is devoted to the generalization of the chaos modeling to random fields in communication channels and its application on the space-time filtering for the incoherent paradigm; that is the purpose of this research. The approach, presented hereafter, is based on the "Markovian" trend in modeling of random fields, and it is applied for the first time to the chaos field modeling through the well-known concept of the random "treatment" of deterministic dynamic systems, first presented by A. Kolmogorov, M. Born, etc. The material presents the generalized Stratonovich-Kushner Equations (SKE) for the optimum filtering of chaotic models of random fields and its simplified quasi-optimum solutions. In addition to this, the application of the multi-moment algorithms for quasi-optimum solutions is considered and, it is shown, that for scenarios, when the covariation interval of the input random field is less than the distance between the antenna elements, the gain of the space-time algorithms against their "time" analogies is significant. This is the general result presented in the following.

Keywords

Chaotic Fields, Variation (Functional) Derivatives, Quasi-Optimum Algorithms for Chaotic Models

1. Introduction

Chaos modeling of real physical processes, particularly in information engineering, is not a new topic and started long ago (see for example [1] and references therein). However, overview of this topic is apart from the goals of this material.

This material is dedicated to extending the ideas of chaos filtering to space-time processing in wireless (MIMO) communications. This intention was inspired by the following reasons:

- Chaos filtering in the time domain and in its multi-moment fashion as well, has recently been demonstrated [2] [3] to be an efficient approach for weak signal filtering (with SNR values less than 0 dB) with rather high accuracy expressed in a normalized mean square error (NMSE) around units of percentage for rather computationally simple algorithms, such as the Extended Kalman Filter (EKF).
- In general, due to its "deterministic" nature, chaos filtering provides "singularity" features for the filtering algorithms. This allows us to obtain surprisingly "spectacular" characteristics for the accuracy as mentioned above.
- Those features, applied for the filtering and detection of "weak" signals of rather "different physical" nature (see [3] [4]) with reduced processing times are a high motivation for recommending such approach for practical implementation.

One must notice that, nowadays, the modern signal processing algorithms in many areas such as: communications (wireless), Radiodetection, hydro location, measurement and instrumentation, etc. are implemented in space-time (see for example [5] [6] and many, many others).

Though, some natural questions arise: how effective the ideas of chaos filtering for space-time processing might be? What kind of corresponding algorithms might be considered? Where and how can the current algorithms (in time domain, with limited computational complexity) be generalized for space-time processing and so on?

All the above questions are absolutely nontrivial and require accurate study of some related results. But, before that, it is necessary to recall and pay attention to the following:

First, the input signal for the space-time processing algorithms is a random field and there are numerous ways ([5] [6], etc.) to describe it, and it was done and published before (see the list of References). So, how they might be interpreted (if possible?!) for the chaotic field dynamics? For reasons that might be clear a bit later, in the following, the main ideas of S. Tzafestas [7] about the Markovian description of random fields will be applied.

Secondly, the Markovian description of a random field contains causality and Markovian properties both in time and space which are always necessary to consider for practice. Then, how this will affect the computational complexity of the processing algorithms and therefore their practical implementation?

Thirdly, what is the reasonable trade-off between the computational complexity of the space-time algorithms and their accuracy? This certainly depends on the initial formulation of the random field model, generally and implicitly, assumed to be a statistically Non-Gaussian process.

All these aspects of the problem will be analyzed in the present material. The paper is organized as follows. Section 2 is completely dedicated to the Markovian model of the random field. Section 3 presents the generalization of the Stratonovich-Kushner Equation (SKE) for the optimum filtering of chaotic fields. Section 4 is devoted to the quasi-optimum algorithms with details about their multi-time formulation. Finally, Section 5 presents the concluding remarks.

2. Chaotic Modeling of the Random Field: Markovian Approach

As it was already mentioned above, the main goal of the outgoing material is to generalize effective filtering algorithms based on the chaos modeling of the input random signals, to the space-time case, *i.e.* to random fields.

The natural way to do it is to assume, that the nonlinear dynamics of a hypothetic generator of the "chaos" field is also a generalization of the continuous dynamic system (in time domain) to the appropriate dissipative non-linear system, *i.e.* space-time "strange attractor". It seems natural as well, to apply to this hypothetical space-time nonlinear dynamics all the basic assumptions for the stochastic motion of the deterministic dynamic systems as well as the ideas of A. N. Kolmogorov, Max Born, etc. [8].

Those ideas are broadly and well described in the existing literature (see [8] [9], etc.) and therefore they will not be repeated hereafter. At the same time, it is necessary to stress one important point. The most frequent approach for chaos modeling of continuous phenomena is related to the Markovian properties of random processes, which for the space-time case implicitly assumes the causality property not only in time, but in space as well. What does this mean, for example, when it is applied to the MIMO receiving terminals?

It means that the causality takes place not only in time, but also in the aperture of the elements of the MIMO antennas. But, in this case, the assumption of causality of the field along the aperture of the antenna elements, clearly might be neglected (approximately) if the spacing between receiving elements is significantly less than the "space correlation parameter" of the receiving field.

Such kind of limiting assumption, obviously, reduces the complexity of the processing algorithms as well: as causality in both, time and space depends on the results of accuracy of the processing in both channels, space and time together, and so, the processing errors might mutually affect the result.

Though, such models preserve causality and Markovian properties only in time, but not in space [7] [10] [11] and the fundamental idea of S. Tzafestas [7] for random fields as a generalization of infinite dimensional Markovian processes with the values taken from the Gilbert space, might be successfully applied in the following. Moreover, the possible "benefit" of those models might simplify the "bridge" between space-time algorithms and the already obtained filtering algorithms (only in time domain).

Now, it is time to clarify, what does the Markovian model of the random field mean?

First, could it be possible to assume that, it is a simple infinite dimensional generalization of the Markov processes? In order to make true this assumption, let us suppose that the radii-vector of the random field $\boldsymbol{r}$ is represented by "m" points $\boldsymbol{r}_i$ $\left(i=\overline{1,m}\right)$ in some area $\boldsymbol{D}$ and values of the field $x\left(r_i,t\right)=\left\{x_i\left(t\right)\right\}^m$ form an m-dimensional Markovian process (see [7] [10] [11]) described by m-dimensional stochastic differential equation SDE of the type [12]:

$$\dot{x} = \boldsymbol{f}\left(\boldsymbol{x},t\right)+\boldsymbol{\xi}\left(t\right) \tag{2.1}$$

where $\boldsymbol{x} = \boldsymbol{x}(t)$, $\boldsymbol{x}(t_0) = \boldsymbol{x}_0$, $\boldsymbol{f}(\cdot)$ is a nonlinear vector function and $\boldsymbol{\xi}\left(t\right)$ is an m-dimensional vector of the exitation "white noises" with the matrix mxm of its intensities $\boldsymbol{N}(t)$.

If "m" in (2.1) tends to infinity in the way that all the above-mentioned points uniformly and continuously cover the area $\boldsymbol{D}$, then (2.1) might be rewritten by the following SDE:

$$\dot{x}\left(t,\boldsymbol{r}\right)=\boldsymbol{f}\left(t,\boldsymbol{r},\boldsymbol{x}\right)+\boldsymbol{\xi}\left(t,\boldsymbol{r}\right) \tag{2.2}$$

with $\boldsymbol{x}(t_0, \boldsymbol{r}) = \boldsymbol{x}_0(\boldsymbol{r})$, $\xi\left(t,\boldsymbol{r}\right)$ is a "hypothetical" field of the external random white noise excitations.

One can see that the model of the random field from (2.2) considers the coordinate vector "$\boldsymbol{r}$" as a **parameter**.

Considering the natural and practical assumption of a limited average energy of the field in the area $\boldsymbol{D}$, then the space of functions $\boldsymbol{x}(t, \boldsymbol{r})$ in a fixed time instant "t" is of Gilbert type and $\boldsymbol{x}(t, \boldsymbol{r})$ is a function of "t".

This is the essence of S. Tzafestas' idea of the Markovian approach of the random field modeling [7], which was further developed and generalized by A. Shmelev in [10] [11].

Thanks to this idea, it was possible hereafter to apply all the "richness" of the Markovian methods of the optimum filtering: Stratonovich-Kushner Equations (SKE), A. Kolmogorov-Max Born ideas of the stochastic dynamics of deterministic systems [8] [9], etc. to the chaotic filtering of the fields (see [2] [4] and references therein).

Applying all this to the chaotic random fields might be done in a rather straightforward way from the previously developed chaotic models for random processes [2] [4] [9]. For chaos models [8] [9] the random force $\boldsymbol{\xi}\left(t,\boldsymbol{r}\right)$ must be delta-correlated (in time) in the way:

$$\overline{\xi\left(t,\boldsymbol{r}\right)\xi\left(t',\boldsymbol{r}'\right)}=\kappa\left(t,\boldsymbol{r},\boldsymbol{r}'\right)\delta\left(t-t'\right), \tag{2.3}$$

with matrix of intensities $\boldsymbol{N}(t, \boldsymbol{r}, \boldsymbol{r}')$ being "very small" according to the idea of Kolmogorov-Born to "provide" the stochastic dynamics of the deterministic continuous nonlinear system:

$$\dot{x}\left(t,\boldsymbol{r}\right)=\boldsymbol{F}\left(t,\boldsymbol{r},\boldsymbol{x}\right), \tag{2.4}$$

and "generate" the stochastic chaos [8] [9] [13].

The idea of Kolomogorov-Born means, that the "coordinates" of the chaos trajectory might not be determined precisely, but only with a small error which is actually "provided" by the process noise. In simple words it is a "statement" of the Kolmogorov-Born stochastic principle,and it is a "bridge" between ODE (2.4) and the SDE (2.2). Though, in the following, everything which depends on SDE (2.2), depends (in the same sense) on (2.4), modified with the small process noise of (2.3) type.

In the following, the small external "noises" which are "artificially" inserted into the chaos model (2.4) are called "process noises". The vector-function in (2.4) and the vector-function in (2.2) are not the same, besides formally those equations are similar.

To clarify this "controversial" issue, imagine that for the Non-Gaussian random field the m-dimensional Markov model, in the form of SDE (2.1), was "synthesized" with a priority given PDF $W(\boldsymbol{x})$. In [2] [3] [13] etc. it was already discussed how to do this and so, it will not be repeated hereafter. By making $m\to\infty$, SDE (2.1) is transformed into SDE (2.2), where $\boldsymbol{r}$ is a fixed parameter.

Then if $\xi(t,\boldsymbol{r}) = 0$ in (2.2), then it is an ordinary differential equation (ODE), which follows from (2.2). One might ask if this ODE is "obliged" to produce the same chaos as it is assumed from (2.4). Definitely not! So, the logic of how to generate chaos must be chosen in a somehow opposite way.

First, assuming "homogeneity" of the field in all $\{r_i\}_1^m$, one has to select the chaotic attractor algorithm for the given PDF on each $\{r_i\}_1^m$ (see examples in [2] [13], etc.)

Secondly, with the hypothesis of a homogeneous field in all $\{r_i\}_1^m$, following (2.2), the nonlinearity of the attractor can be applied as $\boldsymbol{F}(t,\boldsymbol{r},\boldsymbol{x})$ for ODE (2.4), where, $\boldsymbol{F}(\cdot)$ is determined by the attractor type: for example, Lorenz, Chua, Rossler, etc.

Thirdly, by selecting the intensity of the process noise for the "observable" component in the equation of the attractor, as it was recommended in [2] [13], one transforms the ODE (2.4) into the corresponding SDE of the random chaotic field model:

$$\dot{\boldsymbol{x}} = \boldsymbol{F}(t,\boldsymbol{r},\boldsymbol{x}) + \xi(t,\boldsymbol{r}). \tag{2.5}$$

The equation (2.5), which is formally similar to (2.2), might be applied, as well as SDE (2.2), to the optimum synthesis procedure of the chaotic filtering algorithms due to the generalized Stratonovich-Kushner Equation (SKE).

To conclude this part, it might be stated that, in any case, the above material does not pretend to be a "rigorous" explanation for the introduction of chaos into the random field modeling. Not at all!

It is just a qualitative illustration of the basics of Markovian modeling for random chaotic fields and it only provides a natural approach to prepare the synthesis of the optimum filtering algorithms (see next section).

3. Generalization of the Stratonovich-Kushner Equations (SKE) for the Chaotic Filtering: Quasi-Optimum Solutions

3.1. Introductive Comments

The theory of optimum filtering, detection and estimation has a rather long history (see [5] [14]-[16] and references therein) and so, there is no need to repeat it hereafter. Applying as a criterion, for the optimal processing, the well-known criterion of the maximum a posteriori probability density function $W_{ps}(\cdot)$, or its probabilistic functional, the whole theory of the optimum filtering, detection, estimation, etc. (and its quasi-optimum modifications, see [5] [14]-[16] etc.) was created and developed. This theory is based on the differential equation for the time evolution of $W_{ps}(\cdot)$, applying the a priori characteristics of the Markovian model of the observed stochastic process.

This was already done, and it is called the Stratonovich-Kushner Equation (SKE). The interested reader might find numerous references related to it and its quasi-optimum modifications (see also the already mentioned References), for convenience it is introduced in (3.3). For the chaotic filtering based on the Markovian approach, the related modifications of the SKE (3.1.a) and its quasi-optimum solutions were also detailed in rather numerous sources [2] [5] [13], etc. It is worth noticing that all this was done for the time domain.

The current material is dedicated to random fields (space-time) with application of the Markovian approach for their chaotic modeling as well. So, what are the specific features of this case? Actually, they are well known (see [7] [10] [11]) and they will be briefly outlined hereafter. One must notice that when $m \to \infty$, and the SDE (2.2) is transformed into SDE (2.5) the probability density function (PDF) of its solution is transformed as well to the probability density functional $\boldsymbol{P}(\boldsymbol{x}(\boldsymbol{r}, t))$ - (PDF_n).

So, the way of stochastic description of random fields $\boldsymbol{x}(\boldsymbol{r}, t)$ is drastically different from the random processes $\boldsymbol{x}(t)$: it is a "functional" description by means of probabilistic functionals *i.e.* "functions of functions $\boldsymbol{x}(\boldsymbol{r}, t)$". Though, all the derivatives in terms of the Markovian approach: ordinary and partial derivatives, integrals, etc., start to be "functional (variation) derivatives", "functional integrals", etc.—components of the so-called well known long ago functional calculus (see, for example, [17]) for the fields[1].

Note, that for the current material all the forthcoming equations for the a priori and a posteriori probability density functionals (PDF_n) have to be considered somehow in "symbolic" sense in order not to be involved with the "tiny" mathematical details which actually are not so important in the following.

This statement is true both for the scalar and the general vector random fields (for example MIMO case) $\boldsymbol{x}(\boldsymbol{r},t) = \left[x_1(\boldsymbol{r},t), \cdots, x_n(\boldsymbol{r},t)\right]^{\mathrm{T}}$ both stochastic and chaotic. Here "T" means transpose.

Returning to the Markovian approach, one has to remember, that a priori description of the corresponding PDF_n - $\boldsymbol{P}_{pr}(\boldsymbol{x}, t)$, must be presented in the way similar to the Fokker-Plank-Kolmogorov Equation (FPKE) for m-dimensional Markovian processes. This equation, in the following, will also be called as Fokker-Plank-Kolmogorov Equation (FPKE) [12] [14] [15] but with "functional derivatives" and is given by:

[1]Please do not mix this term with the "Functional Analysis in Mathematics".

$$\begin{aligned}\frac{\partial \boldsymbol{P}_{pr}(\boldsymbol{x},t)}{\partial t} &= \sum_{i=1}^{n}\int_D \frac{\delta}{\delta x_i(\boldsymbol{r})}\left[f_i(t,\boldsymbol{r},\boldsymbol{x})\right]\boldsymbol{P}_{pr}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{r} \\ &+\frac{1}{2}\sum_{i,j}^{n}\iint_D \kappa_{ij}(t,\boldsymbol{r}_1,\boldsymbol{r}_2)\frac{\delta^2 \boldsymbol{P}_{pr}(\boldsymbol{x},t)}{\delta x_i(\boldsymbol{r}_1)\delta x_j(\boldsymbol{r}_2)}\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2 \\ \boldsymbol{P}(\boldsymbol{x},t_0) &= \delta\left[\boldsymbol{x}(\boldsymbol{r})-\boldsymbol{x}_0(\boldsymbol{r})\right],\end{aligned} \tag{3.1}$$

where $\frac{\delta}{\delta x_i(\boldsymbol{F})}(\cdot)$ is a functional derivative, and $\int_D(\cdot)$ is a functional integral along the area $\boldsymbol{D}$. The operator for $\boldsymbol{P}_{pr}(\boldsymbol{x}, t)$ in (3.1) is linear though, the interpretation of (3.1) can be seen in the way:

$$\frac{\partial \boldsymbol{P}_{pr}(\boldsymbol{x},t)}{\partial t} = L\left\{\boldsymbol{P}_{pr}(\boldsymbol{x},t)\right\}, \tag{3.2}$$

where $L\{\}$ is a linear operator with functional derivatives. This form of the FPKE might be applied later, but notice once more, that all those formulas (for the moment!) have to be considered as "symbolic" and are just an illustration of the "idea" how the input signal must be treated. That is why, in the following, the way how to make concrete calculus is not discussed. Comparing FPK (3.1) with the FPK for the Markovian processes (see for example [12] [14] [15]) and (3.3) one can easily see the complete formal similarity.

$$\frac{\partial \boldsymbol{P}_{pr}(\boldsymbol{x},t)}{\partial t} = -\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left[f_i(\boldsymbol{x},t)\boldsymbol{P}_{pr}(\boldsymbol{x},t)\right]+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2}{\partial x_i \partial x_j}\left[\kappa_{ij}\boldsymbol{P}_{pr}(\boldsymbol{x},t)\right]. \tag{3.3}$$

3.2. Generalized SKE for the $P_{ps}(x, t)$

In the previous section the complete similarity between FPKE for Markovian processes and fields was presented (in symbolic way for the case of fields). Absolutely the same issue takes place for the SKE for $\boldsymbol{P}_{ps}(\boldsymbol{x}, t)$ which is nothing else, but an algorithm for optimum filtering (see [5] [14]-[16], etc.) with the criterion of maximum a posteriori probabilistic functional. Omitting details about the development of the SKE (which almost coincide with those for the Markovian processes[2]) it finally yields:

$$\frac{\partial \boldsymbol{P}_{ps}(t,\boldsymbol{r},\boldsymbol{x})}{\partial t} = L\left\{\boldsymbol{P}_{ps}(t,\boldsymbol{r},\boldsymbol{x})\right\}+\boldsymbol{P}_{ps}(t,\boldsymbol{r},\boldsymbol{x})\left[\boldsymbol{F}(t,\boldsymbol{x})-\left\langle \boldsymbol{F}(t,\boldsymbol{x})\right\rangle\right], \tag{3.4}$$

where

$$\boldsymbol{F}(t,\boldsymbol{x}) = \iint_D Q_n(\boldsymbol{r},\boldsymbol{r}')\boldsymbol{S}(t,\boldsymbol{r}',\boldsymbol{x})\left[\boldsymbol{y}(t,\boldsymbol{r})-\frac{1}{2}\boldsymbol{S}(t,\boldsymbol{r},\boldsymbol{x})\right]\mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{r}', \tag{3.5}$$

$$\boldsymbol{y}(t,\boldsymbol{r}) = \boldsymbol{S}(t,\boldsymbol{r},\boldsymbol{x}(t,\boldsymbol{r}))+\boldsymbol{n}(t,\boldsymbol{r}), \tag{3.6}$$

$\boldsymbol{n}(t, \boldsymbol{r})$—Gaussian white noise field (δ-correlated in time) with covariation function:

$$\boldsymbol{K}_n(t,t',\boldsymbol{r},\boldsymbol{r}') = \boldsymbol{K}_n(\boldsymbol{r},\boldsymbol{r}')\delta(t-t'), \tag{3.7}$$

[2]Certainly taking into account the difference between partial and functional derivatives [10] [11].

$\boldsymbol{S}(t, \boldsymbol{r}, \boldsymbol{x}(t, \boldsymbol{r}))$—Generally is assumed as a desired signal which depends on the desired field $\boldsymbol{x}(t, \boldsymbol{r})$—object of filtering. $Q_n(\boldsymbol{r}, \boldsymbol{r}')$ can be found from the integral equation:

$$\int_D Q_n(\boldsymbol{r},\boldsymbol{r}')\boldsymbol{K}_n(\boldsymbol{r},\boldsymbol{r}')\mathrm{d}\boldsymbol{r}' = \delta(\boldsymbol{r}-\boldsymbol{r}'), \tag{3.8}$$

$\langle \boldsymbol{F}(t,\boldsymbol{x}) \rangle$ means the averaging procedure for $\boldsymbol{F}(t, \boldsymbol{x})$ over $\boldsymbol{P}_{ps}(t, \boldsymbol{r}, \boldsymbol{x})$.

For convenience let us present the SKE version for Markovian processes in the way

$$\frac{\partial \boldsymbol{P}_{ps}(\boldsymbol{x},t)}{\partial t} = -L_{pr}\{\boldsymbol{P}_{ps}(\boldsymbol{x},t)\} + \frac{1}{2}\left[\boldsymbol{F}(\boldsymbol{x},t) - \int_{-\infty}^{\infty} \boldsymbol{F}(\boldsymbol{x},t)\boldsymbol{P}_{ps}(\boldsymbol{x},t)\mathrm{d}\mathbf{x}\right]\boldsymbol{P}_{ps}(\boldsymbol{x},t), \tag{3.9}$$

where $\boldsymbol{P}_{ps}$ is the a posteriori Probability Density Function which has to be found by solution of the SKE [12] [14] [15].

The reader can compare the above presented sketch of the development of SKE, using $\boldsymbol{P}_{ps}(t, \boldsymbol{r}, \boldsymbol{x})$, for Markovian random fields with the result of the standard procedure for Markov processes or chaos presented in [5] [14]-[16] and find out, that they are completely identical, for sure taking into account that the formulas for processes are "physical" and symbolic for fields. The latter offers really spectacular opportunities to obtain a powerful way to drive many results already obtained for processes, to fields, (with certain caution related to the above made comments regarding derivatives).

Now, it is time to make another main statement. The SKE as an optimum algorithm for filtering, estimation, identification, etc. (either for processes, fields, chaos, etc.) practically never (except for the Gaussian case) offers exact analytical solutions. Therefore, for application in Non-Gaussian cases quasi-optimum algorithms are always recommended.

The final comments here might help to understand why chaotic filtering (for both, processes and fields) offers high accuracy for low SNR. For this matter return once more to SKE (3.1), from where it can be seen that the SKE is made of two parts, the first is related to a priori data and the second to results of the filtering processes (a posteriori data). When the desired signal is a random process (field) then, the instantaneous value of the a priori part is certainly random (uncertain) and the analysis of the a posteriori part "compensates" this uncertainty of the a priori data and gives acceptable accuracy when $\mathrm{SNR} \gg 1$.

When the model of the desired signal is chaotic (almost deterministic) then, the a priori part is somehow "predefined" or almost certain (known). Then, a posterior analysis is "practically" not needed, and SNR might be less than one (0 dB). Anyway, its influence on the filtered signal is practically negligible. This is in short words an illustration of the effect of "singularity" of chaos filtering. A more rigorous explanation can be found in [13].

Note that in the framework of the statistical theory of communications, the singular scenarios are mainly considered to be non-practical when the accuracy does not depend on the SNR. But, if the models of the desired signals are somehow "shifted" from the random concept to the almost deterministic concept, like chaotic, then those scenarios seem to be rather opportunistic. This gives a "chance" to determine (identify, classify) many signals, images, etc. in "hard" conditions such as: low SNR, many targets, many users, etc. (see [18] [19] for example).

As a "last" thought, one might ask: what for these "symbolic equations" are presented if they are actually not real for calculus!? The answer is: to take advantage of the analogy of the real development of their analogues in time domain, which are absolutely real! (see the following) and apply them for concrete scenarios.

3.3. Some Quasi-Optimum Solutions for Field Filtering Problems

As it was mentioned in the previous subsection the SKE is a very important approach for optimum filtering however, it does not give any opportunity to get solutions for non-Gaussian scenarios, for which quasi-optimum (suboptimum) algorithms might be much more useful. The list of suboptimum solutions is rather long[3] and the reader can find a rather "contemporaneous" version in [13]. Also, in [13], an experimental study was presented, and showed that in the framework of the trade-off between computational complexity and filtering accuracy, the Extended Kalman Filter algorithm (EKF) seems to be a reasonable option. In the following, the EKF algorithm is presented for vector fields (both physical and chaotic) with the help of functional derivatives. It follows then:

$$\begin{aligned}\frac{\mathrm{d}\hat{\boldsymbol{x}}_k(\boldsymbol{\rho})}{\mathrm{d}t} &= -f_k(t,\boldsymbol{\rho},\hat{\boldsymbol{x}}_k)+\kappa_1^k(t,\boldsymbol{\rho})+\int_D \frac{\delta F(t,\boldsymbol{x})}{\delta x_i(\boldsymbol{r})}K_{ik}(\boldsymbol{r},\boldsymbol{\rho})\mathrm{d}\boldsymbol{r}\\ &\quad -\frac{1}{2}\iint_D \frac{\delta^2 f_k(t,\boldsymbol{\rho},\hat{x})}{\delta x_i(\boldsymbol{r}_1)\delta x_j(\boldsymbol{r}_2)}K_{ij}(\boldsymbol{r}_1,\boldsymbol{r}_2)\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2\\ \frac{\mathrm{d}K_{l,m}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)}{\mathrm{d}t} &= -\int_D\left[\frac{\delta f_i(t,\boldsymbol{\rho}_1,\hat{x})}{\delta x_i(\boldsymbol{r})}K_{lm}(\boldsymbol{r},\boldsymbol{\rho}_2)+\frac{\delta f_m(t,\boldsymbol{\rho}_2,\hat{x})}{\delta x_i(\boldsymbol{r})}K_{il}(\boldsymbol{r},\boldsymbol{\rho}_1)\right]\mathrm{d}r\\ &\quad +\iint_D \frac{\delta^2 F(t,\hat{x})}{\delta x_i(\boldsymbol{r}_1)\delta x_j(\boldsymbol{r}_2)}K_{il}(\boldsymbol{r}_1,\boldsymbol{\rho}_1)K_{jm}(\boldsymbol{r}_2,\boldsymbol{\rho}_2)\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2\\ &\quad +\kappa_2^{lm}(t,\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\end{aligned} \tag{3.10}$$

where $\hat{\boldsymbol{x}}_k(\boldsymbol{\rho})$ is an averaged a posteriori estimation of the k-th component of the vector field at the "$\boldsymbol{\rho}$" coordinate and the "t" time moment. In other words it is the result of the filtering process; $\kappa_{ij}(t,r_1,r_2)$ is a space covariation function for the external vector stochastic force (noise) delta-correlated in time, in the SDE (2.3) and (2.5); $K_{l,m}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$ is a posteriori covariation matrix of the filtered field (which as usual is an error matrix or an evaluation of the filtering accuracy, for each component of the vector field!).

Once more, the above equations (3.10) are completely symbolic and are presented just to illustrate that they are identical to the EKF algorithm for random processes in continuous time (assuming, of course, the difference in notations

[3]Hereafter the "analogy" between the "physical" representation of the algorithms for processes and "symbolic" for the fields will be significantly applied.

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for the derivatives, see [2] [13] and the references therein). Though, there is a clear opportunity to establish a "bridge" between the results for processes and fields (never mind random of chaotic) and apply the well-known results to new scenarios. In this context the next section will be dedicated to an application of the above results (certainly adapted) to MIMO terminals.

4. Chaos Filtering in MIMO Receiving Antennas

4.1. Problem Statement

One must notice (as it was already stressed above) that all the presented material is considered in the framework of the Incoherent paradigm for 6G in NOMA-RIS-MIMO wireless communications, in the corresponding frequency bands [18] [20] [21]. The purpose of the filtering might be focused on the "identification" of the NOMA transmission users by means of the chaos filtering approach. Note that, the first attempt to do it for SISO case was presented in [18] with a rather high efficiency due to the "determinism" of the chaos modeling of the users' received signals at the terminal. Tentatively, the same idea is behind the MIMO case but for the space time field scenario.

The necessary theoretical results were presented above, including the rational quasi-optimum chaos filtering algorithms as the Extended Kalman Filter. In order to properly adjust the previous theoretical results, it is reasonable to make some preliminary comments.

- In the framework of the incoherent paradigm for 6G, it is reasonable to assume (as a first approximation) that, according to wave propagation conditions (in the GHz band), the fading on each "artificial trajectory" of the MIMO channel after its Orthogonalization (see [19]) can be neglected [18] [20] and the received signal is:

$$y(t,\boldsymbol{r}) = A\cos\left[\omega_0 t + \boldsymbol{x}(\boldsymbol{r})\right] + \boldsymbol{n}(t,\boldsymbol{r}), \tag{4.1}$$

where $\boldsymbol{n}(t, \boldsymbol{r})$ is a noise, delta correlated in time and space; $\boldsymbol{x}(\boldsymbol{r})$ is a vector of random chaos field, modeled as SDE (2.5) with a "weak" processes noise. Approximately SDE (2.5) is:

$$\frac{\mathrm{d}\boldsymbol{x}(\boldsymbol{r})}{\mathrm{d}t} = 0. \tag{4.2}$$

- Regarding $\boldsymbol{x}(\boldsymbol{r})$ and MIMO antenna parameters, two asymptotic assumptions can be taken:

a) If the interval between antenna elements Δ_a for a linear planar antenna (PA) is much less than τ_f—a posteriori space correlation interval for the field $\boldsymbol{x}(\boldsymbol{r})$, then for $\boldsymbol{x}(\boldsymbol{r})$ the antenna aperture is somehow "solid" or "complete" and actually there is no MIMO, only a SISO scenario, and all the results related to chaos filtering (see [2] [13], etc.) are valid for this case.

b) If $\Delta_a \sim \tau_f$, then chaos filtering is actually "space-time" and the latter must influence the filtering accuracy. The interested reader might have already figured out, that the Markovian concept of random field modeling might be applied only for each of the antenna elements of the aperture, if and only if, they are statistically independent. Otherwise, all the volume of the $\boldsymbol{r}_i$ coordinates have to be taken into account, *i.e.* admit field causality. Though, the algorithm (3.10) must be concretized for this case.

An additional assumption also must be taken into account, when $\Delta_a \geq \tau_\beta$, then the MIMO antenna elements are decorrelated and the difference $\boldsymbol{r}_i$ - $\boldsymbol{r}_j$ is nothing else but a difference of the location of the antenna elements.

As processing is assumed to be held on each element of the MIMO antenna (*i.e.* $\{\boldsymbol{r}_j\}$ are known and fixed), then the processing algorithms have to be presented in digital way for the antenna elements and then summarized for all the area $\boldsymbol{D}$. The latter gives a hope, that this "last stage", can be considered as something like to the linear incoherent diversity addition that, might provide an additional increment on the filtering's accuracy (diversity effect). This issue deserves to be detailed.

As was shown in [18], even for the SISO case, the normalized MSE (mean square error) for SNR ~ 0 dB is characterized with small percentage values. With the effect of diversity addition, it might diminish even to fractions of the percentage! Note that, an accurate analytical study of this, is hardly possible, but anyway, some evaluations might be done.

4.2. Quasi-Optimum Algorithms for the MIMO Terminal

Taking into account the concretization scenario in the form (4.1), it is possible to rewrite the quasi-optimum algorithms (3.10) in the following way:

$$\begin{aligned} \frac{\mathrm{d}\hat{\boldsymbol{x}}(t,\boldsymbol{\rho})}{\mathrm{d}t} &= -\frac{A}{N_0}\int_D \boldsymbol{y}(t,\boldsymbol{r})\boldsymbol{B}(\boldsymbol{r},\boldsymbol{\rho})\sin\left[\omega_0 t+\hat{\boldsymbol{x}}(t,\boldsymbol{r})\right]\mathrm{d}\boldsymbol{r} \\ \frac{\mathrm{d}\boldsymbol{B}(\rho_1,\rho_2)}{\mathrm{d}t} &= -\frac{A}{N_0}\int_D \boldsymbol{y}(t,\boldsymbol{r})\boldsymbol{B}(\boldsymbol{r},\rho_1)\boldsymbol{B}(\boldsymbol{r},\rho_2)\cos\left[\omega_0 t+\hat{\boldsymbol{x}}(t,\boldsymbol{r})\right]\mathrm{d}\boldsymbol{r} \end{aligned} \tag{4.3}$$

The second equation in (4.3) can be additionally simplified for the band-pass equivalent signal processing approach, if all "high-frequency" components are neglected.

$$\frac{\mathrm{d}\boldsymbol{B}(\rho_1,\rho_2)}{\mathrm{d}t} = -SNR\int_D \boldsymbol{B}(\boldsymbol{r},\rho_1)\boldsymbol{B}(\boldsymbol{r},\rho_2)\mathrm{d}\boldsymbol{r}\ , \tag{4.4}$$

here $SNR = -\dfrac{A^2}{2N_0}$ is signal to noise ratio on each element.

Then, for each element of the MIMO antenna, the digital version of (4.3) is

$$\begin{aligned} \frac{\mathrm{d}\hat{x}_i(t)}{\mathrm{d}t} &= -\frac{A}{N_0}\sum_j y_j(t)B(t,i-j)\sin\left[\omega_0 t+\hat{x}_j(t)\right] \\ \frac{\mathrm{d}B(t,l)}{\mathrm{d}t} &= -SNR\sum_{m=-\infty}^{\infty} B(t,m)B(t,l-m) \end{aligned} \tag{4.5}$$

Please note, that the "*m*'s" here are simply the number of antenna elements, but "*l*" denotes actually, the "distances Δ_a" taken between the corresponding antenna elements and might be either, positive or negative. The "addition" operator in the second equation in (4.5) is concretely, the linear incoherent diversity addition operation and $B(t,i-j)=Q$ is the number of "effective" diversity "branches" taking part in the addition operation and can substitute "m" in (4.5).

If the "branches" are assumed to be homogeneous and statistically independent, then as a first approximation (roughly!) the resulting MSE can be considered as:

$$\mathrm{MSE}_{\Sigma} \sim \left(\mathrm{MSE}_i\right)^Q . \tag{4.6}$$

If $Q \gg 1$ it can be seen from (4.6), the resulting MSE_{Σ} might be negligible or very small.

In some sense, this statement might be seen beforehand, but one more important question arises: when is it reasonable to apply space-time processing? (see comments above as well). The answer is evaluated according to the degree of reduction of the MSE with the chaos space-time filtering on each antenna element compared with the MSE for the only time filtering case.

In order to make an analytical illustration of this, let us assume that the covariation of the fields, along the antenna elements, is exponential:

$$K(m) \sim \exp\left(-\beta|m|\right), \tag{4.7}$$

where $1/\beta$ is the covariation interval.

As before, considering two limiting cases $\beta^{-1} \gg \Delta_a$ and $\beta^{-1} \ll \Delta_a$, it follows that, physically one is dealing (for the first case) with a complete (continuous) antenna aperture and there is no sense to use space-time processing (see above). But for the second case ($\beta^{-1} \ll \Delta_a$) the situation is completely different.

Introducing for each antenna element the parameter $g^{-1} = \frac{\mathrm{MSE}_{\mathrm{T}}}{\mathrm{MSE}_{\mathrm{S/T}}}$, where

$\mathrm{MSE}_{\mathrm{T}}$ is the mean square error of the filtering for pure time processing and $\mathrm{MSE}_{\mathrm{S/T}}$ is the same, but for the space-time case, one can see, that g^{-1} is nothing else but a metric of how much the MSE is reduced due to the space-time processing. So, when g takes its minimum value, the gain of the space-time processing is maximum.

It might be shown that, when it is possible to neglect the time fluctuations of the field (fadings), one has:

$$g_{\min} = \sqrt{1-\exp(-2\beta)} . \tag{4.8}$$

From (4.8) it follows that, when $\beta \ll 1$ (for example $\beta \sim 0.1$) then, the gain of the space-time filtering (*i.e.* the MSE) might be more than twice, which is really outstanding taking into account that the real gain is $\sim 2^Q$.

4.3. Multi-Moment Approach

As it follows from the earlier presented data, even a rather "modest" (no more than twice) improvement for the MSE_i of the filtering on each antenna element might be a significant improvement on the resulting MSE_Q due to the diversity effect.

The latter might be encouraging for intentions to, even lightly, improve the filtering accuracy of the space-time processing on the antenna elements. Tentatively, one of the possible trends might be the "multi-moment" approach presented in [13], etc., which showed very good results for "pure time" processing.

Remind that the essence of the multi-moment approach is the introduction of the time instants [13]: $t_1 < t_2 < \cdots < t_n$, and the consideration of $\boldsymbol{x} = \boldsymbol{x}_i(t_i)$, $i = \overline{1,n}$ which forms a new vector $\boldsymbol{x}(t) = \left[\boldsymbol{x}(t_1), \cdots, \boldsymbol{x}(t_n)\right]^{\mathrm{T}}$, as a new "input" to be processed in an "aggregate" way.

The possible "improvement" of the filtering accuracy might follow from the aggregate processing of the field $\boldsymbol{x}(t, \boldsymbol{r})$, ($\boldsymbol{r}$ fixed), of "$\boldsymbol{n}$" instants (some kind of "time diversity") (see [2] [13]).

For this matter, it is reasonable to invoke Equations (3.1)-(3.4). The first two are dedicated to the description of the a priori Probability Functional, $\boldsymbol{P}_{PR}(t, \boldsymbol{x})$, of the input chaotic field and the last one to the generalized SRE processing of chaotic space-time filtering.

Applying then the straightforward analogy between the generalized SRE for filtering in partial derivatives and the variation (functional) derivatives (see section 3 above) one can write the multi moment generalized SRE in the way

$$\frac{\partial \boldsymbol{P}_{ps}(t, \boldsymbol{r}, \boldsymbol{x})}{\partial t_1 \cdots \partial t_n} = L_{pr}\left\{\boldsymbol{P}_{ps}(t, \boldsymbol{r}, \boldsymbol{x})\right\} + \boldsymbol{P}_{ps}(t, \boldsymbol{r}, \boldsymbol{x})\left[\boldsymbol{F}(t, \boldsymbol{x}) - \left\langle \boldsymbol{F}(t, \boldsymbol{x})\right\rangle\right], \tag{4.9}$$

with $t = \left[t_1, t_2, \cdots, t_n\right]^{\mathrm{T}}$.

It is obvious (and there is no need for additional comments), that the real multi-time space-time filtering (4.9), even in the quasi-optimum fashion, might be rather complex in computational sense and therefore impractical. But for each antenna element and applying the simplest EKF algorithm (for two-moment scenario) the possible improvement for the MSE_i (and some illustrations about it) can be seen from the following tables [22].

In order to interpret correctly the data from the following tables, some comments are necessary:

- The filtering (identification) experiment for was focused on 2 scenarios: one for filtering 52 orthogonal carriers in presence of AWGN (≠1)—**Table 1**; and the other for 52 non-orthogonal carriers (≠2)—**Table 2**.
- All the signals were processed in a regime of low Signal to Interference plus Noise ratios $\mathrm{SINR} = \dfrac{\mathrm{Signal}}{\mathrm{Interference} + \mathrm{AWGN}}$ with sample values −3 dB and −10 dB; AWGN denotes the white Gaussian noise of the channel; the interferences were considered as the total interference signal of all carriers, which do not depend on the concrete selected signal for filtering.
- For modeling each carrier, the *x*-components of the chaotic attractors Rossler and Chua were considered [13]. For the filtering (by means of the Extended Kalman Filter, EKF) the "process noise" concept was applied with intensity $Q \ll 1$ (see the details in [13]).
- 1MM and 2MM, denote the one moment and the two moment filtering ap-

proaches.

Table 1. Chaos Filtering for 52 orthogonal carriers in presence of AWGN.

SINR	EKF (Rossler)		EKF (Chua)	
	−3 dB	−10 dB	−3 dB	−10 dB
1MM	0.0119	0.0504	0.0143	0.0730
2MM	0.0107	0.0487	0.0134	0.0677

Table 2. Chaos Filtering for 52 non-orthogonal carriers in presence of AWGN.

SINR	EKF (Rossler)		EKF (Chua)	
	−3 dB	−10 dB	−3 dB	−10 dB
1MM	0.0109	0.052	0.0139	0.0705
2MM	0.0103	0.047	0.0138	0.0672

As it can be seen from the tables, for the simple EKF algorithm with low SINR (less than 0 dB), the chaotic filtering operates with an MSE_i of less than 10% (on each antenna element), *i.e.* it gives rather good results for extremely tough scenarios. Next, the difference in the filtering efficiency between 1 MM and 2 MM approaches is not significant (less than 10%), but taking into account the diversity effect for MSE_Q, it might be seen as an important factor (see above) for the MIMO space-time processing with $Q \gg 1$.

5. Concluding Remarks

The above presented material is dedicated to one of the possible theoretical trends for the space-time processing of random fields in communications and related areas. It can be called "Markovian" and it was first proposed by S. Tzafestas in the early '70s of the last century and it was further developed by A. Shmelev, etc. (see the list of references to this material).

In this paper, this approach was applied to the so-called chaotic models, which according to the ideas of A. Kolmogorov, Max Born, etc. might be also "treated" in the framework of the Markovian approach. Therefore, it was found reasonable to apply the ideas of chaotic filtering (rather effective in the time domain) to space-time chaos filtering, particularly for the MIMO-type systems.

Though, this material can be considered as well as an opportunistic tool, which might be adjusted for the Incoherent paradigm in 6G MIMO systems design in the sense, that applying chaos filtering for space-time processing gives a "hope", for example, to considerably reduce the MSE for user's identification in 6G transmissions in NOMA regime, which gives additional "chance" for NOMA to be used in 6G.

The latter, as it was shown, follows from the properties of the space-time chaos filtering together (maybe) with multi-time properties of chaos filtering. These spectacular features were demonstrated above.

Finally, including the space-time chaos filtering into the list of the Incoherent paradigm "tools" [19] [21] could make it more attractive engineering practice.

But, it is very important at the same time, that one has to notice that the presented material is (in some sense) one of the first steps and, for sure, requires further significant development.

In this regard, it is time to make some important remarks which were not touched on above.

- In order to apply all the presented material to the "Incoherent paradigm for 6G" it is necessary to ensure, that for the frequency bands assigned to 6G, the Markovian model for the propagation channel (see above) is valid.
- Next, what it is necessary to prove is, whether or not the concept of the double selectivity of the channel might be applied?

For the moment some physical analysis of those issues shows that this is correct, but careful studies must be carried out as well.

Then, there is real hope for taking advantage of the spectacular characteristics of the chaos filtering accuracy.

As time passes and the demand and requirements for wireless communications from the users grow drastically, the optimum solutions for space-time filtering, as well as its quasi-optimum versions (see sections 2 and 3), have to be constantly and critically revised in order to reduce the processing errors (MSE) and work on optimization strategies to reduce the computational complexity taking into account the broad application and modern advances in the AI and cheap design.

The Incoherent paradigm for 6G and beyond certainly might provide great opportunities in this field.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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