

# Diffusion Equations of the Electric Charges and Magnetic Flux

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## Abstract

Innovative definitions of the electric and magnetic diffusivities through conducting mediums and innovative diffusion equations of the electric charges and magnetic flux are verified in this article. Such innovations depend on the analogy of the governing laws of diffusion of the thermal, electrical, and magnetic energies and newly defined natures of the electric charges and magnetic flux as energy, or as electromagnetic waves, that have electric and magnetic potentials. The introduced diffusion equations of the electric charges and magnetic flux involve Laplacian operator and the introduced diffusivities. Both equations are applied to determine the electric and magnetic fields in conductors as the heat diffusion equation which is applied to determine the thermal field in steady and unsteady heat diffusion conditions. The use of electric networks for experimental modeling of thermal networks represents sufficient proof of similarity of the diffusion equations of both fields. By analysis of the diffusion phenomena of the three considered modes of energy transfer; the rates of flow of these energies are found to be directly proportional to the gradient of their volumetric concentration, or density, and the proportionality constants in such relations are the diffusivity of each energy. Such analysis leads also to find proportionality relations between the potentials of such energies and their volumetric concentrations. Validity of the introduced diffusion equations is verified by correspondence their solutions to the measurement results of the electric and magnetic fields in micro-wave ovens.

## Keywords

Diffusion Coefficient, Diffusion Equation, Electric Charge, Magnetic Flux, Electromagnetic Waves, Electric Field, Magnetic Field

## 1. Introduction

Recent studies found the electric charge and magnetic flux as electromagnetic waves which have electric or magnetic potentials, like the heat radiation which is defined as electromagnetic waves that have a thermal potential [1]. However, the electric current was defined in literature as flow of electrons and its diffusion was wrongly defined as diffusion of electrons [2]. Recognizing the proper nature of flow of the electric charges as flow of energy, or electromagnetic waves, suggests an analogical definition of the electric diffusivity as an energy diffusivity, like the heat diffusivity, whose units are  $\text{m}^2/\text{s}$  [3]. Additionally, the use of electric networks for experimental modeling of thermal networks, in case of unsteady heat conduction processes, represents a sufficient proof of similarity of the equations that should govern the diffusion of heat and the electric charges [4]. The traditional definition of the magnetic flux, as a static region around a magnet in which the magnetic force exists, also led to a wrong definition of the magnetic diffusion as motion of magnetic fields [5]. According to Faraday's induction experiment that found analogical natures of the electric charge and magnetic flux in his induction coil, and the recent definition of the magnetic flux as flow of energy, the definition of the magnetic diffusivity should be also defined as electric diffusivity whose units are  $\text{m}^2/\text{s}$  [6].

In this study, it will be analytically investigated the impact of such concluded diffusivities of the electric charge and magnetic flux as energy diffusivities. Starting by defining the volumetric concentration of each energy in transfer, as the thermal, electric, or magnetic energies, it will be found proportional relations between the potentials of the three considered modes of energy transfer and their volumetric concentration. Then, it will also be investigated the role of the electric and magnetic diffusivities as the coefficient of the proportionality between the rates of transfer of the electric charge and magnetic flux and the gradient of their volumetric concentrations.

Accordingly, it will be derived differential equations that characterize the diffusions of the electric charges and magnetic flux that involves their properly defined diffusivities. The heat diffusion equation involves the Laplacian operator for determination of the temperature field in steady or unsteady conduction [7]. So, it is followed analogical procedures to derive similar diffusion equations that characterize the electric and magnetic fields and involve the Laplacian operator in addition to their energy diffusivities. The newly derived diffusion equations may represent a solution that deletes redundancies in determination of the electric and magnetic fields in previous electromagnetic applications [8]. The validity of the innovative diffusion equations of the electric charges and magnetic flux is realized by comparing their solutions by the measurement results of separate magnetic and electric fields in some applications [9].

## 2. The Thermal Diffusion

Consider a thin conducting slab of thickness " $dx$ ", area " $A$ ", and a thermal con-

ductivity “ $k_{therm}$ ”, with a temperature difference “ $dT$ ” between the opposite faces of such slab, as shown in **Figure 1**. The rate of heat flow across this thin slab is denoted as “ $\frac{dQ_{therm}}{d\tau}$ ”. This flow is driven by a temperature gradient of magnitude “ $\frac{dT}{dx}$ ”, found according to the Fourier’s law of heat conduction as follows [10]:

$$\frac{dQ_{therm}}{d\tau} = k_{therm} A \frac{dT}{dx} \text{ Watt} \quad (1)$$

By dividing and multiplying the R.H.S. of Equation (1) by the product “ $\rho c$ ”, where  $\rho$  is the density and “ $c$ ” is the specific heat of the slab material, we get the following equation:

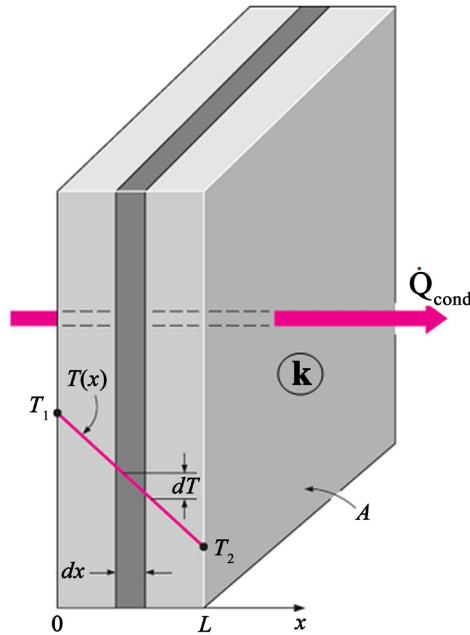
$$\frac{dQ_{therm}}{d\tau} = \frac{k_{therm}}{\rho c} A \frac{d\rho c T}{dx} \text{ Watt} \quad (2)$$

The term “ $\frac{k}{\rho c}$ ” in Equation (2) represent the thermal diffusivity of the conductor denoted as [10]:

$$\alpha_{thermal} = \frac{k_{therm}}{\rho c} \frac{m^2}{sec} \quad (3)$$

The thermal diffusivity,  $\alpha_{thermal}$  is defined as the ability of the material of the conductor to conduct heat energy relative to its ability to store heat energy according to the following relation [11],

$$\alpha_{therm} = \frac{k_{therm}}{\rho c} = \frac{\text{Ability of heat conduction per unit area per unit time}}{\text{Ability to store heat per unit mass per unit degree}} \quad (4)$$



**Figure 1.** Heat transfer rate,  $\dot{Q}_{cond}$ , in a plane wall due to a temperature gradient,  $dT/dx$  [10].

Investigating the product “ $\rho c$ ” in Equation (4), it has the units “Joule/K/m<sup>3</sup>” which measures the heat capacity of unit volume of the conductor due to increase, or decrease, of its temperature by 1 degree Kelvin. So, it is possible to call such product as the volumetric specific heat, denoted as “ $c_{therm,vol}$ ” or as the capacity of unit volume of the substance to store heat per unit rise of its temperature and to have the following units:

$$c_{therm,vol} = \rho \frac{\text{kg}}{\text{m}^3} \times c \frac{\text{Joule}}{\text{kg} \cdot \text{K}} = \rho c \frac{\text{Joule/m}^3}{\text{K}} \quad (5)$$

So, it is also possible to rewrite an alternative definition of the thermal diffusivity as follows:

$$\alpha_{therm} = \frac{k_{therm}}{c_{therm,volum}} \quad (6)$$

$$= \frac{\text{Ability of heat conduction per unit area per unit time}}{\text{Heat capacity per unit volume per unit degree (volumetric specific heat)}}$$

Substituting the units of the terms in Equation (5) into Equation (6), the units of the thermal diffusivity can be found as follows:

$$\alpha_{therm} = \frac{k_{therm}}{\rho c} \left( \frac{\text{J}}{\text{s} \cdot \text{m} \cdot \text{K}} \cdot \frac{1}{\text{J/m}^3 \cdot \text{K}} \right) = \frac{k_{therm}}{\rho c} \frac{\text{m}^2}{\text{sec}} \quad (7)$$

Investigating the product “ $\rho c T$ ” in Equation (2), it has the following units:

$$\rho c T = \rho c T \frac{\text{kg}}{\text{m}^3} \cdot \frac{\text{Joule}}{\text{kg} \cdot \text{K}} \cdot \text{K} = \rho c T \frac{\text{Joule}}{\text{m}^3 \cdot \text{K}} \cdot \text{K} = \rho c T \frac{\text{Joule}}{\text{m}^3} \quad (8)$$

The found units of this product as “Joule/m<sup>3</sup>” represent the thermal energy “ $Q_{therm}$ ” in “Joule” per unit volume “ $v$ ” in “m<sup>3</sup>” [J/m<sup>3</sup>]. So, the term “ $\rho c T$ ” can be interpreted as the volumetric concentration, or density, of the thermal energy in conductors according to the following equation:

$$\rho c T = \frac{Q_{therm}}{v} \quad (9)$$

By substituting the product “ $\rho c T$ ” according to (9) in (2) and the ratio “ $\frac{k_{therm}}{\rho c}$ ” according to (6) in (2)); we get a modified form of Fourier’s law of heat transfer by conduction as follows:

$$\frac{dQ_{therm}}{d\tau} = \alpha_{therm} \frac{d(Q_{therm}/v)}{dx} \quad (10)$$

The term “ $\frac{d(Q_{therm}/v)}{dx}$ ” in Equation (10) represents the gradient of the volumetric concentration of the thermal energy inside the conductor. According to Equation (9), the rate of heat transfer in a conductor is directly proportional to the gradient of the volumetric concentration of the thermal energy in such a conductor. As seen in Equation (9), the proportionality constant in such equation is the thermal diffusivity “ $\alpha_{thermal}$ ”.

According to Equation (9), it is possible to conclude the relation between the

thermal potential “ $T$ ” and the volumetric density or concentration of the thermal energy in a conductor is according to the following equation:

$$T = \frac{1}{c_{therm,volum}} (Q_{therm}/v) \quad (11)$$

Equation (10) indicates the thermal potential is directly proportional to the volumetric concentration of the thermal energy in conductors and the constant of proportionality is the reciprocal of the volumetric specific heat “ $c_{therm,vol}$ ”.

Reviewing the heat conduction equation for determining the temperature field in transient conditions, it reads [11]:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha_{thermal}} \frac{\partial T}{\partial \tau} \quad (12)$$

In case of steady state conditions, we get the known Laplace equation involving the known Laplacian operator “ $\nabla^2$ ” [12]:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \nabla^2 T = 0 \quad (13)$$

Replacing “ $T$ ” in Equation (11) by the volumetric density of heat according to Equation (9), we get:

$$\frac{\partial^2 (Q_{therm}/v)}{\partial x^2} = \frac{1}{\alpha_{therm}} \frac{\partial (Q_{therm}/v)}{\partial \tau} \quad (14)$$

Equation (13) represents an innovative differential equation for determination of one dimensional, volumetric concentration of thermal energy through a conductor. For three-dimensional heat diffusion, we can write the following diffusion equation [12].

$$\frac{\partial^2 (Q_{therm}/v)}{\partial x^2} + \frac{\partial^2 (Q_{therm}/v)}{\partial y^2} + \frac{\partial^2 (Q_{therm}/v)}{\partial z^2} = \frac{1}{\alpha_{thermal}} \frac{\partial (Q_{therm}/v)}{\partial \tau} \quad (15)$$

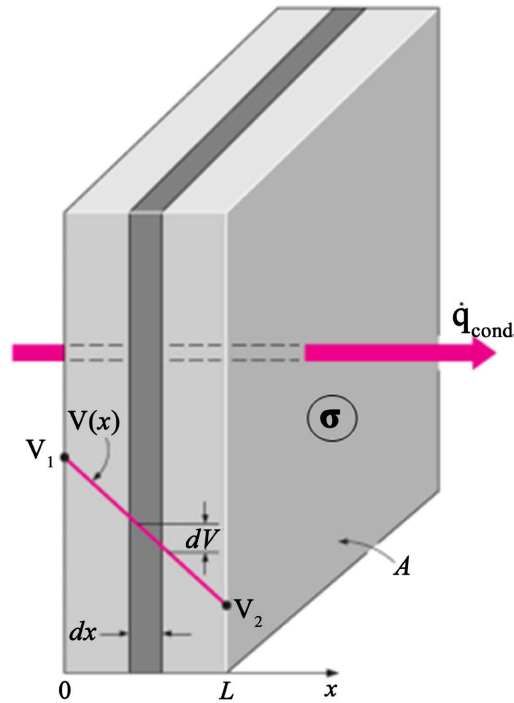
According to the field theory, Equation (14) can be applied to determine the concentration field of thermal energy in conductors [12].

### 3. The Electric Diffusion

Ohm’s law states that the current density in an ohmic conductor is proportional to the electric field. So, the rate of charge flow “ $\frac{dQ_{elect}}{d\tau}$ ” across a thin slab of thickness “ $dX$ ” due to the action of potential difference “ $dV$ ,” as shown in **Figure 2** can be expressed as follows [10]:

$$\frac{dQ_{elect}}{d\tau} = k_{elect} A \frac{dV}{dX} \text{ Watt} \quad (16)$$

where  $k_{elect}$  and  $A$  are the electrical conductivity and the surface area of the slab. Comparing Equation (1) and Equation (16), it is seen the analogy between the laws that govern the flow of thermal energy or heat flow “ $Q_{therm}$ ” and the flow of electrical energy or of charge “ $Q_{elect}$ ” in a conductor [13]. Both energies are defined as forms of electromagnetic waves which have either thermal or



**Figure 2.** Charge transfer rate in a plane conductor due to a potential gradient “ $\frac{dV}{dX}$ ” [14].

electrical potential. As a proof of the similarity of thermal and electric field is the experimental simulation of thermal field by electric field in different applications [14]. So, we may expect the charge diffusion is characterized by analogical diffusion equations, as Equation (11) and Equation (14), that characterize the heat diffusion.

The electric capacitance “ $C_{elect}$ ” is defined in literature as the *capability of material object or device to store electric energy or charge in Joules* [15]. According to this definition, it is possible to introduce a definition of the volumetric electric capacity, like the volumetric thermal capacity, as the capability of a unit volume “ $v$ ” of a conductor to store electric energy that increases its potential by 1 Volt as follows:

$$c_{elect,vol} = \frac{Q_{elect}/v \text{ Joule/m}^3}{V \text{ Volt}} \quad (17)$$

Multiplying and dividing the R.H.S. of Equation (16) by  $c_{elect,vol}$

$$\frac{dQ_{elect}}{d\tau} = \frac{k_{elect}}{c_{elect,vol}} A \left[ \frac{dV \cdot c_{elect,vol}}{dx} \right] \quad (18)$$

Substituting “ $c_{elect,vol}$ ” from (17) into (18), we get:

$$\frac{dQ_{elect}}{d\tau} = \frac{k_{elect}}{c_{elect,vol}} A \left( \frac{d(Q_{elect}/v)}{dx} \right) \quad (19)$$

By the analogy between the laws governing the flows of thermal and electrical energies and the similarity of their natures, it is possible to define the electric

quotient “ $\frac{k_{elect}}{c_{elect,vol}}$ ” as electrical diffusivity like the definition of a similar thermal

quotient “ $\frac{k_{therm}}{c_{therm,vol}}$ ” as thermal diffusivity since both has also the same components and units as follows:

$$\alpha_{elect} = \frac{k_{elect}}{c_{elect,vol}} \frac{m^2}{sec} \quad (20)$$

Then, it is possible to rewrite the electric conduction Equation (18) as follows:

$$\frac{dQ_{elect}}{d\tau} = \alpha_{elect} A \left( \frac{d(Q_{elect}/v)}{dx} \right) \quad (21)$$

where “ $\frac{d(Q_{elect}/v)}{dx}$ ” is the gradient of the volumetric concentration of the electric energy or charge inside the conductor. According to Equation (21), the rate of charge transfer in a conductor is directly proportional to the gradient of the volumetric concentration of the electrical energy or charge in such conductor. The proportionality constant in the Equation (21) is the electrical diffusivity “ $\alpha_{electric}$ ”.

According to Equation (17), it is possible to conclude that the electrical potential “ $V$ ” is directly proportional to the volumetric density or concentration of the electrical energy in a medium according to the following equation:

$$V = \frac{1}{c_{elect,vol}} (Q_{elect}/v) \text{ Volt} \quad (22)$$

Considering the balance of flow of the electric energy, as shown in **Figure 2**, according to the principle of conservation of energy, during an infinitesimal time  $\Delta t$  interval as follows [15]:

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} = \frac{\Delta Q_{element}}{\Delta \tau} \quad (23)$$

According to the definition of the volumetric electric capacity “ $c_{elect,vol}$ ”, the change in the charge content of the element can be expressed as:

$$\Delta Q_{element} = Q_{\tau+\Delta\tau} - Q_{\tau} = c_{elect,vol} \cdot A \Delta x \cdot (V_{\tau+\Delta\tau} - V_{\tau}) \quad (24)$$

According to Ohm’s law, the L.H.S. of Equation (22) can be also expressed:

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} = -k_{elect} \cdot A \frac{\Delta V}{\Delta x} = c_{elect,vol} \cdot A \Delta x \cdot \frac{V_{\tau+\Delta\tau} - V_{\tau}}{\Delta \tau} \quad (25)$$

Dividing the sides of Equation (24) by “ $c_{elect,vol} \cdot A \Delta x$ ” and taking the limits of the equation when the time and coordinate intervals  $\Delta \tau$  and  $\Delta x$  tends to zero, we get:

$$\frac{k_{elect}}{c_{elect,vol}} \frac{\partial^2 V}{\partial x^2} = \frac{\partial V}{\partial \tau} \quad (26)$$

Replacing the quotient  $\frac{k_{elect}}{c_{elect,vol}}$  by the electric diffusivity according to Equation

(20), we get the following charge diffusion equation for determining the electric field in transient diffusion conditions as follows:

$$\frac{\partial^2 V}{\partial x^2} = \frac{1}{\alpha_{elect}} \frac{\partial V}{\partial \tau} \quad (27)$$

According to the similarity of the natures of the thermal and electrical energy as electromagnetic waves of thermal or electric potentials, it is found the analogy between the equations characterizing the thermal and electric fields as expressed by Equation (10) and Equation (27). So, Equation (27) can be also extended to the three-dimension coordinates as follows [12]:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{1}{\alpha_{elect}} \frac{\partial V}{\partial \tau} \quad (28)$$

Equation (27) represents an innovative differential equation for determination of the electric fields in case of transient electric diffusion in analogy to Equation (11) that is traditionally used for determination of the thermal field. The analogy of the diffusion equations of the electric fields and magnetic fields is a must for the applied experimental simulation of thermal field by electric field as a tool for complicated thermal fields [14]. The identity of the values of the measurement results in both fields is proof of the analogy of the equations that govern both fields, Equation (11) and Equation (28), and the similarity of the natures of flow of heat and of electric charges [16] [17].

In case of steady state condition, we get the known Laplace equation for electric field, by substituting  $\frac{\partial V}{\partial \tau} = 0$  in Equation (28), as follows:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad (29)$$

Substituting Equation (21) in Equation (28); we get:

$$\frac{\partial^2 (Q_{elect}/v)}{\partial x^2} + \frac{\partial^2 (Q_{elect}/v)}{\partial y^2} + \frac{\partial^2 (Q_{elect}/v)}{\partial z^2} = \frac{1}{\alpha_{elect}} \frac{\partial (Q_{elect}/v)}{\partial \tau} \quad (30)$$

Equation (30) represents an innovative differential equation for determination of volumetric concentration of electric charges through a conductor during a transient charge-transfer process that fits for electro-magneto hydrodynamic applications [18].

#### 4. The Magnetic Diffusion

Analysis of the magnetic diffusivity in this article depends on a proper definition of the magnetic flux, like the electric charge, as energy whose unit is “**Joule**” and whose magnetic potential is also measured by “**Volt**” [1]. Such definitions also depend on the results of Faraday’s experiments that found the changeability between the electric charge and magnetic flux where the magnetic flux may induce electric potential, hence generate a flow of electric charge, and the flow of electric charges may also induce magnetic potential or generate magnetic flux [19].



Results of these experiments assure the similarity of the natures of the electric charge and magnetic flux as flow of electromagnetic waves of either electric potential or magnetic potential [19]. Hence, the magnetic conductivity is defined by an analogous equation as (21) and (22) for electric conductivity as follows [1]:

$$\frac{dB}{d\tau} = k_{mag} A \frac{dH}{dx} \quad (31)$$

So, the magnetic permeability or conductivity can be defined as follows [20]:

$$k_{mag} = \frac{\text{rate of flow of magnetic flux}}{l \times \text{magnetic potential}} \frac{\text{Watt}}{\text{m} \cdot \text{Volt}} \quad (32)$$

In analogy to the definition of the *volumetric electric capacity* “ $c_{elect,vol}$ ”, it is possible to define the *volumetric magnetic* as the capacity of a magnetic medium to store magnetic energy that increases its magnetic potential by a unit magnetic potential of 1 Volt according to the following equation:

$$c_{mag,vol} = \frac{B/v}{H} \quad (33)$$

According to Equation (32), it is to find the following proportionality between the magnetic potential “ $H$ ” and the volumetric concentration of magnetic energy in a conductor according to the following equation:

$$H = \frac{1}{c_{mag,vol}} (B/v) \quad (34)$$

Multiplying and dividing the R.H.S. of Equation (16) by  $c_{mag,vol}$

$$\frac{dB}{d\tau} = \frac{k_{mag}}{c_{elect,vol}} A \left[ \frac{dH \cdot c_{mag,vol}}{dx} \right] \quad (35)$$

Substituting  $c_{mag,vol}$  from Equation (17) into Equation (18), we get:

$$\frac{dB}{d\tau} = \frac{k_{mag}}{c_{mag,vol}} A \left( \frac{d(B/v)}{dx} \right) \quad (36)$$

Regarding the analogy between the magnetic and electric fields, it is also possible to define the ratio “ $\frac{k_{mag}}{c_{mag,vol}}$ ” as the magnetic diffusivity as follows:

$$\alpha_{mag} = \frac{k_{mag}}{c_{mag,vol}} \quad (37)$$

Substituting Equation (37) in Equation (36) we get the following equation:

$$\frac{dB}{d\tau} = \alpha_{mag} A \left( \frac{d(B/v)}{dx} \right) \quad (38)$$

It is possible, by considering the magnetic flux as energy and starting by the principles of energy conservation during transfer of magnetic energy during a transient process, to derive for the magnetic field an equation like (26), that was derived for the electric field, for transient diffusion of magnetic energy which reads [21]:

$$\frac{\partial^2 H}{\partial x^2} = \frac{1}{\alpha_{\text{magn}}} \frac{\partial H}{\partial \tau} \quad (39)$$

Equation (35) can be also extended for here dimensions as follows:

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = \frac{1}{\alpha_{\text{magn}}} \frac{\partial H}{\partial \tau} \quad (40)$$

Equation (40) represents an innovative differential equation for determination the magnetic field in case of transient diffusion conditions. In case of steady state, we get a new application of the Laplacian operator for determination of the magnetic field “ $H$ ” as follows:

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = 0 \quad (41)$$

Such conclusion is necessary for proper analysis of the electrohydrodynamic flow and microwave analysis and can be inserted to solve found redundancies in such field [22] [23].

Replacing the magnetic potential in Equation (36) by the volumetric concentration of the magnetic flux according to Equation (34), we get an innovative diffusion equation of magnetic-energy concentration:

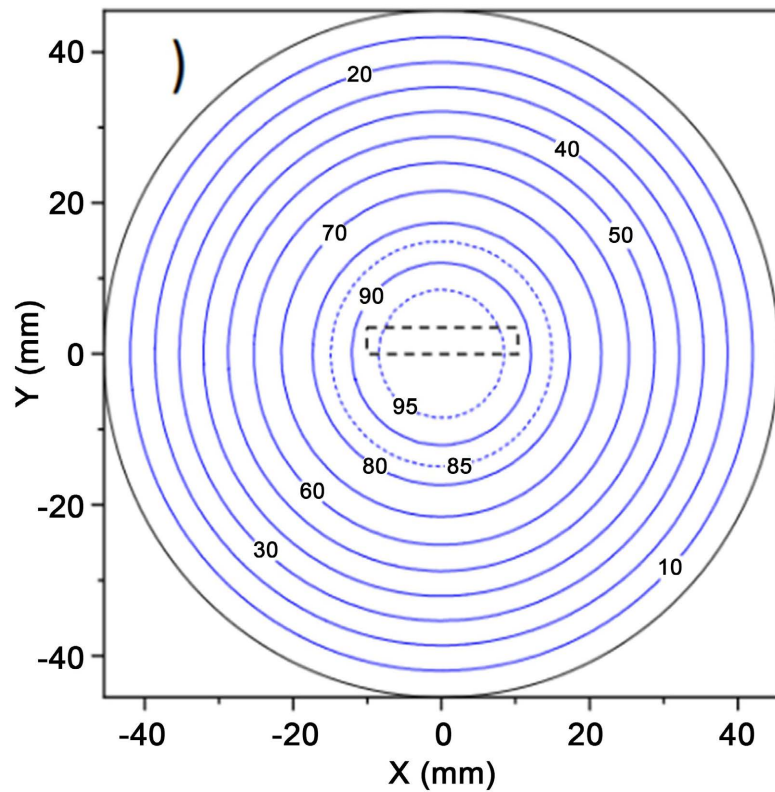
$$\frac{\partial^2 (B/v)}{\partial x^2} + \frac{\partial^2 (B/v)}{\partial y^2} + \frac{\partial^2 (B/v)}{\partial z^2} = \frac{1}{\alpha_{\text{magn}}} \frac{\partial (B/v)}{\partial \tau} \quad (42)$$

According to the field theory, Equation (38) can be applied to determine the concentration field of magnetic energy in conductors [12].

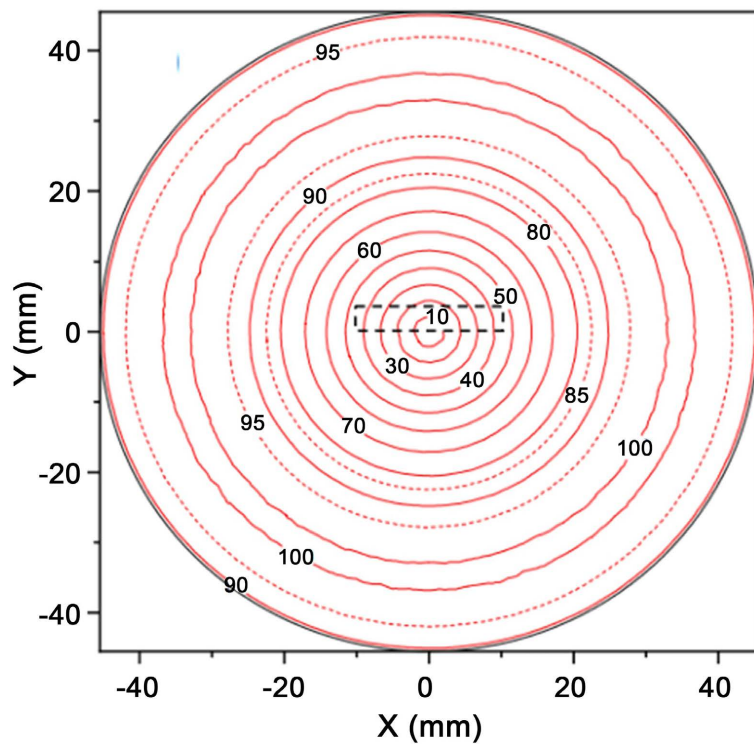
## 5. Experimental Verification

To verify the validity of the innovative diffusion Equations (26) and (35) as a tool for predicting the electric and magnetic fields, it is investigated the measurement results of microwave heating with separate magnetic and electric fields which was carried out by using scanning electron microscopy and energy dispersive X-ray spectroscopy [24]. **Figure 3** and **Figure 4** show the distributions of the strengths of the magnetic and electric fields produced by a microwave resonator, which was used to compare magnetic and electric field heating. These figures show the similarity of the characteristics and contours of both fields which can be concluded from the similarity of the diffusion equations (28) and Equation (36).

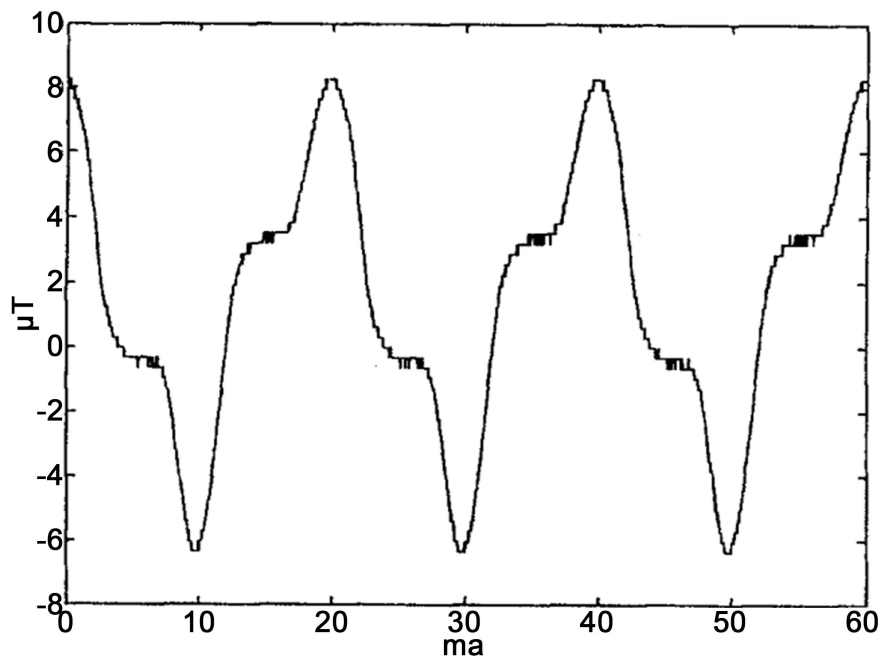
The mathematical solutions of the innovative differential equations, (26) and (35), are normally done by separation of variables as a product of an exponential function of time and a sinusoidal function that determine the pulsating flow of electric and magnetic energies along the coordinates of energy propagations [25]. Such characteristics of solution can be seen in the measurement results of the electric and magnetic fields produced by microwave ovens under different circumstances. **Figure 5** and **Figure 6** show the sinusoidal waveform of the magnetic field and electric field lines measured from the microwave oven. Such



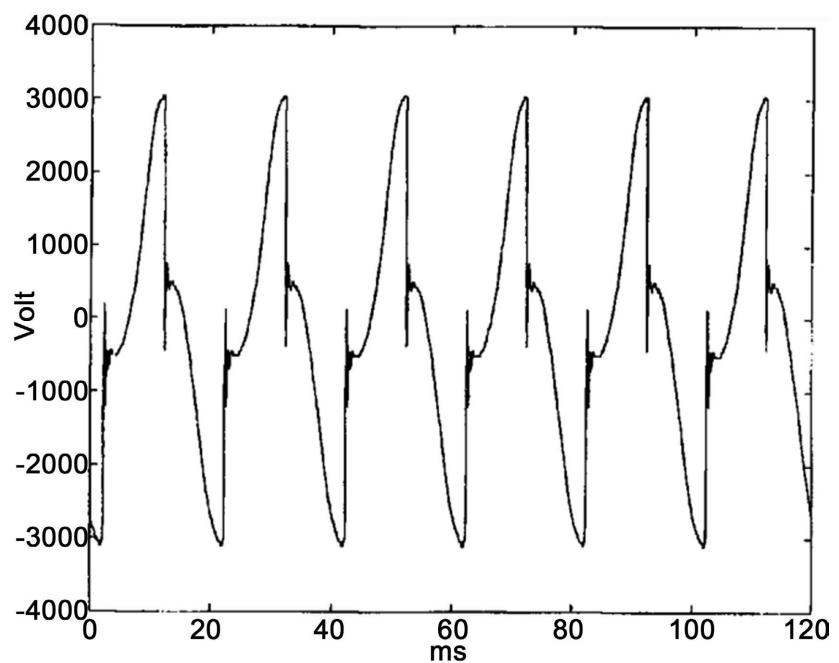
**Figure 3.** The distributions of the strengths of the electric field produced by a microwave resonator [24].



**Figure 4.** The distributions of the strengths of the magnetic field produced by a microwave resonator [24].



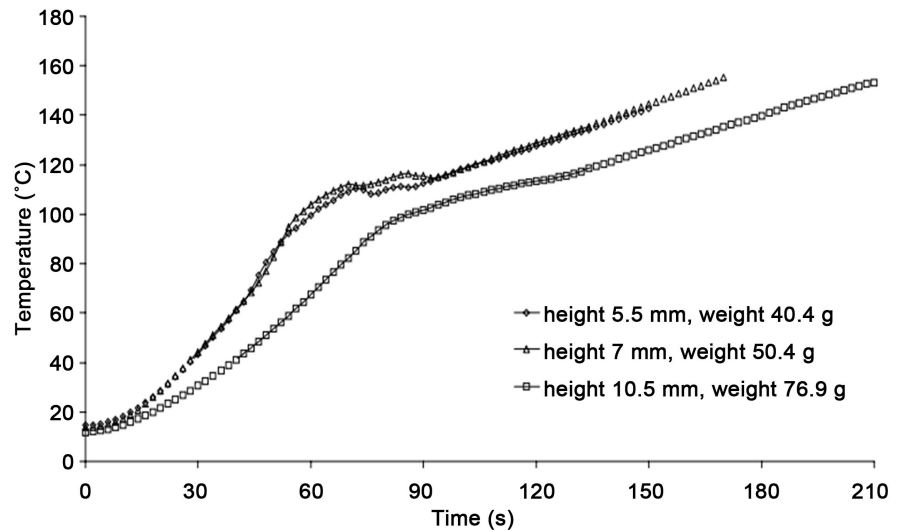
**Figure 5.** Profile of the incident magnetic field  $H$  sampled during 60ms at a distance from a microwave oven [26].



**Figure 6.** Profile of the electric field ( $V$ ) measured between the HV capacitor of the microwave oven [26].

measured sinusoidal wave forms represent solutions that can be found by solving the electric and magnetic diffusion Equations (28) and (36), [26].

**Figure 7** shows the exponential rise of the temperature of the oven during the time of operation of a microwave oven due to the integrated influence of the exponential rise of the electric and magnetic potentials or fields of the incident



**Figure 7.** Temperature Profile during Microwave heating of samples in a Microwave Oven [27].

energy. Such exponential can also be predicted from the mathematical solutions of the innovative diffusion Equations, (28) and (36) [27].

## 6. Conclusions

Depending on the similarity of natures of the thermal, electric, and magnetic energies, as electromagnetic waves of corresponding potentials, it is found an analogy of the equations that characterize their transfer by diffusion. The validity of the innovative diffusion equations, to predict the electric and magnetic fields, is realized by reviewing the measurement results of separate magnetic and electric fields in microwave ovens.

According to this article, it is achieved the following innovations:

- 1) An innovative definition of the electric diffusivity that is expressed in terms of the conductor's magnetic conductivity and a defined conductor's specific volumetric capacity of electric charges whose units are " $\text{m}^2/\text{sec.}$ "
- 2) An innovative definition of the magnetic diffusivity that is expressed in terms of the conductor's magnetic conductivity, or permeability, and a defined conductor's specific volumetric capacity of magnetic flux whose units are " $\text{m}^2/\text{sec.}$ "
- 3) An innovative diffusion equation of the electric charges that involves the Laplacian operator and the defined electric diffusivity.
- 4) An innovative diffusion equation of the magnetic flux that involves the Laplacian operator and the defined electric diffusivity.
- 5) Innovative proportionality relation between the potentials of the three considered energies in transfer, *i.e.* thermal, electrical, or magnetic manergies, to their volumetric concentration where the reciprocals of the specific volumetric concentration of each energy represents the proportionality constant in each relation.
- 6) Innovative proportionality relation between the rate of energy transfer by

the three considered mechanisms and the gradient of their volumetric concentration where the diffusivity of each energy represents the proportionality constant in each relation.

7) Innovative plausible explanations of the measurement results of the electric and magnetic fields produced by microwave ovens under different circumstances.

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## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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