

# C-W Algorithm Based AGV Route Planning and Improvement

Wei Liu, Qi Tang

College of Management, Tianjin polytechnic University, TJPU Tianjin, China

Email: guyang0801@163.com, tangqi65@126.com

**Abstract:** Route optimization for the purpose of reducing transport costs has increasingly become a focus of modern logistics enterprises. In accordance with mileage saving ideas, this article creatively applies C-W algorithm to the route planning of logistics center AGV, and greatly improves operations efficiency of the logistics center, in addition, this article proposes further improvement ideas for AGV route planning, which is to get transport tasks of the logistics center to realize standardization and automation through integration of computer-aided system and C-W algorithm.

**Keywords:** component; logistics; route planning; C\_W algorithm; AGV

## 1 Introduction

As China's economy booms, logistics gets more and more attention as an emerging field; introduction and innovation of various theories and technology greatly prompt the rapid development of China's logistics industry. AGV (Automated Guided Vehicle, AGV) is a new technology needed for the development requirements of logistics, which is often call as the AGV cart, the cart is equipment equipped with magnetic or optical etc automatic guide, and can run along the prescribed guidance path, transport goods to the designated location. AGV now are mostly used in goods inbound and outbound operations of large distribution centers.

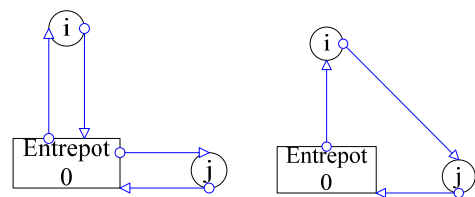
AGV working principle can be described as below: taking inbound for example, after a batch of goods are transported to logistics center from production firm, the logistics center shall store the products in accordance with a certain product catalog, at that time, professional technical personnel gives a instruction to the computer that AGV transports the products to stacking bay according to the prescribed path, in collaboration with electronic control equipment, the instruction will be accepted and implemented by AGV.

For a batch of goods contain a number of tasks, and the final destination of each task is different to each other, this comes down to how to plan the route so that AGV can complete the task and takes the shortest route which means minimum cost. According to study of Traveling Salesman Problem, Clarke and Wright proposed the algorithm in 1964, which is called C-W algorithm for abbreviation, to solve optimal scheduling issues of non-full loads vehicle in goods consolidation or goods delivering, since this algorithm is simple and practical, and can include some constraints within operation into algorithm model simply, the algorithm is widely used in vehicle scheduling and route planning.

## 2 Research and development of C-W algorithm

### 2.1. The basic idea of C-W Algorithm

C-W algorithm is also known as algorithm, as the name suggests, it's a method of obtaining the optimal route eventually by accumulated savings which remove waste. The example is shown in the figure 1.



the original route      the optimized route

Figure 1. The route optimization process

C-W algorithm defines the route which only connects each node and 0 node as original route. As shown in the figure, original route contains "0-i-0" and "0-j-0", this is clearly a circuitous route, which results in waste. Take "s" as mileage, " $c_{ij}$ " as cost from i point to j point which usually can be represent by distance between the two points which is  $c_{ij} = d_{ij}$ .

$$s_1 = 2c_{0i} + 2c_{0j}$$

C-W algorithm uses the mileage saving idea, and can obtain the optimized route after improvement "0-i-j-0", so that

$$s_2 = c_{0i} + c_{ij} + c_{j0}$$

The optimized route is clearly shorter than the original

route, taking  $s(i,j)$  as saved mileage, so that

$$\begin{aligned} s(i,j) &= s_1 - s_2 \\ &= c_{0i} + c_{j0} - c_{ij} \end{aligned}$$

Generally, a batch of goods contain a number of tasks, taking “1” for expression, connecting only the task point and “i” constitutes 1 original routes, the total cost is “s”

$$s = \sum_{i=1}^l c_{0i} + \sum_{i=1}^l c_{i0}$$

Then calculate mileage saving after adding a point one by one with iterative computations, eventually we can obtain one or more combination of routes which are the most efficient routes combinations. C-W algorithm improves cumulating step by step through such successive iteration, and eventually calculates the optimal route.

## 2.2. Time window constraint of C-W algorithm

Along with generation of time-related theory, such as JIT idea, zero inventory idea and agile manufacturing etc, modern logistics industry also gradually emphasizes on time cost, so that taking time factor into C-W algorithm becomes inevitable. The improved C-W algorithm indicates, because of the time window factor, when the vehicle reaches j point from i point, three situations may occur, one situation is that the vehicle reaches j point, but the j point is not ready for loading and unloading, then there will be waiting phenomenon; one situation is that before the vehicle reaches j point, j point has been ready for loading and unloading for the goods for a long time, then there will be delay phenomenon; then the last phenomenon is that, the vehicle arrives j point on time. The improved C-W algorithm defines the advanced time or delayed time for the vehicle to reach j point along the improved route than along the original route as  $EF_j$ :

$$EF_j = s_i + T_i + t_{ij} - s_j$$

$s_i$  indicates the time to reach i point, if  $ET_i \leq t_{0i} \leq LT$ , then  $s_i$  takes  $t_{0i}$ , if  $t_{0i} < ET_i$ , then  $s_i$  takes  $ET_i$ ;  $T_i$  indicates the time to accomplish task i;  $t_{ij}$  indicates the time from i point to j point.

Obviously, when  $EF_j < 0$ , the time for the vehicle to reach j point becomes earlier; when  $EF_j = 0$ , the vehicle reaches j point on time; when  $> 0$ , the time for the vehicle to reach j point delays.

To express the extent of the vehicle's advancing and delay, we define two expressions:

$$\Delta_j^- = \min_{r \geq j} |S_r - ET_r|$$

$\Delta_j^-$  indicates the maximum possible advance amount for the vehicle to reach j point if on the route there is no wait at each task point after j point;  $ET_r$  indicates the earliest time for the vehicle to reach r point.

$$\Delta_j^+ = \min_{r \geq j} |LT_r - S_r|$$

$\Delta_j^+$  indicates the maximum possible delay amount for the vehicle to reach j point if on the route the tasks after j point do not violate time window constraints.  $LT_r$  indicates the latest time for the vehicle to reach r point.

## 3 Use of C-W algorithm in AGV route optimization

In the modern logistics operations, AGV is mainly used in goods inbound and outbound operations of logistics center, to facilitate use of C-W algorithm in AGV route optimization, we describe these issues as follows. Taking inbound for example, in one distribution center (referred to as point 0), AGV cart's capacity is q, now a batch of goods have arrived in the center, these goods shall be stored separately in l cargoes which are referred to as 1, ..., l in accordance with catalogue, the known goods volume of task i is  $g_i$  ( $i=1, \dots, l$ ), and  $g_i < q$ , and seek the AGV route with minimum cost which meets the freight transportation demand.

### 3.1. Mathematical model

As the traffic demands shortest route and lowest cost, the construct objective function is as follow:

$$\min z = \sum_i \sum_j \sum_k c_{ij} x_{ijk}$$

Constraint condition:

$$\sum_i g_i y_{ki} \leq q \quad \forall k; \quad (1)$$

$$\sum_k y_{ki} = 1 \quad i=1, \dots, l \quad (2)$$

$$\sum_i x_{ijk} = y_{kj} \quad j=0, 1, \dots, l; \quad \forall k \quad (3)$$

$$\sum_j x_{ijk} = y_{ki} \quad i=0, 1, \dots, l; \quad \forall k \quad (4)$$

$$x_{ijk} = \begin{cases} 1 \\ 0 \end{cases} \quad (5)$$

$$x_{kj} = \begin{cases} 1 \\ 0 \end{cases} \quad (6)$$

Constraint (1) indicates capacity of each route shall not exceed the capacity of the vehicle; constraint (2) indicates task of  $i$  point must be accomplished by one vehicle; constraint (3) indicates indicated if vehicle  $k$  unloads at  $j$  point,  $k$  must reach  $j$  point from  $i$  point; constraint (4) indicates if vehicle  $k$  unloads at  $i$  point,  $k$  must reach  $j$  point after unloading at  $i$  point; constraint (5) indicates if vehicle  $k$  goes from  $i$  to  $j$ , then  $x_{ijk}=1$ , otherwise,  $x_{ijk}=0$ ; constraint (6) indicates if the task of  $i$  point is achieved by vehicle  $k$ , so  $y_{kj}=1$ , otherwise,  $y_{kj}=0$ .

### 3.2. Solution steps of C-W algorithm

According to C-W algorithm co-sponsored by Clarke and Wright, the solution steps are list as below.

- Step 1, take the cargo port of the logistics center as original point 0, and separately connect stacking bay  $i$  point of each task with 0 point, and form 1 original route, which is "0-i-0".
- Step 2, calculate the route saving amount  $s(i,j)$  by connecting  $i$  point and  $j$  point in a same route. And arrange  $s(i,j)$  in the sequence from big to small, and form set  $M$ .
- Step 3, examine  $s(i,j)$  separately according to the set sequence, and check if it meets one of the following conditions: (1) Stacking bay  $i$  point and  $j$  point are both in the original route; (2) One of stacking bay  $i$  and  $j$  is in other combination route, and is the start or end point of that route, and the other is in the original route; (3)  $i$  point and  $j$  point are in two different routes, but one is the start point and the other is the end point. Continue to the next step, or turn to step 7.
- Step 4, check the total route freight traffic volume after connecting  $i$  and  $j$  point, if  $Q \leq q$ , continue to the next step, or turn to step 7.
- Step 5, calculate  $EF_j$ . If  $EF_j=0$ , turn to step 6; if  $EF_j < 0$ , calculate  $\Delta_j^-$ , when  $|EF_j| \leq \Delta_j^-$ , continue to the next step, or turn to step 7; if  $EF_j > 0$ , calculate  $\Delta_j^+$ , when  $|EF_j| \leq \Delta_j^+$ , continue to the next step, or turn to step 7.
- Step 6, connect  $i$  point and  $j$  point, calculate the new time for the little cart to reach each stacking bay, turn to step 7.
- Step 7, make  $M:=M-s(i,j)$ , turn to step 3, explore new route, or terminate algorithm.

### 3.3. Analysis of a case

One logistics center accepts a batch of goods, which

need to be send to 8 goods yard by AGV separately for stacking, suppose the running speed of the AGV little cart is 50m/min, the loading capacity is 800kg, each discharge time is  $T_i$ , the time frame to start implementation of each task is  $[ET_i, LT_i]$  (Unit: time). As shown in the table I and table II.

TABLE I. THE CHARACTERISTICS OF EACH MISSION

Assignment (i)	1	2	3	4	5	6	7	8
$g_i$ (kg)	200	150	450	300	150	400	250	300
$T_i$	1	2	1	3	2	2.5	3	0.8
$[ET_i, LT_i]$	[1, 4]	[4, 6]	[1, 2]	[4, 7]	[3, 5.5]	[2, 5]	[5, 8]	[1.5, 4]

According to the principle of C-W algorithm, we design procedure of solving the route planning in the following.

1) Separately connect each stacking bay with the cargo port 0 point of logistics center, and get 8 original routes.

$$"0 \rightarrow i \rightarrow 0", i=1, \dots, 8.$$

And we can calculated the total mileage amount  $S_1$  of the original routes.

$$\begin{aligned} S_1 &= \sum_{i=1}^8 c_{0i} + \sum_{i=1}^8 c_{i0} \\ &= 2 \times (50+70+85+100+210+110+170+90) \\ &= 1770 \end{aligned}$$

2) Calculate mileage saving volume. And arrange by the sequence from big to small, and for set  $M$ , as shown in the table III.

3) Construct route. Check the elements in  $M$  by the sequence from big to small, find out whether they meet 1 of 3 conditions in step 3 (as figure 4). If two points are both not in the constructed routes, check  $i \rightarrow j$  and  $j \rightarrow i$ ; if  $i$  is not in the route,  $j$  is the start point of the route, or  $i$  is the end point of the route, and  $j$  is not in the route, check  $i \rightarrow j$  (the same for  $i$  and  $j$  exchanges); if the two point are both exterior points in the route, according two the position relation between the two points, form "the end point of one rout  $\rightarrow$  the start point of another route" sequence. If the two points can not be connected, do not check other items. For example, (5, 7) are both points in the route, then "5 $\rightarrow$ 7" and "7 $\rightarrow$ 5" the two routes shall be considered. For another example, in (5, 6), after checking (5, 7), it's found out that 5 is the

start point of the route, and 6 is an exterior point, then route “6→5” can be constructed.

TABLE II. THE DISTANCE BETWEEN THE GOODS ALLOCATION

Centre 0	Centre 0									
goods allocation 1	50	goods allocation 1								
goods allocation 2	70	75	goods allocation 2							
goods allocation 3	85	50	85	goods allocation 3						
goods allocation 4	100	110	110	110	goods allocation 4					
goods allocation 5	210	60	110	60	110	goods allocation 5				
goods allocation 6	110	85	85	100	85	80	goods allocation 6			
goods allocation 7	170	120	85	100	85	100	80	goods allocation 7		
goods allocation 8	90	110	85	160	110	85	110	110	goods allocation 8	

TABLE III. THE ASSEMBLE OF MILEAGE SAVING

(i,j)	(5,7)	(5,6)	(3,5)	(5,8)	(4,5)	(1,5)	(6,7)
s(i,j)	280	240	235	215	200	200	200
(i,j)	(4,7)	(2,5)	(2,7)	(3,7)	(7,8)	(4,6)	(1,7)
s(i,j)	185	170	155	155	150	125	100
(i,j)	(2,6)	(3,6)	(6,8)	(1,3)	(4,8)	(1,6)	(2,8)
s(i,j)	95	95	90	85	80	75	75
(i,j)	(3,4)	(2,3)	(2,4)	(1,2)	(1,4)	(1,8)	(3,8)
s(i,j)	75	70	60	45	40	30	15

4) Check whether there is overweight. After connecting route, it must be met that the total freight traffic volume in the route shall not exceed load capacity of AGV little cart. If there is overweight, other items are no longer considered (figure 4).

5) Step 5 to step 7 is study on time window constraints. Calculation can be carried out according to the time window constraints expression (as figure 4). For example, for (5, 7), positions of the two points determine connecting “5→7” and connecting “7→5” are both possible, and there is no overweight in this route, choosing which route needs the time window constraints. Route “5→7” meets  $EF_7 < \Delta_7^+$ , so this route can be con-

nected, and for  $EF_5 \geq \Delta_5^+$ , it is indicated that there is no feasible route for the time, so “7→5” is not chosen.

To clearly understand and analyze each step of C-W algorithm, we can express it in the table IV.

Solve in accordance with C-W algorithm steps, we can obtain the following routes combinations:

$$\begin{aligned} &0 \rightarrow 8 \rightarrow 5 \rightarrow 7 \rightarrow 0 \\ &0 \rightarrow 6 \rightarrow 4 \rightarrow 0 \\ &0 \rightarrow 3 \rightarrow 1 \rightarrow 2 \rightarrow 0 \end{aligned}$$

Then, the total mileage volume of the new routes combination is  $S_2$ , and

$$\begin{aligned} S_2 &= c_{08} + c_{85} + c_{57} + c_{70} + c_{06} + c_{64} + c_{40} + c_{03} + c_{31} + c_{12} + c_{20} \\ &= 80 + 75 + 90 + 160 + 100 + 75 + 90 + 75 + 40 + 65 + 60 \\ &= 910 \end{aligned}$$

The mileage saving is  $\Delta S$ , and

$$\begin{aligned} \Delta S &= S_1 - S_2 \\ &= 1610 - 910 \\ &= 700 \end{aligned}$$

Thus, before using C-W algorithm, the AGV cart need to run 1610m and 8 routes to go there and back. With C-W algorithm, the AGV just need to run 910m after route optimization, what's more, 8 original routes are optimized

to 3 routes combinations. Through the route optimization by C-W algorithm, not only make the AGV cart saves trip mileage, but also to improve the operational efficiency of the logistics center.

This paper mainly discusses AGV cart route planning by using the C-W algorithm under time constraint conditions. In logistics practice, the algorithm can handle other constraint conditions, such as the volume of goods shall not

exceed the capacity of the cart, and in a circuit connection, needing to check whether the violation of the length of the routes. In a word, considering the more constraint conditions, the more complex the problem. So integrating C-W algorithm, the factors to be considered in the algorithm and computer, through the computer intelligence is calculated, which can improve the operation scheme efficiency.

TABLE IV. THE TABULAR METHOD OF C-W ALGORITHM

1	2	3	4	5	6	7
$i \rightarrow j$	Location	$Q = \sum g_i$	$EF_j = s_i + T_i + t_{ij} - s_j$	$\Delta_j^+$ or $\Delta_j^-$	Concatenate	$s_k := s_k + EF_j$
5→7	Non-route point	$Q=400 < q$	$EF_7 = 3$	$\Delta_7^+ = 3$	5→7	$s_7 := 8$
6→5	Non-route point; External point	$Q=800 = q$	$EF_5 = 2.1$	$\Delta_5^+ = 0.7$	×	
3→5	Non-route point; External point	$Q > q$			×	
8→5	Non-route point; External point	$Q=700 < q$	$EF_5 = 0.1$	$\Delta_5^- = 1$	8→5 →7	$s_5 := 3.9$ $s_7 := 7.7$
4-5 1-5	Internal point				×	
7→6 7→4	External point; Non-route point	$Q > q$			×	
2-5	Internal point				×	
7→2 7→3	External point; Non-route point	$Q > q$			×	
4-6	Non-route point	$Q=700 < q$	$EF_4 = 2.4$	$\Delta_4^+ = 3$	6→4	$s_4 := 6$
7→2 2→6 3→6	External point; Non-route point	$Q > q$			×	
6→8	Origin				×	
1-3	Non-route point	$Q=650 < q$	$EF_1 = 2.7$	$\Delta_1^+ = 3$	3→1	$s_1 := 3.3$
4→8 1→6 2→8 4→3	External point	$Q > q$			×	
2→3	Non-route point; External point	$Q=800 = q$	$EF_3 = 6$	$\Delta_3^+ = 0.3$	×	
4→2	External point; Non-route point	$Q > q$			×	
1→2	External point; Non-route point	$Q=800 = q$	$EF_2 = -0.5$	$\Delta_2^+ = 2$	1→2	$s_2 := 5.6$

#### 4. Further improvement ideas in AGV route planning based on C-W algorithm.

Though C-W algorithm has good solution to single task in AGV route planning, after arriving of each batch goods to the logistics center, storage locations are differ-

ent, which results changes in driving routes, so to logistic centers with high operating volume, C-W algorithm also forms problems in application. For example, the first batch of goods has m tasks, which means the batch of goods shall be separately stored in m different stacking bays; the second batch has n tasks, which means the

batch of goods shall be separately stored in  $n$  different stacking bays (and the  $m$  stacking bays are different from the  $n$  stacking bays). And C-W algorithm is a task-specific route planning, and can not be applied to all tasks. Therefore, to make C-W algorithm play high productivity features in AGV route planning, C-W algorithm needs to take further improvement.

In order to improve C-W algorithm application in AGV route planning, this article proposes the idea of integrating computer system and C-W algorithm. The idea details are described as below: set all stacking bay coordinates of the logistics center and time window constraint conditions etc for cargo handling as variable set into the database, and program in accordance with C-W algorithm rules and procedures, so that we can obtain general solution for each task through the computer's autonomic computing, in the actual operation, the technicians simply need to input the stacking bay coordinates etc variations into the computer system, the computer will calculate the optimal route.

After integration, operating steps of route planning could be expressed as the figure 2.

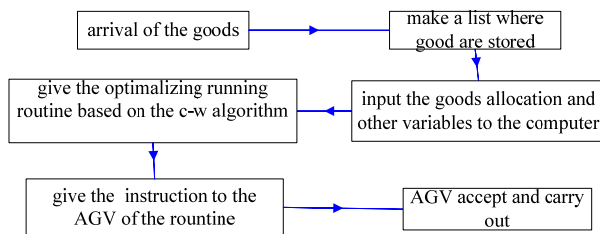


Figure 2. The operating steps of route planning after integration

After integration with computer, solving of AGV travel route is not only simple but also more universal and extensive in application, which get goods inbound and outbound of logistics center realize standardization

and automation, and greatly increases efficiency of logistics operations.

## 5. Conclusion

Planning of logistic vehicle transport rout is a key sector in optimization of logistics system. Route planning on delivery vehicle can improve the logistics economic benefits; realize logistics function of social economy. This article studies planning of logistics center's AGV little cart running routes with C-W algorithm, by application of C-W algorithm, logistics center can scientifically select AGV little cart's guide route to save the little cart's running mileage, and reduce logistics costs; constraints on the time window reflects time efficiency of modern logistics operation, is JIT form of logistics center's operating system. In addition, this article points out problems of C-W algorithm in planning of logistics center AGV running routs, and proposed solution s to those problems, but, as programming and integration etc belong to computer programming field, to fully realization of C-W algorithm-guided AGV running further study is needed.

## References

- [1] Kunpeng Li, Shihua Ma. Modeling and analysis of 3PL transportation scheduling based on JIT delivery [J]. Transportation and computer, 2008(2):73-79.
- [2] Xiaolan Zhu, Yifei Zhao. Application of C-W algorithm in assembly business' procurement logistics [J]. Shanghai Jiaotong University Journal 2007(9):1420-1424.
- [3] Xiaoyu Lin, Jinming Li, Shouwen Ji. Improvement and implementation of Vehicle routing problem Clarke-Wright algorithm [J]. Transportation and computer, 2004(6):72-75
- [4] Jincheng Fang, Qishan Zhang. Algorithm study of logistics vehicle routing problem VRP [J]. Xuzhou Institute of Technology Journal. 2007(2):84-88
- [5] Jun Li, Yaohuang Guo. Optimal scheduling theory and method of logistics vehicle [M]. China Logistics Publishing House. 2001(6)