Application of Dynamic Random Simulation in Supermarket

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Abstract: Queues waiting at a counter in a supermarket are dynamically simulated with Matlab/Simulink 7.0 in Monte Carlo Theory in this paper, and factors thus gained about customers' random arrival show how many cashier's desks should be required accordingly to establish a proper relationship between the customers' waiting cost and the market's running cost.

Keywords: dynamic simulation; Monte Carlo theory; radom values;queuing theory; cashier's desks

1. Introduction

Quequing is a phenomenon commonly seen in our daily life, such as in a bank, at a toll-gate, when a certain service system can not meet the need of a waiting crowd.

In such a system, customers waiting at a counter expects to spend less and less time, thus demanding more and more counters to be opened, which may result in a increasing running cost for the market. On the contrary, less counters may decrease its service quality level, and even lose its customers. So it remains a problem how to scientifically decide the number of necessary counters in a market to reduce costs and to enhance efficiency.

2. Monte carlo Theory

Factors Monte carlo Theory[1], also called a random simulation, by means of experiments through random simulation and statistics, produces a sequence of a group of numbers from certain random variables that accord with probability distribution and use it as an input variable sequence to make particular experiments and find the solution.

In the process of application of the method, random values should meet the features of the probability distribution[2]. The basic steps of this method are as follows:

step1. to establish a probability model, that is, to turn a research problem into a probability problem, or to form a probability model that meets the features;

step2. to generate a random sequence meeting the features of the probability distribution;

step3. to make a lot of digital simulation tests using the random sequence in order to get its simulation results;

step4. to process the test results statistically (For example, to calculate the frequency, mean), and further to explain the research questions.

3.Establish simulation model based on queuing theory

3.1 Service queue model in surpermarket (M/M/C)

Service of the cashier's desks in surpermatket is a random service system, the system has the following characteristics:

1) its clients are customers that have already selected their goods; the customer source is unlimited, independent to each other, and their arrival time are random.

2) the system has several waiters,and every waiter' service time is independent.

3) its service is up to "first come, first serve" rules(FCFS).

4) each cashier's desk has one queue, customers can choose a shorter queue to join, thus forming a single queue with multiple attendants (M / M / C) queuing system. The waiting queue system in a supermarket is as follows:

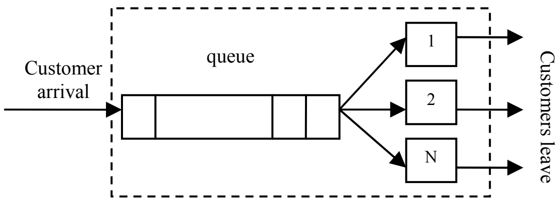

figure 1 one line waiting queue system

3.2 Generate a set of random values

Because the arrival time and the service time subject to negative exponential distribution, let the negative exponential distribution random number to be x, and the negative exponential distribution density function is:

$$f(x)=\lambda e^{-\lambda x}(x\geq 0) \quad (1)$$

its distribution function is:

$$F(x)=\int_0^x \lambda e^{-\lambda x}dx=1-\lambda e^{-\lambda x}(x\geq 0) \quad (2)$$

the inverse function of $F(x)$ is :

$$x=\frac{1}{\lambda}\ln[1-F(x)] \quad (3)$$

Suppose u is an independent and uniform distribution variable in the interval of $[0,1]$, so the random

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value asked for is:

$$x = -\frac{1}{\lambda}\ln(1-u) \qquad (4)$$

Simplified:

$$x = -\frac{1}{\lambda}\ln u \qquad (5)$$

So we can get a negative exponential distribution random number [4].

According to customers' arrival time and service time statistics, two sets of random sequence are generated: the interval of arrival time of customers $a(i)$ and service time $st(i)$, which will be inputed for simulation.

3.3 Variables of the model

$at(i)$: arrival time of the i-th customer

$a(i)$: arrival interval of the i-th customer

$st(i)$: service time for the i-th customer

$sst(i)$: start service time for the i-th customers

$lea(i)$: departure time of the i-th customer

$ls(j)$: departure time of the last customer of the j-th queue

$ls(m)$: the earliest departure time in all $ls(j)$

$freet(j)$: average waiting time of the j-th waiter

$w(i)$: waiting time of the i-th customer in queue

Note: $at(i+1) = at(i) + a(i+1)$

$$sst(i) = \max(at(i), ls(m))$$

$$w(i) = \max(0, ls(m) - at(i))$$

$$ls(m) = \min(lt(j))$$

4. Simulate the simulation system

4.1 Simulation of cashier's desks service

To solve the queuing problem in a supermarket, the key is to determine how many cashier's desks are required under normal condition appropriately. According to the statistics of the cashier's desks, we can measure the efficiency of waiters and the arrival rate of customers, thus we can measure the appropriate number of cashier's desks by simulating. The basic steps are as follows:

To construct the simulation model:

1) the system has become stable after a long time's running.

2) according to the actual inspection, we see the number of customers increase violently on weekends and holidays, and also appears several peaks in a day[5].So we make the following improvement: let one day be divided into four periods（9：00-12：00，12：00-14：00，14：00-18：00，18：00-21：00），firstly survey the working condition of one of those periods through Monday to Friday （the third period, for example）the survey data are shown in table 1.

3) according to the statistic[6], we get the number n of arrival customers in unit time and the service time t for every customer, then, by Chi-square testing[7] we come to a conclusion that the arrival number of customers in unit time subjects to Possion distribution, and the service time subjects to negative exponential distribution.

Table 1. Arrival time and service time for customers in the third period:

Customer number	1	2	3	…	1000	…
interval	3″	16″	23″	…	6″	…
service time	87″	30″	108″	…	74″	…

Statistics above shows that 49 customers were served at a cashier's desk in an hour, namely $\mu = 49$, and 277 customers arrived in an hour, namely $\lambda = 277$.Suppose the average waiting time be less than 5 minutes, simulating time be 1000 times. Simultaneously, the value of μ and λ will change with the actual situation, so as to simulate the working condition under different vaule of μ and λ.

4.2 Design the simulation program

Firstly, to supose customers' arrival time and service time accord to the required model, then to judge weather there are idle cashier's desks or not. If yes, customers will get service without waiting; if no , they will come into a queue waiting for service.The time that a cashier's desk become idle should be calculated before the customer come into the queue earliest. Simulation clock was driven by the event of the arrival time of each customer.The service time should be calculated when a customer come into the queue with the simulation method, and the next event would begin upon the arrival of next customer).

In the process of simulating,the busy rate of cashier's desks ,the length of the queue and the waiting time of customers should be calculated,so as to achieve the simulation aims, as shown in figure 2.

This model(M/M/C) takes the third period as example: the arrival of customers are random, arrival mode subjects to Possion distribution, namely $\lambda = 277$, the service time is random too , which subjects to exponential distribution, namely $\mu = 49$; Simulation would be terminated when 2000 customers has passed, the simulation clock was driven by the events(the arrival of the customers). MATLAB7.0/Simulink6.0 was taken as tool to dynamic simulation[8], 1000 times to simulate, and results to be averaged.

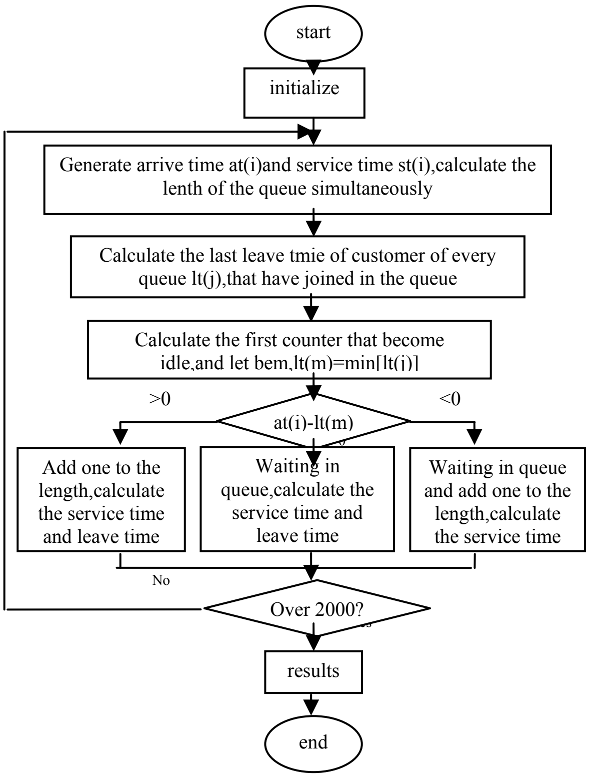

figure 2 Monte carlo dynamic simulating flow

4.3 Simulation results

Table 2 result of dynamic simulation

S	5	6	7	8	10	11
P	99.25	93.57	83.49	70.3	60.37	49.37
W	3627.5	250.92	47.09	20.07	4.84	0.44

Notes：S-the number of cashier's desks
P-busy rate of cashier’s desks(%)
W-average waiting time(seconds)

Analysis of results: under the condition of the arrival of customers 277 per hour and service rate 49 per hour,when five desks were opened, the waiters are busy and the average waiting time of customers will be more than an hour. When eleven desks are open, customers almost wait no time,and the busy rate of the desks is nearly 50 percent.The waiting cost of customers and the service cost of the supermarket were both acceptable,so, seven desks should be opened with an average busy rate of desks of 83.49%,and an average waiting time of customers of 47.09 sceonds.

In the same way,we can simulate the numbers of customers and service levels of different periods ,so as to know how many cashier’s desks and waiters should be required under different conditions to improve service quality and to increase benefit.

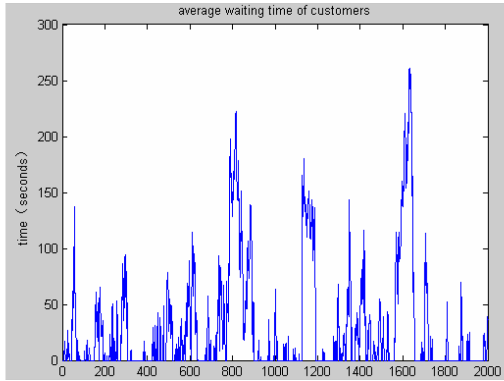

figure 3 average waiting time of customers

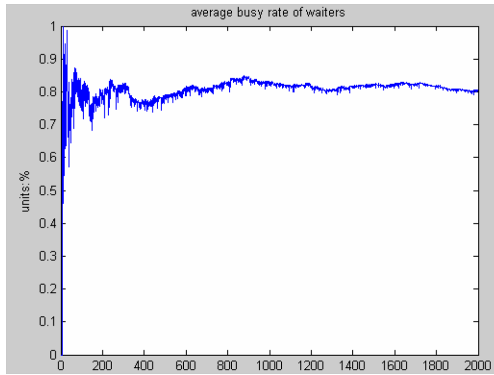

figure 4 average busy rate of counters(%)

5. Conclusions

It is very difficult to solve a complex random service system by analytical method because of the probability laws itself,however,simulation with its own characteristics is an effective method to do this.

The simulation method mentioned in this paper has universality,and it can be used in charging system of supermarkets, banks and hospitals, etc.which has practical significance in guiding life and work.

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