

# A Method to Obtain the Moving Speed of Uncooperative Target Based on Only Two Measurements

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## Abstract

The existing research results show that a fixed single station must conduct three consecutive frequency shift measurements and obtain the target's moving speed by constructing two frequency difference equations. This article proposes a new method that requires only two consecutive measurements. While using the azimuth measurement to obtain the angular difference between two radial distances, it also conducts two consecutive Doppler frequency shift measurements at the same target azimuth. On the basis of this measurement, a frequency difference equation is first constructed and solved jointly with the Doppler frequency shift equation. By eliminating the velocity variable and using the measured angular difference to obtain the target's lead angle, the target's velocity can be solved by using the Doppler frequency shift equation again. The new method avoids the condition that the target must move equidistantly, which not only provides an achievable method for engineering applications but also lays a good foundation for further exploring the use of steady-state signals to achieve passive positioning.

## Keywords

Moving Speed, Non-Cooperative Target, Doppler Frequency Shift, Frequency Difference Equation, Azimuth, Steady-State Signal, Passive Location

## 1. Introduction

In the literature on passive location, there seem to be relatively few papers that study the speed of moving targets independently. One reason why the author cannot provide many references is that due to factors such as the author's current scope of knowledge and websites that allow readers to retrieve information.

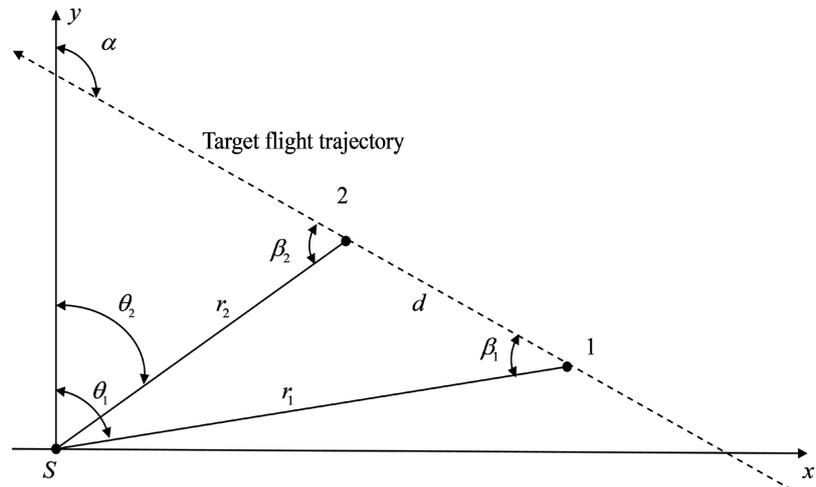
The author often has to focus on retrieving domestic literature. However, on websites where foreign literature can be viewed, there seem to be relatively few papers related to passive location topics. One factor may be that there are many universities in China, and each year many graduates with master's and doctoral degrees in radio location need to complete their graduation papers. Of course, it is certain that there is no research abroad, but there should be relatively few publicly published papers.

Doppler frequency shift is a nonlinear function that includes both velocity and angle. Unlike the application scenarios of passive detection of fixed targets by motion detection stations [1] [2] [3] [4], the difficulty of detecting moving targets by a fixed single station lies in the unknown movement speed and distance of the moving target. Among the three variables of distance, speed, and time, only time can be approximately measured. If we only explain the problem of determining the solution of a fixed single station to a moving target from a mathematical analysis perspective, the difficulty lies in the fact that the unknown angle variables contained in the Doppler function cannot be directly measured. If the angle variable needs to be eliminated, it usually requires multiple measurements to obtain multiple definite solution equations.

Regarding the analysis of the speed of moving targets, the author has published several papers to date [5] [6] [7]. These research papers are all based on multiple measurements. A few years ago, the author studied and proposed a method for obtaining the target moving speed by using a fixed single station, making three consecutive frequency shift measurements, and constructing two frequency difference equations [5]. A paper published this year gives an analytical method that requires four consecutive detections to obtain the target moving speed [7]. This only explains the principle of a method being studied, but it is not practical.

In this article, the author proposes a method for solving the target motion speed by improving the existing analytical calculation method, which requires only two measurements of azimuth angle and two measurements of Doppler frequency shift. One reason for the improvement is that the author found that after obtaining the angle difference through two measurements of azimuth angle, only one frequency difference equation needs to be constructed to correctly solve the target's leading angle in the flight direction. Once the leading angle is obtained, the target's moving speed can be solved using the frequency shift equation.

The new method of only two detections obviously reduces the detection time for moving targets. But more importantly, for targets with unknown moving speeds, it may be difficult to achieve continuous equidistant measurements in three signal detections. If only two signal measurements are required, that is, only a non-cooperative target moves a certain distance, the target's moving speed can be accurately determined. This clearly avoids the condition that the target must move equidistantly, providing an achievable method for engineering applications and laying a good foundation for further exploration and research.



**Figure 1.** Geometric model.

## 2. Measurement Method

Assuming that the target moves at a uniform speed along a straight line in the two-dimensional plane, for the fixed single station azimuth/Doppler frequency shift passive location system shown in **Figure 1**, the azimuth and Doppler frequency shift of the moving target can be obtained by tracking the radiation signal of the detected target for two consecutive times.

The Doppler frequency shift received by the measuring station at two different times:

$$\lambda f_{di} = v \cos \beta_i \quad (i = 1, 2) \quad (1)$$

where:  $\lambda$  is the signal wavelength of target radiation;  $f_{di}$  Doppler shift;  $v$  the moving speed of the target;  $\beta_i$  the leading angle of the target end between the radial distance and the target flight direction.

While receiving the Doppler frequency shift value of the moving target, the measuring station also uses the angle measuring equipment to measure the azimuth angle difference  $\Delta\theta = \theta_1 - \theta_2$  between the two frequency shift measurements. Also, based on the relationship between the inner and outer corners:  $\Delta\theta = \beta_2 - \beta_1 = \Delta\beta$ . So there is:

$$\Delta\beta = \theta_1 - \theta_2 \quad (2)$$

where:  $\Delta\beta$  is the azimuth angle difference between the two radial distances of the detected target frequency shift;  $\theta_i$  the azimuth of the station end.

## 3. Formula Derivation

A frequency difference equation can be obtained from two Doppler measurements

$$\lambda(f_{d2} - f_{d1}) = v(\cos \beta_2 - \cos \beta_1) \quad (3)$$

The ratio between frequency difference equation and Doppler frequency shift equation  $\lambda f_{d1} = v \cos \beta_1$  is:

$$\frac{(f_{d2} - f_{d1})}{f_{d1}} = \frac{(\cos \beta_2 - \cos \beta_1)}{\cos \beta_1} \quad (4)$$

This can eliminate unknown velocity variables. Because:  $\beta_2 = \beta_1 + \Delta\beta$ , a solution formula containing only one unknown quantity  $\beta_1$  can be obtained, from which:

$$\beta_1 = \text{tg}^{-1} \left[ \frac{1}{\sin \Delta\beta} \left( \frac{\Delta f_d}{f_{d1}} - 1 + \cos \Delta\beta \right) \right] \quad (5)$$

And:

$$\beta_2 = \text{tg}^{-1} \left[ \frac{1}{\sin \Delta\beta} \left( \frac{\Delta f_d}{f_{d1}} - 1 + \cos \Delta\beta \right) \right] + \Delta\beta \quad (6)$$

After obtaining the leading angle of the target, the moving speed of the target can be obtained directly by using the Doppler frequency shift equation:

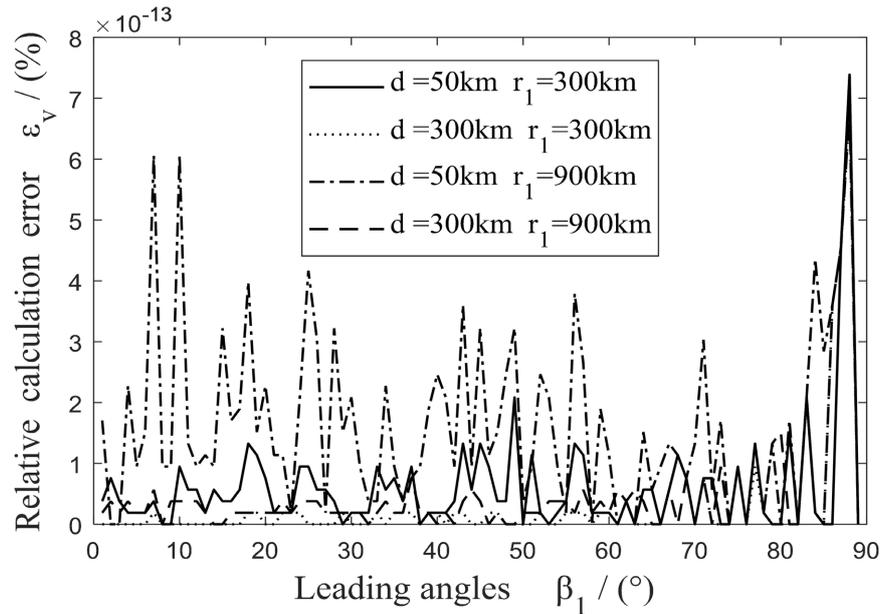
$$v = \frac{\lambda f_{di}}{\cos \beta_i} \quad (i = 1, 2) \quad (7)$$

**Figure 2** shows the relative calculation error of the target moving speed:

$$\varepsilon_v = 100 \frac{|v - v_{th}|}{v_{th}} \quad (8)$$

where:  $v_{th}$  is the theoretical value of the predetermined velocity.

It is obvious that the calculation accuracy of the analytical solution is very good. During the simulation calculation, the default values are:  $v_{th} = 300$  m/s,  $\lambda = 0.3$  m respectively. And let the leading angle change linearly in the range of 0~90 degrees, and then calculate the remaining geometric parameters according to the trigonometric function relationship.



**Figure 2.** Relative calculation error of target moving speed.

## 4. Error Analysis

For simplicity, equation (5) is selected here to analyze the speed measurement error. First set the transition function:

$$P = \lambda f_{d1}$$

$$Q = \cos \beta_1.$$

There are:  $v = \frac{P}{Q}$ .

### 4.1. Speed Measurement Error Caused by Frequency Shift $f_{d1}$

$$\frac{\partial v}{\partial f_{d1}} = \frac{1}{Q^2} \left( Q \frac{\partial P}{\partial f_{d1}} - P \frac{\partial Q}{\partial f_{d1}} \right)$$

Including:  $\frac{\partial P}{\partial f_{d1}} = \lambda$ ,

$$\frac{\partial Q}{\partial f_{d1}} = -\sin \beta_1 \frac{\partial \beta_1}{\partial f_{d1}}.$$

### 4.2. Speed Measurement Error Caused by Frequency Shift $f_{d2}$

$$\frac{\partial v}{\partial f_{d2}} = \frac{1}{Q^2} \left( Q \frac{\partial P}{\partial f_{d2}} - P \frac{\partial Q}{\partial f_{d2}} \right)$$

Including:  $\frac{\partial P}{\partial f_{d2}} = 0$ ,

$$\frac{\partial Q}{\partial f_{d2}} = -\sin \beta_1 \frac{\partial \beta_1}{\partial f_{d2}}.$$

### 4.3. Speed Measurement Error Caused by Included Angle $\Delta\beta$

$$\frac{\partial v}{\partial \Delta\beta} = \frac{1}{Q^2} \left( Q \frac{\partial P}{\partial \Delta\beta} - P \frac{\partial Q}{\partial \Delta\beta} \right).$$

Including:  $\frac{\partial P}{\partial \Delta\beta} = 0$ ,

$$\frac{\partial Q}{\partial \Delta\beta} = -\sin \beta_1 \frac{\partial \beta_1}{\partial \Delta\beta}.$$

### 4.4. Angle Measurement Error Caused by Frequency Shift and Included Angle on Leading Angle $\beta_1$

1. Basic formula

According to the differential rule:

$$\frac{\partial \beta_1}{\partial f_{d1}} = \frac{1}{1+A^2} \frac{\partial A}{\partial f_{d1}}$$

$$\frac{\partial \beta_1}{\partial f_{d2}} = \frac{1}{1+A^2} \frac{\partial A}{\partial f_{d2}}$$

$$\frac{\partial \beta_1}{\partial \Delta \beta} = \frac{1}{1+A^2} \frac{\partial A}{\partial \Delta \beta}.$$

Where A is the intermediate transition function:

$$A = \frac{1}{\sin \Delta \beta} \left( \frac{\Delta f_d}{f_{d1}} - 1 + \cos \Delta \beta \right).$$

2. Differentiation of intermediate transition function A

$$\text{Set: } A = \frac{p}{q}.$$

$$\text{Including: } p = \left( \frac{\Delta f_d}{f_{d1}} - 1 + \cos \Delta \beta \right),$$

$$q = \sin \Delta \beta.$$

Differential of transition function A to each measured parameter:

$$\frac{\partial A}{\partial f_{d1}} = \frac{1}{q^2} \left( q \frac{\partial p}{\partial f_{d1}} - p \frac{\partial q}{\partial f_{d1}} \right)$$

$$\frac{\partial A}{\partial f_{d2}} = \frac{1}{q^2} \left( q \frac{\partial p}{\partial f_{d2}} - p \frac{\partial q}{\partial f_{d2}} \right)$$

$$\frac{\partial A}{\partial \Delta \beta} = \frac{1}{q^2} \left( q \frac{\partial p}{\partial \Delta \beta} - p \frac{\partial q}{\partial \Delta \beta} \right).$$

Including:

$$\frac{\partial p}{\partial f_{d1}} = \frac{1}{f_{d1}^2} (-f_{d1} - \Delta f_d)$$

$$\frac{\partial q}{\partial f_{d1}} = 0$$

$$\frac{\partial p}{\partial f_{d2}} = \frac{1}{f_{d1}}$$

$$\frac{\partial q}{\partial f_{d2}} = 0$$

$$\frac{\partial p}{\partial \Delta \beta} = -\sin \Delta \beta$$

$$\frac{\partial q}{\partial \Delta \beta} = \cos \Delta \beta.$$

#### 4.5. Simulations

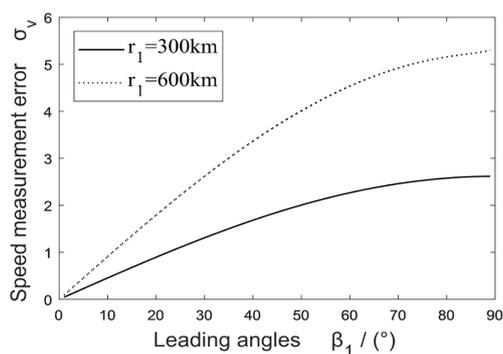
When the errors of each observation are zero mean and independent of each other, there are velocity measurement errors:

$$\sigma_v = \sqrt{\sum_{i=1}^2 \left( \sigma_f \frac{\partial v}{\partial f_i} \right)^2 + \left( \sigma_\beta \frac{\partial v}{\partial \Delta \beta} \right)^2} \quad (9)$$

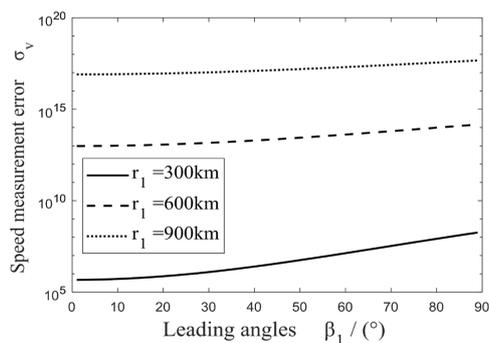
where:  $\sigma_f$  and  $\sigma_\beta$  are root mean square errors of frequency shift and angle measurement errors respectively. Take:  $\sigma_f = 10\text{Hz}$ ,  $\sigma_\beta = 0.5^\circ \frac{\pi}{180^\circ}$  when

calculating.

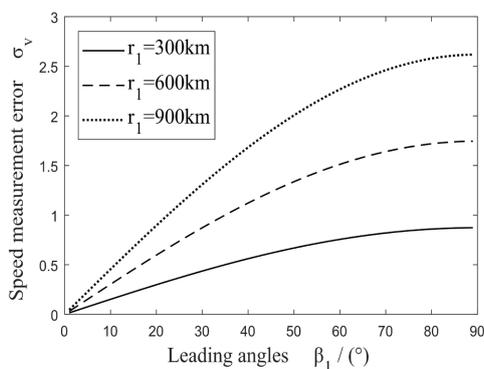
**Figure 3** shows the speed measurement error at different radial distances when moving distances  $d = 150$  km. **Figure 4** shows the speed measurement error curve at different radial distances when moving distance  $d = 50$  km. The simulation calculation shows that the ratio of the radial distance to the moving distance cannot be too large, if it is too large, there will be divergence, and at this time, the root mean square error of the measurement error of adjusting each parameter has little effect. **Figure 5** further shows the velocity measurement error curve when the moving distance  $d = 900$  km, which is much larger than the radial distance. This shows that in order to obtain good speed measurement accuracy, the moving distance must be long enough.



**Figure 3.** Speed measurement error curve with large moving distance.



**Figure 4.** Speed measurement error curve with small moving distance.



**Figure 5.** Results when the movement is far greater than the radial distance.

## 5. Conclusions

The contribution of this article is that it achieves the detection of the moving speed of non-cooperative targets based on only two consecutive frequency shift/angle measurements based on steady-state signals. This is a breakthrough result in engineering applications. Prior to this, research results based on three measurements mainly reflected the value of academic exploration, as three measurements cannot solve the problem of equidistant detection.

However, the author's research also indicates that it is necessary to construct a frequency difference equation using two frequency measurements. Without using a frequency difference equation, based on only two Doppler frequency shift equations, it seems that the solving conditions have been met on the surface, but in fact, it is still impossible to solve. This is because for non-cooperative targets, in the case where both distance and speed cannot be directly measured, due to the ratio relationship between distance and speed. There will be some uncertainty in solving the flight speed.

Obviously, if only using steady-state signals without using detection methods related to the rate of change, at least two frequency measurements are still required based on the current research results of the author. Based on classic direction finding/time difference/frequency shift techniques, a fixed single base station can obtain the location and motion information of the target by using a continuous multi-point tracking method [8]. In order to achieve real-time positioning of a target moving at approximately constant speed, it is generally necessary to detect transient signals [9] [10]. Although theoretically, using transient signal information can improve the positioning convergence speed and positioning accuracy [11] [12] [13], they come at the cost of increasing the complexity and difficulty of measurement, which in fact restricts practical applicability, not only placing high demands on system design, but also increasing costs. Therefore, the study of using steady-state signals to achieve single station passive Doppler/angle detection still has practical engineering significance.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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