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The Performance of Option-Based Portfolio Insurance on a Dividend Payment Stock

Paulina Nangolo[1], **Elias Rabson Offen**[2], **Othusitse Basmanebothe**[2]

[1]Department of Computing, Mathematical and Statistical, University of Namibia, Windhoek, Namibia
[2]Department of Mathematics, University of Botswana, Gaborone, Botswana
Email: paulinanangolo2@gmail.com, eliasoffen@gmail.com, othusitsebasimanebotlhe@ub.bw

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Abstract

Portfolio insurance is a type of hedging which is a dynamic investment strategy that is designed to guarantee the portfolio value at maturity or up to maturity to be greater or equal to a given lower bond (floor). We analyse the efficiency and the performance of option-based portfolio insurance, by employing two strategies to determine the numbers of stocks, put options, bond value and call options in such a way that the floor value is always protected. Furthermore, we compare the insured versus the non-insured portfolio value.

Keywords

Portfolio Insurance, Option-Based Portfolio Insurance, Black-Scholes Model, European Option

1. Introduction

Portfolio Insurance refers to a particular investment approach that independently requires using financial products. It is a dynamic investment strategy that is designed to guarantee the portfolio value at maturity to be greater or equal to a given lower bound (floor), normally predetermined as a percentage of the initial investment. This approach permits the investor to limit downside risk while employing some potential in case of an advanced upside market which is however lowered in comparison with the unprotected portfolio as pointed out by [1].

Furthering, a portfolio insurance strategy is a designed protection, based on the definition of a secured threshold such that the terminal portfolio value always lies above it. This technique passes all the risks of the actual return being below the expected return, or the uncertainty about the magnitude of that difference as stated by [2] [3]. The portfolio insurance strategy as an investment strategy that guarantees a minimum level of wealth at some specified time horizon all with participating in the potential gains of some reference portfolio is defined by [4].

The proposal of portfolio insurance strategies was first introduced by [5], succeeding the collapse of stock markets (the New York Stock Exchange's Dow Jones Industrial Average and the London Stock Exchange FT30) that insinuated the pension fund withdrawal. The risk-averse conduct of pension funds prevented from them the subsequent rally on the market in the preceding year. The presence of insurance for that type of risk could have convinced the investors not to leave the market, guaranteeing them later the opportunity to take advantage of the advance of the same.

There are different portfolio strategies, but for the purpose of this paper, we only look at one of the most important types of portfolio strategy—Option-Based Portfolio Insurance (OBPI). The OBPI strategy is a portfolio insurance computational characterized by ensuring a minimum terminal portfolio value [6].

Introduced by [5], Option-Based Portfolio Insurance (OBPI) strategy fundamentally consists of purchasing, a $t = 0, q$ shares of a risky asset S and P shares of a European call option written on S with a maturity T and a strike price K. At the terminal date T, if the value of the risky asset S_T was inferior to the strike price K, the investor will choose to exercise the put options lock up in her portfolio and get the strike value qK. Otherwise, if S_T was superior to K, the investor will choose not to exercise the put options and benefit from the excess of the risky asset price above the strike K. In the process, the value of OBPI portfolio at maturity T will be superior, or at least equal, to the fixed amount qK which represents the insured amount at maturity:

The main objective of this paper is to find the possible number of stocks q and the number of puts p that guarantee the portfolio value at any time $[t, T]$, will not fall below the designed floor.

Portfolio insurance outlines a possible set of strategies that allow investors to reduce their exposure to market risk by guaranteeing the value of the portfolio to exceed a certain value at the end of the investment period while allowing for an engagement in the rising stock market.

Different researchers have tried to solve this mystery in terms of portfolio insurance strategies, at least starting from the early 70s of the century, as conceptualized by [7] [8] [9] [10].

Option-Based Portfolio Insurance (OBPI) refers to a set of strategies in which either a conventional put option (protective put) or a replicated put option (synthetic put) is used to insure a portfolio against unfavourable price movements.

The design for protection insurance was first conceptualized by [5] in the mid-1970. He understands that an equality portfolio can be insured by purchasing a put option on it [11]. This strategy, protective put, avert the value of the portfolio to end below the strike price of the option at the money at the end of the investment period, the option is exercised, by guaranteeing the floor value. Provided that suitable exchange-traded put options did not exist at the time, Leland used the same arbitrage argument underlying the Black-Scholes option pricing formally to replicate the option.

[12] was the first to present a model for option pricing that did not depend on one or more arbitrary variables. Instead, fully necessary inputs for the model are either observable or exogenously given.

Later, extensions and further implementations to the model were developed, for instance by [13]. This established possibilities for new and innovative investment strategies, including OBPI.

In addition, [4] [14] define a portfolio insurance trading strategy as a strategy that guarantees a minimum level of wealth at a specified time horizon, but also participates in the potential gains of a reference portfolio. The outline of this paper is as follows. Section 1: Introduction. Section 2: Statement of the problem. Section 3: Main results. In Section 4, conclusion and suggestions are given.

2. Statement of the Problem

Initiated by [5], Option-Based Portfolio Insurance (OBPI) strategy primarily consists of purchasing, at $t = 0, q$ shares of risky asset S and p shares of European put option P written on S with a maturity T and a strike price K.

At the terminal date T, if the value of the risky asset S_T was inferior to the strike price K, the investor will choose to exercise the put options keep in her portfolio and to get the strike value qK. Or else, if S_T was superior to K, the investor will choose not to exercise the put options and to profit from the excess of the risky asset price above the strike K. In such manner, the value of OBPI portfolio at maturity T will be superior, or at least equal, to the predetermined amount qK which represents the insured amount at maturity:

The OBPI strategy is a portfolio insurance procedure characterized by guarantee a minimum terminal portfolio value [6]. According to [1], the value of OBPI portfolio

$$V^{OBPI} = \left\{V_t^{OBPI}\right\}_t \in [0,T],$$

with initial value V_0^{OBPI}, as follows

$$V_t^{OBPI} = qB_t + pcall\left(t, S_t, K\right),$$

for all $t \in [0,T]$

where q stand for the number of risk-less assets acquired by the investor to protect the capital, $call\left(t, S_t, K\right)$ is the call option at the time t, written on S_t, having strike price K and maturity T, while $p > 0$ is the number of calls which can be purchased at time $t = 0$, given the risk budget.

The OBPI technique is said to be static in the sense that no trading occurs in $(0,T)$, so that the distinctive portfolio value we are interest in are

$$V_t^{OBPI} = qB_0 + pcall\left(0, S_0, K\right)$$

and

$$V_t^{OBPI} = qK + pput \max\left\{S_T - K, 0\right\},$$

consequently, at maturity, the client gets the capital qK plus p times any positive performance of S_T greater than K. In case $q = 1, p = 1$ and $S_T \geq K$, the client gets completely the performance of the underlying asset.

The value of OBPI portfolio at maturity, $V_T^{OBPI} = q\left(S_T + \left(K - S_T\right)^+\right) \geq qK$. Due to the relation between Black-Scholes European put and call prices "the call-put parity", this strategy is equivalent to investing, at time $t = 0$, an amount $qK\mathrm{e}^{-rT}$ in a risk-free asset and buying q shares of an European call option written on a risky asset S with a strike price K and maturity T. In this case, the value of OBPI portfolio maturity can be written as follows:

$$V_T^{OBPI} = q\left(K + \left(S_T - K\right)^+\right). \tag{1}$$

And the value of OBPI portfolio at any time $t \leq T$ is given by

$$V_t^{OBPI} = q\left(S_t + P\left(t, S_t, K\right)\right) = q\left(K\mathrm{e}^{-r(T-t)} + C\left(t, S_t, K\right)\right), \tag{2}$$

where $P\left(t, S_t, K\right)$ and $C\left(t, S_t, K\right)$ are respectively the time t no-arbitrage Black-Scholes price of European put and call options written on S with a maturity T and a strike price K.

It is clear from the previous equation that the value of OBPI portfolio will, at any time $t \leq T$, be superior to the amount $qK\mathrm{e}^{-r(T-t)}$.

In general, investors are willing to recover a percentage $p < \mathrm{e}^{rT}$ of their initial investment V_0 at maturity T. In this case, the following relation must be verified:

$$pV_0 = pq\left[S_0 + P\left(0, S_0, K\right)\right] = pq\left[K\mathrm{e}^{-rT} + C\left(0, S_0, K\right)\right] = qK.$$

Which implies that:

$$\frac{C\left(0, S_0, K\right)}{K} = \frac{1 - p\mathrm{e}^{-rT}}{p}. \tag{3}$$

And that:

$$q = \frac{V_0}{S_0 + P\left(0, S_0, K\right)} = \frac{V_0}{K\mathrm{e}^{-rT} + C\left(0, S_0, K\right)}. \tag{4}$$

Equation (3) shows that the strike price K is an increasing function. Equation (4) gives the respective number of options to be held in OBPI portfolio.

Finally, we can deduce the total return of OBPI portfolio at maturity T:

$$\frac{V_T}{V_0} = \begin{cases} p & \text{if } S_T < K, \\ \left(\frac{S_T}{S_0}\right)\left(\frac{S_0}{S_0 + P\left(0, S_0, K\right)}\right) & \text{otherwise.} \end{cases}$$

Due to a dynamic allocation strategy the Portfolio is protected against market falls by guaranteed floor, which preserve a minimum level of wealth that a specific time horizon.

3. Main Results

Portfolio Insurance is a hedging strategy that protects the investor from excessive capital losses depending on the quantity of risk he or she is willing to take during the trade-off between risk and expected return. In these strategies the goal is to limit the downside risk while continue to benefit partially from the upside potential from risky assets [11]. In such a manner that at a certain point of time, the Portfolio manager look at and change the structure of a Portfolio in such a way that the value does not fall below a designed floor.

There are numerous possible strategies to insure portfolio classification by the frequency with which the positions in the portfolio have to rebalance. Two techniques are distinguished. The static strategies rebalance the portfolio positions solely before expiration of the investment horizon, and dynamic strategies rebalance the portfolio positions frequently, preferably continuously, according to certain rules.

We consider an investor buying different number of stocks and puts. Or alternatively he buys with a face value equal to the floor he is aiming at and for the remaining money buys calls on the stock.

All strategies regularly practiced depend on modifications of the above stated options. While following the first alternative it is the put which guarantees that the invested capital does not drop below the floor, applying the second alternative, it is the bond which insures the investor against falling prices.

The stock respectively calls make up for the profit presuming of rising prices. Before deciding about what type of portfolio insurance will be used some points have to be classified: which financial instruments are provided by the market, and what are their characteristics (coupons, volatilities, correlation with the market etc.)? And which idea does the investor have about.

The composition of the portfolio (which financial instrument), amount of capital to invest and the floor (lower bond of the portfolio value) or rather the minimum return he is aiming at the end of the investment.

Given the floor F and the capital invested V the possibly negative minimum return of a one year invested is given by

$$\rho = \frac{F - V}{V}.$$

We may illustrate, the two alternatives described above in the following example.

Supposed the investor has decided to invest in stock. Depending on the type of return of the object we distinguish two cases (for negative return, as storage costs of real values for example, the approach can be applied analogously):

1) continuous dividend yield l.

2) ex ante known discrete yields with a time 0 discounted total value of L_0.

Placing our consideration at the beginning $t = 0$ of the investment, the time to maturity is $\tau = T - t = T$. For both strategies, the goal is to determine the number q of stocks and p of (European put) options. The data of the example is in Table 1. The volatility can be interpreted as a measure of variability of the stock price.

Table 1. Financial instruments.

Current point of time t	0
Available capital V	\$200,000
Target floor F	\$195,000
Investment horizon T	2 years
Current stock price S_0	\$100
Continuously compounded annual interest r	0.10
Annual stock volatility σ	0.30
Dividends during time to maturity	
Case 1: continuous dividends l	0.02
Case 2: dividends with present value L_0	\$10

Case 1: All stochastic processes are defined on a stochastic basis $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, \mathbb{P}\right)$ which satisfies the usual hypotheses. We consider two assets, the evolution of the risk asset S, that satisfies the Black-Scholes model,

$$dS_t = S_t\left(\mu d_t + \sigma dw_t\right), S_0 = s,$$

where $W = (W_t)_{0\le t\le T}$ denotes a standard Brownian motion with respect to the real world measure $\mathbb{P}$ and μ and σ are constant and we assume that $\mu > 0$ and $\sigma > 0$ and also another asset the put option written on S_t such that

$$V^{OBPI} = q\mathrm{e}^{l\tau} \cdot S_t + p\mathrm{e}^{l\tau} \cdot P_{K,T}\left(S_t, \tau\right).$$

The stock pays a continuous divided at rate $l = 2\%$ p.a. which he reinvested immediately. At maturity T the position in the stock grew from q to $q\mathrm{e}^{l\tau}$ with $\tau = T - 0 = T$. Thus, for strategy one he has to buys in $t = 0$ some number of put options also. The investor chooses the put options delivery price K such that his capital after two years does not drop below the floor F he is aiming at. That is, exersing the puts in time T (if $S_T \ge K$) must give the floor F which gives the second condition. $qK = F$ which is

$$q\mathrm{e}^{l\tau} \cdot K = F \Rightarrow q = \frac{F}{K}\mathrm{e}^{-l\tau}.$$

If the investor choose the strike price K = 95, to make sure that the capital does not drop below the floor \$195,000 he buys

$$q = \frac{F}{K}\mathrm{e}^{-l\tau} = 1970$$

number of stocks
and

$$p\mathrm{e}^{l\tau} = 2050$$

number of put options.

According to the Black-Scholes formula with dividend payment

$$P(S,t) = K\mathrm{e}^{-r\tau} N(-d_2) - S\mathrm{e}^{-l\tau} N(-d_1),$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - l + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}},$$

$$d_2 = d_1 - \sigma\sqrt{\tau},$$

and $N(d)$ is a standard normal cumulative distribution function. Therefore the price of the put option is \$8.87/put.

All stochastic processes are defined on a stochastic basis $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, \mathbb{P}\right)$ which satisfies the usual hypotheses. We consider a riskless bond B grows at a constant interest rate r, $dB_t = B_t r dt$ where $B_0 = b$ and a call option written on B_t such that

$$V^{OBPI} = q \cdot B_0 + p\mathrm{e}^{l\tau} \cdot C_{K,T}(B_0, \tau).$$

Following the corresponding strategy two he invests $F\mathrm{e}^{-r\tau} = \$159652.5$ in bonds at time 0 which gives compounded to time T exactly the floor, $F = \$195000$. For the remaining capital of $V - F\mathrm{e}^{-r\tau} = \$40347.5$, he buys 2050 put, which have a price of \$23.08/call according to the Black-Scholes formula

$$C(S,t) = S\mathrm{e}^{-l\tau} N(d_1) - K\mathrm{e}^{-r\tau} N(d_2),$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - l + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}},$$

$$d_2 = d_1 - \sigma\sqrt{\tau}.$$

and $N(d)$ is a standard normal cumulative distribution function.

From the put-call parity follows the equivalence of both strategies *i.e.* both portfolio consisting of stocks and puts respectively zero bonds and calls have at each time t the same value:

$$q \cdot B_0 + p\mathrm{e}^{l\tau} \cdot C_{K,T}(B_0, \tau) = q \cdot K\mathrm{e}^{-b\tau} + p\mathrm{e}^{l\tau} \cdot P_{K,T}(S_t, \tau),$$

where $\tau = T - t, b = r - l$ and $qK\mathrm{e}^{-br} = F\mathrm{e}^{-r\tau}$.

Table 2 shows the impact of the stock price ST in one year on both the insured and the non-insured Portfolio value and returns, when the insured Portfolio is at least worth \$195, 000. In Figure 1, the insured portfolio the value is at least worth \$195,000 when $S_T < 100$ and thus protect the portfolio against downside losses to fall below the floor when it compared to a non-insured portfolio. Case 2: Until maturity the stock pays dividends with a time 0 discounted total values. $L_0 = \$10$ which are after distribution immediately invested in bonds. At time T, the dividend yield has a compounded value of $L_T = L_0\mathrm{e}^{r\tau} = \$12.2$ where $\tau = T$ denotes the time to maturity. Reasoning as in case 1) and taking the dividend L_T into account he buys q stocks respectively p puts and obtains the following equation

Table 2. The effect of a portfolio insurance on portfolio value and return as adapted from @SFEexerput.xpl.

Stock price S_T $	Non-insured portfolio value $	Return % p.a.	Insured portfolio value $	Return % p.a.	Insured portfolio in % of the non-insured portfolio
60	124,897	−38	195,000	−3	156
70	145,713	−27	195,000	−3	134
80	166,530	−17	195,000	−3	117
90	187,346	−6	195,000	−3	104
100	208,162	+4	195,000	−3	94
110	228,978	+14	225,500	+13	98
120	249,794	+25	246,000	+23	98
130	270,611	+35	266,500	+33	98
140	291,427	+46	287,000	+44	98
150	312,243	+56	307,500	+54	98

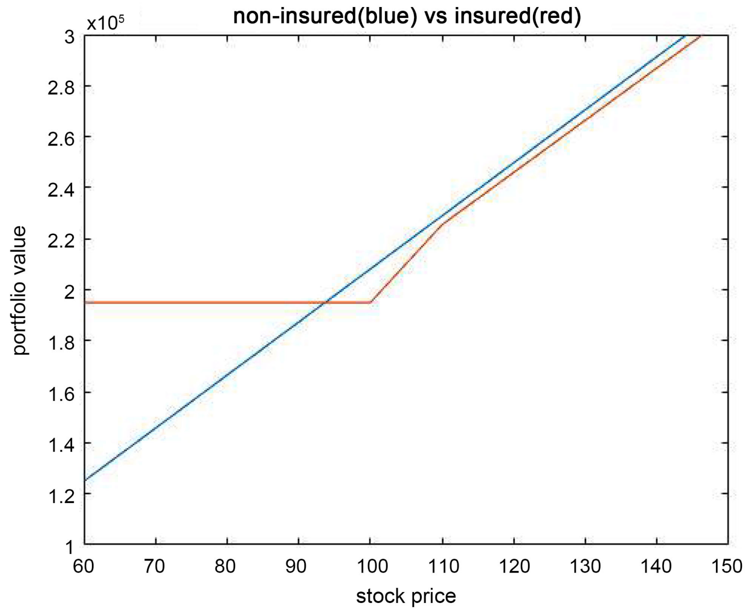

Figure 1. The effects of a portfolio insurance when $S_0 = 100$, $K = 95$, $F = 195000$, $T = 2$, and $l = 0.02$: while the blue line represents the value of the non-insured, the red line represents the value of the insured portfolio.

$$V^{OBPI} = q \cdot S_t + pP_{K,T}\left(S_t - L_T, \tau\right).$$

Solving the equations analogously as in 1) the number q of stocks and p puts for delivery price $K = 95$ for strategy one are obtained

$$q = \frac{F}{K + L_T} = 1819.2$$

stocks and $qe^{lT} = 1893.2$ puts. For strategy two he buys, 1893.2 calls at a price of $23.08/call. He invests $195000e^{-r\tau} = 159652.5$ in bonds.

Table 3 shows the risk increase effect of the insurance, when the insured portfolio is at least worth \$202,951. In **Figure 2**, the insured portfolio the value is at least worth \$202,951 when $S_T < 100$ and thus protect the portfolio against downside losses to fall below the floor when it compared to a non-insured portfolio. We can also illustrate the payoffs of the insured portfolio of a non-dividend payment in the following figure.

It is clear in **Figure 3**, that OBPI portfolio is protected against losses when the value of the underlying risky asset at maturity S_T is lower than the strike price $K = 95$. On the other hand, OBPI portfolio insurance can participate in market gains when S_T is higher than the strike price K.

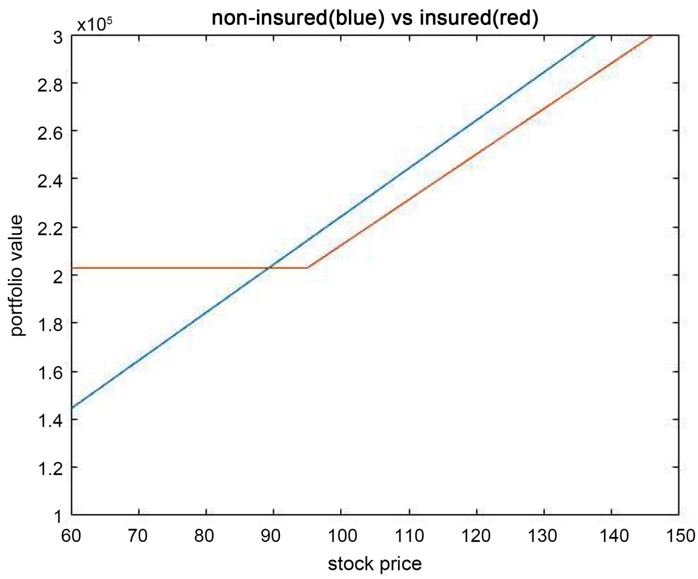

Figure 2. The effects of a portfolio insurance when $S_0 = 100$, $K = 95$, $F = 195000$, $T = 2$, $l = 0.02$, $L_0 = 10$ and $L_T = 12.2$: while the blue line represents the value of the non-insured, the red line represents the value of the insured portfolio.

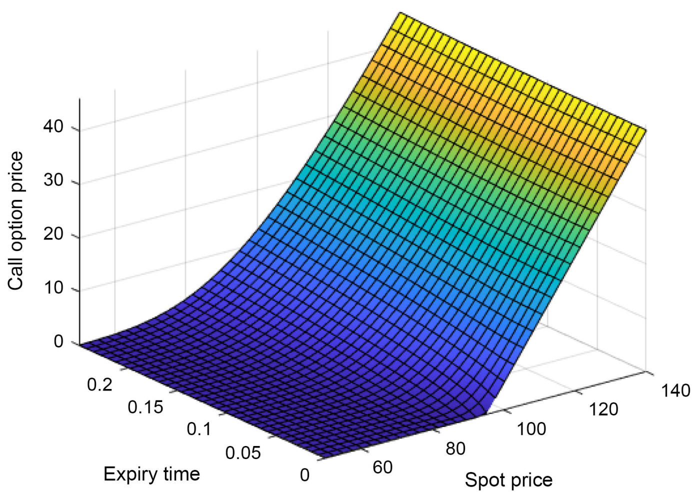

Figure 3. Payoff of insured portfolio as a function of the price of the underlying risky asset S_T when $S = 100$, $S = 100$, $\sigma = 0.30$, $T = 2$, $r = 0.01$.

Table 3. The effect of a insured portfolio value on a non-insured portfolio value and return as adapted from @SFEexerput.xpl.

Stock price S_T $	Non-insured portfolio value $	Return % p.a.	Insured portfolio value $	Return % p.a.	Insured portfolio in % of the non-insured portfolio
60	144,400	−28	202,951	+1	141
70	164,400	−18	202,951	+1	123
80	184,400	−8	202,951	+1	110
90	204,400	+2	202,951	+1	99
95	214,400	+7	202,951	+1	95
100	224,400	+12	212,417	+6	95
110	244,400	+22	231,349	+16	95
120	264,400	+32	250,281	+25	95
130	284,400	+42	269,213	+35	95
140	304,400	+52	288,145	+44	95
150	324,400	+62	307,077	+54	95

4. Conclusion and Suggestions

The dynamic application and analysis of OBPI were the main goal to find the numbers of stocks, put options, bond value and call options in this paper. It gives a guarantee that at any time t, the value of the portfolio will never fall below the given floor. Thus, the OBPI portfolio is protected against losses when the value of the underlying risk asset at maturity is lower than the strike price and gains when the underlying risky asset is higher than the strike price. No matter how low the price for the underlying portfolio falls, the payoff is at least equal to the floor return. If the price of the underlying increases over the investment period, the payoff increases as well. The difference in the payoff from an uninsured portfolio in the case of favourable price movement can be interpreted as the cost of insurance or insurance premiums. The level of the floor is similar to the deductible of conventional insurance. Choosing a lower floor decreases the guaranteed amount for the investor at the end of the investment period. Further study can be contacted to optimize different consumer appetites for risks and therefore determine the optimal utility.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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