

An Omni-directional Multi-angle Structuring Element Weighted Combination Morphological Filter

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Abstract: To improve the performance of morphological filters in image denoising, a new morphological filter is constructed by using omni-directional multi-angle structuring elements. Comparing with median filter, omni-direction structuring elements morphological filter and nonlinear filter, it can not only efficiently suppress image noise, but also preferably preserve geometrical features of images and improve the image visual effect.

Keywords: mathematical morphology; image processing; morphological filter; structuring element;

1 Introduction

In the digital image processing, filtering is a main method of denoising. It demands that the detailed information of original signals should be preserved as far as possible, when noise is filtered. Most of noise is impulse noise or Gaussian noise in the processing of image capture and transmission, and contains many high frequency components. Using the method of median or linear filtering to filter noise always seriously blurs detailed information. Nonlinear filtering can not only filter noise, but also preserve image high frequency information at the same time. This makes image clearer and verisimilitude, so it gets more broad study and application.

Morphological filters are a class of new nonlinear filtering methods that grow from mathematical morphology [1-2]. They are based on signal geometrical characteristics, by matching signals with the predefined constitutional unit to extract signal and reduce noise. The common Morphological filters only use single constitutional unit, so they blur image detailed information when filtering noise so that this makes recovering geometrical information impossible. This paper proposes a new morphology operation method that is omni-directional multi-angle structuring element weighted combination morphological filter. Simulations prove that the filter has high performance for reducing noise and preserving image detailed information and is easily realized.

2 Fundamental Transformation of Morphology

Fundamental transformations of mathematical morphology include erosion, dilation, open and close operation.

Definition 1. $f(n)$ and $g(n)$ are defined as two discrete functions in two dimensional discrete spaces of F and G . $f(n)$ is an input signal, $g(n)$ is structuring elements. So the four fundamental morphology transformation definitions are as follows:

Erosion and dilation of $f(n)$ by $g(n)$ are:

$$(f \ominus g)(n) = \min_{m \in G} \{f(n+m) - g(m)\} \quad (1)$$

$$(f \oplus g)(n) = \max_{m \in G} \{f(n-m) + g(m)\} \quad (2)$$

Morphological opening and closing operations of $f(n)$ by $g(n)$ are:

$$(f \circ g)(n) = [(f \ominus g) \oplus g](n) \quad (3)$$

$$(f \bullet g)(n) = [(f \oplus g) \ominus g](n) \quad (4)$$

The information that morphological opening and closing processes is relative to image gravure and convex, so they are all sided operators. Morphological opening can reduce signal positive pulse noise, and morphological closing can reduce signal negative pulse. To reduce signal positive and negative pulse noise at the same time, it usually uses cascade form of morphological opening and closing filters. Margos defined morphological open-closing and morphological close-opening filters by using the same structuring element [3].

Definition 2. $f(n)$ is input signal, $g(n)$ is structuring element, so the filters of morphological open-closing and close-opening are separately defined as follows:

$$OC(f(n)) = (f \circ g \bullet)(n) \tag{5}$$

$$CO(f(n)) = (f \bullet g \circ)(n) \tag{6}$$

3 Omni-directional Multi-angle Weighted Mean Morphological Filter

The output of morphological filter depends on not only transformation, but also size and shape of structuring element. Open-closing and close-opening filters only use a kind of structuring element, so the output of filters only contain a kind of geometrical information, and the others are filtered with noise, this prejudices preserving of image geometrical information. In literature [4], multi-structuring element morphological filter of open-closing maximal and close-opening minimal are defined, their definitions are as follows:

Definition 3. Omni-directional morphological open-closing (OOC) maximal filter and closing-opening minimum filter (OCO) are defined as follows:

$$OOC(f(n_1, n_2)) = \max_{k \in \{0, 1, \dots, 4N-1\}} (f \circ W_k \bullet W_k)(n_1, n_2) \tag{7}$$

$$OCO(f(n_1, n_2)) = \min_{k \in \{0, 1, \dots, 4N-1\}} (f \bullet W_k \circ W_k)(n_1, n_2) \tag{8}$$

Where $f(n_1, n_2)$ is input image, $W_k = \{W_0, W_1, \dots, W_{4N-1}\}$ is omni-directional structuring element of $(2N+1) \times (2N+1)$ rectangular window, they separately correspond to linear structuring elements that use center pixel as midpoint with rotation angle $\theta = ka (k = 0, \dots, 4N-1; a = 180^\circ / 4N)$. Simulation indicates denoising effect is not ideal, it needs to be improved in terms of preserving detailed information and reducing noise.

Definition 4. $\{X(n_1, n_2)\} (n_1, n_2) \in Z, Z = \{\dots, -1, 0, 1, \dots\}$ is a digital image. The size of two-dimensional filter window W is $(2N+1) \times (2N+1)$, and its pixel denotes as $\{X(n_1 + l_1, n_2 + l_2) | -N \leq l_1, l_2 \leq N\}$. The flowing subset $W_k (k = 0, 1, \dots, 8N-1)$ is defined as omni-directional multi-angle structuring element [5-6]:

$$W_k = \{X(n_1 + l_1, n_2 + l_2) | \theta_k = ka, -N \leq l_1, l_2 \leq N\} \forall k = 0, 1, \dots, 8N-1$$

Where $a = 180^\circ / 4N$, a is identity rotate angle. For example: when $N = 1$, omni-directional multi-angle structuring elements are shown in Figure 1. The previous two sets $(W_1 : W_8)$ respectively correspond

$\theta_k = 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ$. The third set $(W_9 : W_{12})$ is directional protection structuring element.

Based on the proposed omni-directional multi-angle structuring elements in definition 4, we propose an omni-directional multi-angle structuring elements weighted mean morphological filter. It alternately uses close-opening and open-closing filters to remove image noise, then computes weighted mean, its structure is shown in Figure 2.

Definition 5. Setting input image is f , omni-directional multi-angle structuring elements are $\{W_1, W_2, \dots, W_N\}$, output is Y . The output of new algorithm of omni-directional multi-angle weighted mean filter is

$$Y = \frac{1}{N} \sum_{i=1}^N x_i \tag{9}$$

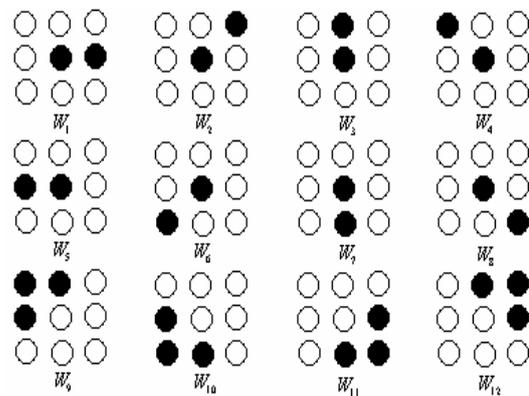


Figure 1. Omni-directional multi-angle structuring elements

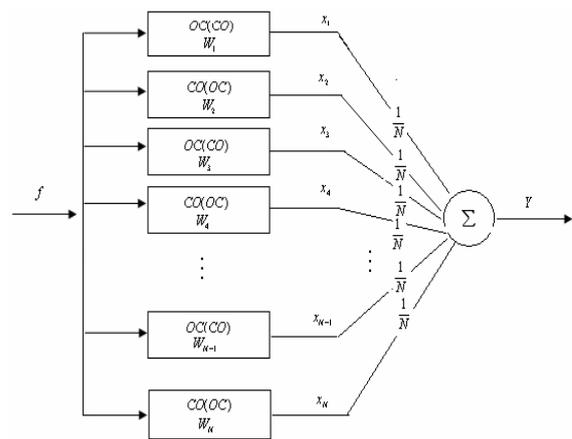


Figure 2. Omni-directional multi-angle structuring element weighted mean filter structure of new algorithm

4 Simulation and Discussion

To prove validity of the proposed filtering algorithm, we use Lena image as original image that pixel size is 256×256 and pixel depth is 8bits. Employing structuring

element mentioned in Figure 1 as omni-directional structuring element. Figure 3 are all kinds of filtering results of the image added 5% salt and pepper noise. Figure 4 is Peak signal to noise ratio of image added salt and pepper noise, its abscissa 1-7 correspond to the filtering types that are noise, open-closing, close-opening, multi-structuring element open-closing maximum, Multi-structuring element close-opening minimum, median and the new filtering method. Table 1 is the main performance parameter of image that added salt and pepper noise and removed noise, the following filtering all use 3×3 filter window.



Figure3.1Original image



Figure3.2 Noise i image



Figure3.3 open-closing filtering



Figure3.4 close-opening filtering



Figure3.5 multi-structuring Open-closing



Figure3.6 multi-structuring close-opening min max



Figure3.7 median filtering



Figure3.8 Proposed filtering

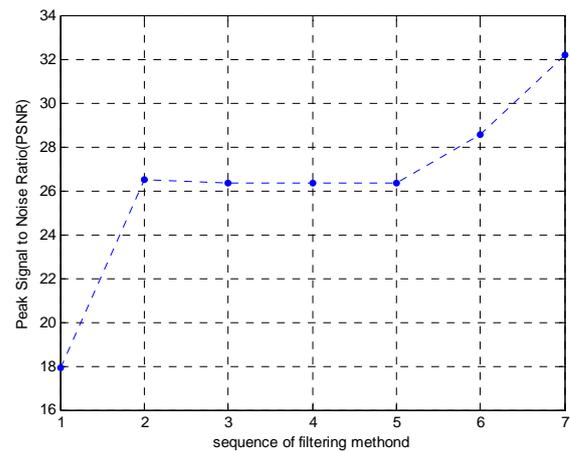


Figure 4. Peak signal to noise ratio of image added salt and pepper noise

Table 1. The main performance parameter of image that added salt and pepper noise and removed noise

Filter type	Minimum mean square error	Mean Absolute error	Peak signal to noise ratio
Noise image	940.08	6.43	17.943
Open-closing	131.66	4.83	26.483
Close-opening	136.52	4.88	26.323
Multi-structuring element open-closing maximum	136.21	2.78	26.33
Multi-structuring element close-opening minimum	135.95	2.71	26.34
Median filtering	81.91	3.20	28.54
proposed filtering algorithm	35.39	1.58	32.19

By comparing the performance parameters, we can see that the filtering performance of tradition and multi-structuring element minim filters of opening-closing and

closing-opening are not ideal, and they blur image detailed information. The main performance parameters of image after being removed noise by the new filtering algorithm are superior to median filter algorithm. Especially the new algorithm not only filters noise effectively, but also well preserves image detailed information and makes visual effects better in filtering noise of the image with added salt and pepper noise.

4 Conclusion

Because the proposed filtering algorithm combines omni-directional multi-angle structuring element property of morphology filter, so the performances of removing noise, preserving image geometry and others detailed information have been improved with better effect of filtering, comparing with tradition filters. The algorithm structure is also simple and easy to be realized.

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