

# An Optimal Method for Fault Location Based on Trust Region Algorithm

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**Abstract:** The traditional time-domain algorithms may cause many errors if the line parameters often change because of environmental factors and so on. This paper presents an optimal method for fault location without requiring line parameters. The voltage and current measurement at both ends of a transmission line during a fault are assumed to be available and the unknowns can be obtained by utilizing the trust region algorithm and the result of fault location can also be obtained. Evaluation studies using field data show that the proposed algorithm is effective and reliable.

**Keywords:** optimal method; transmission line; fault location; trust region

## 1 Introduction

Transmission line is an important component of the power system, which is responsible for power transmission. As the voltage level increased and the transmission distance added, the operating environment become complex and the faults are increased. Speedy and precise location of faults in a transmission system can help the maintenance engineers to quickly pinpoint the faulted component, accelerate system restoration, reduce the outage time and improve system reliability [1, 2].

A number of researchers have worked in fault location and developed a valuable set of algorithms [3-9]. Based on different signals, one-terminal, two-terminal, or multi-terminal algorithms have been proposed in the past. Based on different math models, the algorithms use algebraic equations or differential equations. Based on the signals in different transient time, the algorithms can be divided into traveling waves and the analysis of fault method. The analysis of fault method can also be divided into time-domain and frequency-domain method.

In order to locate fault in absence of synchronized data and line parameters, this paper presents an optimal method that employs unsynchronized voltages and currents from two ends of the line. A set of equations involving fault location, synchronized angle, and the line parameters have been established, from which six real equations with six unknown variables can be obtained. Then, the unknowns are solved by applying the trust region method. By using the field data in [9], this method is tested to be correct and efficient.

## 2 Trust region method

Trust region method [10] is a well-established technique in nonlinear optimization. The traditional methods such as Gauss-Newton method and the least square method may cause some problems if the chose values are far from the actual values. There are many advantages in trust region method, for example, the stability, robustness

and convergence of this method are all strong.

Consider the unconstrained minimization problem:

$$\min f(x) \quad (1)$$

Where  $x = (x_1, x_2, \dots, x_n)^T$  is a vector, suppose you are at a point  $x_k$  and you want to improve behavior of function  $f$  with a simpler function  $q$ , which reasonably reflects the behavior of function  $f$  in a neighborhood  $N$  around the point  $x_k$ . This neighborhood is the trust region.

$$\text{Define: } \min_d [q(d), d \in N] \quad (2)$$

Where  $d$  is a trial step in a neighborhood  $N$ .

Mathematically the trust region sub-problem is typically stated.

$$\min_d \left[ \frac{1}{2} d^T H_k d + g_k^T d \right] = \phi(d) \quad (3)$$

$$\text{and } \|d\|_2 \leq \Delta_k \quad (4)$$

Where  $g_k$  is the gradient of  $f(x)$  at the current point  $x_k$ ,  $H_k$  is the Hessian matrix,  $\Delta_k$  is a positive scalar.

$$\text{Define: } \begin{cases} J(x_k)d_k = -f(x_k) \\ x_{k+1} = x_k + d_k \end{cases} \quad (5)$$

$$\text{Where } d_k \text{ is a trial step, } J(x_k) = \begin{bmatrix} \nabla f_1(x_k)^T \\ \nabla f_2(x_k)^T \\ \vdots \\ \nabla f_n(x_k)^T \end{bmatrix} \text{ is}$$

the n-by-n Jacobian, and so the trust region sub-problem is

$$\min_{d_k} \left\{ \frac{1}{2} f(x_k)^T f(x_k) + d_k^T J(x_k)^T f(x_k) + \frac{1}{2} d_k^T [J(x_k)^T J(x_k)] d_k \right\} \quad (6)$$

### 3 Method for fault location

#### 3.1 Equations

Consider the two-terminal line as shown in Fig.1. It is assumed that three phase voltages and currents at terminals M and N are available, and the line is a transposed line.  $E_M$  and  $E_N$  represent the equivalent voltage sources at buses M and N. F indicates the fault point.

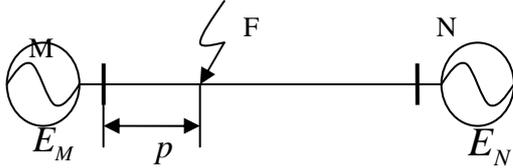


Figure 1. System diagram with two-terminal power

The following notations are used:

$\dot{V}_{ma}, \dot{V}_{mb}, \dot{V}_{mc}$ : phase a, b, c voltages during the fault at terminal M, respectively;

$\dot{I}_{ma}, \dot{I}_{mb}, \dot{I}_{mc}$ : phase a, b, c currents during the fault at terminal M, respectively;

$\dot{V}_{na}, \dot{V}_{nb}, \dot{V}_{nc}$ : phase a, b, c voltages during the fault at terminal N, respectively;

$\dot{I}_{na}, \dot{I}_{nb}, \dot{I}_{nc}$ : phase a, b, c currents during the fault at terminal N, respectively;

$p$ : per-unit fault distance from terminal M.

We have the equations when there is fault [8, 9].

$$\begin{bmatrix} \dot{V}_{ma} \\ \dot{V}_{mb} \\ \dot{V}_{mc} \end{bmatrix} - p \begin{bmatrix} R_s + jX_s & R_m + jX_m & R_m + jX_m \\ R_m + jX_m & R_s + jX_s & R_m + jX_m \\ R_m + jX_m & R_m + jX_m & R_s + jX_s \end{bmatrix} \cdot \begin{bmatrix} \dot{I}_{ma} \\ \dot{I}_{mb} \\ \dot{I}_{mc} \end{bmatrix} = \begin{bmatrix} \dot{V}_{na} \\ \dot{V}_{nb} \\ \dot{V}_{nc} \end{bmatrix} - (1-p) \begin{bmatrix} R_s + jX_s & R_m + jX_m \\ R_m + jX_m & R_s + jX_s \\ R_m + jX_m & R_m + jX_m \end{bmatrix} \begin{bmatrix} \dot{I}_{na} \\ \dot{I}_{nb} \\ \dot{I}_{nc} \end{bmatrix} \quad (7)$$

Where  $R_s + jX_s$  are the total self resistance and reactance,  $R_m + jX_m$  are the total mutual resistance and

reactance,  $\delta$  is the synchronization angle between measurement at terminal M and N.

Then, six equations with six unknown variables can be obtained as

$$f_i(x) = 0, i = 1, \dots, 6 \quad (8)$$

#### 3.2. Proposed approach for faults

Define:  $x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T$

$$= [p \ \delta \ R_s \ X_s \ R_m \ X_m]^T \quad (9)$$

$$f(x) = [f_1(x) \ f_2(x) \ \dots \ f_6(x)]^T \quad (10)$$

The Jacobian matrix  $J(x)$  is calculated as

$$J_{ij}(x) = \frac{\partial f_i(x)}{\partial x_j}, i = 1, \dots, 6, \quad j = 1, \dots, 6 \quad (11)$$

Where  $J_{ij}(x)$  is the element in the  $i$ th row and  $j$ th column of  $J(x)$ .

The unknowns can be obtained by the proposed approach.

1) The initial value  $x^{(0)}$  and accepted error  $\varepsilon \geq 0$  are given. Make  $k = 0$ ;

2) Compute  $J_{ij}(x^{(k)}) = \frac{\partial f_i(x^{(k)})}{\partial x_j}$ ,

$i = 1, \dots, 6, j = 1, \dots, 6$ ;

3) Get  $\Delta x^{(k)}$  from

$$\min_{\Delta x^{(k)}} \left\{ \frac{1}{2} f^T(x^{(k)}) f(x^{(k)}) + \Delta x^{(k)T} J(x^{(k)}) f(x^{(k)}) + \frac{1}{2} \Delta x^{(k)T} [J(x^{(k)})^T J(x^{(k)})] \Delta x^{(k)} \right\}$$

Compute  $x^{(k+1)} = x^{(k)} + \Delta x^{(k)}$ ;

4) Terminate the program, if  $\|\Delta x^{(k)}\| < \varepsilon$ ; or else make  $k = k + 1$  and turn to 2).

#### 3.3. Errors in Fault Location

Assuming the total length of the line is  $L$ , the actual fault distance is  $L_a$  and the fault distance using the proposed approach is  $L_f$ .

Define the error:  $\frac{|L_a - L_f|}{L} \times 100 \quad (12)$

#### 4 Evaluation Studies

In order to test the efficiency and reliability of this

method, the field data in [9] are used. Considering the convergence and reliability, the value of  $\delta$  is set to 0 and the initial value of  $p$  is set to 0.5 at each iteration. The fault location is kept between 0 and 1 by setting it to 0 when it becomes negative and setting it to 1 when it exceeds 1.

The system is a 500kV, 138.69km transmission line showed in Fig.1. The actual fault distance is 57.82km from terminal M. The recorded data of the voltage and current are shown in Table 1.

Table 1. The voltage and current estimates in case 1

Quantities	Values
$\dot{V}_{ma}$	0.71253+j0.71835
$\dot{V}_{mb}$	0.027296-j0.46566
$\dot{V}_{mc}$	-0.96676+j0.35402
$\dot{I}_{ma}$	-4.9741-j3.0075
$\dot{I}_{mb}$	-39.1991-j13.7695
$\dot{I}_{mc}$	1.7197-j1.7414
$\dot{V}_{na}$	0.054162+j0.99844
$\dot{V}_{nb}$	0.54877-j0.38948
$\dot{V}_{nc}$	-0.90144-j0.42359
$\dot{I}_{na}$	0.32874+j5.6612
$\dot{I}_{nb}$	-15.2273-j48.4533
$\dot{I}_{nc}$	-1.7765-j1.0783

Without any knowledge of the values of line parameters, the proposed method is applied to estimate the fault location. Starting values of 0.5, 0, 0.5, 0.5, 0.35 and 0.35 for  $p, \delta, R_s, X_s, R_m$  and  $X_m$ . After several iterations, the result is shown in Table 2.

Table 2. The unknowns estimates in case 1

$p$	$\delta$	$R_s$	$X_s$	$R_m$
$X_m$				
0.4027	-0.7187	0.0030	0.0179	0.0018
0.0063				

The fault location is estimated as 55.85km, with an error of 1.97km. The result shows that the proposed method is very efficient.

The convergence behavior of the algorithm is studied by varying the starting values. Table 3 shows the required iteration number by using various starting values.

From Table 3, the trust region algorithm is more robust than the least square method, and converges quickly to the correct solution within about 15 or fewer iterations.

### 5 Conclusion

This paper presents a new solution for fault location when the line parameters are not available. The studies show that this optimal method is quite robust and insensitive to starting values, and converges quickly. As transmission line is assumed a transposed line, there may be some errors in some degrees.

Table 3. Effects of starting values on algorithm convergence characteristics in case 1

$p$	$\delta$	$R_s$	$X_s$	$R_m$	$X_m$	Trust region algorithm		The least square method	
						Iteration number	fault location	Iteration number	fault location
0.5	0	0.02	0.2	0.01	0.1	14	0.4027	75	0.4437
0.5	0	0.5	0.5	0.25	0.25	15	0.4027	85	0.4506
0.5	0	0.5	0.5	0.35	0.35	13	0.4027	85	0.4669
0.5	0	0.3	0.3	0.15	0.15	13	0.4027	80	0.4591
0.5	0	0.2	0.2	0.1	0.1	12	0.4027	85	0.4660
0.5	0	0.2	0.2	0.15	0.15	10	0.4027	85	0.4691
0.5	0	0.2	0.15	0.1	0.1	12	0.4027	85	0.4636
0.5	0	0.2	0.15	0.15	0.1	13	0.4027	85	0.4655
0.5	0	0.25	0.25	0.15	0.15	12	0.4027	67	0.4583
0.5	0	0.15	0.15	0.1	0.1	10	0.4027	85	0.4559

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