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Fuzzy Inventory Model with Variable Production and Selling Price Dependent Demand under Inflation for Deteriorating Items

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Abstract

The main purpose of this paper is to develop an inventory model under fuzzy approach by considering the effect of inflation and time value of money, to determine the optimal time period for inventory cycle and minimum total average costs. The model is integrated production inventory model developed where; the Demand has a direct linear impact on production rate. The model can be divided into four stages. In the first two stages with original production rate and subsequent change in production rate, inventory level rises. Third stage is time after the accumulation of inventory and before the deterioration starts, where demand which selling price dependent is depreciating the inventory level, while in the fourth stage deterioration occurs, which is considered to follow two parameter Weibull distribution. The back-order is not considered. Hexagonal fuzzy numbers are used to derive optimum solution and defuzzification by graded mean integration representation method. A numerical example is given to demonstrate the applicability of the purposed model and sensitivity analysis is carried out to reveal the impact of change in parameter values.

Keywords

Weibull Distribution Deterioration, Variable Production Rate, Hexagonal Fuzzy Number, Selling Price Dependent Demand, Inflation

1. Introduction

Any inventory system should focus on maintaining and increase levels of customer satisfaction while keeping inventory costs within predetermined time frames.

The only way to increase the profit is to manage the product demand in accordance with market ups and downs and fluctuations. When it comes to low-life products that deteriorate quickly, such as milk products and vegetables. Additionally, it is not feasible to fix the demand for such things during the course of the products lifecycle. The stability of the production process, uncertainty regarding the magnitude of future requests, uncertainty regarding inventory costs, uncertainty regarding deterioration, etc., are some key factors that determine whether inventory control is successful or not.

In practice, parameters change over time, depending on the circumstances. In this research, a production inventory model for deteriorating products under the influence of inflation is considered. In today's unstable economy, especially for long-term investments, the effects of inflation cannot be ignored because uncertainty regarding future inflation may also affect the ordering strategy. As inflation devalues currency, this effect of inflation should also be taken into account. Tayal *et al.* [1] studied production model where demand rate is exponential and shortages are not allowed, holding cost is time dependent with constant deteriorating rate. Ghasemi [2] developed economic production quantity model for deteriorating products with and without shortages where holding cost depends on ordering run length. Krishnaraj & Ishwarya [3] developed an inventory model with Weibull demand rate for deteriorating items where shortage is considered during lead time. Ardak & Borade [4] studied optimal policy for deteriorating products where demand pattern changes during buildup time and during depletion period also deterioration starts after a certain time and it varies as well. S. Singh *et al.* [5] studied partially backlogged inventory model for deteriorating items where demand is time dependent with shortages taken into account and partially backlogged at a rate of decreasing function of waiting time for next replenishment. Tripathi *et al.* [6] established inventory model with and without shortages allowed and demand is exponential time dependent with variable deterioration. Sahoo & Tripathy [7] studied time dependent holding cost with deterioration following three parameter Weibull distribution, also salvage value is considered in this model. S. R. Singh *et al.* [8] developed inventory model where production rate is time dependent and demand is function of production rate. Ardak [9] investigated production inventory model with constant deterioration rate and checked the effect on holding cost by change in demand rate. D. Singh [10] constructed production inventory model with constant deterioration rate and stock as well as selling price dependent demand. Also proposed solution-search process to determine the preservation technology and ideal production time. Sinha & Modak [11] developed a production inventory model that takes into account the issues of carbon emission and carbon trading. S. R. Singh & Rani [12] developed an inventory model under inflation where demand is multivariate with markdown policy with shortages for deteriorating item. Abdul Halim *et al.* [13] studied inventory model with an overtime production opportunity for deteriorating items. K. Kumar *et al.* [14] proposed an inventory model for healthcare medicinal products with deterioration rate following three parameter Weibull distribution under the inflation and partial backlogging. Barman *et al.* [15] analysed optimal production policy for supply chain model with two levels for deteriorating products considering both cases, with and without shortage. Roy Chowdhury [16] formulated production inventory model with time dependent demand and time dependent holding cost for constant deteriorating rate and shortages are avoided. Sharma *et al.* [17] studied an economic production quantity model with time dependent deterioration and different demands assumed at the different stages of the model to improve the profit for low-life items and shortages are partially satisfied.

Some parameters have ambiguous definitions or not clearly defined; their values are approximated based on subjective beliefs. In order to evaluate the optimal solution for the model in various diverse circumstances, the inventory model is solved in a fuzzy environment. Shekarian *et al.* [18] performed a survey as a scientific and complete evaluation in the subject of fuzzy inventory model, figuring out the principal achievements attained. In total, 210 paper samples are diagnosed and labeled in line with the common characteristics of the model.

Roy *et al.* [19] formulated fuzzy inventory model with stock dependent demand under inflation and time value of money. Pal *et al.* [20] studied fuzzy production inventory model with two parameter Weibull deterioration rate under inflation where shortage are not considered and demand is ramp type. Pal *et al.* [21] developed fuzzy economic order quantity model under inflation with ramp type demand and shortages with Weibull deterioration rate. Jaggi *et al.* [22] studied optimal ordering policy in fuzzy environment with constant demand under inflation over fixed planning horizon. Behera & Tripathy [23] investigated inventory model under fuzzy environment where demand which is function of time and depends on reliability for deteriorating items. Sen & Saha [24] investigated negative exponential demand rate with fuzzy lead time with partial backlogging for deteriorating items. The model has distinctive design due to probabilistic deterioration. K. Kumar *et al.* [14] formulated an inventory model where demand is time dependent and ordering cost is function of time as well, where trapezoidal fuzzy numbers are used and partial backlogging are allowed for deteriorating items. S. Kumar [25] developed a production inventory model with exponential time dependent demand under fuzzy environment and shortages are partially backlogged. The backlog of undersupply is regarded as a function of waiting time. Chaudhary & Kumar [26] studied a model under Intuitionistic fuzzy set theory to reduce the uncertainty with constant deterioration rate, and the demand is considered to be quadratic with shortage. Choudhury *et al.* [27] investigated adverse effects of environmental contamination brought on by production under fuzzy approach. The model is considered for deteriorating products having expiration date. Malumfashi *et al.* [28] constructed a production model with two stages of production and exponential demand with time dependent holding cost for deteriorating products

In this paper, an inventory model using a fuzzy approach was built to ascertain the ideal time period for the inventory cycle and the lowest possible total average costs. It is the production inventory model created for deteriorating products in which the production rate linearly dependent on the demand. There are four stages in the model. In the first two stages, with starting production rate and following change in production rate, inventory level rises. The third stage occurs when demand, which is based on selling price, is depreciating the inventory level. This stage occurs after inventory accumulates but before deterioration begins. Deterioration occurs in the fourth stage, which is two-parameter Weibull deterioration. Backorders are not taken into account. The optimum solution is determined using hexagonal fuzzy numbers, and the defuzzification process is handled using the graded mean integration representation approach.

2. Definition and Preliminaries

Definition 2.1. [29]

A fuzzy set $\tilde{A}$ on the given universal set is a set of order pairs $\tilde{A}=\{(x,\mu_{\tilde{A}}(x)):x\in X\}$, where, $\mu_{\tilde{A}}:X\to[0,1]$ is a mapping called membership function. The membership function is also a degree of compatibility or a degree of truth of x in $\tilde{A}$.

Definition 2.2. [29]

The α-cut of $\tilde{A}$ is defined by, $A_\alpha=\{x:\mu_{\tilde{A}}(x)=\alpha,\alpha\ge 0\}$

If R is a real line, then a fuzzy number is a fuzzy set $\tilde{A}$ with membership function $\mu_{\tilde{A}}:X\to[0,1]$, having following properties,

1) $\tilde{A}$ is normal *i.e.*, there exists $x\in R$ such that $\mu_{\tilde{A}}(x)=1$;
2) $\tilde{A}$ is piecewise continuous;
3) $\sup p(\tilde{A})=cl\{x\in R:\mu_{\tilde{A}}(x)>0\}$;
4) $\tilde{A}$ is a convex fuzzy set.

Definition 2.3. [30]

The fuzzy number set $\tilde{A}=(a,b,c,d,e,f)$ where, $a\le b\le c\le d\le e\le f$ and defined on R, is called the Hexagonal fuzzy number, if the membership function of $\tilde{A}$ is given by,

$$\mu_{\tilde{A}}(x)=\begin{cases} L_1(x)=\frac{1}{2}\left(\frac{x-a}{b-a}\right), & a\le x\le b\\ L_2(x)=\frac{1}{2}+\frac{1}{2}\left(\frac{x-b}{c-d}\right), & b\le x\le c\\ 1, & c\le x\le d\\ R_1(x)=1-\frac{1}{2}\left(\frac{x-d}{e-d}\right), & d\le x\le e\\ R(x)=\frac{1}{2}\left(\frac{f-x}{f-e}\right), & e\le x\le f\\ 0, & \text{Otherwise}\end{cases}$$

The α-cut of $\tilde{A}=(a,b,c,d,e,f)$, $0\le\alpha\le 1$ is $A(\alpha)=[A_L(\alpha),A_R(\alpha)]$ where,

$$A_{L_1}(\alpha)=a+(b-a)\alpha=L_1^{-1}(\alpha),$$
$$A_{L_2}(\alpha)=b+(c-b)\alpha=L_2^{-1}(\alpha),$$
$$A_{R_1}(\alpha)=e+(e-d)\alpha=R_1^{-1}(\alpha),$$
$$A_{R_2}(\alpha)=f+(f-e)\alpha=R_2^{-1}(\alpha),$$

And,

$$L^{-1}(\alpha)=\frac{L_1^{-1}(\alpha)+L_2^{-1}(\alpha)}{2}=\frac{a+b+(c-a)\alpha}{2}$$
$$R^{-1}(\alpha)=\frac{R_1^{-1}(\alpha)+R_2^{-1}(\alpha)}{2}=\frac{e+f+(d-f)\alpha}{2}$$

Definition 2.4. [30]

Suppose $\tilde{A}=(a_1,a_2,a_3,a_4,a_5,a_6)$ and $\tilde{B}=(b_1,b_2,b_3,b_4,b_5,b_6)$ are two hexagonal fuzzy numbers, then arithmetical operations are defined as,

1) $\tilde{A}\oplus\tilde{B}=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4,a_5+b_5,a_6+b_6)$

2) $\tilde{A}\otimes\tilde{B}=(a_1b_1,a_2b_2,a_3b_3,a_4b_4,a_5b_5,a_6b_6)$

3) $\tilde{A}-\tilde{B}=(a_1-b_1,a_2-b_2,a_3-b_3,a_4-b_4,a_5-b_5,a_6-b_6)$

4) $\tilde{A}\oslash\tilde{B}=\left(\frac{a_1}{b_1},\frac{a_2}{b_2},\frac{a_3}{b_3},\frac{a_4}{b_4},\frac{a_5}{b_5},\frac{a_6}{b_6}\right)$

5) $\alpha\oplus\tilde{A}=\begin{cases}(\alpha a_1,\alpha a_2,\alpha a_3,\alpha a_4,\alpha a_5,\alpha a_6) & \alpha\geq 0\\ (\alpha a_6,\alpha a_5,\alpha a_4,\alpha a_3,\alpha a_2,\alpha a_1) & \alpha<0\end{cases}$

Definition 2.5. [30]

If $\tilde{A}=(a,b,c,d,e,f)$ is a hexagonal fuzzy number, then the graded mean integration representation (GMIR) method of $\tilde{A}$ is defined as,

$$P(\tilde{A})=\frac{\int_0^{W_A}\frac{h}{2}\left(\frac{L^{-1}(h)+R^{-1}(h)}{2}\right)\mathrm{d}h}{\int_0^{W_A}h\mathrm{d}h}, \text{ with } 0\leq W_A\leq 1.$$

$$P(\tilde{A})=\frac{a+3b+2c+2d+3e+f}{12}$$

3. Notations and Assumptions

The following notations and assumptions are considered throughout the paper:

3.1. Notations

η	:	Demand coefficient
γ	:	Demand constant
p	:	Selling-price
T	:	Duration of cycle
r	:	Discount rate which represents time value of money

Continued

i	:	Inflation rate per unit time
C_{HC}	:	Holding cost
C_{PC}	:	Production cost
C_{DC}	:	Deterioration cost
A	:	Setup cost
Q_1	:	Maximum inventory level at time t_1
Q_2	:	Maximum inventory level at time t_2
Q_3	:	Maximum inventory level at time t_d
$I_1(t)$	:	Inventory level, at any time t, during $[0, t_1]$
$I_2(t)$	:	Inventory level, at any time t, during $[t_1, t_2]$
$I_3(t)$	:	Inventory level, at any time t, during $[t_2, t_d]$
$I_4(t)$	:	Inventory level, at any time t, during $[t_d, T]$
$TC(T)$	:	Total inventory cost
$\tilde{C}_{HC}$	:	Fuzzy holding cost
$\tilde{C}_{DC}$	:	Fuzzy deterioration cost
$\tilde{C}_{PC}$	:	Fuzzy production cost
$\tilde{A}$	:	Fuzzy Setup cost

3.2. Assumptions

1) Inventory cycle for single product is considered.

2) The demand rate $D(p)$ is dependent on selling price p *i.e.* $D(p) = \eta p^{-\gamma}$ where, $\eta > 0$, $\gamma > 0$, where, η is scaling factor, γ is index of price elasticity.

3) The production rate is linearly dependent on demand, that is, $P(p) = \lambda \cdot D(p)$ where, $\lambda > 1$ and production rate is greater than demand rate $D(p)$.

4) Lead time is considered to be negligible.

5) The inflationary effects and time value of money are taken into consideration.

6) The setup cost for inventory is constant.

7) No Shortages are allowed.

8) The deterioration of the products starts after a certain fix time. The rate of deterioration at time (t_d, T) is $\theta(t) = \alpha\beta t^{\beta-1}$ which is two parameter Weibull distribution where, α represents scale parameter and β represents shape parameter. There is no deterioration before time t_d.

4. Model formulation

As shown in **Figure 1**, the inventory cycle is formulated with two rates of production and demand dependent on selling price. In this inventory model, the production started at time $t = 0$, during the time interval $(0, t_1)$ production rate and the demand rate are $\lambda D(p)$ and $D(p)$ respectively. At a rate of $(\lambda - 1)D(p)$ level of inventory reaches to Q_1 at the time $t = t_1$, then in the time interval (t_1, t_2),

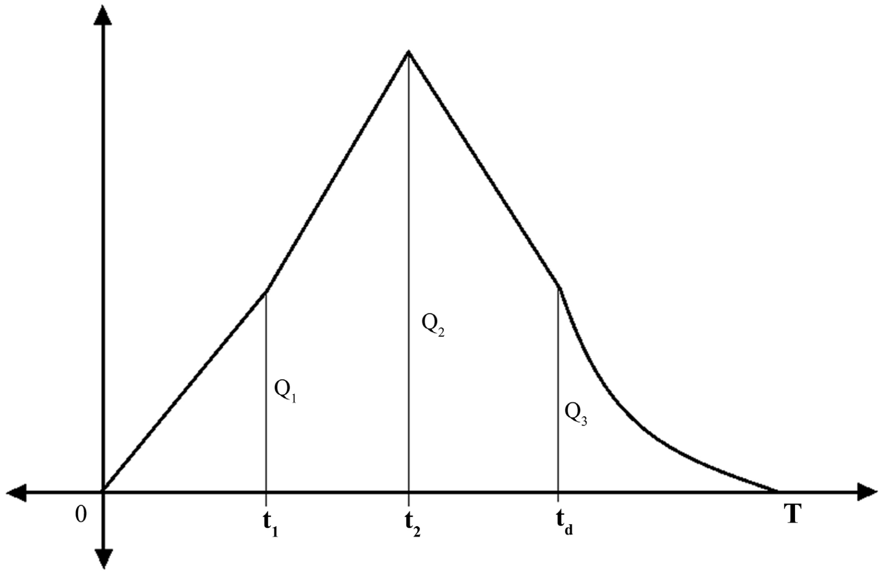

Figure 1. Graphical representation of inventory model.

inventory levels start rising at a rate $a(\lambda - 1)D(p)$. When the inventory level becomes Q_2 at time $t = t_2$, production stopped. The inventory level is depleted due to demand alone during the time interval (t_2, t_d) and at $t = t_d$ inventory level becomes Q_3. In the time interval (t_d, T), inventory level starts decreasing due to both deterioration as well as demand rate and then reaches to zero level at time $t = T$.

The differential equations representing the inventory model are,

$$\frac{\mathrm{d}I_1(t)}{\mathrm{d}t} = (\lambda - 1)D(p), \quad 0 \le t \le t_1 \tag{1}$$

$$\frac{\mathrm{d}I_2(t)}{\mathrm{d}t} = a(\lambda - 1)D(p), \quad t_1 \le t \le t_2 \tag{2}$$

$$\frac{\mathrm{d}I_3(t)}{\mathrm{d}t} = -D(p), \quad t_2 \le t \le t_d \tag{3}$$

$$\frac{\mathrm{d}I_4(t)}{\mathrm{d}t} + \alpha\beta t^{\beta-1} I_4(t) = -D(p), \quad t_d \le t \le T \tag{4}$$

The boundary conditions are,

$$\begin{aligned} &I_1(0) = 0,\ I_1(t_1) = I_2(t_1) = Q_1,\ I_2(t_2) = I_3(t_2) = Q_2, \\ &I_3(t_d) = I_4(t_d) = Q_3,\ I_1(T) = 0 \end{aligned} \tag{5}$$

The solutions of differential equations above are given by,

$$I_1(t) = \frac{\eta(\lambda - 1)t}{p^\gamma} \tag{6}$$

$$I_2(t) = \frac{\eta a(\lambda - 1)t}{p^\gamma} + \frac{\eta(1-a)(\lambda - 1)t_1}{p^\gamma} \tag{7}$$

$$I_3(t) = \frac{\eta}{p^\gamma}(t_d - t) + \frac{\eta}{p^\gamma}\left[(T - t_d) + \frac{\alpha}{\beta+1}\left(T^{\beta+1} - t_d^{\beta+1}\right)\right]\mathrm{e}^{-\alpha t_d^\beta} \tag{8}$$

$$I_4(t)=\frac{\eta}{p^{\gamma}}\left[(T-t)+\frac{\alpha}{\beta+1}\left(T^{\beta+1}-t^{\beta+1}\right)\right]e^{-\alpha t^{\beta}} \tag{9}$$

Using initial boundary conditions,

$$Q_1=\frac{\eta(\lambda-1)t_1}{p^{\gamma}} \tag{10}$$

$$Q_2=\frac{\eta(\lambda-1)t_1}{p^{\gamma}}+\frac{\eta a(\lambda-1)(t_2-t_1)}{p^{\gamma}}$$

$$Q_3=\frac{\eta}{p^{\gamma}}\left[(T-t_d)+\frac{\alpha}{\beta+1}\left(T^{\beta+1}-t_d^{\beta+1}\right)\right]e^{-\alpha t_d^{\beta}} \tag{11}$$

The different costs included in total cost are as follows, considering the influence of inflation and time value of money.

Total cost of inventory cycle per unit time is,

$$TC=\frac{1}{T}\left[\text{Production Cost}+\text{Holding Cost}+\text{Deterioration Cost}+\text{Setup Cost}\right]$$

i)
$$\begin{aligned}\text{Production Cost}&=C_{PC}\left[\int_0^{t_1}\frac{\eta\lambda}{p^{\gamma}}e^{-(r-i)t}dt+\int_{t_1}^{t_2}\frac{a\eta\lambda}{p^{\gamma}}e^{-(r-i)t}dt\right]\\&=C_{PC}\frac{\eta\lambda}{p^{\gamma}}\left[a\left(t_2-\frac{(r-i)t_2^2}{2}\right)+(1-a)\left(t_1-\frac{(r-i)t_1^2}{2}\right)\right]\end{aligned} \tag{12}$$

ii)
$$\begin{aligned}\text{Holding Cost}&=C_{HC}\left[\int_0^{t_1}I_1(t)e^{-(r-i)t}dt+\int_{t_1}^{t_2}I_2(t)e^{-(r-i)t}dt\right.\\&\qquad\left.+\int_{t_2}^{t_d}I_3(t)e^{-(r-i)t}dt+\int_{t_d}^{T}I_4(t)e^{-(r-i)t}dt\right]\\&=C_{HC}\left\{\frac{\eta a(\lambda-1)}{p^{\gamma}}\left(\frac{t_2^2}{2}-\frac{(r-i)t_2^3}{3}\right)+\frac{\eta(r-i)}{p^{\gamma}}\left(\frac{t_d^3-t_2^3}{3}-\frac{t_d\left(t_d^2-t_2^2\right)}{2}\right)\right.\\&\quad+\frac{\eta(\lambda-1)(1-a)}{p^{\gamma}}\left(t_1t_2-\frac{t_1^2}{2}+\frac{(r-i)t_1^3}{6}-\frac{(r-i)t_1t_2^2}{2}\right)\\&\quad+\frac{\eta}{p^{\gamma}}\left(t_d-t_2-\frac{r-i}{2}\left(t_d^2-t_2^2\right)\right)\left[(T-t_d)+\frac{\alpha}{\beta+1}\left(T^{\beta+1}-t_d^{\beta+1}\right)\right]e^{-\alpha t_d^{\beta}}\\&\quad+\frac{\eta(t_d-t_2)^2}{2p^{\gamma}}+\frac{\eta}{p^{\gamma}}\left[\frac{(T-t_d)^2}{2}+\frac{\alpha\beta\left(T^{\beta+2}-t_d^{\beta+2}\right)}{(\beta+1)(\beta+2)}-\frac{\alpha T t_d\left(T^{\beta}-t_d^{\beta}\right)}{\beta+1}\right.\\&\quad-(r-i)\left(\frac{T^3}{6}-\frac{Tt_d^2}{2}+\frac{t_d^3}{3}\right)-\frac{\alpha(r-i)T^{\beta+3}}{2(\beta+3)}+\frac{\alpha(r-i)T^{\beta+1}t_d^2}{2(\beta+1)}\\&\quad\left.\left.-\frac{\alpha(r-i)t_2^{\beta+3}}{(\beta+1)(\beta+3)}\right]\right\}\end{aligned} \tag{13}$$

iii)
$$\begin{aligned}\text{Deterioration Cost}&=C_{DC}\left[\int_{t_d}^{T}\theta(t)I_4(t)e^{-(r-i)t}dt\right]\\&=C_{DC}\left\{\frac{\eta\alpha\beta}{p^{\gamma}}\left[\frac{T^{\beta+1}}{\beta(\beta+1)}-\frac{Tt_d^{\beta}}{\beta}+\frac{t_d^{\beta+1}}{\beta+1}+\frac{(r-i)\left(T^{\beta+2}-t_d^{\beta+2}\right)}{\beta+2}\right.\right.\end{aligned}$$

$$+\frac{(r-i)T\left(T^{\beta+1}-t_d^{\beta+1}\right)}{\beta+1}\Bigg]\Bigg\} \tag{14}$$

iv) Setup Cost = A (15)

Then, The Total cost per unit time for inventory cycle is,

$$\begin{aligned}
TC(T)=\frac{1}{T}\Bigg[&C_{PC}\frac{\eta\lambda}{p^{\gamma}}\left[a\left(t_2-\frac{(r-i)t_2^2}{2}\right)+(1-a)\left(t_1-\frac{(r-i)t_1^2}{2}\right)\right]\\
&+C_{HC}\left\{\frac{\eta a(\lambda-1)}{p^{\gamma}}\left(\frac{t_2^2}{2}-\frac{(r-i)t_2^3}{3}\right)+\frac{\eta(\lambda-1)(1-a)}{p^{\gamma}}\left(t_1t_2-\frac{t_1^2}{2}\right.\right.\\
&\left.+\frac{(r-i)t_1^3}{6}-\frac{(r-i)t_1t_2^2}{2}\right)+\frac{\eta(r-i)}{p^{\gamma}}\left(\frac{t_d^3-t_2^3}{3}-\frac{t_d\left(t_d^2-t_2^2\right)}{2}\right)\\
&+\frac{\eta(t_d-t_2)^2}{2p^{\gamma}}+\frac{\eta}{p^{\gamma}}\left(t_d-t_2-\frac{r-i}{2}\left(t_d^2-t_2^2\right)\right)\left[(T-t_d)+\frac{\alpha}{\beta+1}\left(T^{\beta+1}\right.\right.\\
&\left.\left.-t_d^{\beta+1}\right)\right]e^{-\alpha t_d^{\beta}}+\frac{\eta}{p^{\gamma}}\left[\frac{(T-t_d)^2}{2}+\frac{\alpha\beta\left(T^{\beta+2}-t_d^{\beta+2}\right)}{(\beta+1)(\beta+2)}-\frac{\alpha Tt_d\left(T^{\beta}-t_d^{\beta}\right)}{(\beta+1)}\right.\\
&-(r-i)\left(\frac{T^3}{6}-\frac{Tt_d^2}{2}+\frac{t_d^3}{3}\right)-\frac{\alpha(r-i)T^{\beta+3}}{2(\beta+3)}+\frac{\alpha(r-i)T^{\beta+1}t_d^2}{2(\beta+1)}\\
&\left.\left.-\frac{\alpha(r-i)t_2^{\beta+3}}{(\beta+1)(\beta+3)}\right]\right\}+C_{DC}\left\{\frac{\eta\alpha\beta}{p^{\gamma}}\left[\frac{T^{\beta+1}}{\beta(\beta+1)}-\frac{Tt_d^{\beta}}{\beta}+\frac{t_d^{\beta+1}}{\beta+1}\right.\right.\\
&\left.\left.+\frac{(r-i)\left(T^{\beta+2}-t_d^{\beta+2}\right)}{\beta+2}+\frac{(r-i)T\left(T^{\beta+1}-t_d^{\beta+1}\right)}{\beta+1}\right]\right\}+A\Bigg]
\end{aligned} \tag{16}$$

Let $t_1=c_1T$, $t_2=c_2T$, $t_d=c_3T$ such that, $0<c_1,c_2,c_3<1$ and $T>t_d>t_2>t_1$

$$\begin{aligned}
TC(T)=\frac{1}{T}\Bigg[&C_{PC}\frac{\eta\lambda}{p^{\gamma}}\left[a\left(c_2T-\frac{(r-i)c_2^2T^2}{2}\right)+(1-a)\left(c_1T-\frac{(r-i)c_1^2T^2}{2}\right)\right]\\
&+C_{HC}\left\{\frac{\eta a(\lambda-1)}{p^{\gamma}}\left(\frac{c_2^2T^2}{2}-\frac{(r-i)c_2^3T^3}{3}\right)\right.\\
&+\frac{\eta(\lambda-1)(1-a)}{p^{\gamma}}\left(c_1c_2T^2-\frac{c_1^2T^2}{2}+\frac{(r-i)c_1^3T^3}{6}-\frac{(r-i)c_1T\left(c_2^2T^2\right)}{2}\right)\\
&+\frac{\eta(r-i)}{p^{\gamma}}\left(\frac{c_3^3T^3-c_2^3T^3}{3}-\frac{t_d\left(c_3^2T^2-c_2^2T^2\right)}{2}\right)+\frac{\eta(c_3T-c_2T)^2}{2p^{\gamma}}\\
&+\frac{\eta}{p^{\gamma}}\left(c_3T-c_2T-\frac{r-i}{2}\left(c_3^2T^2-c_2^2T^2\right)\right)\left[(T-c_3T)+\frac{\alpha}{\beta+1}\left(T^{\beta+1}\right.\right.\\
&\left.\left.-c_3^{\beta+1}T^{\beta+1}\right)\right]e^{-\alpha c_3^{\beta}T^{\beta}}+\frac{\eta}{p^{\gamma}}\left[\frac{(T-c_3T)^2}{2}+\frac{\alpha\beta\left(T^{\beta+2}-c_3^{\beta+2}T^{\beta+2}\right)}{(\beta+1)(\beta+2)}\right.
\end{aligned}$$

$$-\frac{\alpha c_3 T^2\left(T^\beta-c_3^\beta T^\beta\right)}{\beta+1}-(r-i)\left(\frac{T^3}{6}-\frac{T\left(c_3^2T^2\right)}{2}+\frac{c_3^3T^3}{3}\right)$$
$$-\frac{\alpha(r-i)T^{\beta+3}}{2(\beta+3)}+\frac{\alpha(r-i)T^{\beta+1}\left(c_3^2T^2\right)}{2(\beta+1)}-\frac{\alpha(r-i)c_2^{\beta+3}T^{\beta+3}}{(\beta+1)(\beta+3)}\Bigg]\Bigg\}$$
$$+C_{DC}\left\{\frac{\eta\alpha\beta}{p^\gamma}\left[\frac{T^{\beta+1}}{\beta(\beta+1)}-\frac{c_3^\beta T^{\beta+1}}{\beta}+\frac{c_3^{\beta+1}T^{\beta+1}}{\beta+1}\right.\right. \tag{17}$$
$$\left.\left.+\frac{(r-i)\left(T^{\beta+2}-c_3^{\beta+2}T^{\beta+2}\right)}{\beta+2}+\frac{(r-i)T\left(T^{\beta+1}-c_3^{\beta+1}T^{\beta+1}\right)}{\beta+1}\right]\right\}+A\Bigg]$$

By minimizing the total cost $TC(T)$, the following differential equation can be solved to determine the optimum value of T.

$$\frac{\mathrm{d}TC(T)}{\mathrm{d}T}=0 \quad \text{satisfying the condition,} \quad \frac{\mathrm{d}^2TC(T)}{\mathrm{d}T^2}>0$$

Fuzzy Model

Due to uncertainty in the market, all parameters cannot be defined precisely, hence considering $\tilde{A},\tilde{C}_{HC},\tilde{C}_{DC},\tilde{C}_{PC}$ may change within some limits.

Let $\tilde{A}=(A_1,A_2,A_3,A_4,A_5,A_6)$, $\tilde{C}_{HC}=(C_{HC1},C_{HC2},C_{HC3},C_{HC4},C_{HC5},C_{HC6})$, $\tilde{C}_{DC}=(C_{DC1},C_{DC2},C_{DC3},C_{DC4},C_{DC5},C_{DC6})$, $\tilde{C}_{PC}=(C_{PC1},C_{PC2},C_{PC3},C_{PC4},C_{PC5},C_{PC6})$, are Hexagonal fuzzy numbers.

In a fuzzy sense, the total cost of the model per unit of time is given by,

$$\widetilde{TC}(T)=\frac{1}{T}\left[\tilde{C}_{PC}\frac{\eta\lambda}{p^\gamma}\left[a\left(c_2T-\frac{(r-i)c_2^2T^2}{2}\right)+(1-a)\left(c_1T-\frac{(r-i)c_1^2T^2}{2}\right)\right]\right]$$
$$+\tilde{C}_{HC}\left\{\frac{\eta a(\lambda-1)}{p^\gamma}\left(\frac{c_2^2T^2}{2}-\frac{(r-i)c_2^3T^3}{3}\right)\right.$$
$$+\frac{\eta(\lambda-1)(1-a)}{p^\gamma}\left(c_1c_2T^2-\frac{c_1^2T^2}{2}+\frac{(r-i)c_1^3T^3}{6}-\frac{(r-i)c_1T\left(c_2^2T^2\right)}{2}\right)$$
$$+\frac{\eta(r-i)}{p^\gamma}\left(\frac{c_3^3T^3-c_2^3T^3}{3}-\frac{t_d\left(c_3^2T^2-c_2^2T^2\right)}{2}\right)+\frac{\eta(c_3T-c_2T)^2}{2p^\gamma}$$
$$+\frac{\eta}{p^\gamma}\left(c_3T-c_2T-\frac{r-i}{2}\left(c_3^2T^2-c_2^2T^2\right)\right)\left[(T-c_3T)+\frac{\alpha}{\beta+1}\left(T^{\beta+1}\right.\right.$$
$$\left.\left.-c_3^{\beta+1}T^{\beta+1}\right)\right]\mathrm{e}^{-\alpha c_3^\beta T^\beta}+\frac{\eta}{p^\gamma}\left[\frac{(T-c_3T)^2}{2}+\frac{\alpha\beta\left(T^{\beta+2}-c_3^{\beta+2}T^{\beta+2}\right)}{(\beta+1)(\beta+2)}\right.$$
$$-\frac{\alpha c_3T^2\left(T^\beta-c_3^\beta T^\beta\right)}{\beta+1}-(r-i)\left(\frac{T^3}{6}-\frac{T\left(c_3^2T^2\right)}{2}+\frac{c_3^3T^3}{3}\right)$$
$$-\frac{\alpha(r-i)T^{\beta+3}}{2(\beta+3)}+\frac{\alpha(r-i)T^{\beta+1}\left(c_3^2T^2\right)}{2(\beta+1)}-\frac{\alpha(r-i)c_2^{\beta+3}T^{\beta+3}}{(\beta+1)(\beta+3)}\Bigg]\Bigg\}$$

$$+\tilde{C}_{DC}\left\{\frac{\eta\alpha\beta}{p^{\gamma}}\left[\frac{T^{\beta+1}}{\beta(\beta+1)}-\frac{c_3^{\beta}T^{\beta+1}}{\beta}+\frac{c_3^{\beta+1}T^{\beta+1}}{\beta+1}\right.\right.$$
$$\left.\left.\left.+\frac{(r-i)\left(T^{\beta+2}-c_3^{\beta+2}T^{\beta+2}\right)}{\beta+2}+\frac{(r-i)T\left(T^{\beta+1}-c_3^{\beta+1}T^{\beta+1}\right)}{\beta+1}\right]\right\}+\tilde{A}\right] \tag{18}$$

Let $\widetilde{TC}_i(T)$ be the corresponding total cost obtained by replacing $\tilde{A}_i, \tilde{C}_{HCi}, \tilde{C}_{DCi}, \tilde{C}_{PCi}$ in Equation (17) for $i=1,2,3,4,5,6$. *Using graded mean representation to defuzzify the fuzzy total cost* $\widetilde{TC}(T)$.

We get,

$$\widetilde{TC}(T)=\frac{1}{12}\left[\widetilde{TC}_1(T)+2\widetilde{TC}_2(T)+3\widetilde{TC}_3(T)+3\widetilde{TC}_4(T)+2\widetilde{TC}_5(T)+\widetilde{TC}_6(T)\right]$$

By minimizing the total cost $\widetilde{TC}(T)$, the following differential equation can be solved to determine the optimum value of *T*.

$$\frac{\mathrm{d}\widetilde{TC}(T)}{\mathrm{d}T}=0 \text{ satisfying the condition, } \frac{\mathrm{d}^2\widetilde{TC}(T)}{\mathrm{d}T^2}>0$$

The Economic Production Quantity (*EPQ*) for inventory model with inventory cycle length (*T*) can obtained as,

EPQ^* = Total demand during production period + total demand after production stopped + total demand during deterioration + total number of deteriorated items

$$Q^*=\frac{\eta(\lambda-1)t_1}{p^{\gamma}}+\frac{\eta a(\lambda-1)(t_2-t_1)}{p^{\gamma}}+\frac{\eta(\lambda-1)(t_d-t_2)}{p^{\gamma}}$$
$$+\frac{\eta}{p^{\gamma}}\left[(T-t_d)+\frac{\alpha}{\beta+1}\left(T^{\beta+1}-t_d^{\beta+1}\right)-\alpha t_d^{\beta}(T-t_d)\right] \tag{19}$$

5. Numerical Example

5.1. Crisp Model

Consider following parametric values.

C_{PC} = Rs 10/unit, C_{DC} = Rs 12/unit, C_{HC} = Rs 7/unit, A = Rs 4000/order, α = 0.01, β =2, p = 20, γ = 2.1, λ = 4, a = 1.5, η = 20,000, c_1 = 0.3, c_2 = 0.5, c_3 = 0.7, r = 0.5, i = 1.2.

The solution of crisp model is

T = 2.7220, $TC(T)$ = 3507.74, t_1 = 0.8166, t_2 = 1.3610, t_d = 1.9054, Q^* = 232.5356

5.2. Fuzzy Model

$\tilde{C}_{PC}$ = (7, 8, 9, 11, 12, 13), $\tilde{C}_{HC}$ = (4, 5, 6, 8, 9, 10), $\tilde{C}_{DC}$ = (9, 10, 11, 13, 14, 15), $\tilde{A}$ = (1000, 2000, 3500, 4500, 5000, 6000), α = 0.01, β =2, p = 20, γ = 2.1, λ = 4, a = 1.5, η = 20,000, c_1 = 0.3, c_2 = 0.5, c_3 = 0.7, r = 0.5, i = 1.2.

The solution of fuzzy model is given by,

T = 2.6270, $\widetilde{TC}(T)$ = 3383.11, t_1 = 0.7881, t_2 = 1.3135, t_d = 1.8390, Q^* = 224.38

The Time parameters of inventory cycle for crisp and fuzzy model are compared in Figure 2. The total Inventory time cycle for fuzzy model is smaller than crisp model. The total cost of crisp model and fuzzy model over a period of time T is shown in Figure 3. It can be observed from the figure that as the time increases the total cost for both crisp and fuzzy decreases till it hit minimum then starts increasing again. It can be seen that, the minimum point of total cost of fuzzy model is lesser than that of crisp model. As a result, the fuzzy model is advantageous since it lowers costs, which raises profits.

6. Sensitivity Analysis

Taking into account the above numerical example of the fuzzy model for sensitivity analysis to examine the impact of changing various inventory model parameters.

As shown in Table 1, the data can be interpreted as,

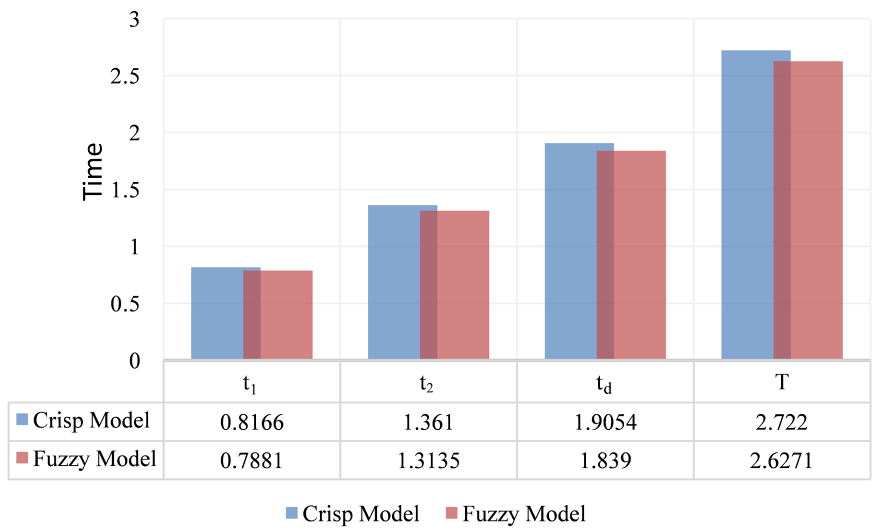

Figure 2. Comparison between time parameters of Crisp model and Fuzzy model.

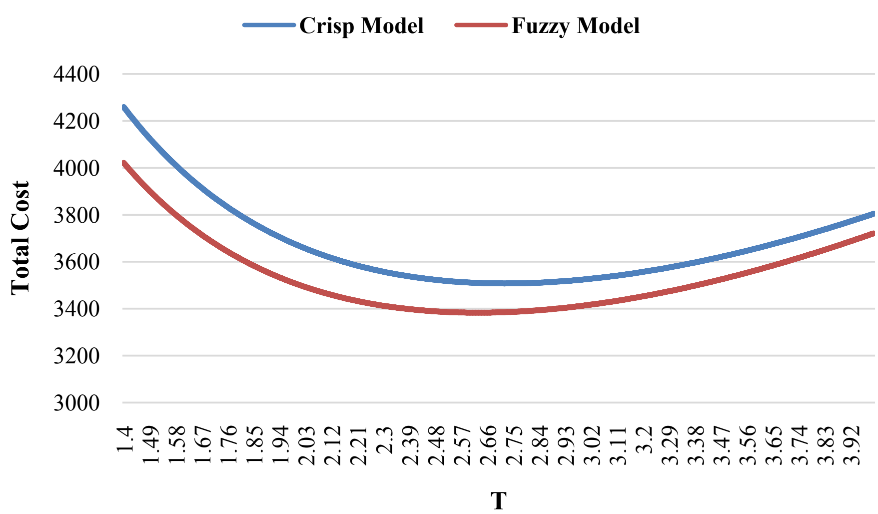

Figure 3. Comparison between total cost of crisp model and fuzzy model.

Table 1. Sensitivity analysis for various parameters.

		T	TC	Q	t_1	t_2	t_d	Q_1	Q_2	Q_3
A	−20%	2.4591	3419.97	251.98	0.7377	1.2296	1.7214	98.42	196.83	33.25
	−10%	2.5824	3578.64	264.67	0.7747	1.2912	1.8077	103.35	206.7	34.97
	0%	2.6972	3730.15	276.49	0.8092	1.3486	1.888	107.95	215.89	36.57
	10%	2.8049	3875.53	287.58	0.8415	1.4025	1.9634	112.26	224.51	38.07
	20%	2.9065	4015.59	298.05	0.8719	1.4533	2.0346	116.32	232.64	39.5
C_{PC}	−20%	2.6695	3304.28	273.64	0.8009	1.3348	1.8687	106.84	213.67	36.18
	−10%	2.6353	3454.6	270.12	0.7906	1.3177	1.8447	105.47	210.94	35.7
	0%	2.6021	3524.01	266.7	0.7806	1.3011	1.8215	104.14	208.28	35.24
	10%	2.5699	3753.58	263.38	0.771	1.285	1.7989	102.85	205.7	34.79
	20%	2.5387	3902.29	260.17	0.7616	1.2694	1.7771	101.6	203.2	34.36
C_{DC}	−20%	2.6043	3603.22	266.92	0.7813	1.3022	1.823	104.23	208.45	35.27
	−10%	2.6032	3603.79	266.81	0.7809	1.3016	1.8222	104.18	208.37	35.26
	0%	2.6021	3604.36	266.7	0.7806	1.3011	1.8215	104.14	208.28	35.24
	10%	2.601	3604.92	266.58	0.7803	1.3005	1.8207	104.09	208.19	35.22
	20%	2.5999	3605.49	266.47	0.7799	1.2999	1.8199	104.05	208.1	35.21
C_{HC}	−20%	2.7672	3459.45	283.7	0.8302	1.3836	1.937	110.75	221.49	37.54
	−10%	2.68	3533.46	274.72	0.804	1.34	1.876	107.26	214.51	36.33
	0%	2.6021	3604.36	266.7	0.7806	1.3011	1.8215	104.14	208.28	35.24
	10%	2.5318	3672.51	259.46	0.7595	1.2659	1.7723	101.33	202.65	34.26
	20%	2.4679	3738.2	252.89	0.7404	1.2339	1.7275	98.77	197.54	33.38
η	−20%	2.8764	2972.88	196.63	0.8629	1.4382	2.0135	76.74	153.49	26.05
	−10%	2.7423	3181.36	210.85	0.8227	1.3712	1.9196	82.21	164.63	27.9
	0%	2.627	3383.11	224.38	0.7881	1.3135	1.8389	87.61	175.23	29.66
	10%	2.5264	3579.14	237.33	0.7579	1.2632	1.7685	92.68	185.37	31.34
	20%	2.4375	3770.22	249.76	0.7313	1.2188	1.7063	97.55	196.1	32.95
λ	−20%	2.8506	2985.05	192.87	0.8551	1.4253	1.9954	69.72	139.44	32.26
	−10%	2.7317	3186.84	209.07	0.8195	1.3659	1.9122	78.96	157.92	30.87
	0%	2.627	3383.11	224.38	0.7881	1.3135	1.8389	87.61	175.23	29.66
	10%	2.5341	3574.59	238.95	0.7602	1.2671	1.7739	95.78	191.57	28.58
	20%	2.4507	3761.83	252.86	0.7352	1.2254	1.7155	103.53	207.06	27.61

1) An increase in set-up cost A, increases the total average cost $TC(T)$, production time (t_1 and t_2), non-production time (t_3), optimum inventory cycle time (T) and economic production quantity (Q), Maximum level of inventory (Q_1, Q_2 and Q_3) also increases.

2) An increase in the purchase cost (C_{PC}), decreses economic production quantity (Q), optimum inventory cycle time (T), production time (t_1 and t_2), non-

production time (t_3) and Maximum level of inventory (Q_1, Q_2 and Q_3), but the total average cost $TC(T)$ increases.

3) An increase in the Deteriorating cost (C_{DC}), decreses economic production quantity (Q), optimum inventory cycle time (T) production time (t_1 and t_2), non-production time (t_3) and Maximum level of inventory (Q_1, Q_2 and Q_3), but the total average cost $TC(T)$ increases.

4) With the increase in holding cost (C_{HC}), it is observed that, economic production quantity (Q), optimum inventory cycle time (T) production time (t_1 and t_2), non-production time (t_3) and Maximum level of inventory (Q_1, Q_2 and Q_3) decreses, but the total average cost $TC(T)$ increases.

5) An increase in demand coefficient η, increases the total average cost $TC(T)$, economic production quantity (Q) and Maximum level of inventory (Q_1, Q_2 and Q_3) but optimum inventory cycle time (T), production time (t_1 and t_2), non-production time (t_3) decreses.

6) With increase in the value of λ, the total average cost $TC(T)$, economic production quantity (Q) and Maximum level of inventory (Q_1, Q_2 and Q_3) increases but optimum inventory cycle time (T), production time (t_1 and t_2), non-production time (t_3) decreses.

7. Conclusions

In the developed production inventory model, inflation and time value of money under fuzzy environment is considered, where demand is a function of selling price. Production rate, demand rate and deterioration rate are the three important factors in the inventory model, whereas production rate is considered to be dependent on demand rate, and inventory level increases with two production rates at two stages of inventory cycle. As the production stops, the inventory level diminishes only due to demand, before the deterioration period starts. In the last stage, inventory reaches zero level due to demand and deterioration rate which is following two parameter Weibull distribution. As the inventory level reaches zero production is started again instantly. Shortages are not considered in this model.

The optimum solution for total average cost, economic production quantity and Maximum level of inventory (Q_1, Q_2 and Q_3), inventory cycle time (T), production time (t_1 and t_2), non-production time (t_3) is obtained for crisp model as well as fuzzy model. Hexagonal fuzzy numbers and for defuzzification graded mean integration representation method are used for the fuzzy model. The back-order is not considered. Hexagonal fuzzy numbers are used to derive optimum solution and defuzzification by graded mean integration representation method. A numerical example is given to demonstrate the applicability of the purposed model. By comparing the results of crisp model and fuzzy model, it can be concluded that, Fuzzy model is more beneficial.

In future aspect, one can develop this paper by adding shortages with fully backlogging or with partial backlogging.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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