Research on Algorithms and Modeling for a Transportation Problem Based on Sale Volume

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Abstract: This paper mainly discusses production and transportation based on a certain sales volume as variables for a corporation, namely, for various sale status in different places, how to determine the optimum stock volume, production and transportation in order to maximize the corporation profit. The paper also gives the modeling and solution for such problems.

Key word: transportation problem; sale profit; tableau method

1. Present

If there are m subsidiaries in m different areas as $A_{i,}$ the output of a product is $[a_i, d_i]$ (i=1, 2, …, m), respectively. The product are sent to n different areas where have n shops as B_j(j=1, 2, …, n), and the sale volume $y_j \sim N(\mu_j, \sigma_j^2)$. If the product can be sold in a unit term, the profit is l_j for each product, or else, every product lost h_j, and the unit freight is c_{ij} from A_i to B_j. Sup pose there is no stock cost, and in order to maximize the corporation profit and minimize the freight, how many products should be produced in every subsidiaries and how many products should be sent to the shops?

2. Analysis the problem

Suppose the transportation is x_{ij} from subsidiaries A_i to shop B_j where i=1, 2, …, m and j=1, 2, …, n, then the total freight is

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \qquad (1)$$

And suppose the total stock volume of B_j is $S_j\ (j=1,2,\cdots,n)$, then $S_j=\sum_{i=1}^{m}x_{ij}$, the sale volume is y_j in sale areas B_j, and we have

$$f(y_j)=\frac{1}{\sqrt{2\pi}\sigma_j}\exp\left(-\frac{(y_j-\mu_j)^2}{2\sigma_j^2}\right)$$

And the profit function is $u_j(y_j, S_j)$

$$u_j(y_j,S_j)=\begin{cases} l_jS_j & S_j\le y_j \\ l_jy_j-h_j(S_j-y_j) & y_j\le S_j \end{cases}$$
$$(j=1,2,\cdots,n)$$

The expectation of profit in each shop is

$$E[u_j(y_j,S_j)]=\int_{-\infty}^{+\infty}u_j(y_j)\frac{1}{\sqrt{2\pi}\sigma_j}\exp\left(-\frac{(y_j-\mu_j)^2}{2\sigma_j^2}\right)dy_j$$

$$=\int_0^{S_j}[l_jy_j-h_j(S_j-y_j)]\frac{1}{\sqrt{2\pi}\sigma_j}\exp\left(-\frac{(y_j-\mu_j)^2}{2\sigma_j^2}\right)dy_j$$

$$+\int_{S_j}^{+\infty}l_jS_j\frac{1}{\sqrt{2\pi}\sigma_j}\exp\left(-\frac{(y_j-\mu_j)^2}{2\sigma_j^2}\right)dy_j$$

Because

$$\frac{d\{E[u_j(y_j\ S_j)]\}}{dS_j}=l_j-(l_j+h_j)\Phi\left(\frac{S_j-\mu_j}{\sigma_j}\right)+h_j\Phi\left(\frac{\mu_j}{\sigma_j}\right)$$

where $\Phi(t)$ is Standard normal distribution function, and make

$$\beta_j=l_j-(l_j+h_j)\Phi\left(\frac{S_j-\mu_j}{\sigma_j}\right)+h_j\Phi\left(\frac{\mu_j}{\sigma_j}\right)$$

which reflects the expected profit margin.

Suppose $\frac{d\{E[u_j(y_j\ S_j)]\}}{dS_j}=0$,

and we can easily solve the function and get the optimal stock volume, signed as b_j

then we can get

$$\beta_j^* = l_j - (l_j + h_j)\Phi\left(\frac{b_j - \mu_j}{\sigma_j}\right) + h_j\Phi\left(\frac{\mu_j}{\sigma_j}\right),$$

so the maximized expect value of shop B_j's profit is

$$g_j(b_j) = \beta_j^* b_j$$

The total profit is

$$w = \sum_{j=1}^{n} g_j(b_j) \tag{2}$$

and

$$b_j = \sum_{i=1}^{m} x_{ij} \tag{3}$$

Now the problem converts into: under the condition of (3), how to solve the transportation scheme in order to minimize the total freight (1).

3. Modeling

We know that the total transportation from subsidiaries to shop B_j is $b_j = \sum_{i=1}^{m} x_{ij}$, so the total profit is $w = \sum_{j=1}^{n} g_j(b_j) = \beta_j^* b_j$, where

$$\beta_j^* = l_j - (l_j + h_j)\Phi\left(\frac{b_j - \mu_j}{\sigma_j}\right) + h_j\Phi\left(\frac{\mu_j}{\sigma_j}\right)$$

So we have

$$w = \sum_{j=1}^{n} \beta_j^* \sum_{i=1}^{m} x_{ij} \tag{4}$$

Now we solve the minimum of freight (1), under the condition of (3), we can get the model:

$$\min z = \sum_{j=1}^{n} c_{ij} x_{ij}$$
$$\begin{cases} a_i \le \sum_{j=1}^{n} x_{ij} \le d_i, & i = 1, 2, \cdots, m \\ \sum_{i=1}^{m} x_{ij} = b_j, & j = 1, 2, \cdots, n \\ x_{ij} \ge 0, & i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, n \end{cases} \tag{5}$$

Mode (5) becomes the transportation problem of product-sale imbalance.

4. Solution

Because model (5) is the problem of product-sale imbalance, the virtual subsidiaries A_{m+1} are added, whose production are

$$a_{m+i} = d_i - a_i \quad (i = 1, 2, \cdots, m)$$

Another virtual shop B_{n+1} is added, whose sale volume is

$$b_{n+1} = \sum_{i=1}^{m} d_i - \sum_{j=1}^{n} b_j$$

and $c_{m+i,\,j} = c_{ij}$, $c_{m+i,\,n+1} = 0$, $i = 1, 2, \cdots, m$.

So, model (5) can be transformed into the transportation problem of product-sale balance:

$$\min z = \sum_{j=1}^{n+1} \sum_{i=1}^{2m} c_{ij} x_{ij}$$
$$\begin{cases} \sum_{j=1}^{n+1} x_{ij} = a_i, & i = 1, 2, \cdots, 2m \\ \sum_{i=1}^{2m} x_{ij} = b_j, & j = 1, 2, \cdots, n, n+1 \\ x_{ij} \ge 0, & i = 1, 2, \cdots, 2m;\ j = 1, 2, \cdots, n+1 \end{cases}$$

(6)

Model (6) has been converted into the transportation problem of product-sale balance, thus tableau method can be used to solve.

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