Journal of Geoscience and Environment Protection, 2022, 10, 309-326
https://www.scirp.org/journal/gep
ISSN Online: 2327-4344
ISSN Print: 2327-4336

Assessment of Hydrological Drought Using the Standardized Streamflow Index (SSFI): A Case Study of the Tien Yen River Basin of Quang Ninh Province, Vietnam

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How to cite this paper: Van Hieu, N., Van Tuan, N., Bang, N. K., Hai, P. H., Ha, L. V., & Hoa, T. T. (2022). Assessment of Hydrological Drought Using the Standardized Streamflow Index (SSFI): A Case Study of the Tien Yen River Basin of Quang Ninh Province, Vietnam. *Journal of Geoscience and Environment Protection, 10,* 309-326.
https://doi.org/10.4236/gep.2022.108019

Received: July 22, 2022
Accepted: August 28, 2022
Published: August 31, 2022

Abstract

Probabilistic assessment of drought plays an important role in providing valuable information for evaluating water resources systems under drought conditions, and bivariate copulas are effective and efficient for the probabilistic assessment of drought based on joint distributions and/or joint return periods of drought characteristics. In this study, hydrological drought events and their characteristics (including duration and severity) in the Tien Yen River Basin of Quang Ninh province are detected using the Standardized Streamflow Index (SSFI). The BB8Copula is selected as the best-fit copula for hydrological drought duration and severity. Joint probabilities and joint return periods of drought duration and severity in the cases "and" and "or" are calculated based on the BB8Copula, which are employed for drought assessment. The results show that the drought events with 1-season or cross-quarter duration were more popular than others; joint probabilities and joint return periods of the detected drought events from 1962 to 2009, ranged from 0.2% to 92.2% and from 0.782 years to 315.414 years, respectively, in the case "and", and ranged from 3.8% to 99.6% and from 0.724 years to 18.785 years, respectively, in the case "or".

Keywords

Probabilistic Assessment, Standardized Streamflow Index (SSFI), Hydrological Drought, Drought Characteristics, Copula

1. Introduction

Global climate change has been causing many problems to hydrology and water resources, the most common of which are droughts and floods. For coastal areas, inland freshwater resources play an important role and any change in this water source has rather sensitive impacts not only on water users, but also on the environment, ecological and environmental equilibrium. The North-Eastern coastal region of Vietnam, including the provinces of Quang Ninh, Hai Phong and Thai Binh, is one of such areas.

Drought is a costly and complex natural disaster, causing huge impacts on environmental degradation and human lives (Li et al., 2020; Mishra & Singh, 2010; Buzin, 2008). Drought is known as a creeping phenomenon, which occurs when a lack of precipitation results in prolonged shortages in the demands of human activities and the environment (Du, Bui, Nguyen, & Lee, 2018; WMO, 2006). The particular characteristics of drought events with their slow onset and development lasting weeks to years make their effects cumulative and damaging (WMO, 2006; Wang, Ertsen, Svoboda, & Hafeez, 2016). There are four main types of drought, including meteorological, hydrological, agricultural, and socio-economic droughts (WMO, 2006; Heim, 2002). Based on the main influence factors of a type of drought, there are a lot of drought indices that have been built to characterize and quantify different types of drought (Shukla & Wood, 2008; Zargar, Sadiq, Naser, & Khan, 2011; Wilhite, Svoboda, & Hayes, 2007). Drought indices are characteristically calculated numerical representations of drought severity using variables or parameters, which are used to describe drought conditions (Hao & Singh, 2015). These indices are useful in planning and designing applications of irrigation infrastructure construction (WMO & GWP, 2016). Several drought indices have been built using the standardized methods for different drought variables (Hao et al., 2016), such as Standardized Precipitation Index (SPI) (Mckee, Doesken, & Kleist, 1993), Standardized Water-level Index (SWI) (Bhuiyan, 2004), Standardized Runoff Index (SRI) (Shukla & Wood, 2008), Streamflow Drought Index (SDI) (Nalbantis & Tsakiris, 2009), Standardized Precipitation Evapotranspiration Index (SPEI) (Vicente-Serrano, Beguería, & López-Moreno, 2010), Standardized Snowmelt and Rain Index (SMRI) (Staudinger, Stahl, & Seibert, 2014), and Standardized Streamflow Index (SSFI) (Modarres, 2007; Vicente-Serrano et al., 2011). In the calculation of SSFI, since the streamflow at different hydrological stations is affected by several factors to different extents, including precipitation, vegetation cover, water resources management, etc., the spatio-temporal variability of streamflow is fierce. Therefore, the SSFI at every hydrological station cannot be obtained by the use of a unique probability distribution (Vicente-Serrano et al., 2011). As a result, it is necessary to apply different probability distributions for each month at each station to calculate an SSFI (Vicente-Serrano et al., 2011; Telesca, Lovallo, Lopez-Moreno, & Vicente-Serrano, 2012).

On the other hand, drought is one of the stochastic phenomena, so probabilistic theories and stochastic methods are suitable for research on drought (Shiau, 2006). Probabilistic assessment of drought provides valuable information for evaluating water resources systems under drought conditions (Ayantobo, Li, Song, Javed, & Yao, 2018). Drought is featured by its characteristics, such as drought duration, severity (Mishra & Singh, 2011; Ladimirov, 2009). In general, the probabilistic assessment of drought usually relates to the estimation of joint probabilities and/or return periods of more drought characteristics (Mishra & Singh, 2011). It can be used as a standard to design hydraulic systems (Shiau, 2006). Copulas are flexible to describe the joint or conditional distributions of random variables (Shiau, 2006; Hao & Singh, 2016). Thus, there have been a lot of studies that used copulas for probabilistic assessment of drought based on the joint probabilities and/or joint return periods of drought characteristics. In the assessment of meteorological drought in Wushantou of Taiwan, Shiau (Shiau, 2006) detected drought events and their characteristics based on SPI. Then the Galambos copula was employed to determine the joint or conditional probabilities and the joint or conditional return periods of drought duration and severity, which were used for drought assessment. Liu et al. (Liu, Zhang, Singh, & Cui, 2011) used the SPI at a timescale of 12 months to identify drought events, and drought duration and severity in Guangdong province of China. Then, the Gumbel, Clayton, and Frank copulas were utilized to describe the joint probabilities of drought duration and severity, which were used for the analysis of drought. Mirabbasi et al. (Mirabbasi, Fakheri-Fard, & Dinpashoh, 2012) detected drought events and characteristics in the Northwest of Iran based on SPI, then the Galambos copula was employed to calculate the joint probabilities, joint return periods, conditional probabilities, and conditional return periods of drought duration and severity, which were used for meteorological drought assessment. Based on the drought events detected by SPI, Yusof et al. (Yusof, Hui-Mean, Suhaila, & Yusof, 2013) utilized the Galambos copula to calculate the conditional probability and the conditional return period, which were used to assess drought characteristics in Peninsular Malaysia. Vergni et al. (Vergni, Todisco, & Mannocchi, 2015) used the water volume in the root zone to characterize agricultural drought, and identified drought events and characteristics in Perugia of Central Italy. Then, the Student's t copula was employed to describe the joint probabilities and joint return periods of the relative onset and the relative severity. Finally, the agricultural drought assessment was carried out based on the obtained joint probabilities and joint return periods. Nabaei et al. (Nabaei, Sharafati, Yaseen, & Shahid, 2019) evaluated meteorological drought in Iran using SPI to identified drought events. Then, the Archimedean Copulas (Clayton, Frank, and Gumbel) were used to calculate the joint return periods of drought characteristic pairs, including drought duration and severity, drought duration and peak, and drought severity and peak, which were used for drought assessment. Das et al. (Das, Jha, & Goyal, 2020) identified drought events and their characteristics based on the non-stationary drought SPI index. Then the joint return periods of drought characteristics were calculated using the bivariate copulas (Normal and Frank), which were employed to assess drought characteristics over the Himalayan states in India. The previous studies demonstrated that copulas are effective and efficient in using for the probabilistic assessment of drought based on the joint distribution and/or joint return period of drought characteristics.

This study selected the Tien Yen River Basin in Quang Ninh province as the case study site. The Tien Yen River plays an important role in irrigation, transportation, and domestic water supply for districts of Tien Yen, Ba Che, Dam Ha and a part of Cam Pha city of Quang Ninh province. There have been very few studies on hydrological drought in this area. Therefore, research on hydrological drought assessment is urgently needed for drought management in this study area. In this study, hydrological drought events, drought duration and severity are detected based on SSFI, then the best-fit bivariate copula is selected to calculate joint probabilities and joint return periods of drought characteristics, which are employed for drought assessment. This study will provide a scientific basis for drought prevention and mitigation measures to reduce social and economic losses induced by droughts in the Tien Yen River Basin.

2. Materials and Methods

2.1. Study Area

The Tien Yen River Basin is located in Northern Vietnam with latitudes from 21˚32'N to 21˚33'N, and longitudes from 107˚25'E to 107˚31'E (see **Figure 1**). This River basin has a total catchment area of 1070 km^2, a length of 80 km, and its elevation varies from sea level to 1460 m above sea level. This river plays an important role in irrigation, transportation, and domestic water supply.

There are two distinct seasons in a hydrological year in the Tien Yen River Basin, including the rainy season (from May to October) and the dry season (from November to April). The streamflow data collected at Binh Lieu Station showed quiet a big difference between the two seasons in a hydrological year. The annual streamflow ranged from 10.6 m^3/s to 39.6 m^3/s in the period from 1962 to 2019. The multi-year average streamflow was 39.4 m^3/s and 7.3 m^3/s in the in the rainy season and the dry season, respectively. It was an uneven inter-annual and intra-annual distribution of streamflow that caused the frequent occurrence of droughts and floods in the Tien Yen River Basin (Pham, n.d.).

2.2. Streamflow Data

In the Tien Yen River Basin, a long series of monthly streamflow observations is available at Binh Lieu Hydrological Station (see **Figure 1**). The quality of the collected streamflow data from January 1962 to December 2019, which was obtained from the National Hydro-Meteorological Information and Data Center of the Vietnam Meteorological and Hydrological Administration, has been checked for analysis.

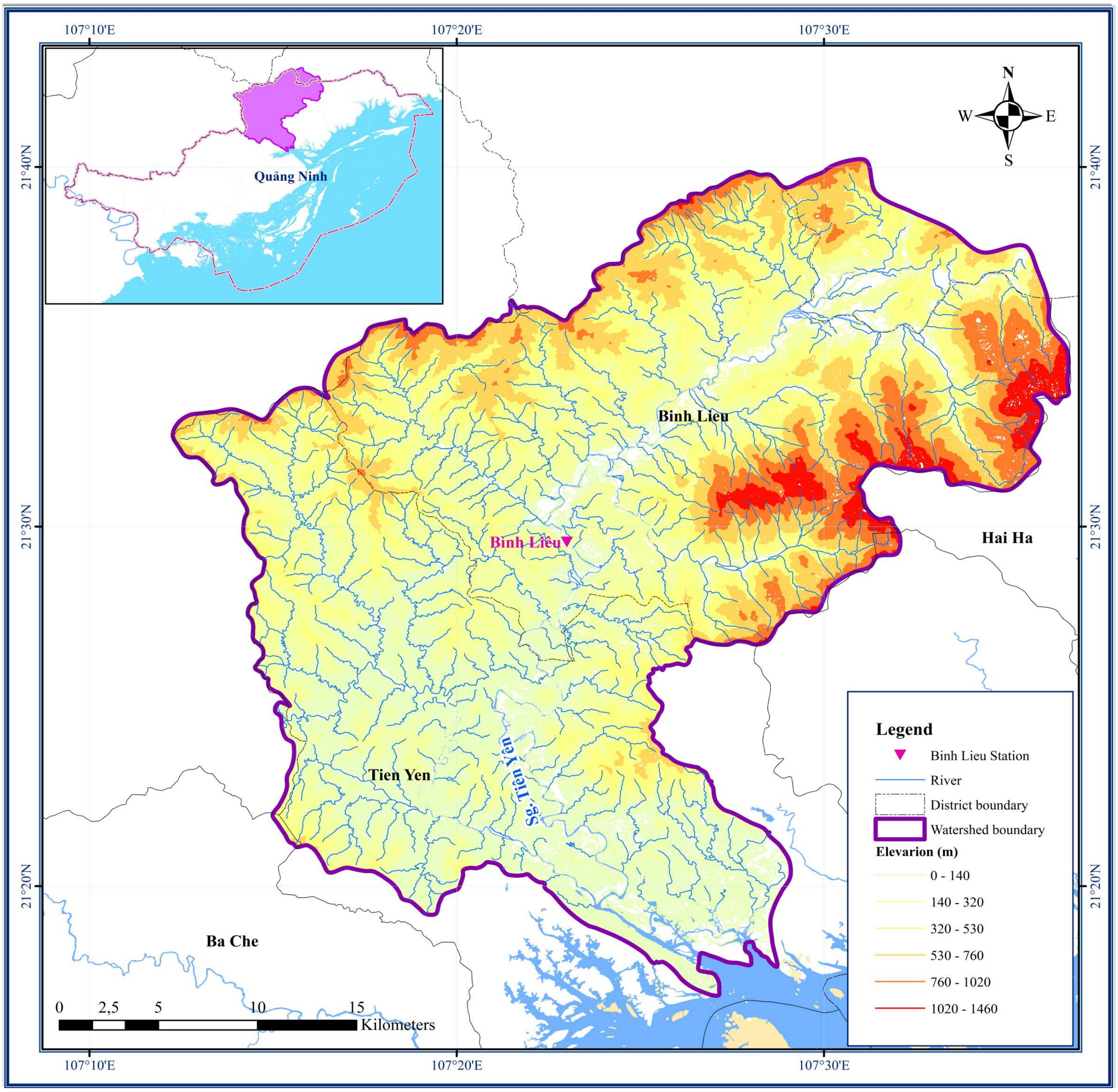

Figure 1. Tien Yen River Basin, Quang Ninh Province.

2.3. Calculation of the Standardized Streamflow Index (SSFI)

In this study, SSFI is used to detect hydrological drought events, duration, and severity. To calculate SSFI at Binh Lieu Station in the Tien Yen River Basin, different distribution functions are used for the calculation of SSFI in different months. Therefore, the SSFI value of a month is calculated using the best-fit distribution for the streamflow series of that month. To obtain the best-fit distribution function of a month, 7 three-parameter univariate distributions are examined to fit the streamflow series data of that month.

Firstly, the L-moment method is applied to estimate parameters of 7 distributions (Vicente-Serrano et al., 2011; Ganora & Laio, 2015), including the Log-

Normal, Pearson type III, Log-Logistic, Generalized Extreme Value (GEV), Generalized Pareto, Weibull, and the Burr XII distributions. Then, based on the estimated parameters, the Maximum Likelihood Estimation (MLE) method (Venables & Ripley, 2002) is employed to re-estimate the distribution parameters. Finally, the Anderson-Darling Test of Goodness-of-Fit (GOF) (Anderson & Darling, 1952) is used to select the best-fit distribution by the smallest value of the test statistic (An) and the p-value accepting the null hypothesis of the GOF test at the significance level of 0.05. After selecting the best-fit distribution for each month of the year, the method of normalization introduced by McKee et al. (Mckee et al., 1993), is applied to calculate the SSFI value of a month as follows:

$$\text{SSFI} = N^{(-1)} F(.) \tag{1}$$

where $F(.)$ is the best-fit distribution function for a month of the year. SSFI is a standardized drought index, thus its drought classification follows the classification for Standardized Precipitation Index (SPI) (Madadgar & Moradkhani, 2014), as shown in Table 1.

2.4. Copula and Selection of a Bivariate Copula

2.4.1. Copula

A copula is a multivariate distribution with all uniform marginal distributions on the interval [0, 1] (Joe, 1997). Based on Sklar's theorem (Sklar, 1959), an n-variate distribution $F(x_1,\cdots,x_n)$ with univariate marginal distributions $F_1(x_1),\cdots,F_n(x_n)$ can be represented by a copula as:

$$F(x_1,\cdots,x_n) = C\{F_1(x_1),\cdots,F_n(x_n)\} = C(u_1,\cdots,u_n), x,u \in R^n. \tag{2}$$

A copula can also be expressed as follows:

$$C(u_1,\cdots,u_n) = F\{F_1^{-1}(u_1),\cdots,F_n^{-1}(u_n)\} \tag{3}$$

where $F_1^{-1}(u_1),\cdots,F_n^{-1}(u_n)$ are the inverse distribution functions of the marginal $F(x_1,\cdots,x_n)$. If the marginal distribution functions are continuous, then copula C is unique. Under the assumption that F is a continuous distribution with strictly increasing, continuous marginal distributions, joint density function f is determined by Sklar (Sklar, 1996):

$$f(x_1,\cdots,x_n) = c(u_1,\cdots,u_n)\cdot\prod_{i=1}^{n} f_i(x_i) \tag{4}$$

where $c(u_1,\cdots,u_n)$ is the copula density and $f_i(x_i)$ is the marginal density.

Table 1. Classification of drought for Standardized Precipitation Index (SPI).

Classification	Value
No drought	>0.00
Mild drought	0.00 to −0.99
Moderate drought	−1.00 to −1.49
Severe drought	−1.50 to −1.99
Extreme drought	≤−2.00

Generally, there are four types of parametric copula including Archimedean type (including Clayton, Frank, Gumbel, Joe, Ali-Mikhail-Haq, BB1, BB6, BB7, and BB8Copulas), Elliptical type (including Normal and Student's t copulas), Extreme value type (including Galambos, Husler-Reiss, Tawn, t-EV, and Gumbel copulas), and others (including Plackett and Farlie-Gumbel-Morgenstern copulas) (Khedun, Mishra, Singh, & Giardino, 2014). Copula is used to describe the dependence structures between random variables. All copulas together can capture several dependence structures like positive, negative, symmetric, asymmetric, and tail dependences (Hao & Singh, 2016). However, each one-parameter copula can only catch one type of dependence (Hao & Singh, 2016), while each two-parameter bivariate copula such as BB1 (Clayton-Gumbel), BB6 (Joe-Gumbel), BB7 (Joe-Clayton), BB8 (Joe-Frank) can capture more than one dependence type between random variables (Khedun et al., 2014).

2.4.2. Selection of a Bivariate Copula

The best-fit marginal distributions are chosen before selecting the best-fit bivariate copulas. And, marginal distributions are used to transform original data of variables to uniform data on the interval [0, 1], which are used for fitting copulas. In this study, to select the best-fit marginal distribution for a variable, the parameter univariate distributions, such as the Normal, Logistic, Gumbel distributions, etc., are tested to fit the selected variables data. Then the MLE method (Venables & Ripley, 2002) is used to estimate the parameters of these distributions. The best-fit marginal distribution for each variable is selected based on the smallest values of the Akaike Information Criterion (AIC) test (Akaike, 1974), the greatest value of log-likelihood, and passing of the Anderson-Darling test at the significance level of 0.05. Besides, the best fit marginal distributions are also selected by the best match between the observed marginal distributions with the theoretical distributions.

After selecting the best-fit marginal distributions, bivariate copulas from the Archimedean type (including Clayton, Frank, Gumbel, Joe, BB1, BB6, BB7, and BB8 copulas), and Elliptical type (including Normal and Student's t copulas) are tested to fit the best-fit marginal distributions. The MLE method (Hofert, Mächler, & McNeil, 2012) is used to estimate the parameters of these copulas. Based on the GOF test (using Kendall's transform (Genest, Rémillard, & Beaudoin, 2009)) and the AIC test, the best-fit copula is selected. The test statistics of the GOF test include the Cramér-von Mises (Sn) and Kolmogorov-Smirnov (Ks) statistics. The p-value of the GOF test can be calculated by using a parametric bootstrap (set for 1000 replications) (Genest & Rémillard, 2008). After the theoretical copulas pass the GOF test at a significant level of 0.05, the best-fit one is chosen by the smallest values of AIC and the greatest value of log-likelihood.

2.5. Assessment of Drought Using SSFI

2.5.1. Determination of Drought Events and Characteristics Using SSFI

In this study, drought events and characteristics are detected using drought index, which are the basis for drought assessment. A drought event occurs when drought index values fall below a threshold (Brito et al., 2018), which associates with a state of drought based on the drought classification of drought indices. Since the drought classification of SSFI, a drought event is defined as a continuous period in which the SSFI value is below 0, meaning that a drought event starts when the SSFI value is less than 0 and ends when the SSFI value is greater than 0.

Drought characteristics are used in this study, including drought duration (D) and drought severity (S). Drought duration is the occurrence period of a drought event, which is determined by the number of months between the start month (included) and the end month (not included). Drought severity is the accumulated magnitude of a drought event, which is calculated by the absolute value of the sum of SSFI values during a drought duration. Furthermore, this study used the Classification of drought duration, which was used in the previous studies (Nabaei et al., 2019; Zuo, Feng, Zhang, & Hou, 2018), the classification of drought duration is shown in **Table 2**.

2.5.2. Calculation of Joint Probabilities of Drought Duration and Severity

The results of the probabilistic assessment of drought differ depending on the two cases: "and ($\wedge$)" and "or ($\vee$)" (Chang, Li, Wang, & Yuan, 2016; Salvadori & De Michele, 2004). Thus, based on the joint distribution of drought duration and severity, a drought event is dangerous when both drought duration and severity are greater than or equal to given thresholds, or either drought duration or severity is greater than or equal to given thresholds (Salvadori & De Michele, 2004). Further, joint probabilities of both drought duration and severity greater than or equal to certain thresholds provide useful information for a water supply plan and improving water resources management (Shiau, 2006; Saghafian & Mehdikhani, 2014). Therefore, since the selected copulas for drought duration and severity, joint probabilities of both drought duration and severity greater than or equal to certain thresholds can be determined as follows (Shiau, 2006):

$$P\left(D \geq d \wedge S \geq s\right) = 1 - F\left(d\right) - F\left(s\right) + C\left[F\left(d\right), F\left(s\right)\right]. \tag{5}$$

And the probabilities of either drought duration or drought severity greater than or equal to certain thresholds can also be expressed as:

$$P\left(D \geq d \vee S \geq s\right) = 1 - C\left[F\left(d\right), F\left(s\right)\right]. \tag{6}$$

Table 2. Classification of drought duration by Zuo et al. (Zuo et al., 2018) and Nabaei et al. (Nabaei et al., 2019).

Drought category	Drought duration (month)
1-month	1
1-season	2 to 3
Cross-quarter	4 to 6
Long-term	>6

2.5.3. Calculation of Joint Return Periods of Drought Duration and Severity

The return period of a drought event expresses the average elapsed time or the average time interval between two adjacent drought events (Cancelliere & Salas, 2010). According to Shiau (Shiau, 2006), the return period of the drought events with their duration and severity values greater than or equal to the certain thresholds and the return period of the drought events with either their duration or their severity values greater than or equal to the certain thresholds can be determined based on the joint probabilities of drought duration and severity. These joint return periods can be calculated as follows:

$$T_{D\wedge S} = \frac{E(L)}{P(D \ge d \wedge S \ge s)} = \frac{E(L)}{1 - F_D(d) - F_S(s) - C\left[F(d), F(s)\right]} \tag{7}$$

$$T_{D\vee S} = \frac{E(L)}{P(D \ge d \vee S \ge s)} = \frac{E(L)}{1 - C\left[F(d), F(s)\right]} \tag{8}$$

where L is the drought inter-arrival time, which is determined as a period between two start times of two adjacent drought events; $E(L)$ is the expected drought inter-arrival time, which can be estimated from observed drought events. For detailed determination of L and $E(L)$ can refer to Shiau (Shiau, 2006).

3. Results

3.1. The Selected Distributions for 12 Months and the Calculated Monthly SSFI

SSFI of a month was calculated using the inverse standard normal distribution function for the best-fit distribution for the streamflow series (from January 1962 to December 2019 at Binh Lieu Station) of that month. **Table 3** shows the distributions that were selected for 12 streamflow series of the 12 months. It can be seen that the Burr XII, GEV, and Log-normal distributions were selected for 7 months, 4 months, and 1 month, respectively. The values determined by all the best-fit distributions can cover all non-negative values of streamflow. The values of the test statistic (An) varied from 0.206 to 0.501, and the p-value changed from 0.745 to 0.989, indicating that the selected distributions well matched the observed distributions for each month. Among 7 tested three-parameter univariate distributions, 3 distributions were selected, of which the Burr XII and GEV distributions were more frequently selected than the others.

After the best-fit distributions of 12 months were selected, monthly SSFI was calculated using the inverse standard normal distribution function for the selected distributions and streamflow data at Binh Lieu Station from January 1962 to December 2019 (see **Figure 2**). Based on the drought classification for SSFI in **Table 1**, it can be seen from **Figure 2** that the severe drought occurred in the years 1962, 1965, 1969, 1977, 1979, 1988, 1990, 1991, and 2008 and the extreme drought occurred in the years 1963, 1985, 1999, 2003, 2004, 2005, 2006, and 2007. In the last decade, severe drought has not been observed in this river basin. The comparison with the recorded drought events (Pham, n.d.) indicated that

Table 3. The best-fit distributions of 12 months with the estimated parameters and the values of the Anderson-Darling test of goodness-of-fit.

Month	Distribution	Parameters	Method	Values of the A-D test	
				An	p-value
January	BURR XII	shape 1 = −2.170 shape 2 = 5.923 scale = 4.619	MLE	0.501	0.745
February	BURR XII	shape 1 = −2.959 shape 2 = 7.532 scale = 3.726	MLE	0.231	0.979
March	BURR XII	shape 1 = −2.638 shape 2 = 5.755 scale = 3.494	MLE	0.266	0.961
April	GEV	location = 5.311 scale = 3.300 shape = 0.186	L-moment	0.215	0.986
May	GEV	location = 9.590 scale = 6.998 shape = 0.309	L-moment	0.369	0.879
June	BURR XII	shape 1 = −0.370 shape 2 = 1.910 scale = 32.119	MLE	0.215	0.986
July	Log Normal	shape = 0.533 scale = 4.053 threshold = −6.012	MLE	0.264	0.962
August	BURR XII	shape 1 = 0.046 shape 2 = 2.043 scale = 63.253	L-moment	0.297	0.940
September	GEV	location = 27.665 scale = 18.919 shape = 0.236	L-moment	0.352	0.894
October	GEV	location = 14.740 scale = 9.261 shape = 0.386	MLE	0.216	0.985
November	BURR XII	shape 1 = −1.747 shape 2 = 4.296 scale = 9.501	MLE	0.293	0.943
December	BURR XII	shape 1 = −1.796, shape 2 = 5.743 scale = 5.799	L-moment	0.206	0.989

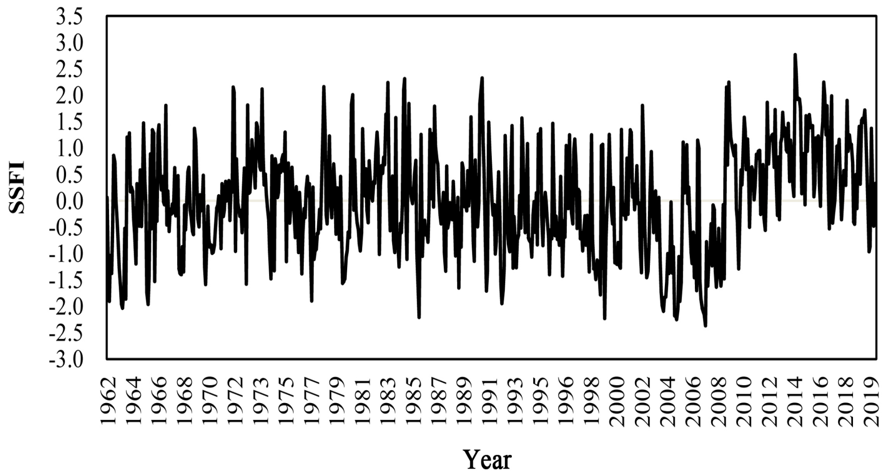

Figure 2. Monthly SSFI from 1962 to 2019 at Binh Lieu Station.

SSFI is capable of capturing hydrological drought and reliable to use for drought assessment in the study area.

3.2. Drought Events and Characteristics Detected by SSFI

To assess hydrological drought in the Tien Yen River Basin, hydrological drought events and their characteristics (including drought duration and severity) were detected, using the identification of drought events and characteristics introduced in Section 2.5.1. The detected drought events and their characteristics based on SSFI were summarized in Table 4. It can be seen from Table 4 that corresponding to drought durations of 1-month, 1-season, cross-quarter, and long-term, the average values of drought severities were equal to 0.83, 1.34, 3.61, and 11.25, respectively, and the numbers of drought events were equal to 13, 33, 22, and 12 events, respectively. The numbers of drought events with drought durations of 1-season and cross-quarter were equal to 55 events (68.8%), indicating that the drought events with 1-season and cross-quarter durations were more popular than others in the Tien Yen River Basin from 1962 to 2019.

3.3. The Selected Distributions and Copula for Drought Duration and Severity

To select the best-fit copula, the best-fit distributions for drought duration, and drought severity were selected (see Table 5). Table 5 shows that based on the smallest values of the AIC and the greatest values of the log-likelihood, the Log-normal and the GEV distributions were selected for drought duration and severity respectively among 9 tested distributions, and BB8Copula was selected for drought duration and severity among 10 tested copulas. The p-values of the Anderson-Darling test of goodness-of-fit (for selection of marginal distribution), and the p-values of the GOF test using the Cramer-von Mises (CM) and the Kolmogorov-Smirnov (KS) statistics (for selection of copula) indicated that the selected distributions passed the test at a significant level of 0.05.

Table 4. Summary of drought events and characteristics based on SSFI at Binh Lieu Station from 1962 to 2019.

Drought duration category	Drought severity	Average drought severity	Numbers of drought events
1-month	0.52 ÷ 1.53	0.83	13
1-season	0.57 ÷ 3.91	1.34	33
Cross-quarter	1.28 ÷ 6.81	3.61	22
Long-term	3.96 ÷ 31.30	11.25	12

Table 5. The selected marginal distributions and copula for drought duration and severity with their estimated parameters, the AIC, the Log-likelihood (Loglik), the p-values of the Anderson-Darling test of goodness-of-fit (for selection of marginal distribution), and the p-values of the GOF test using the Cramer-von Mises (CM) and the Kolmogorov-Smirnov (KS) statistics (for selection of copula).

Variable	Distribution/ Copula	Parameters	AIC	Loglik	p-value (A-D test)	p-value (CM)	p-value (KS)
D	Log-normal	mean = 0.113, sd = 0.733	359.3	−177.6	0.185		
S	GEV	location = 1.078, scale = 0.806, shape = 1.139	320.1	−157.1	0.436		
D, S	BB8	par 1 = 4.53, par 2 = 0.94	−91.5	47.7		0.104	0.106

3.4. Joint Probabilities and Joint Return Periods of Drought Duration and Severity

As introduced before, the assessment of drought events is considered based on the cases "and ($\wedge$)" and "or ($\vee$)". A joint probability of drought characteristics implies a chance for drought events to occur. Figure 3 and Figure 4 respectively show joint probabilities and joint return periods of the detected drought duration and severity in the cases "and" and "or" at Binh Lieu Station from 1962 to 2019.

Consider the case "and": joint probability $P(S \geq s \wedge D \geq d)$ of a drought event indicates the occurrence chance of drought events with their duration greater than or equal to a duration threshold (d) and their severity greater than or equal to a severity threshold (s). It can be seen from Figure 3 that, in the Tien Yen River Basin from 1962 to 2019, the chances to occur the drought events with 1-month, 1-season, cross-quarter, and long-term durations respectively were (47.5 ÷ 92.2)%, (21.1 ÷ 70.4)%, (11.6 ÷ 31.4)%, and (0.2 ÷ 11.7)%. Figure 4 shows that joint return periods of the drought events with 1-month, 1-season, cross-quarter, and long-term durations were (0.782 ÷ 1.519) years, (1.024 ÷ 3.412) years, (2.297 ÷ 6.214) years, and (6.156 ÷ 315.414) years, respectively. Furthermore, in the Tien Yen River Basin from 1962 to 2019, the number

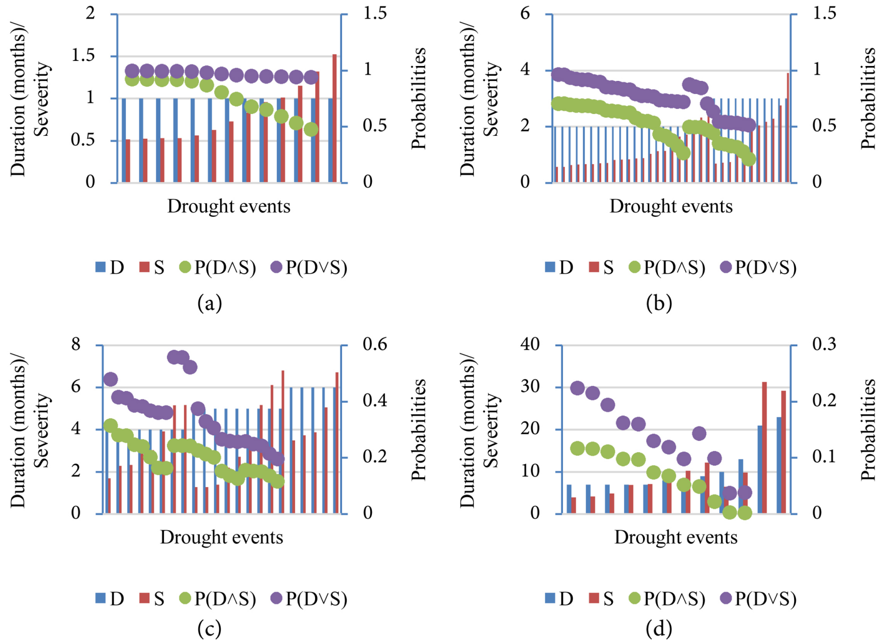

Figure 3. Joint probabilities of drought duration and severity of the detected drought events based on SSFI at Binh Lieu Station from 1962 to 2019: (a) 1-month duration; (b) 1-season duration; (c) cross-quarter duration; (d) long-term duration.

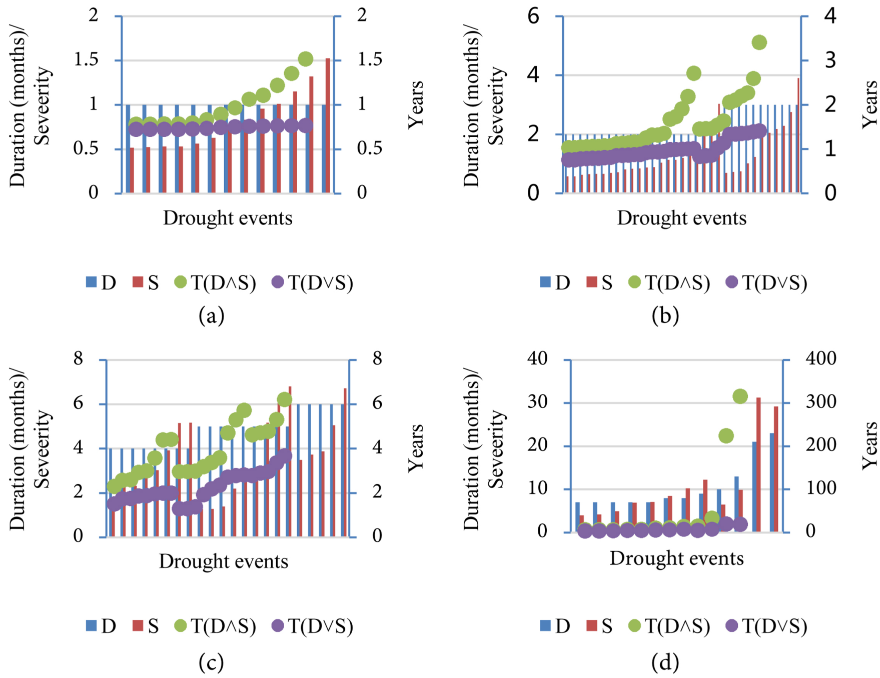

Figure 4. Joint return periods of drought duration and severity of the detected drought events based on SSFI at Binh Lieu Station from 1962 to 2019: (a) 1-month duration; (b) 1-season duration; (c) cross-quarter duration; (d) long-term duration.

of drought events with 1-season duration was the greatest (see **Table 4**), indicating that in the case "and", the drought events with the occurrence chances in the range (21.1 ÷ 70.4)% or with the joint return periods in the range (1.024 ÷

3.412) years were more than others.

Consider the case "or": joint probability $P(S \geq s \vee D \geq d)$ of a drought event implies the occurrence chance of the drought events with either their durations greater than or equal to a threshold of duration (d) or their severities greater than or equal to a threshold of severity (s). In the Tien Yen River Basin from 1962 to 2019, the occurrence chances of the drought events with 1-month, 1-season, cross-quarter, and long-term duration were (93.9 ÷ 99.6)%, (51.2 ÷ 96.4)%, (19.6 ÷ 47.9)%, and (3.8 ÷ 22.4)% respectively (see **Figure 3**), and joint return periods of these drought events respectively were (0.724 ÷ 0.768) years, (0.748 ÷ 1.408) years, (1.504 ÷ 3.671) years, and (3.217 ÷ 18.785) years (see **Figure 4**). **Table 4** and **Figure 3** and **Figure 4** show that the drought events with the occurrence chances in the ranges (51.2 ÷ 96.4)% or with the joint return periods in the ranges (0.748 ÷ 1.408) years were higher than others.

Generally, in two cases, joint probabilities in case "or" were greater than in case "and", joint return periods in case "or" were smaller than in the case "and". These indicated that the occurrence chances of the drought events in case "or" were more likely than in case "and", the drought events in case "and" were less frequent in case "or".

4. Conclusion

In this study, the Standardized Streamflow Index (SSFI) was calculated and used to detect hydrological drought events and their duration and severity. Joint probabilities and joint return periods of drought duration and severity in the cases "and" and "or" were calculated based on the selected BB8Copula, which were employed to assess hydrological drought in the Tien Yen River Basin from 1962 to 2019. The results revealed: 1) the drought events with 1-season or cross-quarter duration were more popular than the others in the Tien Yen River Basin from 1962 to 2019; 2) the occurrence chances of the drought events with 1-month, 1-season, cross-quarter, and long-term durations were (47.5 ÷ 92.2)%, (21.1 ÷ 70.4)%, (11.6 ÷ 31.4)%, and (0.2 ÷ 11.7)%, respectively, in case "and", and were (93.9 ÷ 99.6)%, (51.2 ÷ 96.4)%, (19.6 ÷ 47.9)%, and (3.8 ÷ 22.4)%, respectively, in case "or"; 3) the return periods of the drought events with 1-month, 1-season, cross-quarter, and long-term durations were (0.782 ÷ 1.519) years, (1.024 ÷ 3.412) years, (2.297 ÷ 6.214) years, and (6.156 ÷ 315.414) years, respectively in case "and", and (0.724 ÷ 0.768) years, (0.748 ÷ 1.408) years, (1.504 ÷ 3.671) years, and (3.217 ÷ 18.785) years, respectively, in case "or".

This work shows the characteristics and rules of hydrological drought in the Tien Yen River basin and can provide useful information for a water supply management plan and improving water resources management in context of climate change in Quang Ninh province, a coastal province in North-Eastern region of Vietnam.

Acknowledgements

This paper used data and information generated under the research grant "Researching on scientific basic, bring forward views of technic-economic solutions in order to improve of capacity on the environmental monitoring-bservation and health care for coastal communities in the northern-east sea-side zone to pilot them in Quang Ninh province". The research is led by Institute of Environment and Public Health (VUSTA), Vietnam, funded by the National Research Programme under the Vietnamese Ministry of Science and Technology (MOST) and supported by an International Cooperation Program of the Moscow Academy of Economics and Law (MAEP), the Russian State Hydrometeorological University (RSHU), and the Institute of Water Resources Planning (IWRP), MARD, Vietnam.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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