

# Computing Method Research for Induced Voltage Protection on Overhead Transmission Lines

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**Abstract:** The influence of lightning-induced over-voltage is not negligible in low-voltage systems. This paper includes a full analysis of the waveform, peak value and rate of increase of induced voltage, and evaluates its destructive features. The difficulty in calculating the fault rate caused by induced lightning voltage is to find an effective width equivalent to the width of direct strikes, in which a flashover can be triggered. Using the methods of striking distance theory, first, the effective width of each lightning current can be found. Then, with the lightning current probability density function, the dropout rate caused by induced voltages can be obtained by integrating over all lightning currents.

**Keywords:** overhead transmission lines; study on the lightning protection; lightning induced voltage; tripping rate

## 1 The Formation of Lightning Induced Over-Voltage

In the initial stage of lightning discharge, the lightning stroke begins with the streaming of the cloud areas and then transitions to step leader. When the step leader decreases, the negative charge is left on the way. As the head of leader is closer to the earth, and the surface induced charges increase, until the leader reaches a determined "striking distance", the hit point by lightning is still uncertain.

When the downward leader and the up streamer develop to a certain degree, during which there will be a strong discharge, the positive charge above will neutralize the negative charge below. Because neutralization is a bottom-up development, the direction of which is opposite to that of leader's initial stage of development, it is known as return stroke.

## 2 The Analysis and Calculation of Lightning Induced Over-Voltage

Induced over-voltage mainly gives rise to the flashover of contaminated insulators on lines. The tolerance of induced over-voltage of power lines is mainly reflected in the withstand voltage properties of insulators of the power line.

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Two half waves will develop to the opposite directions after the formation of induced over-voltage, which means the further away from the centre point of lightning the insulator is, the smaller the peak value of induced over-voltage is.

If suppose  $S$  is the horizontal distance between the wire and lightning leader,  $h_d$  is the height of the wire, and  $H$  is the height of the thundercloud, the vertical component of a point A between the wire and the ground.

$$\begin{aligned}
 E_{Ay} &= \frac{\rho_l}{4\pi\epsilon_0} \int_{h-y}^{H-y} \frac{y'dy'}{(y'^2 + S^2)^{3/2}} \\
 &+ \frac{\rho_l}{4\pi\epsilon_0} \int_{h+y}^{H+y} \frac{h''dy''}{(y''^2 + S^2)^{3/2}} \\
 &= \frac{\rho_l}{4\pi\epsilon_0} \left[ -\frac{1}{\sqrt{(H-y)^2 + S^2}} \right. \\
 &+ \frac{1}{\sqrt{(h-y)^2 + S^2}} - \frac{1}{\sqrt{(H+y)^2 + S^2}} \\
 &\left. + \frac{1}{\sqrt{(h+y)^2 + S^2}} \right] \tag{1}
 \end{aligned}$$

$\rho_l$ : The charge linear density of lightning leader. In view of there are  $H \gg S$  and  $H \gg h_d$ , the equation above can be simplified as follows.

$$E_{Ay} = \frac{\rho_l}{2\pi\epsilon_0} \cdot \frac{1}{\sqrt{y^2 + S^2}} \tag{2}$$

Then the induced over-voltage on the power line is given as below.

$$U_g = \int_0^{h_d} E_{Ay} dy = \frac{\rho_l}{2\pi\epsilon_0} \ln \left[ \frac{h_d}{S} + \sqrt{1 + \left(\frac{h_d}{S}\right)^2} \right] \quad (3)$$

Suppose the main discharge speed of lightning is  $v$ ; then lightning current  $I$  is  $I = \rho_l v$ . Therefore, the equation above can be given as follows.

$$U_g = \frac{I}{2\pi\epsilon_0 v} \ln \left[ \frac{h_d}{S} + \sqrt{1 + \left(\frac{h_d}{S}\right)^2} \right] \quad (4)$$

When  $S^2 \gg h_d^2$ , and  $\frac{1}{2\pi\epsilon_0 v} = 60$ , the equation above can be simplified as below.

$$U_g \approx 60 \frac{I h_d}{S} \quad (5)$$

The calculating method of the over-voltage at the point, the distance from which to the centre point is  $x$ , is the same to the centre point.  $S$  is only needed to be changed by  $\sqrt{S^2 + x^2}$ .

$$U_g(x) = \frac{60 I h_d}{\sqrt{S^2 + x^2}} \quad (6)$$

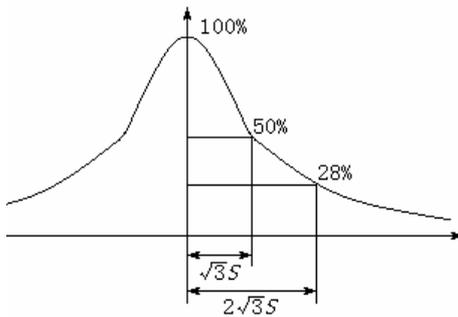


Figure 1. Waveform graph of the induced over-voltage on transmission line

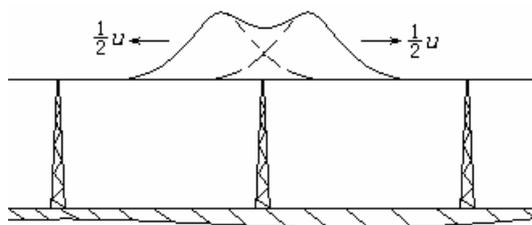


Figure 2. Schematic diagram of over-voltage transmission

Its waveform is shown in Fig.1. The induced over-voltage reduces to 28% the maximum on the point  $2\sqrt{3}S$  away from the central point, so the induced over-voltage can be considered as symmetrical waveform the width of which is  $4\sqrt{3}S$ . When analysis is made, the

voltage wave showed in Figure 1 can be decomposed into two half waves which moves to the opposite direction. And its voltage value is  $U_g(x)/2$ . As to the transmission line shown in Figure 2, the voltage value at a certain time  $t$  can be expressed as followed.

$$\begin{aligned} u_g(x,t) &= \frac{1}{2} U_g(x-vt) + \frac{1}{2} U_g(x+vt) \\ &= 30 I h_d \left[ \frac{1}{\sqrt{S^2 + (x-vt)^2}} + \frac{1}{\sqrt{S^2 + (x+vt)^2}} \right] \end{aligned} \quad (7)$$

If the wave impedance of the wire is supposed to be  $Z = 377\Omega$ , induced lightning current is

$$\begin{aligned} i_g(x,t) &= \frac{u_g(x,t)}{Z} \\ &= 0.08 I h_d \left[ \frac{1}{\sqrt{S^2 + (x-vt)^2}} + \frac{1}{\sqrt{S^2 + (x+vt)^2}} \right] \end{aligned} \quad (8)$$

The corresponding current steepness is

$$\begin{aligned} \frac{\partial i_g}{\partial t} &= 0.08 I h_d v \left\{ \frac{x-vt}{\left[ S^2 + (x-vt)^2 \right]^{3/2}} \right. \\ &\quad \left. - \frac{x+vt}{\left[ S^2 + (x+vt)^2 \right]^{3/2}} \right\} \end{aligned} \quad (9)$$

Then current supreme steepness can be calculated as follows.

$$\begin{aligned} \left( \frac{\partial i_g}{\partial t} \right)_{\max} &= -0.08 I h_d v \times \frac{4}{3\sqrt{3}S^2} = -\left( \frac{0.8 h_d v}{S^2} \right) \frac{I}{2.6} \\ \text{A/s} &= -0.08 \times \frac{h_d}{S^2} \times \frac{I}{2.6} \text{ kA}/\mu\text{A} \end{aligned} \quad (10)$$

It will be seen from the above that induced lightning steepness is far less than the corresponding lightning steepness.

### 3 The Calculation of the Tripping Due to Induced Lightning

As Figure 3 shows, the peak value of the induced over-voltage developed at the point, the distance from which to the pole is  $x$  is:

$$U_{g0} = 60 \frac{I h_d}{S} \quad (11)$$

If the lightning current is supposed to be 100kA, and the height of the pole 10m, the corresponding induced over-voltage can be calculated. The calculating result is

shown in Figure 4.

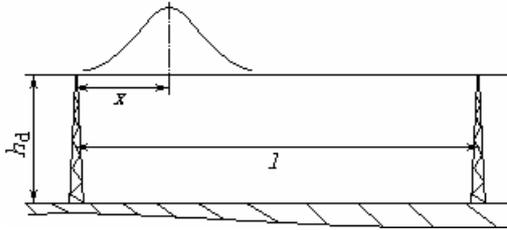


Figure 3. Case diagram of induced over-voltage calculation

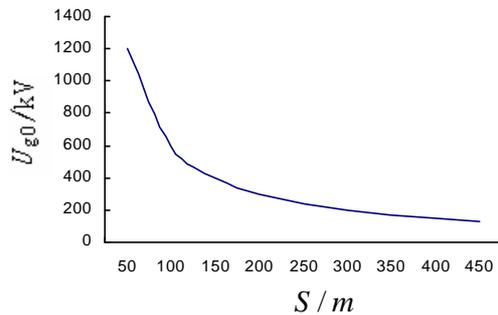


Figure 4. Distribution curve along the line of induced over-voltage

The calculating result demonstrates that induced over-voltage can reach up to 200~400kV, which is high enough to make low-voltage lines and communication lines generate the flashover.

The equation of the voltage instantaneous value of the pole insulator is:

$$u_g(x,t) = 30Ih_d \left[ \frac{1}{\sqrt{S^2 + (x-vt)^2}} + \frac{1}{\sqrt{S^2 + (x+vt)^2}} \right] \tag{12}$$

The occurring time of the voltage peak value of the point which is  $x$  away from the centre of the induced lightning is almost identical to the arriving time of the peak value of the half wave which moves forward to this point. So there is  $t = x/v$ . So the peak value of the over-voltage formed on the insulator is:

$$U_{g0}(x) = 30Ih_d \left( \frac{1}{S} + \frac{1}{\sqrt{S^2 + 4x^2}} \right) = \frac{1}{2}U_0 \left( 1 + \frac{S}{\sqrt{S^2 + 4x^2}} \right) \tag{13}$$

Where  $U_0 = 60 \frac{Ih_d}{S}$ , it is the peak value of the centre of

the induced lightning. It is not difficult to see that when the distance to  $x$  is long enough, there is  $U_{g0}(x) = \frac{1}{2}U_0$ .

This is the situation when the inverse kinematics two half waves separate entirely.

The withstand voltage level  $U_{50\%}$  can be used to analyze and calculate the trip rate of transmission line due to induced lightning.

The condition of causing flashover at different points of lines is

$$U_{g0}(x) = \frac{1}{2}U_0 \left( 1 + \frac{S}{\sqrt{S^2 + 4x^2}} \right) \geq U_{50\%} \tag{14}$$

So the condition of flashover due to induced over-voltage at arbitrary point  $x$  is

$$U_0(x)_{\min} = \frac{2U_{50\%}}{1 + S/\sqrt{S^2 + 4x^2}} \tag{15}$$

Obviously, it is necessary to find out the maximum and minimum distance  $S_{\max}$ ,  $S_{\min}$ , respectively, which can generate induced voltage corresponding to a lightning. After that, the scope of flashover can be determined.

$S_{\max}$  should be determined by  $U_0(x)_{\min}$ , so  $S_{\max}$  can be expressed as below.

$$S_{\max} = 60 \frac{Ih_d}{U_0(x)_{\min}} \tag{16}$$

As to  $S_{\min}$ , it should be the maximum distance that lightning strikes lines. According to strike distance theory, the following equation can be got.

$$S_{\min} = \begin{cases} \sqrt{r_s^2 - (r_s - h_d)^2} = \sqrt{142h_dI^{0.75} - h_d^2} & r_s > h_d \\ r_s = 7.1I^{0.75} & r_s \leq h_d \end{cases} \tag{17}$$

If there is  $S_{\max} > S_{\min}$ , flashover will be generated in the width of  $S_{\max} - S_{\min}$ . When the lightning current is on the small side, there will be  $S_{\max} < S_{\min}$ , which demonstrates such a small lightning current can not give rise to flashover due to induced lightning voltage.

In the area with the width of  $dx$ , the lightning current can give rise to trip between  $S_{\min}$  and  $S_{\max}$ . Therefore, in case of lightning current  $I$ , effective area (both sides of the wire are contained) of flashover at the point of  $x$  is  $2(S_{\max} - S_{\min})dx$ . The number of times of trip-out generated by this current on the scope of the effective

area above-mentioned is expressed as followed.

$$dn = \gamma T \lambda_p(I) dI \times 2 \frac{S_{\max} - S_{\min}}{1000} \times \frac{dx}{1000} \eta \quad (18)$$

Where  $\gamma$  is the lightning strike density, and  $\gamma = 0.015 / km^2 \square lightningday$ ;  $T$  is the average annual lightning day, and  $T = 40$ ;  $\lambda_p(I)$  is the probability of lightning current, and  $\lambda_p(I) = 0.0213 \times 10^{-I/108}$ .

In this case, there is:

$$dn = 2.556 \times 10^{-8} * 10^{-I/108} (S_{\max} - S_{\min}) \eta dI dx \quad (19)$$

The number of times of trip-out which is generated by all the lightning current bigger than  $I_0$  on the scope of width of  $dx$  is:

$$\Delta n = \int_{I_0}^{\infty} dn = 2.556 \times 10^{-8} \times \eta \left[ \int_{I_0}^{\infty} 10^{-I/108} (S_{\max} - S_{\min}) dI \right] dx \quad (20)$$

In half a span, the number of times of tripping per year caused by induced lightning should be

$$\begin{aligned} n_{0.5l} &= \int_0^{0.5l} dn \\ &= \int_0^{0.5l} 2.556 \times 10^{-8} \eta \left[ \int_{I_0}^{\infty} 10^{-I/108} (S_{\max} - S_{\min}) dI \right] dx \\ &= 2.256 \times 10^{-8} \eta \int_0^{0.5l} \int_{I_0}^{\infty} 10^{-I/108} (S_{\max} - S_{\min}) dI dx \end{aligned} \quad (21)$$

The number of times of tripping per 100 kilometers is

$$\begin{aligned} n_g &= \frac{100 \times 1000}{0.5l} \times n_{0.5l} = \\ &= 5.112 \times 10^{-3} \frac{\eta}{l} \int_0^{0.5l} \int_{I_0}^{\infty} (S_{\max} - S_{\min}) \times 10^{-I/108} dI dx \end{aligned} \quad (22)$$

According to the equation above, the induced lightning trip rate of the transmission line can be calculated. The calculating results demonstrate that the induced lightning over-voltage can not be ignored for the communication line and the low voltage line whose voltage is

equal to or below 35kv, but the probability of trip generated by induced lightning is slight.

## 4 Conclusions

Because induced lightning forms approximate equivalent over-oltage in three-phase circuit, it is impossible to induce inter-phase flashover. Induced over-oltage mainly causes single-phase flashover, and the same induced lightning current develops different induced over-voltage due to different distance to power lines, which is different from direct lightning stroke. The over-voltage due to the induced lightning is related to not only the distance between the lightning stroke point and the line, but also the one between the lightning stroke points to the pole along the line. According to the method proposed in this paper, the quantitative analysis and evaluation of the impact of induced over-voltage on the transmission line can be made. Induced lightning trip rate is related to the lightning withstand level, the pole spacing and the length of lightning shield line.

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