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How Would Leveraged Exchange-Traded Funds Perform in Chinese A-Share Market?

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Abstract

This paper examines the performance of Leveraged Exchange-Traded Funds (LETFs) on Chinese A-share indices. We find that bull funds deliver positive average returns and that both the bull and bear Chinese LETFs on average (but less often) exceed the naively expected returns over multiple holding days. Compared to the LETFs on the equity indices of the United States, Chinese bull funds have higher returns over short holding periods, but not over long holding periods. During financial crises, Chinese LETFs suffer more erosion than in normal time and bear funds likely experience positive returns. Lastly, we derive a model to guide investors on when LETFs are likely to perform better or worse than naïve expectations. We not only reveal the potential performance of Chinese LETFs to market participants and regulators for the first time, but also expand the existing LETF literature by documenting new evidence from the unique A-share market.

Keywords

Leveraged Exchange-Traded Funds (LETFs), Fund Performance, LETF Pricing, Chinese A-Share Market

1. Introduction

Exchange-Traded Funds (ETFs) are among the most successful financial innovations in recent years and have grown substantially in trading volume and market size worldwide[1]. An ETF is an investment vehicle that is traded like a stock but tracks an underlying index. Among ETFs, Leveraged Exchange-Traded Funds

[1]According to Blackrock, Exchange Traded Products (ETPs) have more than 5268 billion U.S. dollars of the Asset Under Management (AUM) globally, with the number of ETPs reaching 8921 at the end of the first quarter 2020, despite the financial turmoil then [1]. In the U.S., as shown by [2], ETF daily trading volume exceeded 36% of overall stock market trading volume in the first half of 2016.

(LETFs) have aroused widespread attention recently due to their popularity and complexity. They are designed to deliver a constant multiple, such as 3×, of the underlying index returns. These relatively new products have been highly embraced by investors in the United States (U.S.) and Europe. However, as the second largest economy, China does not have LETFs traded in the mainland financial markets, lagging behind many countries in the development of LETFs in both theory and practice. Fortunately, China has been experiencing astonishing development in its economy and financial system in recent years, and the discussion on the introduction of LETFs leveraging Chinese mainland indices has been put on the agenda. In particular, with the rapid development of Chinese derivatives market, the fundamental financial instruments of LETFs, including swaps and futures which are employed to attain the leverage effect of a benchmark return, are getting complete and mature. These new and ongoing changes in Chinese financial market, together with the intensive needs of Chinese investors on leveraged products, motivate us to study how LETFs leveraging Chinese A-share market would perform if they were listed in the exchanges of Chinese mainland.

This paper examines the performance of potential LETFs leveraging Chinese A-share market. The research work is conducted from three aspects. First, we examine the performance of LETFs on Chinese mainland stock indices during the entire history of the indices and three recent financial crisis periods, as if they were traded in Chinese mainland exchanges. Second, to provide a benchmark for Chinese LETFs, we select LETFs leveraging the most prevailing stock indices in the U.S. market as the comparative counterparts to Chinese LETFs, making a comprehensive comparison between them in fund performance. Furthermore, as our findings show that compounding effect is important for LETF performance across markets and in general, to help guide investors on this effect, we derive a model to investigate the quantitative relationship between index performance and LETF performance, disclosing the essential factors and explicit conditions affecting LETF performances.

Specifically, we use the return data of stock indices in the mainland of China to simulate the return, return deviation, and return multiple of LETFs over various holding periods. Our results show that bull Chinese LETFs deliver positive returns during various holding periods on average. These returns increase as holding period lengthens. For example, for 3× fund, the average target return increases from 0.895% to 213.8% during 5 to 250 trading days[2]. On the other hand, bear LETFs deliver negative returns on average. On average, both bull and bear LETFs perform better than their naively expected counterparts during the

[2]The target return, as defined in Section 3, is the most relevant return of hypothetical LETFs for investors. This is because the actual return of an LETF over any holding period is based on target return. More specifically, after deducting the management fee, financing costs, the extra costs arising from market frictions and inefficiency, and the management's tracking error from the target return, the remaining is equivalent to the actual return of the LETF, which is the final return that investors can receive in each trading day [3].

holding periods of a week, a month and a year. However, in terms of frequency, both bull and bear funds are likely to underperform, relative to their naively expected counterparts, more often for holding periods longer than 5 trading days. These results disclose the complicated behaviors of Chinese LETFs' performance, suggesting that neither the performance enhancement documented in [4] and [5] nor the value destruction warned by the SEC is applicable to the performance of Chinese LETFs during our sample period.

To provide some context for the Chinese LETFs study, we compare their performance to the U.S. counterpart. In particular, we make a comprehensive comparison in return, return deviation, return multiple, Sharpe ratio, and excess return. Within a long common sample period, the comparison across the two markets shows that bull Chinese LETFs perform better than the U.S. counterparts on average. The median return of bull Chinese LETFs is higher than the U.S. counterpart when holding period is between one day and one week. However, the median return of bull Chinese LETFs is actually lower than the U.S. counterpart when holding period lengthens to a month or a year. We further find that the returns of funds tracking the CSI 300 Index are more frequently dragged down than enhanced by return deviation, especially in long holding periods. The effective multiple of fund return deviates from stated multiple when holding period increases, and the discrepancy between the effective multiple and the stated multiple of CSI 300 LETFs is greater than that of S&P 500 LETFs. The Sharpe ratio and the excess return also indicate that the bull Chinese LETFs perform better than their U.S. counterparts in short holding periods but the result reverses over long holding periods. Overall, these results disclose the substantial difference of LETF performance across the two markets and alleviate the concern that high volatility of index is necessarily linked with poor LETF performance, and the difference in LETF performance between the two markets may be associated to the distribution of index performance, especially the relative magnitude of kurtosis.

As the probability to trade LETFs and their trading volume typically increases when the market is volatile, we examine the performance of LETFs based on the CSI 300 Index during three periods of market turbulence, including the 2008 financial crisis, the 2015 Chinese stock market crash, and the 2020 COVID-19 pandemic period[3]. The results show that a majority of the mean and median target returns are positive (negative) for bear (bull) funds, contrary to the observations during the long period of index. Among the three crisis periods, the performance of CSI 300 LETFs seems to be dragged down most during the 2015 Chinese stock market crash period rather than the COVID-19 pandemic. Surprisingly, Chinese LETFs do not seem to be affected as severely as the U.S. funds during the COVID-19 pandemic period. These results reveal the high dependence of LETF performance on the index which is consistent with the previous

[3]In the empirical literature examining ETF performance, a number of studies have also paid attention to crisis periods [6]-[11].

studies.

Our findings show that Chinese LETFs can sometimes perform better than a typical non-professional investor expects, but can also perform worse. To explore the main determinants for this out of expectation or disappointment, we investigate the explicit conditions under which the index performance is beneficial to the fund performance and investor profits. Building our model on the basis of [12], we confirm a U-shaped functional graph of return deviations on the returns of underlying index theoretically and then verify this implication by numerical examples. Advancing the analysis, we compute the corresponding values of return that index needs to accumulate to offset the negative effect of these given variances. A more general conclusion is that, ceteris paribus, to benefit LETF investors and fund performance: 1) Bull (Bear) LETFs have to accumulate a return higher (lower) than the positive (negative) critical return; 2) LETFs with higher leverage require more cumulative index return; 3) Bear LETFs require a higher magnitude of index return than the paired bull LETFs; 4) LETFs with longer holding periods require more cumulative index return. Based on the historical returns of index, our result shows that Chinese LETFs need to accumulate a high return to offset the negative effect of index volatility on fund performance. This is due to the high volatility of Chinese market.

Our study makes some innovative contributions to the literature. First, to the best of our knowledge, we are the first to study the performance of domestic LETFs tracking the stock indices of Chinese A-share markets[4]. With the rapid development of Chinese economy and financial system, our examination on the performance of Chinese potential LETFs is timely and has practical significance for fund issuers and regulators in China, especially as some Chinese fund corporations submitted application materials of LETFs to the China Securities Regulatory Commission recently. Given that most of the current work regarding the pricing and performance of LETFs is based on the indices from developed market, our investigation can provide additional evidence for testing whether the general conclusions from the previous studies on the developed markets can be applied to emerging markets, shedding light upon the arguments for or against introducing and developing more LETFs leveraging Chinese market and other markets with similar characteristics in index performance. Moreover, it does take some time for investors to establish and amend objective perceptions of LETF performance. The education on LETFs is more necessary than on many other securities since LETF performances are complicated and can be counterintuitive even though LETFs are well traded by both institutional and individual

[4]Reference [13] is the only existing study that examines a sample of U.S.-traded Chinese LETFs. However, their study is totally different from ours. First, their sample funds are U.S. traded. In particular, their funds actually track the U.S. based underlying indices, not the China based indices, due to nonsynchronous trading between the markets of the two countries. Second, the indices underlying their funds are Hong Kong-based FTSE China 25 index and U.S.-based BNY/Mellon China Select ADR Index, not the indices from the mainland of China. Third, their fund daily performance was found to be significantly affected by the U.S. market sentiment. This U.S. market sentiment influence can often dominate the impact from underlying index return, making these LETFs like a placebo to fund holders without tracking the underlying index.

investors. Particularly, for investors interested in Chinese LETFs, we wish to provide intuitive impression and details of LETF performance, especially for those who do not have enough knowledge of LETF's performance properties.

Second, our study is innovative to the extant LETF performance literature by adding comparative evidence from two representative markets (China and the U.S.). Due to the difference of market maturity and the respective features of institutional background of financial markets, the indices underlying LETFs can perform differently across markets. The prior studies carry out their tests mainly using U.S.-traded LETFs ([3] [4] [14] [15]) and LETFs leveraging the Canadian equity market [16]. Unlike these papers studying the performance of LETFs based on a single market, our study carries out a comparison across two representative markets. The performance comparison between Chinese LETFs and the U.S. LETFs strikingly manifests that the market differences can affect LETF performance substantially, revealing that there may be some market-specific features of indices underlying LETFs, which could add additional uncertainty to LETF performance. Consequently, the difference in market index is one of the key factors that investors should carefully consider, especially as LETFs on various market indices have become increasingly popular in recent years. Our findings also show that we can't take the findings in the U.S. markets for granted when applying to a different market like Chinese A-share market.

Last, but not least, our study expands the literature on the relationship between index and LETF performance by offering an explicit formula for the critical level of cumulative index return to benefit LETF performance over various situations. It is well documented that the index underlying a daily LETF may add additional uncertainty to fund performance over a multi-day period. More particularly, the extant literature states that, the performance of an LETF will be dragged down when its index experiences high volatility and low cumulative return, or be improved when the index experiences low volatility and high cumulative return (e.g., [17] [18] and [19]). These studies are generally qualitative without detailed quantitative guidance, and there is little discussion on how large the index cumulative return has to be to offset the negative effect of index volatility. There are two studies [12] [20], as far as we know, modelling the performance of LETFs with clear and concise mathematical expressions and produced plentiful intriguing findings. Reference [20] discussed the break-even level of index return on which investors' profit is not eroded by the index performance. Building on the work of [12], our study proposes an explicit formula for the critical level of cumulative index return over various holding periods and various stated multiples of LETFs. The formula offers a simple expression of the approximate quantitative value corresponding to the break-even level of index return. This expression can be generalized to estimate the relationship between index performance and LETF performance in any stated multiples and over any holding periods, and the given numerical cases of break-even levels for index return as a function of the accumulated realized variance in [20] are confirmed to be included in our more extensively applicable formula[5].

The remainder of our study is organized as follows. In the next section, we provide some background information on LETFs and institutional details of Chinese A-share market. In Section 3, we introduce the commonly used variables of LETF performance and describe the data. In Section 4, we present the empirical results. In Section 5, we present the results of additional tests. Finally, we conclude and state the policy implications of this study in Section 6.

2. Background Information and Literature Review

2.1. Background Information on LETFs

ETFs provide the benefits of diversification, liquidity, transparency, easy access and tax efficiency [21]. As a subset of ETFs, LETFs intend to deliver a given multiple, such as 2×, 3×, −1×, −2×, or −3×, of underlying index within a certain period. This multiple is also referred to as fund multiple, stated multiple, promised multiple, or product multiple. The most popular LETFs are the ones to achieve an intended daily multiple. To deliver a constant leverage on the return of underlying index, fund managers rebalance the exposure on the index at the end of each trading day[6]. More specifically, making a deal with counterparties in the swaps market or using the futures to attain a certain level of exposure are the two major ways. The final return received by an LETF investor is the remaining after deducting the financing costs to deliver the leverage effect and the management fees paid to fund management companies from the net earnings of all the assets of LETFs during each trading day. Apparently, the multiple return on the index is not the original "cake" that is distributed to the participants related to LETF investing, but is the perfect mold of the cake. When it comes to multiple holding days, the actual return that investors can receive from their holding LETFs will be determined by the path of index return, which is highly related to market characteristics. Therefore, the stock indices from different markets may contribute to the return behaviors of LETFs in a substantially different way.

2.2. Institutional Background of Chinese A-Share Market

Some institutional background pertinent to Chinese A-share market can make the performance of Chinese LETFs more complicated than the performance of LETFs leveraging other markets. Note that the impact of institutional backgrounds is not directly exerted to LETF performance but is through the perfor-

[5]Reference [20] also discussed the relationship between index performance and the break-even level of investors. Our model or formula to this relationship complements theirs in two ways. First, developing our model on the basis of [12] and by a totally different derivation process, our results can be the mutual verification to their analyses in this relationship. More importantly, our final expressions are more extensively applicable and we find that their numerical examples are special cases of our formula.

[6]These daily rebalanced LETFs are the focus of this paper. There are some LETFs, or LETPs more generally, rebalanced in a non-daily frequency, such as monthly-rebalanced LETPs, quarterly-rebalanced LETPs, etc. See [4] [22] [23] and [24] for more details about monthly rebalanced LETPs.

mance of the indices underlying LETFs, and these market-specific features of indices could not be reflected by the simulated returns of hypothetical indices with the normal distribution[7]. In the context of Chinese A-share market, there are two evident features relevant to the index performance. First, although the A-share market is on the way of being more mature and effective, the market maturity and effectiveness of the market is still at a relatively low level. This feature could be taken along with few years in the future, especially under the limitations pertaining to the short selling which could impede the price efficiency in Chinese market[8]. The other feature relates to the structures and behaviors of investors in Chinese financial market. From the shareholding structure based on the circulation market value, the individual investors have accounted for more than 75% of the total investors in Chinese A-share market, resulting in a market with highly speculative atmosphere. Such a phenomenon will be more evident due to the frequent policy intervention imposed on A-share market. All these mentioned features contribute to the complicated behaviors of securities trading in Chinese A-share market, such as high volatility and strong comovement. As a result, there is likely more uncertainty in the performance of LETFs leveraging Chinese A-share market, which couldn't be explained by the extant literature nor be disclosed by any simulation based on a given index return distribution, such as a log-Normal distribution.

2.3. Literature Review on LETF Performance

In the academic literature, performance is one of the most notable issues of LETFs. The payoff function of an LETF is a multiple of the underlying asset's return over a certain rebalancing period. Since there are often multiple rebalancing periods within a holding period, LETF performance is path dependent. Therefore, pricing LETFs and the relationship between returns of LETFs and the returns of underlying index can be analyzed mathematically [12] [16] [20].

There are two popular simulation methods in the literature used to examine LETF performance. One method is to simulate using the actual historical index returns [4] and the other is to use the Monte Carlo simulation based on an assumed distribution, such as log normal returns [30]. The historical index return method has the advantage to be objective as the historical returns are unique. This method also has the advantage that it does not need any distribution assumption of returns. It is well known that stock returns have fatter tails than typical log-normal distribution, affecting LETF performances. For these reasons, we adopt the simulation method by using historical index returns.

Simulating the performance of LETFs by employing the return data of the en-

[7]In our later analyses, we observe high similarity of LETF performance across funds based on the CSI 300 Index, the Shanghai Composite Index, the Shenzhen Component Index, and the CSI 1000 Index. Moreover, the chart patterns of the historical prices of these four stock indices are highly similar to each other. We thus conjecture that there may be some market-specific features of Chinese markets affecting the performance of Chinese LETFs through the channel of index performance.

[8]For literature on the relationship between short selling and market quality, see [25] [26] [27], and [28].

tire history of Dow Jones Industry Average, [4] confirmed that the general perception regarding the value destruction arising from index return's path is not substantiated. Similarly, [5] find bull LETFs benchmarked to the S&P 500 Index produced much higher return multiple than their given leverage over long investment horizon. As shown in our later empirical tests, there is much to say about the performance properties of Chinese LETFs, which are different from either the performance enhancement documented in [4] and [5] or the value destruction warned by the U.S. Securities and Exchange Commission (SEC) and financial media.

Since LETF performance is highly path dependent, the prior studies on the performance of LETFs, mainly focusing on the developed markets, could not reflect the performance properties of LETFs leveraging the emerging markets[9]. This is particularly relevant as the stock indices of emerging markets perform with more risks and uncertainty than those from developed markets. Even without taking into account the delivery of leveraged effect of index return, the index performance can still significantly affect the return deviation of LETFs over any holding period different from the rebalancing period. Therefore, given the high possibility of being more volatile for stock indices from emerging markets than the stock indices from developed markets, the investigation on the performance of Chinese LETFs can produce different results, which are not documented in the extant literature. In addition, due to the relatively short history of LETFs and the increasing availability of LETFs leveraging different markets, the new evidence of LETF performance based on a rarely explored market can provide new insight for regulators and potential investors.

3. Data and Variables

3.1. Data Collection

The data for this study are obtained from Wind database and Bloomberg. The prices of stock indices in the mainland of China are collected from Wind database, and the American indices data are collected from Bloomberg.

Because the CSI 300 Index is the most popular stock index of Chinese mainland and the most commonly used index of Chinese ETFs, we use it as the underlying index to simulate a set of hypothetical LETFs. When it comes to the U.S. market, due to the similar reasons and for comparison purposes, we use the S&P 500 Index as the underlying index of LETFs. In addition, we also use some other representative stock indices of Chinese market and the U.S. market, including the Shanghai Composite Index, the Shenzhen Component Index, and the CSI 1000 Index from Chinese market, as well as the Dow Jones Industrial Average Index, the NASDAQ Composite Index, and the Russell 2000 Index from the U.S. market.

[9]For instance, [3] [4] [14] [15] [16] [29], and [31] carry out their tests of LETF performance based on the funds listed on developed countries. Actually, most of these studies use the data of U.S. LETFs.

3.2. Variable Definitions

We use the return data of indices underlying LETFs of Chinese and the U.S. markets. To measure LETF performance, we adopt returns, return deviations, and leverage of LETFs, following previous literature.

We compute the target return, the naively expected return and the return deviation of LETFs following [3][10]. As for the leverage, we calculate the effective multiple following [4]. In particular, the daily target return is the product between the target multiple of the LETF and the return on the underlying index. As shown in Equation (1), we compound the daily target return geometrically to obtain the target return over multiple holding days.

$$\text{Target Return} = \prod_{k=1}^{T}\left(1+mr_k\right)-1, \tag{1}$$

where r_k is the index return on day k, T is the number of trading days during the holding period, and m is the target multiple of an LETF.

The naively expected return of an LETF is calculated as the product between the daily target multiple of the LETF and the cumulative return of the underlying index for any investment horizon, which is given as:

$$\text{Expected Return} = m\times\left(\text{Index Return}\right) = m\times\left(\prod_{k=1}^{T}\left(1+r_k\right)-1\right). \tag{2}$$

The return deviation is computed by using the target return less the naively expected return, and the effective multiple is the ratio of target return to underlying index return. Equations (3) and (4) specify the calculation of return deviation and effective multiple, respectively.

$$\text{Return Deviation} = \text{Target Return} - \text{Expected Return}. \tag{3}$$

$$\text{Effective Multiple} = \frac{\text{Target Return}}{\text{Underlying Index Return}}. \tag{4}$$

To clearly show the calculation of the above performance metrics of LETFs, we give a simple example as follows. To simplify the expression, we use prefix of 2×, 3×, −1×, −2× and −3× to indicate the promised multiple of an LETF, corresponding to the return multiple of 2, 3, −1, −2 and −3, respectively. If the underlying index of an LETF experiences a return of 10% in a trading day and then a return of −10% in the next trading day, the index cumulative return is (1 + 10%) × (1 − 10%) − 1 = −1%, the daily target returns of the 2× LETF for the first day and the next day are 20% and −20%, respectively, and the daily expected return is the same as the daily target return. However, the target returns over the two trading days is (1 + 2 × 10%) × (1 − 2 × 10%) − 1 = −4%, differing from the expected return, 2 × ((1 + 10%) × (1 − 10%) − 1) = −2%. Consequently, the re-

[10]Reference [3] constructs a unified model to measure the performance of LETFs. It decomposed the total return deviation of LETFs into three portions, including the compounding deviation, the NAV deviation and the residual deviation. Since we only consider the impact of index performance on the fund performance (the management fees, financing costs and transaction costs are neglected), the total return deviation in our study is the same as the compounding deviation of [3]. Also, the return multiple, or the leverage of LETFs, does not equal the actual return multiple stated in [3], but is computed as the effective multiple stated in [4].

turn deviation for the fund during this holding period is −4% − (−2%) = −2%, and the effective multiple is −4% ÷ (−1%) = 4, totally different from the stated multiple.

Another alternative way to measure the performance of LETFs is to regress the fund returns on index returns. In this study, LETF return is approximated by the target return, so the regression equation is expressed by:

$$\text{Target return}_k = \alpha + \beta r_k + \varepsilon . \tag{5}$$

Based on naïve expectation, an LETF should deliver the target multiple of underlying index return in any investment horizon. If the expectation is true, then α will be zero, and β will equal the promised return multiple of the LETF. In this study, since the fund returns are over multiple days during which the index performance will distort the fund performance, there may be discrepancy between constant and zero, and also between β and the promised multiple. How well an LETF performs relative to the expectation is measured by the significance of α and the closeness of β to the promised multiple. For each LETF, we run the regression as shown in Equation (5).

4. Performance of Hypothetical LETFs

In this section, we first examine the performance of LETFs leveraging the CSI 300 Index during the entire index history. To provide a benchmark for Chinese LETFs, we then compare the performance of LETFs across China and U.S. After that, we test the performance of Chinese LETFs during three recent crisis periods. Finally, we investigate the relationship between LETFs and their underlying indices.

4.1. Evidence from CSI 300 LETFs

Table 1 shows the summary statistics for the performance of hypothetical LETFs tracking the CSI 300 Index. We compute the index return, the target return, the naively expected return, the return deviation and the effective multiple of LETFs. From April 11, 2005 to June 30, 2020, there are 3700 daily return observations. As shown in Panel A, the average daily return of the CSI 300 Index is 5.3 basis points. The standard deviation of the daily return of the CSI 300 Index is 1.719%, showing a high volatility over the history, which is also confirmed by the wide range of the index returns, from the minimum of −9.24% to the maximum of 9.342%. The median return is 9 basis points, showing that the CSI 300 Index experiences positive returns in a majority of trading days. We find a positive return in 1981 out of the 3700 observations or 53.5% of the time. Due to the positive mean and median of the index return, the bull funds, on average and often, experience positive returns in various holding periods, while the bear funds experience negative average and median returns. It should be noted that, due to the daily rebalancing nature of LETFs, the mean (median) of an LETF's daily return is closely associated with the mean (median) of underlying index's daily return, but is not affected by the volatility of the index. However, as we will explain later,

Table 1. Performance of LETFs tracking the CSI 300 index during the index's entire history.

Funds	Variables	Mean	SD	Min	p25	Median	p75	Max	N	N > 0 (%)
					Panel A: 1 trading days					
	R_index	0.053	1.719	−9.240	−0.707	0.090	0.877	9.342	3700	53.5
					Panel B: 5 trading days					
	R_index	0.285	4.042	−22.142	−2.011	0.582	2.642	17.275	3512	55.4
2×	R_target	0.583	8.089	−40.493	−4.108	1.079	5.248	35.607	3512	55.4
	R_expected	0.570	8.085	−44.283	−4.023	1.163	5.284	34.551	3512	55.4
	Deviation	0.013	0.250	−1.238	−0.052	−0.007	0.048	3.790	3512	44.4
	Multiple	2.000	0.149	−1.180	1.977	2.002	2.025	3.836	3512	99.9
3×	R_target	0.893	12.149	−55.505	−6.235	1.522	7.823	54.838	3512	55.3
	R_expected	0.855	12.127	−66.425	−6.034	1.745	7.926	51.826	3512	55.4
	Deviation	0.039	0.749	−3.720	−0.155	−0.021	0.145	10.919	3512	44.4
	Multiple	2.999	0.450	−6.863	2.929	3.004	3.073	8.409	3512	99.9
−1×	R_target	−0.272	4.048	−16.089	−2.655	−0.645	1.915	26.413	3512	44.4
	R_expected	−0.285	4.042	−17.275	−2.642	−0.582	2.011	22.142	3512	44.6
	Deviation	0.012	0.252	−1.420	−0.052	−0.007	0.048	4.272	3512	44.5
	Multiple	−0.999	0.144	−3.977	−1.022	−0.998	−0.975	0.936	3512	0.2
−2×	R_target	−0.533	8.112	−30.887	−5.328	−1.355	3.771	57.612	3512	44.1
	R_expected	−0.570	8.085	−34.551	−5.284	−1.163	4.023	44.283	3512	44.6
	Deviation	0.037	0.761	−4.408	−0.157	−0.021	0.144	13.329	3512	44.6
	Multiple	−1.995	0.429	−10.715	−2.066	−1.993	−1.925	3.911	3512	0.5
−3×	R_target	−0.782	12.203	−44.315	−8.019	−2.169	5.568	94.141	3512	43.9
	R_expected	−0.855	12.127	−51.826	−7.926	−1.745	6.034	66.425	3512	44.6
	Deviation	0.073	1.529	−9.074	−0.316	−0.041	0.287	27.717	3512	44.5
	Multiple	−2.988	0.851	−19.983	−3.130	−2.983	−2.849	9.030	3512	0.7
					Panel C: 22 trading days					
	R_index	1.350	9.344	−29.968	−4.236	1.541	6.067	33.794	3496	56.9
2×	R_target	2.925	19.127	−52.727	−8.917	2.679	11.893	76.932	3496	56.5
	R_expected	2.701	18.688	−59.937	−8.472	3.082	12.133	67.587	3496	56.9
	Deviation	0.225	1.346	−3.196	−0.305	−0.058	0.317	9.344	3496	42.4
	Multiple	1.999	0.337	−3.148	1.937	2.004	2.075	4.952	3496	99.6
3×	R_target	4.754	29.499	−69.350	−13.818	3.401	17.606	131.344	3496	55.8
	R_expected	4.051	28.033	−89.905	−12.708	4.624	18.200	101.381	3496	56.9
	Deviation	0.703	4.131	−9.681	−0.912	−0.169	0.962	29.963	3496	42.4
	Multiple	2.989	1.009	−12.744	2.803	3.004	3.216	11.835	3496	98.9
−1×	R_target	−1.161	9.029	−26.157	−6.240	−1.877	3.926	37.999	3496	42.0
	R_expected	−1.350	9.344	−33.794	−6.067	−1.541	4.236	29.968	3496	43.1

Continued

	Deviation	0.189	1.264	−3.451	−0.312	−0.062	0.317	8.601	3496	42.2
	Multiple	−0.991	0.327	−5.618	−1.053	−0.990	−0.917	1.811	3496	1.1
−2×	R_target	−2.174	17.840	−46.157	−12.547	−4.128	7.553	85.378	3496	41.2
	R_expected	−2.701	18.688	−67.587	−12.133	−3.082	8.472	59.937	3496	43.1
	Deviation	0.527	3.723	−10.601	−0.925	−0.196	0.934	27.799	3496	41.9
	Multiple	−1.963	0.963	−15.163	−2.147	−1.962	−1.746	6.159	3496	2.0
−3×	R_target	−3.084	26.502	−61.272	−18.906	−6.600	10.737	144.275	3496	40.1
	R_expected	−4.051	28.033	−101.381	−18.200	−4.624	12.708	89.905	3496	43.1
	Deviation	0.967	7.318	−21.382	−1.878	−0.386	1.822	59.888	3496	41.8
	Multiple	−2.907	1.882	−27.843	−3.277	−2.913	−2.486	12.671	3496	3.0
Panel D: 250 trading days										
	R_index	24.278	66.747	−70.535	−14.362	6.237	36.899	324.576	3279	56.0
2×	R_target	82.438	239.174	−93.032	−31.364	7.632	70.654	1485.898	3279	54.3
	R_expected	48.556	133.494	−141.070	−28.724	12.474	73.798	649.153	3279	56.0
	Deviation	33.882	114.719	−19.048	−4.006	−0.577	9.706	836.745	3279	46.6
	Multiple	2.154	0.882	−5.531	1.822	2.061	2.447	10.752	3279	98.3
3×	R_target	213.832	694.151	−98.690	−48.081	2.357	92.072	5089.649	3279	50.9
	R_expected	72.833	200.241	−211.606	−43.086	18.710	110.697	973.729	3279	56.0
	Deviation	140.999	518.837	−58.233	−11.527	−0.958	26.651	4115.920	3279	48.4
	Multiple	3.541	2.926	−16.734	2.360	3.096	4.261	25.036	3279	94.9
−1×	R_target	−9.362	36.924	−79.234	−33.029	−10.023	9.305	173.641	3279	36.9
	R_expected	−24.278	66.747	−324.576	−36.899	−6.237	14.362	70.535	3279	44.0
	Deviation	14.916	42.754	−18.083	−4.276	−0.739	9.767	245.343	3279	44.6
	Multiple	−0.831	0.759	−7.920	−1.061	−0.847	−0.513	8.069	3279	7.1
−2×	R_target	−12.760	71.597	−96.196	−58.836	−22.559	12.032	504.045	3279	32.1
	R_expected	−48.556	133.494	−649.153	−73.798	−12.474	28.724	141.070	3279	44.0
	Deviation	35.796	102.818	−56.914	−12.419	−2.718	27.680	552.957	3279	44.0
	Multiple	−1.446	2.082	−19.224	−2.063	−1.468	−0.626	21.502	3279	12.0
−3×	R_target	−15.423	106.446	−99.386	−77.508	−36.696	5.639	971.458	3279	27.7
	R_expected	−72.833	200.241	−973.729	−110.697	−18.710	43.086	211.606	3279	44.0
	Deviation	57.410	170.537	−112.130	−23.993	−6.516	51.788	874.343	3279	43.9
	Multiple	−1.808	3.697	−29.879	−2.910	−1.825	−0.552	35.196	3279	16.3

Note: This table reports the summary statistics for the performance of a set of hypothetical LETFs tracking the CSI 300 Index within the holding period of 1 trading day (Panel A), 5 trading days (Panel B), 22 trading days (Panel C) and 250 trading days (Panel D). The sample period is from the inception date of the CSI 300 Index (April 11, 2005), to June 30, 2020. R_index, R_target, and R_expected are the index's return, the target return and the expected return of investors, respectively. Deviation is the return deviation defined by Equation (3), and multiple is the effective multiple defined by Equation (4). N is the number of observations, and N > 0 (%) is the ratio of the number of positive observations to N. All returns and return deviations are denoted in percentage terms.

for a holding period longer than a trading day, the case will be different and complicated.

In Panels B, C and D, we extend the holding period to 5, 22 and 250 trading days. As shown in Panel B, the returns of underlying index return range from −22.142% to 17.275%, with the average return of 0.285% and the median return of 0.582% during 5 trading days. The standard deviation of the index's return is 4.042%, displaying a high volatility. As shown in the second row of Panel B, the target returns of the 2× LETF averages at 0.583%, ranging from −40.493% to 35.607%. At the same time, the expected return is 0.57%. These results show that the target return is actually different from what a naïve investor would expect, which leads to a return deviation. As shown in the fourth row of Panel B, the return deviation has a positive mean of 0.013%, but a negative median of −0.07%. These results show that the return deviation, in terms of frequency, is more likely to be detrimental to the performance of the 2× LETF during the sample period. As the holding period is not too longer than a trading day, the effective multiples of all the five LETFs in Panel B are close to their stated multiples. The rest of Panel B illustrates that the positive mean and negative median of return deviations also apply to the 3×, −1×, −2× and −3× LETFs. For LETFs in Panels C and D, a similar conclusion applies to their performance.

Comparing across the panels, as the number of holding days increases, the returns of bull LETFs increase and those of bear funds decrease. For example, for 3× fund, the average target return increases from 0.16% to 0.895%, 4.75%, and 213.8% during 1, 5, 22, and 250 trading days respectively. Without leverage, the average returns of 1× fund would be 0.053%, 0.285%, 1.35%, 24.8% during 1, 5, 22, and 250 trading days respectively. Based on these findings, it is clear that the gap between the 3× fund and the 1× fund gets larger when the holding period lengthens. The existence of this gap is due to the fact that the LETFs have a daily rebalancing. It is a bit surprising to observe such a large gap between the average return of 24.8% for 1× fund and 213.8% for 3× fund. This gap is much larger than the product multiple of 3 times would predict. To accurately measure this gap, as previously mentioned, we define the variable of return deviation between the cumulative return of 3× fund and 3× of the cumulative index returns of the index. When this return deviation is positive, there is a positive surprise to the LETF fund holder. For example, as shown in Panel D, the highest return deviation during the 250-trading-day periods is 4115.9%. This return deviation can also be detrimental to the LETF returns. Actually, the worst return deviation is −58.2%, showing that the return of 3× fund is much lower than its naively expected counterpart during the 250 trading days. Across Panels B, C and D, we find that the return deviation increases as the holding period lengthens. For example, the average return deviations of −3× fund change from 0.073% to 0.967%, and further to 57.41% for 5, 22, and 250 holding days respectively, showing that, on average, the LETFs perform better than the naive expectation during these holding periods respectively.

When comparing the effective multiple of LETFs across Panels B, C, and D, we find that the discrepancy between the effective multiple and stated multiple is increasing with the holding period length. For example, the average effective multiple for −3× fund changes from −2.988, to −2.907, and further to −1.808 for 5, 22, and 250 holding days respectively, suggesting a larger deviation from the target multiple of −3× as the holding period lengthens.

Collectively, bull Chinese LETFs deliver positive returns during various holding periods on average. These returns increase as the holding period lengthens. On the other hand, bear LETFs deliver negative returns on average. Moreover, both bull and bear LETFs perform better than their naively expected counterparts on average during the holding period of a week, a month, and a year. However, in terms of frequency, both bull and bear funds are likely to underperform, relative to their naively expected counterparts, more often for holding periods of 5 trading days or longer. These results indicate that the performance of CSI 300 LETFs based on the entire history of index is complicated and cannot be outlined directly by the performance enhancement documented in [4] and [5] nor the potential value destruction often warned by regulators and financial media.

4.2. Using S&P 500 LETFs as the Comparative Counterparts

To provide some context to the Chinese LETFs, in this subsection, we investigate the relative performance of CSI 300 LETFs benchmarked on the S&P 500 LETFs, as the U.S. LETFs are mostly studied in the literature. To ease the inconsistency concern of sample period between the two indices, we match the two samples to the same period of January 2006 to June 2020 and present the comparison in **Table 2**[11].

Across the two markets, the results of **Table 2** show that, the average target returns and return deviations of all the bull funds written on the CSI 300 Index are always higher than their counterparts leveraging the S&P 500 Index. However, we should not conclude that these Chinese LETFs perform better than U.S. LETFs, because the means of returns and deviations are easily affected by the extreme observations. In this case, the medians could be more instructive for investors when judging the investment decisions on the leveraged funds. As shown in Panels A and B, Chinese bull LETFs deliver higher target returns than the U.S. counterparts when the holding period is one day or 5 trading days. For example, the 5-day target returns for 3× Chinese LETF have a median of 1.554%, which is higher than the median of 1.239% for 3× U.S. LETF counterpart. In contrast, as shown in Panels C and D, when the holding period lengthens to 22 or 250 trading days, U.S. bull LETFs actually deliver higher median target returns than their Chinese counterparts. For example, the 250-day target returns for 3× Chinese LETF have a median of −5.03%, which is lower than the median of 31.476% for 3× U.S. LETF counterpart. These findings suggest that a 3× Chinese LETF holder

[11]The first LETF was introduced to the equity market in 2006, so we choose 2006 as the starting year of the sample period.

Table 2. Performance comparison of LETFs from January 2006 to June 2020.

Funds	Variables	CSI 300 LETFs			S&P 500 LETFs			Difference test	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		Mean	SD	Median	Mean	SD	Median	Mean	Median
		Panel A: 1 trading day							
	R_index	0.058	1.737	0.092	0.033	1.283	0.069	0.025	0.023
		Panel B: 5 trading days							
	R_index	0.303	4.083	0.597	0.161	2.574	0.440	0.142*	0.157**
2×	R_target	0.619	8.171	1.117	0.304	5.142	0.847	0.315*	0.270**
	R_expected	0.606	8.166	1.194	0.322	5.147	0.881	0.285*	0.313**
	Deviation	0.013	0.255	−0.007	−0.017	0.204	−0.003	0.030***	−0.004***
	Multiple	2.000	0.150	2.002	1.997	0.360	2.002	0.003	0.000
3×	R_target	0.949	12.274	1.554	0.432	7.709	1.239	0.516**	0.315
	R_expected	0.909	12.250	1.791	0.482	7.721	1.321	0.427*	0.470**
	Deviation	0.039	0.765	−0.021	−0.050	0.600	−0.011	0.089***	−0.01***
	Multiple	2.999	0.455	3.005	2.991	1.071	3.006	0.008	−0.001
−1×	R_target	−0.290	4.089	−0.676	−0.179	2.585	−0.476	−0.111	−0.200***
	R_expected	−0.303	4.083	−0.597	−0.161	2.574	−0.440	−0.142*	−0.157**
	Deviation	0.013	0.258	−0.007	−0.018	0.217	−0.004	0.031***	−0.003***
	Multiple	−0.999	0.146	−0.998	−1.002	0.363	−0.998	0.003	0.000
−2×	R_target	−0.569	8.193	−1.408	−0.378	5.189	−0.981	−0.19	−0.427***
	R_expected	−0.606	8.166	−1.194	−0.322	5.147	−0.881	−0.285*	−0.313**
	Deviation	0.037	0.777	−0.021	−0.057	0.667	−0.011	0.094***	−0.010***
	Multiple	−1.994	0.433	−1.992	−2.005	1.089	−1.993	0.011	0.001
−3×	R_target	−0.835	12.325	−2.245	−0.598	7.825	−1.523	−0.237	−0.722***
	R_expected	−0.909	12.250	−1.791	−0.482	7.721	−1.321	−0.427*	−0.470**
	Deviation	0.074	1.562	−0.041	−0.116	1.365	−0.022	0.190***	−0.019***
	Multiple	−2.986	0.859	−2.983	−3.009	2.174	−2.986	0.023	0.003
		Panel C: 22 trading days							
	R_index	1.376	9.452	1.569	0.701	5.058	1.582	0.675***	−0.013
2×	R_target	2.982	19.356	2.719	1.322	10.068	3.067	1.659***	−0.348
	R_expected	2.752	18.903	3.139	1.402	10.116	3.163	1.350***	−0.024
	Deviation	0.229	1.373	−0.059	−0.080	0.633	−0.047	0.309***	−0.012*
	Multiple	2.000	0.337	2.004	1.988	0.305	2.002	0.012	0.002
3×	R_target	4.847	29.867	3.462	1.887	14.977	4.391	2.960***	−0.929**
	R_expected	4.128	28.355	4.708	2.103	15.174	4.745	2.026***	−0.037
	Deviation	0.719	4.215	−0.174	−0.215	1.854	−0.137	0.934***	−0.037**
	Multiple	2.991	1.013	3.004	2.957	0.901	3.003	0.034	0.001
−1×	R_target	−1.184	9.126	−1.923	−0.807	5.032	−1.701	−0.376**	−0.222
	R_expected	−1.376	9.452	−1.569	−0.701	5.058	−1.582	−0.675***	0.013

Continued

	Deviation	0.193	1.289	−0.065	−0.106	0.677	−0.045	0.299***	−0.020***
	Multiple	−0.990	0.326	−0.989	−1.004	0.299	−0.995	0.014*	0.006***
−2×	R_target	−2.217	18.023	−4.160	−1.749	9.950	−3.537	−0.468	−0.623**
	R_expected	−2.752	18.903	−3.139	−1.402	10.116	−3.163	−1.350***	0.024
	Deviation	0.535	3.795	−0.201	−0.347	2.070	−0.138	0.883***	−0.063***
	Multiple	−1.960	0.958	−1.962	−2.005	0.872	−1.982	0.045**	0.020***
−3×	R_target	−3.147	26.764	−6.634	−2.852	14.663	−5.525	−0.296	−1.109***
	R_expected	−4.128	28.355	−4.708	−2.103	15.174	−4.745	−2.026***	0.037
	Deviation	0.981	7.458	−0.399	−0.749	4.213	−0.276	1.730***	−0.123***
	Multiple	−2.899	1.866	−2.911	−2.995	1.681	−2.959	0.095**	0.048***
Panel D: 250 trading days									
	R_index	21.958	67.539	3.865	8.085	16.203	11.777	13.872***	−7.912***
2×	R_target	77.543	243.770	2.368	15.946	32.766	22.399	61.597***	−20.031***
	R_expected	43.915	135.078	7.730	16.171	32.405	23.553	27.744***	−15.823***
	Deviation	33.628	117.438	−1.003	−0.225	4.406	−0.473	33.852***	−0.530***
	Multiple	2.135	0.918	2.037	1.888	0.728	1.997	0.247***	0.040***
3×	R_target	207.174	710.181	−5.030	23.751	49.411	31.476	183.423***	−36.506***
	R_expected	65.873	202.617	11.595	24.256	48.608	35.330	41.617***	−23.735***
	Deviation	141.301	531.563	−2.347	−0.505	13.194	−1.800	141.806***	−0.547
	Multiple	3.478	3.026	3.012	2.609	2.080	2.943	0.869***	0.069**
−1×	R_target	−7.597	37.108	−8.256	−8.760	14.420	−12.379	1.163*	4.123***
	R_expected	−21.958	67.539	−3.865	−8.085	16.203	−11.777	−13.872***	7.912***
	Deviation	14.361	43.625	−1.118	−0.675	4.345	−0.209	15.036***	−0.909***
	Multiple	−0.841	0.797	−0.873	−1.033	0.711	−0.955	0.192***	0.082***
−2×	R_target	−9.785	72.384	−20.220	−18.643	25.121	−24.616	8.857***	4.396***
	R_expected	−43.915	135.078	−7.730	−16.171	32.405	−23.553	−27.744***	15.823***
	Deviation	34.130	104.828	−4.010	−2.472	13.368	−0.360	36.602***	−3.650***
	Multiple	−1.469	2.189	−1.547	−2.020	1.999	−1.833	0.550***	0.286***
−3×	R_target	−11.761	108.171	−34.379	−29.682	30.968	−36.794	17.921***	2.415***
	R_expected	−65.873	202.617	−11.595	−24.256	48.608	−35.330	−41.617***	23.735***
	Deviation	54.112	173.728	−9.335	−5.425	27.399	−0.124	59.538***	−9.211***
	Multiple	−1.839	3.894	−1.941	−2.902	3.662	−2.611	1.063***	0.670***

Note: This table reports the mean, standard deviation and the median of the performance metrics of hypothetical LETFs tracking the CSI 300 Index (columns (1) - (3)) and the S&P 500 Index (columns (4) - (6)) within the holding period of 1 trading day (Panel A), 5 trading days (Panel B), 22 trading days (Panel C) and 250 trading days (Panel D). We also conduct the difference tests of mean and medians of LETF performance between the CSI 300 LETFs and S&P 500 LETFs (see columns (7) and (8)). The sample period is from January 2006 to June 2020. For CSI 300 LETFs, the number of observations for Panels A, B, C and D are 3521, 3341, 3325 and 3108, respectively. For S&P 500 LETFs, the number of observations for Panels A, B, C and D are 3647, 3461, 3445 and 3228, respectively. R_index, R_target, and R_expected are the index's return, the target return and the expected return of investors, respectively. Deviation is the return deviation defined by Equation (3), and multiple is the effective multiple defined by Equation (4). All returns and return deviations are denoted in percentage terms. *significant at the 10% level; **significant at the 5% level; ***significant at the 1% level.

is more likely to experience a negative cumulative return for a holding period of one year. In contrary, a 3× U.S. LETF holder is more likely to experience a positive cumulative return for a holding period of one year.

Examining the medians of return deviations in Table 2, we find that all the median return deviations are negative for all bull and bear Chinese LETFs, showing that the target returns are more likely to be less than the naively expected returns for all Chinese LETFs. In addition, the median return deviation of Chinese funds is always more negative than that of the corresponding U.S. funds, indicating that, from January 2006 to June 2020, Chinese LETFs are more likely to suffer losses from the path of underlying index return, resulting in poor performance over a long holding period. For the −3× funds, the median return deviations are −9.335% for Chinese LETF and −0.124% for the U.S. LETF, differing by more than nine percentage points, as shown in Panel D. Consequently, the effective multiples of Chinese LETFs display larger deviations to the stated multiples than those of U.S. LETFs.

To formally test the difference of performance between CSI 300 LETFs and S&P 500 LETFs, we conduct the mean-difference tests and median-difference tests of performance metrics of LETFs between the two markets. The results are shown in the last two columns of Table 2. As shown in Panel A, the difference tests show that, no matter on means or medians, all the target returns of Chinese funds are higher than those of U.S. funds, suggesting a better performance in Chinese funds than in U.S. funds over short holding periods. However, these results do not hold any more as the holding period lengthens to 22 and 250 trading days. As for the return deviations, almost all the means (medians) of return deviations of CSI 300 LETFs are significantly more positive (negative) than those of S&P 500 LETFs, within a holding period of 5, 22 and 250 trading days. Collectively, the difference test in LETF performance shows that there are indeed many differences in LETF performance across the two markets. In particular, Chinese leveraged funds are more appropriate for short-term investment than their U.S. counterparts but suffer more distortions in fund performance over long holding periods.

Observing that the results of average return deviations and median return deviations are opposite, we conduct further analysis to compare the performance across CSI 300 LETFs and S&P 500 LETFs. Specifically, we regress the target returns of LETFs on the cumulative return of index over LETFs' holding period of 5, 22 and 250 trading days, respectively. The results are presented in Table 3. As shown in Panel A, over a holding period of 5 trading days, all ten funds have superior performance, indicated by a coefficient very close to the promised return multiple and an R-squared very close to 1. However, when it comes to the constant, there is a significant difference between each CSI 300 LETF and its counterpart tracking the S&P 500 Index. The constants of all the five CSI 300 LETFs are positive and significant at 10% level, and the results of S&P 500 LETFs are significantly negative at the level of 1%. Collectively, these findings suggest that, over a short holding period, the LETFs tracking the CSI 300 Index can deliver

Table 3. Regression results of target returns on index returns over various holding periods.

Variables	CSI 300 LETFs					S&P 500 LETFs				
	2×	3×	−1×	−2×	−3×	2×	3×	−1×	−2×	−3×
Panel A: 5 trading days										
R_index	2.000***	3.000***	−0.999***	−1.998***	−2.994***	1.996***	2.986***	−1.001***	−2.000***	−2.994***
	(400.75)	(202.58)	(−193.17)	(−127.05)	(−93.94)	(307.21)	(160.35)	(−131.83)	(−83.32)	(−59.30)
Constant	0.013*	0.039*	0.012*	0.037*	0.072*	−0.017***	−0.048***	−0.018***	−0.057***	−0.117***
	(1.88)	(1.92)	(1.76)	(1.72)	(1.67)	(−2.99)	(−3.00)	(−2.91)	(−2.87)	(−2.84)
R-squared	0.999	0.996	0.996	0.991	0.984	0.998	0.994	0.993	0.983	0.970
Obs.	3341	3341	3341	3341	3341	3461	3461	3461	3461	3461
Panel B: 22 trading days										
R_index	2.043***	3.131***	−0.957***	−1.869***	−2.733***	1.987***	2.939***	−0.986***	−1.926***	−2.785***
	(149.86)	(74.41)	(−76.66)	(−51.44)	(−38.78)	(134.25)	(66.68)	(−69.25)	(−46.13)	(−33.89)
Constant	0.170***	0.539***	0.133**	0.355**	0.613*	−0.070**	−0.172*	−0.116***	−0.400***	−0.900***
	(2.86)	(2.98)	(2.34)	(2.10)	(1.84)	(−2.18)	(−1.85)	(−3.34)	(−3.76)	(−4.17)
R-squared	0.995	0.982	0.982	0.960	0.931	0.996	0.985	0.982	0.958	0.923
Obs.	3325	3325	3325	3325	3325	3445	3445	3445	3445	3445
Panel C: 250 trading days										
R_index	3.501***	9.604***	−0.442***	−0.685***	−0.825***	2.004***	2.939***	−0.860***	−1.431***	−1.632***
	(27.98)	(14.37)	(−14.29)	(−9.69)	(−7.47)	(69.03)	(34.60)	(−33.12)	(−20.89)	(−14.28)
Constant	0.671	−3.707	2.115	5.253	6.349	−0.256	−0.016	−1.806***	−7.074***	−16.484***
	(0.24)	(−0.31)	(1.47)	(1.37)	(0.97)	(−0.71)	(−0.02)	(−5.00)	(−7.23)	(−9.91)
R-squared	0.941	0.834	0.648	0.408	0.265	0.982	0.929	0.934	0.852	0.729
Obs.	3108	3108	3108	3108	3108	3228	3228	3228	3228	3228

Note: This table presents the regression results of LETF target returns on index returns over 5 trading days (Panel A), 22 trading days (Panel B) and 250 trading days (Panel C). The sample period is from January 2006 to June 2020. R_index is the return of LETFs' underlying index. Newey-West robust t-statistics are given in parentheses; *significant at the 10% level; **significant at the 5% level; ***significant at the 1% level.

positive excess returns, while the LETFs leveraging the S&P 500 Index deliver negative excess returns. When holding period lengthens to 22 or 250 trading days, the coefficients of LETFs seem to deviate from their respective stated multiples, especially for CSI 300 LETFs. Therefore, since the results in Panels B and C don't have the intended tracking multiple, the differences in the constants between CSI 300 LETFs and S&P 500 LETFs cannot be regarded as the excess returns like the case in Panel A. Overall, the regression results in Table 3 are consistent with the findings in Table 2.

To test whether the results are robust, we compute the Sharpe ratios of LETFs, one of the risk-adjusted performance metrics of funds, and then again compare between CSI 300 LETFs and S&P 500 LETFs. The results are presented in Table 4. For the purpose of comparison across different holding periods, we annualize

Table 4. Sharpe ratios of LETFs over a long history.

Funds	CSI 300 LETFs				S&P 500 LETFs			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	1 day	5 days	22 days	250 days	1 day	5 days	22 days	250 days
Panel A: based on rolling observations								
2×	0.485	0.478	0.466	0.299	0.378	0.377	0.395	0.434
3×	0.500	0.500	0.503	0.280	0.387	0.364	0.385	0.432
−1×	−0.618	−0.585	−0.516	−0.277	−0.461	−0.548	−0.607	−0.678
−2×	−0.574	−0.534	−0.457	−0.178	−0.433	−0.543	−0.628	−0.787
−3×	−0.559	−0.512	−0.430	−0.143	−0.424	−0.558	−0.682	−1.000
Panel B: based on non-rolling observations								
2×	0.485	0.490	0.489	0.401	0.378	0.380	0.396	0.439
3×	0.500	0.516	0.518	0.409	0.387	0.361	0.367	0.444
−1×	−0.618	−0.575	−0.541	−0.339	−0.461	−0.581	−0.724	−0.607
−2×	−0.574	−0.518	−0.472	−0.300	−0.433	−0.586	−0.794	−0.695
−3×	−0.559	−0.488	−0.429	−0.370	−0.424	−0.614	−0.907	−0.943

Note: This table presents the annual Sharpe ratio of hypothetical LETFs tracking the CSI 300 Index (columns (1) - (4)) and the S&P 500 Index (columns (5) - (8)) within the holding period of 1, 5, 22 and 250 trading day/days. The sample period is from January 2006 to June 2020. Panel A is based on rolling observations and Panel B is based on non-rolling observations. The risk-free rate of Chinese market is attained from RESSET database and for U.S. market, the risk-free rate is collected from Kenneth French's web site at Dartmouth.

all the ratios. As shown in the table, the results computed by rolling returns are very close to those based on non-rolling results. The results in Panel B show that, only the bull LETFs have positive Sharpe ratios and all the bear funds have negative Sharpe ratios. Across the two types of funds leveraging different market indices, the bull Chinese funds seem to have higher positive risk-adjusted returns than their U.S. counterparts over the holding periods of 1, 5, and 22 trading day/days. Interestingly, this result reverses when the holding period lengthens to 250 trading days. Nevertheless, these results are consistent with our previous analyses, which support that Chinese LETFs tend to perform better than U.S. LETFs in short holding periods while the result reverses in long holding periods.

The better performance of CSI 300 LETFs over short holding period is due to the higher possibility of extreme observations in the returns of the CSI 300 Index relative to those of the S&P 500 Index. As documented in [4], compared to the normal distribution, the more leptokurtic distribution of actual index results in more performance enhancement of LETFs[12]. Applying this proof to our cases, since the observations of kurtosis of daily returns on the CSI 300 Index during 5-trading-day holding period are more likely to be higher than that those on the

[12]This beneficial effect is generated from the high density of distribution around the positive mean of daily index return and the positive return following the increased exposure which is required by the previous positive return on the underlying index.

S&P 500 Index, it is reasonable to expect a better performance of CSI 300 LETFs over that 5 holding period[13]. Similarly, it is natural to observe an inversed result over the holding period of 250 trading days since we do observe a lower mean kurtosis of the daily returns for CSI 300 in this long holding period.

Based on the empirical analysis in this subsection, we draw the following conclusions On Chinese LETFs. A high volatility of underlying index does not necessarily correspond to poor LETF performance, and the cumulative return of index can also play a key role on the net effect on LETF performance. Therefore, estimating the net effect of index on LETF performance is not a simple nor easy task. In our performance comparison, although the volatility of the CSI 300 Index is higher than that of the S&P 500 Index, CSI 300 LETFs perform better than S&P 500 LETFs for short holding period. In contrast, S&P 500 LETFs perform better than CSI 300 LETFs for longer holding time periods. On one hand, these findings alleviate the concern that high volatility of index is necessarily linked with LETF poor performance, and LETFs leveraging the Chinese A-share market seem to display a complicated performance due to the characteristics of stock market index. On the other hand, it inspires us to investigate the explicit quantitative relationship between index return and index volatility, which is instructive to market participants.

4.3. Performance of Chinese LETFs during Crisis Periods

As the demand to trade LETFs often increases when there could be large changes in the index returns, we examine the performance of Chinese LETFs during crisis periods. In particular, the entire history of the underlying indices allows us to investigate the 2008 U.S. financial crisis period, the 2015 Chinese stock market crash period, and the 2020 COVID-19 pandemic period[14]. Unlike the performance of LETFs based on the entire history of index, the tests based on a short interval don't always allow for the holding period of 250 trading days during the crisis period. Therefore, in this subsection, we only conduct the tests on fund performance over the holding periods of 1, 5, and 22 trading day/days.

The results during the 2008 financial crisis period are shown in columns (1) - (3) of **Table 5**. Following [4], this crisis period is set from October 2007 to March 2009. The standard deviation of daily return for the CSI 300 Index is 2.816%, higher than that based on a long history of index (as shown in **Table 1**: 1.719%). This higher volatility of index is also associated with a more negative return deviation of LETFs in the same holding period. For example, except for the 3× fund during a holding period of 22 trading days, all the other medians are negative, suggesting that index performance is more likely to be detrimental to the performance of these LETFs during the financial crisis of 2008.

[13]In our untabulated results, the mean kurtosis of daily index return on the CSI 300 Index during 5 trading days is 3.291, and 3.172 for the counterparts on the S&P 500 Index, both of which are higher than 3, the kurtosis of the normal distribution. The corresponding values are 5.23 and 5.849 when the holding period lengthens to 250 trading days.

[14]We choose February 2020 to be the beginning month because the pandemic was known to the public in the end of January.

Table 5. Performance of Chinese LETFs during financial crisis periods.

Funds	Variables	2008 financial crisis			2015 Chinese market crash			2020 COVID-19 pandemic		
		Mean	SD	Median	Mean	SD	Median	Mean	SD	Median
Panel A: 1 trading day										
	Index return	−0.181	2.816	−0.114	−0.246	2.801	0.018	0.051	1.575	0.180
Panel B: 5 trading days										
	R_index	−0.999	6.463	−1.978	−1.357	6.158	−0.287	0.447	3.196	1.115
2×	R_target	−1.965	12.856	−4.416	−2.698	12.093	−0.614	0.896	6.341	2.216
	R_expected	−1.997	12.926	−3.956	−2.713	12.316	−0.573	0.893	6.393	2.230
	Deviation	0.032	0.466	−0.025	0.015	0.555	−0.037	0.003	0.168	−0.003
	Multiple	2.011	0.163	2.000	1.973	0.136	1.994	1.999	0.070	2.002
3×	R_target	−2.901	19.197	−7.095	−4.018	17.827	−0.980	1.348	9.441	3.303
	R_expected	−2.996	19.390	−5.934	−4.070	18.474	−0.860	1.340	9.589	3.345
	Deviation	0.095	1.390	−0.075	0.052	1.638	−0.111	0.008	0.504	−0.010
	Multiple	3.028	0.486	2.992	2.916	0.409	2.981	2.997	0.212	3.005
−1×	R_target	1.030	6.553	1.701	1.366	6.404	0.245	−0.442	3.255	−1.174
	R_expected	0.999	6.463	1.978	1.357	6.158	0.287	−0.447	3.196	−1.115
	Deviation	0.032	0.476	−0.034	0.009	0.586	−0.041	0.005	0.168	−0.003
	Multiple	−0.986	0.164	−0.997	−1.023	0.136	−1.005	−1.000	0.069	−0.998
−2×	R_target	2.092	13.216	3.001	2.733	13.080	0.401	−0.878	6.577	−2.369
	R_expected	1.997	12.926	3.956	2.713	12.316	0.573	−0.893	6.393	−2.230
	Deviation	0.095	1.441	−0.098	0.020	1.792	−0.129	0.015	0.506	−0.008
	Multiple	−1.953	0.494	−1.988	−2.066	0.407	−2.013	−2.000	0.208	−1.994
−3×	R_target	3.186	20.011	3.968	4.094	20.058	0.249	−1.307	9.971	−3.590
	R_expected	2.996	19.390	5.934	4.070	18.474	0.860	−1.340	9.589	−3.345
	Deviation	0.191	2.908	−0.188	0.024	3.661	−0.280	0.033	1.015	−0.014
	Multiple	−2.900	0.991	−2.969	−3.123	0.814	−3.025	−2.998	0.414	−2.986
	R_index	−4.942	12.877	−6.251	−6.312	11.463	−6.617	0.464	6.449	2.485
Panel C: 22 trading days										
	R_index	−4.942	12.877	−6.251	−4.226	7.291	−4.060	−0.716	−2.191**	−4.226
2×	R_target	−9.599	25.117	−13.613	−12.409	21.978	−14.499	0.906	12.865	4.736
	R_expected	−9.885	25.754	−12.502	−12.623	22.925	−13.233	0.928	12.898	4.969
	Deviation	0.286	2.073	−0.061	0.214	1.749	−0.262	−0.022	0.376	−0.047
	Multiple	1.954	0.450	1.953	1.986	0.346	1.968	1.972	0.111	1.989
3×	R_target	−13.899	36.865	−22.405	−18.122	31.525	−22.428	1.344	19.208	6.740
	R_expected	−14.827	38.631	−18.754	−18.935	34.388	−19.850	1.391	19.347	7.454
	Deviation	0.928	6.160	0.117	0.812	5.175	−0.725	−0.048	1.143	−0.155
	Multiple	2.837	1.329	2.836	2.931	1.035	2.883	2.906	0.336	2.956

Continued

−1×	R_target	5.126	13.626	5.033	6.304	12.332	5.632	−0.508	6.436	−2.779
	R_expected	4.942	12.877	6.251	6.312	11.463	6.617	−0.464	6.449	−2.485
	Deviation	0.184	2.139	−0.392	−0.007	1.787	−0.345	−0.044	0.364	−0.090
	Multiple	−1.016	0.452	−1.022	−0.987	0.339	−1.010	−1.019	0.106	−1.006
−2×	R_target	10.301	28.083	8.207	12.338	25.395	10.421	−1.085	12.808	−5.679
	R_expected	9.885	25.754	12.502	12.623	22.925	13.233	−0.928	12.898	−4.969
	Deviation	0.416	6.490	−1.475	−0.285	5.366	−1.191	−0.158	1.086	−0.277
	Multiple	−2.015	1.337	−2.033	−1.932	0.998	−1.996	−2.049	0.315	−2.008
−3×	R_target	15.354	43.423	10.147	17.812	38.998	13.875	−1.760	19.060	−8.687
	R_expected	14.827	38.631	18.754	18.935	34.388	19.850	−1.391	19.347	−7.454
	Deviation	0.527	13.121	−3.124	−1.123	10.709	−2.538	−0.369	2.162	−0.495
	Multiple	−2.962	2.623	−3.008	−2.809	1.949	−2.937	−3.080	0.623	−2.987

The results during the 2015 Chinese market crash period (June 2015 to February 2016) are similar to those during the 2008 financial crisis period. As shown in columns (4) - (6), the median daily return of the CSI 300 Index is positive. In most cases, the bull (bear) LETFs on the CSI 300 Index register negative (positive) returns. As for the return deviation, all the median values are negative but seven of the ten average values are positive, similar to the results based on the history of index.

When it comes to the 2020 COVID-19 pandemic period (June 2015 to February 2016), the results seem to be quite different with those during the 2008 financial crisis period and the 2015 Chinese stock market crash period. As shown columns (7) - (9), the CSI 300 Index cumulative returns over 1, 5 and 22 trading day/days are positive on average and in terms of frequency, resulting in positive mean and median returns of LETFs. However, the positive return does not mean positive return deviations. Actually, due to a high volatility of index return, all the median return deviations of CSI 300 LETFs are negative. In our untabulated results related to the performance of the S&P 500 Index LETFs during the same period, we observe that the mean return of each bull (bear) fund is negative (positive), against the results of CSI 300 LETFs. It suggests that Chinese market indices are not affected as severely as the U.S. market indices by the pandemic.

As shown in Table 6, our further tests on the Sharpe ratios of Chinese LETFs during the three financial crisis periods reveal that it is highly profitable for investors to substitute the bull LETFs with bear LETFs during the 2008 financial crisis period and the 2015 Chinese market crash period. Very high Sharpe ratios occur to the bear funds during the two periods. Specifically, the highest Sharpe ratio during a long history of the CSI 300 Index is 0.518 (3× CSI 300 funds in Table 4 over the holding period 22 trading days based on the non-rolling observations), while the three bear funds during the two crises registered Sharpe ratios around 1. However, the results during the 2020 COVID-19 pandemic period are

Table 6. Sharpe ratio of Chinese LETFs during crisis periods.

Funds	2008 financial crisis			2015 Chinese market crash			2020 COVID-19 pandemic		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	1 day	5 days	22 days	1 day	5 days	22 days	1 day	5 days	22 days
Panel A: based on rolling observations									
2×	−1.048	−1.100	−1.302	−1.421	−1.582	−1.900	0.493	0.944	0.194
3×	−1.038	−1.090	−1.289	−1.413	−1.596	−1.935	0.502	0.959	0.199
−1×	0.962	1.009	1.155	1.349	1.423	1.619	−0.573	−1.006	−0.323
−2×	0.990	1.034	1.143	1.373	1.413	1.556	−0.546	−0.965	−0.316
−3×	1.000	1.038	1.100	1.381	1.382	1.463	−0.537	−0.942	−0.335
Panel B: based on non-rolling observations									
2×	−1.048	−1.116	−0.633	−1.108	−1.319	−1.808	0.493	0.484	0.500
3×	−1.038	−1.147	−0.546	−1.102	−1.356	−1.882	0.502	0.456	0.427
−1×	0.962	0.919	0.767	1.052	1.123	1.315	−0.573	−0.677	−0.846
−2×	0.990	0.918	0.841	1.071	1.111	1.175	−0.546	−0.678	−0.890
−3×	1.000	0.900	0.886	1.077	1.085	0.971	−0.537	−0.697	−0.961

Note: This table presents the annual Sharpe ratio of hypothetical LETFs tracking the CSI 300 Index within the holding period of 1, 5, and 22 trading day/days during three typical crisis periods. Columns (1) - (3), (4) - (6), and (7) - (9) present the results during the 2008 financial crisis, 2015 Chinese stock market crash, and 2020 COVID-19 pandemic, respectively. Panel A is based on rolling observations and Panel B is based on non-rolling observations. The risk-free rate of Chinese market is attained from RESSET database and for U.S. market, the risk-free rate is collected from Kenneth French's web site at Dartmouth.

quite different, with positive Sharpe ratios in bull funds and negative Sharpe ratios in bear funds. These results are consistent with the evidence shown in **Table 5** implying that the performance of Chinese market indices does not experience too many changes during the pandemic.

Collectively, the tests of LETF performance during the three crisis periods show that LETF performance will be distorted severely if the underlying index is highly fluctuating. Moreover, contrary to the results based on a long history of index, bull funds tend to have negative returns and bear funds tend to have positive returns during the crisis periods. Among the three crisis periods, the returns of CSI 300 LETFs seem to be dragged down most during the 2015 Chinese stock market crash period.

4.4. Index Performance and LETF Performance

In the previous analyses, we find that the sample LETFs perform better than naïve expectation sometimes, but worse in other time, due to the compounding effect. This compounding effect is also important to the comparison across the two countries. In this subsection, to provide some guidance to investors, we further explore the explicit conditions of index returns for them to benefit the fund performance in general.

Before carrying out the further investigation on the association between LETF performance and index performance, we wonder whether the impact of index performance accounts for a significant part of the total return deviation in the actual LETFs. To address this issue, we compute the absolute ratio of compounding deviations to the NAV return deviation of a batch of the earliest listed LETFs in the world. The sample period is from 6/20/2006 to 6/30/2020. The NAV deviation is the difference between NAV return and target return of LETFs. Therefore, the ratio we computed can tell which deviation dominates in the total return deviation. We compute the ratios in 5, 22, 250 holding periods, respectively, and all results are winsorized at the 1% tails. The results are shown in Table 7.

As shown in Table 7, all the mean ratios are greater than one, indicating that the compounding deviation over multiple days is the main component of return deviation. As the holding periods lengthen, the compounding deviations seem to account for more of the total return deviations. With that in mind, it is likely that the compounding deviations in Chinese potential LETFs are likely to be more pronounced given the higher volatility of stock indices in Chinese A-share market.

LETF performance vitally depends on the return path of the underlying index [20]. When the target return exceeds the expected return, or when the return deviation is positive, the index return's path benefits the fund performance and vice versa. Building on the analyses of [12], we model the relationship between LETF performance and index performance, and present the detailed derivation in **Appendix**.

According to the derivation in **Appendix**, there must be just two critical values of index returns corresponding to the zero return deviation[15]. In order to figure out these two critical values, we let Equation (A6) be zero:

$$(1+R_I)^m \exp\left(\frac{1}{2}(m-m^2)\sigma^2 t\right)-1-mR_I=0\,, \tag{6}$$

where R_I and σ_2 are the cumulative return and the variance of the underlying index return during time t, respectively.

The approximate values of the two critical returns[16], which are denoted as R_{c1} and R_{c2}, are given as the following two expressions, respectively:

$$R_{c1}=\frac{-m(a-1)+\sqrt{m(a-1)\left[m(a-1)-2a(m-1)\right]}}{am(m-1)}\,, \tag{7}$$

$$R_{c2}=\frac{-m(a-1)-\sqrt{m(a-1)\left[m(a-1)-2a(m-1)\right]}}{am(m-1)}\,, \tag{8}$$

[15]Although the derivations presented in **Appendix** are based on the assumption that rebalancing happens continuously over time, we can use the results to approximately estimate the threshold levels of index returns, above which the LETFs can deliver a return higher than the naively expected level given a specific return volatility, since the daily rebalancing period is very short relative to the multi-day holding period.

[16]There is no explicit root for Equation (6). We hence use a second-order Taylor expansion to get the two approximate real roots of Equation (6). A detailed derivation of these two approximate roots is available upon request. Although these two values are not the precise roots, they help us judge the sign of return deviation.

Table 7. The ratio of compounding deviation to NAV deviation of the LETFs.

Holding days	Mean	S.D.	Min	p25	p50	p75	Max	N
SSO: multiple = 2, benchmarked on the S&P 500 Index								
5	2.50	7.20	0.00	0.14	0.46	1.55	74.92	3457
22	2.43	6.44	0.01	0.25	0.63	1.77	73.51	3441
250	6.50	17.25	0.02	0.41	0.93	2.24	173.30	3217
DDM: multiple = 2, benchmarked on the Dow Jones Industrial Average Index								
5	1.57	4.17	0.00	0.09	0.32	1.03	47.03	3456
22	1.73	4.35	0.01	0.19	0.51	1.26	48.55	3440
250	2.06	3.78	0.01	0.27	0.63	1.57	27.93	3217
QLD: multiple = 2, benchmarked on the NASDAQ-100 Index								
5	3.65	9.34	0.01	0.28	0.86	2.86	111.60	3457
22	5.31	11.81	0.02	0.52	1.45	4.50	126.40	3439
250	6.49	8.77	0.06	1.37	3.41	6.92	59.36	3217
MVV: multiple = 2, benchmarked on the S&P MidCap 400 Index								
5	3.28	7.68	0.01	0.27	0.81	2.53	76.53	3458
22	4.77	10.96	0.03	0.56	1.43	4.06	111.40	3441
250	5.77	14.88	0.03	0.74	1.61	4.39	157.10	3218
SH: multiple = −1, benchmarked on the S&P 500 Index								
5	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71
22	1.30	1.30	1.30	1.30	1.30	1.30	1.30	1.30
250	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75
DOG: multiple = −1, benchmarked on the Dow Jones Industrial Average Index								
5	1.49	1.49	1.49	1.49	1.49	1.49	1.49	1.49
22	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47
250	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.69
PSQ: multiple = −1, benchmarked on the NASDAQ-100 Index								
5	3.23	3.23	3.23	3.23	3.23	3.23	3.23	3.23
22	2.22	2.22	2.22	2.22	2.22	2.22	2.22	2.22
250	4.82	4.82	4.82	4.82	4.82	4.82	4.82	4.82
MYY: multiple = −1, benchmarked on the S&P MidCap 400 Index								
5	2.36	2.36	2.36	2.36	2.36	2.36	2.36	2.36
22	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.69
250	2.63	2.63	2.63	2.63	2.63	2.63	2.63	2.63

Note: This table presents the summary statistics of the ratio of absolute compounding deviation to absolute NAV deviation of a batch of the earliest listed LETFs in the world. All these eight LETFs are listed in the U.S. market. The sample period is from 6/20/2006 to 6/30/2020. The compounding deviation is the discrepancy between target return and the expected return of naïve investors, and the NAV deviation is the difference between NAV return and target return of LETFs. Therefore, the ratio we computed can tell which deviation dominates in the total return deviation computed by the difference between NAV return and expected return. We compute the ratios in 5, 22, 250 holding periods, respectively, and all results are winsorized in at the 1% tails. All deviations are denoted in percentage terms.

where a is given as:

$$a = \exp\left(\frac{1}{2}\left(m - m^2\right)t\sigma^2\right). \tag{9}$$

Given that $m > 1$ or $m < 0$, Equation (7) is always positive and Equation (8) is always negative, implying that there are always two critical returns corresponding to the zero-value of LETF return deviation, one of which is positive and the other is negative. It also means that, in order to benefit LETF performance, the underlying index has to exceed the positive critical return (R_{c1}) for bull funds, or accumulate a return more negative than the negative critical return (R_{c2}) for bear funds.

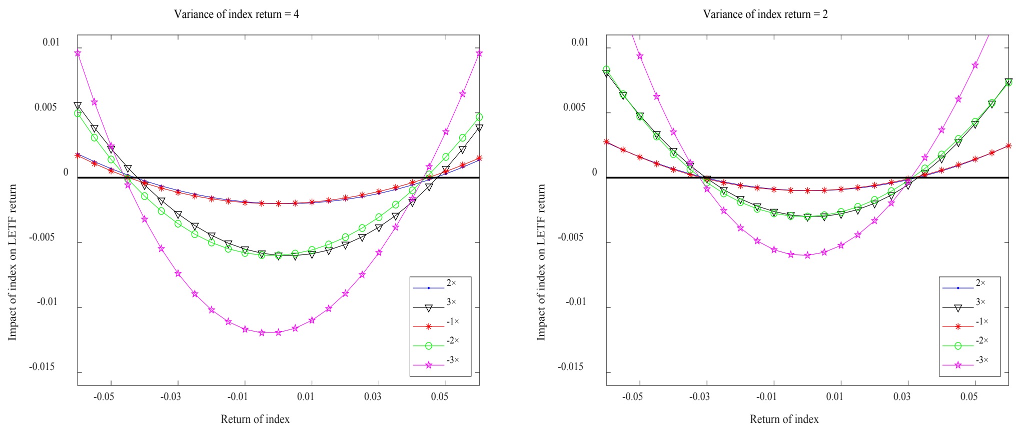
Let m equal 2 or −2, we find that our formulas are equivalent to the expressions of break-even levels of index proposed by [20], indicating that our formulas of critical returns are applicable and general, and the two given numerical expressions of [20] can be included in our extensive formulas.

As shown by the derivation and analysis above, now we can assume that the graph function of return deviation with respect to the index return is approximately U-shaped when the index returns range from −1 to 1[17]. This property of return deviation makes it very easy to understand the clear quantitative relationship between index performance and LETF performance.

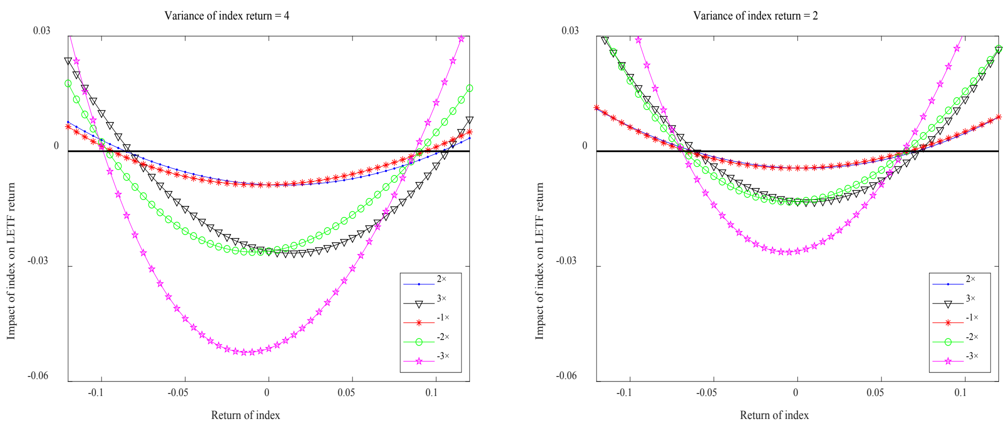
Figure 1 helps us intuitively understand the impact of index performance on LETF performance. In this figure, we set the variances of the index daily returns to 4 in the left three graphs and 2 in the right three graphs. Indeed, as shown in **Figure 1**, each line is U-shaped and crosses the zero-line of return deviation twice when the return of index ranges from −1 to 1. The two critical returns corresponding to the break-even level of return deviation divide the graph of return deviation into three parts. These geometrical characteristics reveal that the magnitude of return deviation depends on the return difference between the index return during the holding period and the apex return of the graph, and the direction of return deviation depends on the range where the return of underlying index lies and which is specified by the two critical returns. In addition, the lines in **Figure 1** are not symmetric around the zero-line of index return, indicating that the two critical returns are not the inverse to each other. More importantly, **Figure 1** intuitively shows that a bull (bear) fund has to accumulate a positive (negative) return higher (less) than the positive (negative) critical return to benefit the fund performance.

To provide market participants general cases about the impact of index on LETF performance for references, given a set of variances of index daily return (0.5, 1, …, 9.5, 10) during the holding period of 5, 22 and 250 trading days, we compute the corresponding values of return that index needs to accumulate to offset the negative effect of these given variances. Since most of the variances of

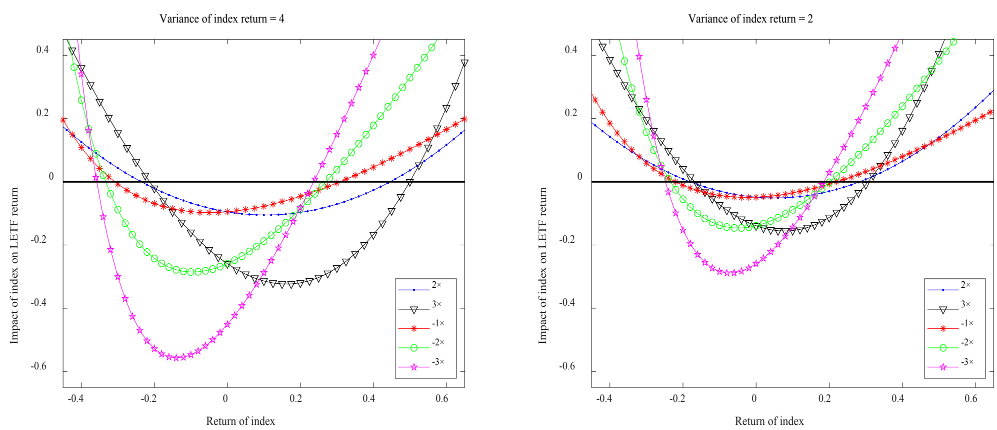
[17]We specify the range of index return from −1 to 1, because we use Taylor expansion to solve Equation (6) and the actual return is no less than −1. In our later empirical analysis, all the actual returns of our sample indices are not exceeding 100% during the holding period, indicating that the range which we specify is very applicable to the actual returns of index.



Graph A: 5 trading days.



Graph B: 22 trading days.



Graph C: 250trading days.

Figure 1. Plot of return deviation over the return of index. Note: This figure illustrates the relationship between the return deviations of LETFs and the returns of index, given the standard deviations of index return (4 percent for left plots and 2 percent for right plots). The holding periods are 5, 25, and 250 trading days for Graphs A, B, and C, respectively. The horizontal axis shows the return of the index, and the vertical axis shows the return deviation. Given the return of index, the target return, the holding period and a constant variance of index daily return during the holding period, the return deviation is computed by the following equation: Return deviation $= (1+R_I)^m \exp\left(\left(m-m^2\right)\sigma^2 t/2\right) - 1 - mR_I$, where m is the stated multiple of LETFs, t is LETF holding period of LETFs, R_I is the index return over t, and σ is the standard deviation of LETFs.

index daily returns are always in the range from 0 to 10, we compute the critical returns based on the variances in this range. Because investors who wish to trade on the upward (downward) movement of index will buy bull (bear) LETFs, we only report the positive (negative) critical returns for bull (bear) LETFs. These critical returns are computed via Equations (7) or (8). The results are presented in Table 8. In Panel A, for the 2× funds, the critical returns for variances of 0.5 and 10 are 1.61% and 7.6%, respectively, with a difference of 5.99%. Comparing the critical returns across 2×, 3× and −3× funds with the same volatility during the same holding period, such as 5 in Panel C, the absolute values increase from 28.34% to 32.69%, and then to 34.43%. Given the same target leverage and the same volatility, such as 3× fund with a variance of 5, when the holding periods

Table 8. What extent of return should the underlying index reach to benefit LETF performance?

Variance	2×	3×	−1×	−2×	−3×	2×	3×	−1×	−2×	−3×	2×	3×	−1×	−2×	−3×
	Panel A: 5 trading days					Panel B: 22 trading days					Panel C: 250 trading days				
0.5	1.61	1.62	−1.59	−1.61	−1.62	3.43	3.49	−3.37	−3.43	−3.49	7.60	7.89	−7.33	−7.62	−7.93
1.0	2.29	2.31	−2.26	−2.29	−2.31	4.92	5.04	−4.80	−4.92	−5.05	11.08	11.71	−10.54	−11.14	−11.81
1.5	2.82	2.86	−2.78	−2.82	−2.86	6.09	6.28	−5.92	−6.10	−6.29	13.90	14.90	−13.07	−14.02	−15.10
2.0	3.26	3.32	−3.21	−3.27	−3.32	7.10	7.35	−6.86	−7.11	−7.38	16.38	17.78	−15.26	−16.57	−18.11
2.5	3.66	3.73	−3.60	−3.67	−3.74	8.00	8.33	−7.71	−8.02	−8.36	18.64	20.47	−17.23	−18.92	−20.96
3.0	4.03	4.11	−3.95	−4.03	−4.11	8.83	9.23	−8.48	−8.86	−9.28	20.76	23.04	−19.04	−21.13	−23.72
3.5	4.36	4.46	−4.27	−4.37	−4.47	9.60	10.07	−9.19	−9.64	−10.14	22.77	25.53	−20.74	−23.25	−26.42
4.0	4.68	4.79	−4.58	−4.68	−4.80	10.33	10.87	−9.85	−10.38	−10.96	24.69	27.95	−22.35	−25.30	−29.10
4.5	4.98	5.10	−4.86	−4.98	−5.11	11.02	11.64	−10.48	−11.08	−11.74	26.55	30.34	−23.88	−27.29	−31.76
5.0	5.26	5.40	−5.13	−5.27	−5.41	11.68	12.38	−11.08	−11.75	−12.50	28.34	32.69	−25.35	−29.24	−34.43
5.5	5.53	5.69	−5.39	−5.54	−5.70	12.32	13.10	−11.66	−12.40	−13.24	30.10	35.02	−26.77	−31.15	−37.11
6.0	5.79	5.96	−5.63	−5.80	−5.97	12.93	13.80	−12.21	−13.03	−13.96	31.81	37.33	−28.15	−33.04	−39.81
6.5	6.04	6.23	−5.87	−6.05	−6.24	13.53	14.48	−12.74	−13.64	−14.66	33.49	39.64	−29.49	−34.91	−42.54
7.0	6.28	6.48	−6.10	−6.29	−6.50	14.11	15.14	−13.26	−14.23	−15.35	35.14	41.94	−30.80	−36.77	−45.30
7.5	6.52	6.73	−6.32	−6.53	−6.75	14.67	15.79	−13.76	−14.81	−16.03	36.76	44.24	−32.07	−38.61	−48.11
8.0	6.74	6.98	−6.53	−6.76	−7.00	15.22	16.43	−14.24	−15.37	−16.69	38.37	46.55	−33.32	−40.44	−50.95
8.5	6.97	7.21	−6.74	−6.98	−7.24	15.76	17.05	−14.72	−15.93	−17.35	39.95	48.86	−34.55	−42.27	−53.85
9.0	7.18	7.44	−6.95	−7.20	−7.47	16.28	17.67	−15.18	−16.47	−17.99	41.52	51.18	−35.76	−44.10	−56.81
9.5	7.39	7.67	−7.14	−7.41	−7.70	16.80	18.27	−15.63	−17.00	−18.63	43.07	53.52	−36.94	−45.92	−59.82
10.0	7.60	7.89	−7.33	−7.62	−7.93	17.30	18.87	−16.07	−17.53	−19.26	44.61	55.86	−38.11	−47.75	−62.89

Note: This table presents the positive (negative) critical return of the underlying index for bull (bear) LETFs, given the target multiple, the holding period of fund and the expected daily return volatility of the underlying index. The results based on a holding period of 5, 22 and 250 trading days are shown in Panels A, B and C, respectively. The expected variance of index return is given at twenty different level, from 0.5 to 10. The critical return is the extent of return that an index underlying an LETF should reach in order to benefit the fund performance. All returns are denoted in percentage terms.

change from 5 to 22, and then to 250 trading days, the critical returns change from 5.40% to 12.38%, and then to 32.69%[18].

Collectively, based on the above analyses in subsection 4.4, we can draw the following conclusions. First, to benefit LETF investors and LETF performance, a bull (bear) fund has to accumulate a return higher (lower) than the positive (negative) critical return. Second, to offset the negative effect of index volatility during the holding period of LETFs, ceteris paribus: 1) LETFs with higher leverage require more cumulative return of index; 2) With the same magnitude of promised multiple, bear funds require more absolute return of index than the corresponding bull funds; 3) LETFs over longer holding periods require more cumulative return of index. Last, within a long holding period, even a small difference between index volatility can cause a substantial negative effect on LETF performance.

5. Additional Tests

5.1. Performance of LETFs on Other Chinese Indices

We also simulate the performance of LETFs on some other representative stock indices in Chinese A-share market, including the Shanghai Composite Index, the Shenzhen Component Index, and the CSI 1000 Index. The results, which are available upon request, are consistent with the results based on the CSI 300 Index. In addition, the chart patterns of the historical prices of these four Chinese stock indices are highly similar to each other. We thus conjecture that there may be some market-specific features of Chinese markets affecting the performance of Chinese LETFs through the channel of index performance.

5.2. Performance of U.S. LETFs during Financial Crises

We also examine the performance of LETFs on the S&P 500 Index during these crisis periods and compare the results between CSI 300 LETFs and S&P 500 LETFs. The results are available upon request. For S&P 500 LETFs, the impact of index path during the 2020 COVID-19 crisis period on LETF return is no less than the impact during the 2008 financial crisis period, which can be explained by the substantial turbulence in U.S. market during the first half of 2020.

5.3. Multi-Day Return and Return Variance

As cumulative index return and the variance of daily returns are the two most important factors to determine whether an LETF will outperformance its native expectation during a multiple-day holding period, we present the key summary statistics for these two variables on the most prevailing stock indices in the two

[18]We compute the critical returns and compare the results between the two markets. We use the historical mean and median of index return variance respectively during the period from January 2006 to June 2020 to represent the expected index return volatility. The results are not reported but available upon request, and they suggest that the characteristics in Chinese LETFs and the differences of fund performance between Chinese and U.S. LETFs are the reflections of the quantitative relationship between the index performance and LETF performance.

countries. In particular, these indices include the CSI 300 Index, the Shanghai Composite Index, the Shenzhen Component Index, and the CSI 1000 Index from Chinese market, as well as the S&P 500 Index, the Dow Jones Industrial Average Index, the NASDAQ Composite Index, and the Russell 2000 Index from the U.S. market. The results are available upon request. The most important result that is critical to multiple-day LETF performance is that all four Chinese stock indices experience a much higher volatility than the four U.S. stock indices within 5, 22, and 250 trading days.

6. Conclusions and Policy Implications

The main objective of this study is to examine the performance behaviors of Chinese LETFs which are designed to track totally different underlying indices relative to the extant literature on LETFs tracking indices from developed financial markets. Overall, our study not only provides a comprehensive and detailed understanding on the performance of LETFs leveraging Chinese A-share market, but also yields several interesting results. In particular, the performance of Chinese LETFs cannot be outlined directly by the performance enhancement documented in [4] and [5] nor the potential value destruction often warned by regulators and financial media. Moreover, Chinese LETFs seem to have more superior performance than U.S. LETFs over short holding periods but this result reverses over long holding periods, thereby alleviating the concern that high volatility of index is necessarily linked with poor LETF performance. The tests during the crisis periods show bear LETFs are indeed very likely to register positive returns in the crises while the bull funds tend to register negative returns, contrary to the results in normal time. However, the tests over the recent COVID-19 pandemic show that the performance of Chinese LETFs did not seem to be affected as severely as the U.S. LETFs. All these results disclose the unique performance properties of Chinese LETFs, which may be related to the market-specific features of Chinese A-share market.

Since Chinese financial market has been experiencing rapid reform recently and there is no ETF designed to leverage or inversely leverage the underlying assets[19], we believe this study is interesting to investors, regulators, and fund managers. For investors, our results intuitively show that there can be a large difference in the performances of LETFs leveraging different markets, so investors should have enough knowledge of the return and volatility of market indices. This is important and should be fully noted by investors since LETFs leveraging different markets are increasingly available. In addition, investing in LETFs over short periods is often safer than in long periods, because a tendency of high return magnitude and low volatility in indices over long period rarely appears in

[19]Leveraged trading and short selling are currently two of the most investor-demanded investment strategies in Chinese financial market. However, there is no leveraged traded product in the mainland of China, and the currently short-selling instruments are very limited, such as several stock index futures and securities margin trading, which are rarely available to retail investors. LETFs happen to be the alternatives that can satisfy the strategies of leveraged trading (bull LETFs) and short selling (bear LETFs) in more convenient and low-costs ways.

recent years.

For regulators and potential fund issuers, our study can help them comprehensively understand the performance behaviors of LETFs leveraging Chinese A-share market before introducing them to Chinese mainland. Second, since the short-term performance of daily LETFs is very close to the multiple times of index return, even in LETFs leveraging an index with high volatility (e.g., the CSI 300 Index), more LETFs leveraging can be introduced and developed for short-term investors in the future. Meanwhile, regulators still need to remind investors of the long-term performance, especially for LETFs leveraging a market with high volatility like Chinese market. Last, regulators should also be aware of the abrupt changes of rebalancing needs of Chinese LETFs since Chinese stock indices often have higher volatility than the U.S. stock indices.

There are some potential future research areas to further explore. One area is to examine the LETF performance based on the indexes traded in Hong Kong. Although the indices in Hong Kong often track the same underling stocks as some A-share indices, their volatility could be quite different. These differences may have influences on LETF performances. Another area is to investigate the detailed sector level of A-share markets. Different sectors are likely to have different returns and volatilities, which will impact the LETF performances.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix

In order to figure out the conditions under which the return deviation is positive or negative, we need to model the return dynamics of securities in continuous time. It should be noted that the return deviation in this study does not include the fees and financing costs. According to [12], the relationship between the NAV of an LETF and the price of the underlying index is as follows:

$$A_t = A_0\left(\frac{S_t}{S_0}\right)^m \exp\left(\frac{1}{2}\left(m-m^2\right)\sigma^2 t\right), \tag{A1}$$

where A_t and S_t represent the NAV of an LETF and the underlying index level at time t, respectively, m is the daily target multiple, and σ^2 is the variance of underlying index return during time t. Note that A_t is the NAV that is depending on the level of underlying index, and the risk free rate implicit in the costs of the total return swaps does not enter into Equation (A1), so the target return of an LETF with an investment horizon of t, denoted by R_{target}, can be shown algebraically that:

$$R_{target} = A_t/A_0 - 1. \tag{A2}$$

Similarly, the underlying index return of an LETF with an investment horizon of t, denoted by R_I, can be given that:

$$R_I = S_t/S_0 - 1. \tag{A3}$$

Applying Equations (A2) and (A3) to Equation (A1), we have:

$$R_{target} = \left(1+R_I\right)^m \exp\left(\frac{1}{2}\left(m-m^2\right)\sigma^2 t\right) - 1. \tag{A4}$$

According to Equation (A4), as $(m - m^2) < 0$, the scalar $\exp((m - m^2) \times \sigma^2 t/2)$ is less than one, so the target return will always be eroded by the variance of underlying index's return. If the price variables are interpreted in natural logs, Equation (A1) can be converted to the following expression:

$$LR_{target} = m \cdot LR_I - \frac{1}{2}\left(m^2 - m\right)\sigma^2 t, \tag{A5}$$

where LR_{target} and LR_I are the target return and underlying index's return in natural logs, respectively. Equation (A5) implies that the target return in natural logs is m times of underlying index's return in natural logs, subtracted a scalar $(m - m^2) \times \sigma^2 t/2$. As $(m - m^2) \times \sigma^2 t/2 > 0$, LR_{target} is always less than m times of LR_I. Moreover, compared to the bullish ETF with the same magnitude of leverage, the target return in natural logs of a bearish ETF is more prone to be eroded by the variance of underlying index's return, consistent with the findings of [20].

With Equations (A1), (A2), (A3) and (A4), the continuous time's expression of an LETF's return deviation denoted by RD can be shown that:

$$RD = R_{target} - mR_I = \left(1+R_I\right)^m \exp\left(\frac{1}{2}\left(m-m^2\right)\sigma^2 t\right) - 1 - mR_I. \tag{A6}$$

Conceptually, this model predicts that the return deviation is determined by underlying index return, target multiple, the volatility of the index return and holding period. In addition, when the return of underlying index is close to zero, the return deviation will be negative because the variance of underlying index's return is always greater than zero. To deeply disclose the relationship between the return deviation and its determinants, we take the derivative of Equation (A6) and then we have:

$$\frac{\partial RD}{\partial R_I}=m\left(\left(1+R_I\right)^{m-1}\exp\left(\frac{1}{2}\left(m-m^2\right)\sigma^2 t\right)-1\right), \tag{A7}$$

$$\frac{\partial RD}{\partial t}=\frac{1}{2}\sigma^2\left(m-m^2\right)\left(1+R_I\right)^m\exp\left(\frac{1}{2}\left(m-m^2\right)\sigma^2 t\right), \tag{A8}$$

$$\frac{\partial RD}{\partial \sigma^2}=\frac{1}{2}t\left(m-m^2\right)\left(1+R_I\right)^m\exp\left(\frac{1}{2}\left(m-m^2\right)\sigma^2 t\right). \tag{A9}$$

Equation (A8) and (A9) are always negative, because the holding period, the variance of underlying index's return and $(1 + R_I)$ are all positive, and $(m - m^2)$ is negative. The return deviation of an LETF is thus adversely affected by holding period and the volatility of underlying index's return, all else being equal. But it does not mean that, the longer the holding period or the larger the variance of underlying index's returns is, the smaller the return deviation will be, since the cumulative return of underlying index and target multiple will also affect return deviation, and sometimes they will even offset the negative impact of holding period and the variance of underlying index's returns.

To find more useful information about the relationship between return deviation and the cumulative return of underlying index, we let Equation (A7) equal zero and then obtain the only stationary point of Equation (A6) with respect to R_I, denoted by R_{I0}, given as following:

$$R_{I0}=\exp\left(\frac{1}{2}m\sigma^2 t\right)-1. \tag{A10}$$

Therefore, the return deviation, denoted as RD_0, corresponding to R_{I0}, can be given as following:

$$RD_0=\left(m-1\right)\left(1-\exp\left(\frac{1}{2}m\sigma^2 t\right)\right). \tag{A11}$$

Equation (A11) is always negative because the target multiple is greater than 1 or no more than −1. Then we take the second derivative of Equation (A6) with respect to R_I:

$$\frac{\partial^2 RD}{\partial R_I^2}=m\left(m-1\right)\left(1+R_I\right)^{m-2}\exp\left(\frac{1}{2}\left(m-m^2\right)\sigma^2 t\right). \tag{A12}$$

Apparently, Equation (A12) is always positive because the target multiple is greater than 1 or no more than −1. Because the expression of RD has only one stationary point with respect to R_I, RD_0 is the minimum of the return deviation and the graph of RD function with respect to R_I is U-shaped. Because R_{I0} is the index return corresponding to the apex point of *RD* function, we name it the apex return. To investigate whether it is possible to gain a positive value for the return deviation, we attempt to find out the maximum of Equation (A6). According to the previous analysis, if there is a maximum of Equation (A6), the return deviation must reach the maximum value when underlying index's return reaches the maximum value or minimum value. In other words, we need to examine the value of return deviation when underlying index's return verges to −1 and +∞. When the target multiple is positive[20], we have:

$$\lim_{R_I \to -1^+} \left(\left(1+R_I\right)^m \exp\left(\frac{1}{2}\left(m-m^2\right)\sigma^2 t\right) - 1 - mR_I \right) = m - 1 > 0, \tag{A13}$$

$$\lim_{R_I \to +\infty} \left(\left(1+R_I\right)^m \exp\left(\frac{1}{2}\left(m-m^2\right)\sigma^2 t\right) - 1 - mR_I \right) = +\infty > 0. \tag{A14}$$

when the target multiple is negative, we have:

$$\lim_{R_I \to -1^+} \left(\left(1+R_I\right)^m \exp\left(\frac{1}{2}\left(m-m^2\right)\sigma^2 t\right) - 1 - mR_I \right) = +\infty > 0, \tag{A15}$$

$$\lim_{R_I \to +\infty} \left(\left(1+R_I\right)^m \exp\left(\frac{1}{2}\left(m-m^2\right)\sigma^2 t\right) - 1 - mR_I \right) = +\infty > 0. \tag{A16}$$

Therefore, there is always an interval of positive value for return deviation, regardless of whether the target multiple and underlying index's return are positive or not. More specifically, one case is that, when underlying index's return is greater than the apex return given by Equation (A10), the larger the former is, the more possibility of being positive the return deviation will have. The other case is that, when underlying index's return is less than the apex return, the smaller the former is, the more possibility of being positive the return deviation will have. In sum, the return deviation is decreasing first and then increasing with the increasing of index return and it will reach the minimum value when underlying index's return verges to the apex return.

[20]In Equation (A13), $R_I \to -1^+$ means that R_I verges to −1 from the right, and similarly hereinafter.