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Mathematical Methods Applied to Economy and Sustainable Development Goals

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Abstract

Mathematics is a key factor in achieving the Sustainable Development Goals (SDGs), because of its applicability to real situations. To achieve the set goals in SDG, this paper suggests some mathematical methods that will be useful for solving real situations in relation to goals 2 and 12 of SDGs approved by UN when modeled mathematically. The Northwest Corner Method (NWCM), Least Cost Method (LCM), and Vogel Approximation Method (VAM), which are the initial solution methods were examined to ascertain the ideal route of transporting commodities from production facilities to requirement destination while the optimal solution methods involve Stepping Stone Method (SSM), and Modified Distribution Method (MDM), that give the feasible solution which will enhance minimum transportation cost were also thoroughly defined. Subsequent research shall focus on application of the methods in relation to SDGs problems in comparison with other existing methods.

Keywords

Transportation Problems, Modeling, Optimal Solution, Initial Solution, Sustainable Development Goals, Demand and Supply

1. Introduction

In 2020, Brundtland is of the opinion that sustainability can be said to mean the ability to achieve the current needs without necessarily interfering with the future generations [1]. We can subdivide sustainability into three which are: economic, environmental and social. Watling, *et al.*, 2020 opined that development can be achieved and sustainable if the following can be properly addressed: economic viability, social justice and environmental impacts [2]. Achieving the 17 Goals of SDGs signed and approved by the United Nation for 2030 Agenda [3] which aimed at putting an end to poverty, elevating quality education, gender equality and communities, etc. needs mathematical techniques. This paper suggests some mathematical methods with clear algorithms in relation to the two most important goals of SGDs which are:

Goal 2: "Zero Hunger". This aimed at putting an end to hunger and ensuring access to safe, nutritious and sufficient foodstuff for all citizens all around the year, and an end to all forms of malnutrition, ensuring the sustainability of food production system. It must be noted that extreme hunger and malnutrition are serious issues to sustainable development that needs ultimate attention through the knowledge of mathematics.

Goal 12: "Responsible Production and Consumption". this is a serious issue to focus on in other to maximize the little resources to reach a large reasonable number of people in the society as well increasing the net welfare gain from economic activities in such a way that the resources used are reduced in achieving a considerably quality life. These goals can be achieved only if the limited resources are properly and evenly transported through the manufacturer to the final consumer which calls for mathematical methods. A study titled "The distribution of a product from several sources to numerous localities" was considered to be the first important attribute to the solution of Transportation Problem (TP). The structure of TP involves a large number of shipping routes from supply (origin) to several demands (destinations) [4]. This problem can be expressed mathematically in terms of linear programming model, which can then be solved using mathematical techniques suitable for it. This paper was motivated by the goals 2 and 12 of SDGs by UN, forecasting for 2030 while the major contribution is looking into the mathematical applicability of these goals.

2. Mathematical Model of Transportation Problem

Mathematical Model evolved from symbolic models which involve the use of mathematical symbols, letters, numbers and mathematical operators such as +, −, ÷, *, etc. to present relationships among various variables of the system and to describe its properties or behavior. In TP, mathematical model involves structuring of large number of shipping routes from several supply origins to several destinations with the aid of variables, parameters and mathematical operators.

3. Mathematical Model Formulation

Olaosebikan (2019) and (2014) respectively opined that single commodity can be transported from three sources of supply to four demands (destinations) as a typical illustration to better the understanding of Mathematical Model Formulation. The sources may be: Production facilities, Warehouses or supply points characterized by available capacities while the assumed destinations are: Consumption facilities and warehouses points characterized by required levels of demand [5] [6].

Goals 2 & 12 (Zero Hunger and Responsible Production and Consumption):

Suppose a company (Government/Private Individual) has three production facilities: S_1, S_2, S_3 with a production capacity: a_1, a_2, a_3 units of a production respectively. It is desired that these units are to be shipped to warehouses D_1, D_2, D_3 and D_4 with requirement of b_1, b_2, b_3 and b_4 units respectively. The information can be tabulated as follows (Table 1).

3.1. Model Formulation

It is aimed to formulate the table above as a linear programming model to minimize the total transportation cost.

Let X_{ij} be the number of units of products to be transported from the factory $i\left(i=1,2,3\right)$ to warehouse $j\left(j=1,2,3,4\right)$.

Using the general formula:

$$\begin{aligned}\text{Min}\left(\text{total transportation cost}\right) = {} & a_{11}x_{11}+a_{12}x_{12}+a_{13}x_{13}+a_{14}x_{14} \\ & +a_{21}x_{21}+a_{22}x_{22}+a_{23}x_{23}+a_{24}x_{24} \\ & +a_{31}x_{31}+a_{32}x_{32}+a_{33}x_{33}+a_{34}x_{34}\end{aligned}$$

Subject to the constraints

$$x_{11}+x_{12}+x_{13}+x_{14}=a_1$$

$$x_{21}+x_{22}+x_{23}+x_{24}=a_2$$

$$x_{31}+x_{32}+x_{33}+x_{34}=a_3$$

$$x_{11}+x_{21}+x_{31}=b_1$$

$$x_{12}+x_{22}+x_{32}=b_2$$

$$x_{13}+x_{23}+x_{33}=b_3$$

$$x_{14}+x_{24}+x_{34}=b_4$$

The linear programming model contains twelve decision variables, *i.e.* $m\times n=3\times 4=12$ decision variables. Where m and n are the number of rows and column respectively and there are $m+n=7$ constraints.

3.2. Generalized Mathematical Model Formulation

In 2019, Kek, S., *et al.* and Kienle Garrido, *et al.* (2018), worked extensively on modelling both in couple tank and cancer therapy [7] [8]. Let m be the source of supply $S_1,S_2,S_3,\cdots,S_m$ having $a_i\left(i=1,2,\cdots,m\right)$ units of supply to be transported among n destinations $D_1,D_2,D_3,\cdots,D_m$ with $b_j\left(j=1,2,\cdots,n\right)$. Let C_{ij} be the

Table 1. Table showing production facilities and capacities of a company.

	D_1	D_2	D_3	D_4	Capacity
S_1	a_{11}	a_{12}	a_{13}	a_{14}	a_1
S_2	a_{21}	a_{22}	a_{23}	a_{24}	a_2
S_3	a_{31}	a_{32}	a_{33}	a_{34}	a_3
Demand	b_1	b_2	b_3	b_4	

cost of transporting one unit of the commodity from source i to destination j for each route. X_{ij} represents the numbers of units of commodities being transported per route from source i to destination j for each route. The aim is to determine the transportation schedule so as to minimize transportation cost satisfying the demand and supply constraints. This can be translated mathematically as (Table 2):

Minimize (Total Cost):

$$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject to the Constraints:

(Supply Constraints)

$$\sum_{j=1}^{n} X_{ij} = a_i, i = 1, 2, 3, \cdots, m$$

(Demand Constraints)

$$\sum_{j=1}^{m} X_{ij} = b_i, i = 1, 2, 3, \cdots, n$$

(Decision Variables)

$$X_{ij} \geq 0 \ \forall i, j$$

Total Supply = Total Demand, so:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

4. Mathematical Methods

Suggested mathematical methods for the mathematical model formulation are summarized as follows:

Table 2. Table showing the cost effect of demand and supply of a company.

To/From	D_1	D_2	D_3	…	…	…	D_n	Supply
S_1	$c_{11}x_{11}$	$c_{12}x_{12}$	$c_{13}x_{13}$	…	…	…	$c_{1n}x_{1n}$	a_1
S_2	$c_{21}x_{21}$	$c_{22}x_{22}$	$c_{23}x_{23}$	…	…	…	$c_{2n}x_{2n}$	a_2
S_3	$c_{31}x_{31}$	$c_{32}x_{32}$	$c_{33}x_{33}$	…	…	…	$c_{3n}x_{3n}$	a_3
…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…
S_m	$c_{m1}x_{m1}$	$c_{m2}x_{m2}$	$c_{m3}x_{m3}$	…	…	…	$c_{mn}x_{mn}$	a_m
Demand	b_1	b_2	b_3	…	…	…	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

Step 1: Formulate the problem and set it up in the matrix form: the formulation of the transportation problem is similar to the linear programming problem formulation. Here the objective function is the total transportation cost and the constraints are the supply and demand available at each source and destination respectively.

Step 2: Obtain an initial basic feasible solution: there are three different methods to obtain an initial solution which are:

1) Northwest Corner Rule.

2) Least Cost Method.

3) Vogel's Approximation Method.

The initial solution obtained by any of the three methods must satisfy the following condition:

4) The solution must be feasible.

5) The number of positive allocation must be equal to $m+n-1$, where m is the number of rows and n is the number of columns.

Step 3: Test the initial solution for optimality: to test the optimality of the solution obtained in Step 2 using the Modified Distribution Method. If the current solution is optimal, then stop, otherwise, determine a new improved solution.

Step 4: Updating the solution: Repeat Step 3, until an optimal solution is reached.

4.1. Method of Generating Initial Solution

There are three methods of finding initial solution as stated in Step 2 above, which are: Northwest Corner Method, Least Cost Method and Vogel Approximation Method yield the best starting basic solution.

4.1.1. Northwest Corner Method (NWCM)

The method starts at the Northwest Corner cell (route) of the table. It is a simple and efficient method to obtain an initial solution.

1) Characteristics features of NWCM

a) It involves least computation.

b) It does not take cost into consideration.

c) It is simple and efficient.

d) It is a quick and easy method.

e) The solution obtained is always to form optimal.

2) Algorithm of NWCM

Step 1: Allocate as much as possible to the selected cell, and adjust the associated amounts of supply and demand by subtracting the allocation amount.

Step 2: Cross out the row or column with zero supply or demand to indicate that no further assignments can be made in that row or column. If both a row and column net to zero simultaneously cross out row (column).

Step 3: If exactly one row or column is left uncrossed out, stop, otherwise, move to the cell to the right of a column that has just been crossed out or below if a row has been crossed out. Go to Step 1.

4.1.2. Least Cost Method (LCM)

The LCM finds a better starting solution by concentrating on the cheapest routes. This method starts by assigning as many as possible calls with the smallest unit cost (ties are broken arbitrarily).

1) Characteristics Features of LCM

a) It concentrated on the cheapest routes.

b) It is considered to be the better method of finding starting solution.

c) It is a simple and efficient method.

d) It starts by assigning or allocating to possible cells with the smallest unit cost.

2) Algorithm of LCM

Step 1: Select the cell with the lowest unit cost in the entire transportation tableau and allocate as much as possible to this cell and eliminate that row or column in which either supply or demand is exhausted. If a row and column are satisfied simultaneously, only one way is to cross out. In case the smallest unit cost is not unique then select the cell where maximum allocation can be made.

Step 2: After adjusting the supply and demand for all uncrossed out rows and column, repeat the procedure with the next lowest unit cost among the remaining rows and columns of the transportation tableau and allocate as much as possible to the cell and eliminate that row or column in which either supply or demand is exhausted.

Step 3: Repeat the possible until the entire available supply at various sources and demand at various destinations are satisfied.

4.1.3. Vogel Approximation Method (VAM)

VAM is a heuristic method and is preferred to the other two methods described above in this method; each allocation is made on the basis of the opportunity cost that would have been incurred if allocations in certain cells with minimum unit transportation cost were missed. In this method, allocations are made so that the penalty cost is minimized. The advantage of this method is that it gives an initial solution which is nearer to an optimal solution itself.

1) Characteristics Features of VAM

a) The solution obtained by VAM is always nearer to the optimal solution.

b) It is relatively easy to implement by hand.

c) It is sometimes referred to as optimal.

d) It takes cost into account. *i.e.* into consideration in relative way.

e) It yields the best starting basic solution.

2) Algorithm of VAM

Step 1: Calculate the difference between the two lowest distribution cost for each row and column.

Step 2: Select the row or column with the great differences and circle the value. In case of tie, select the row or column allowing the greatest movement of units.

Step 3: Assign the largest possible allocation within the restrictions of a row or column to the lowest cost cell for the row or column selected.

Step 4: Cross out any row or column satisfied by the assignment made in Step 3.

Step 5: Repeat Steps 1 to 4 except for rows and columns that have been crossed out until all the assignment have been made.

5. Methods for Generating Optimal Solution

The three methods studied above cannot necessarily guarantee optimal solution, even though they give feasible solution. Hence, they are referred to as initial solution methods. Optimal solution, therefore, starts where initial solution stops. To check for optimality, the following methods are used.

1) Stepping Stone Method (SSM).

2) Modified Distribution Method (MODI).

5.1. Stepping Stone Method (SSM)

The SSM is one of the methods for generating optimal solution that involves allocation of limit to the unoccupied cells in the initial basic solution so as to minimize transportation cost. It utilizes unoccupied cells with negative sign allocation in the change and in a situation where two cells have allocation (tie) the first allocation goes to the specific one with the highest allocation.

5.1.1. Characteristics Features of SSM

1) It utilizes unoccupied cells with the negative sign.

2) It is one of the methods used in generating an optimal solution.

3) It first allocation goes to the specific one with the highest negative cell or sign.

4) It makes use of the final solution *i.e.* final tableau of these methods: NWCM, LCM and VAM.

5.1.2. Algorithm of SSM

Step 1: Start with the initial basic solution of the three methods discussed and confirm that the final tableau form the initial method satisfies $m+n-1$ cells where m and n are columns and rows respectively.

Step 2: Identify the occupied and unoccupied cells and for unoccupied cell, compute the net change.

Step 3: If the net change is negative, allocate minimum unit to the value otherwise no need for allocation.

Step 4: If two unoccupied cells have the same negative sign, start with one with the highest unit or make arbitrary choice.

Step 5: Update your optimality and checkmate your result if the unoccupied cells are positive.

Step 6: If they are positive, stop, otherwise, go to Step 2.

5.1.3. Modified Distribution Method (MDM)

The modified distribution method of ascertaining optimal solution in transportation problems is an efficient technique, which helps in comparing the relative advantage of alternative allocations for all the unoccupied cells simultaneously. The advantage of this method is that, it provides a flat form of minimizing transportation cost associated with transporting goods from production facilities to requirement facilities. If function with the aid of any of the three basic solutions discussed previously. For a given basic feasible solution we associate numbers U_i and V_i with row $i = 1(1)m$, and column $j = 1(1)n$ of the transportation table respectively, U_i and V_i must satisfy the equations:

$$U_i + V_j = C_{ij} \quad \text{(Occupied Cells)}$$

$$e_{ij} = C_{ij} - U_i - V_j \quad \text{(Unoccupied Cells)}$$

The preceding computations are usually done directly on the transportation tableaus, meaning that, it is not necessary to write the equations explicitly, instead, we start by setting $U_i = 0$. Then, we can compute V_j values of all the columns that have been determined, we can evaluate the non basic X_{ij}.

5.1.4. Algorithm for MDM

Step 1: For an initial basic feasible solution with $m+n-1$ occupied cells, calculate U_i and V_j for rows and columns. The initial solution can be obtained by any of the three methods discussed. To start with, any one of U_i's and V_j's assign zero for a particular U_i or V_j where there are maximum number of allocation in a row or column respectively as it will reduce arithmetic work considerably. The complete the calculation of U_i's and V_j's for other rows and columns by using the relation $U_i + V_j = C_{ij}$ fo all occupied cells (*ij*).

Step 2: For unoccupied cells, calculate opportunity cost by using the relation

$$e_{ij} = C_{ij} - U_i - V_j$$

Step 3: Examine sign of each stage in Step 2. *i.e.*

1) If $e_{ij} > 0$, then current basic solution is optimal.

2) If $e_{ij} = 0$, then current basic solution will remain unaffected but an alternate solution exist.

3) If one or more $e_{ij} < 0$, then an improved solution can be obtained by entering unoccupied cell (C_{ij}) in the basis.

Step 4: Construct a closed path for the unoccupied cell with largest negative opportunity cost. Start the close path with selected unoccupied cell and mark a plus (+) in the cell, trace a path along the row or column to an occupied cell, mark the corner with a minus sign (−) and continue down the column (row) to an occupied cell and mark the corner with plus sign (+) and minus sign (−) alternatively. Then, back to the selected unoccupied cell.

Step 5: Select the smallest quality among the cells marked with minus sign on the corners of closed path. Allocate value to the selected unoccupied cell and add it to occupied cell marked with plus and subtract it from the occupied cell marked with signs.

Step 6: Obtain a new improved solution by allocating units to the unoccupied cell to step and calculate the new transportation cost.

Step 7: Test the revised solution further for optimality. The procedure terminate when all $e_{ij} > 0$ for unoccupied cells.

6. Conclusion

It is aimed to suggest some mathematical methods which will be used to solve some real-life problems in relation to SDGs 2 and 12 in particular. Clearly, this paper has been able to show some methods both for initial solution and optimal solution as proposed. In the subsequence research, focus shall be on simulation experiment of these suggested methods in comparison with existing methods for solving the said problems.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Bureau Brundtland, G.H. (1987) World Commission on Environment and Development: Our Common Future. http://www.un-documents.net/our-common-future.pdf

[2] Watling, A. and Zhou, E. (2011) Attitudes towards Sustainability: A Quantitative Study of Sustainable Ålidhem. Bachelor's Thesis, Umea Universitet, Umea, Sweden. http://umu.diva-portal.org/smash/record.jsf?pid=diva2%3A430152&dswid=-6754

[3] United Nation Development Programme. https://www.undp.org/sustainable-development-goals?utm_source=EN&utm_medium=GSR&utm_content=US_UNDP_PaidSearch_Brand_English&utm_campaign=CENTRAL&c_src=CENTRAL&c_src2=GSR&gclid=Cj0KCQjwqKuKBhCxARIsACf4XuG5ahIybQA-BFGI3UTl9STEIwVA4B2MoSiUj2fXggw85omw7OloI5AaAuRWEALw_wcB

[4] Lafuente-Lechuga, M., Cifuentes-Faura, J. and Faura-Martinez, U. (2020) Mathematics Applied to the Economy and Sustainable Development Goals: A Necessary Relationship of Dependence. *Education Sciences*, **10**, Article No. 339. https://doi.org/10.3390/educsci10110339

[5] Olaosebikan, T.E. (2019) Application of the Euler-Lagrange-Method for Solving Optimal Control Problem. GRIN Verlag, Munich.

[6] Olorunsola, S.A., Olaosebikan, T.E. and Adebayo, K.J. (2014) On Application of Modified Lagrange Multipliers in Both Equality and Inequality Constraints. *International Organization of Scientific Research Journal of Mathematics*, **10**, 6-9.

[7] Kek, S., Sim, S. and Chen, C. (2019) An Expanded Optimal Control Policy for a Coupled Tanks System with Random Disturbance. *Advances in Pure Mathematics*, **9**, 317-329. https://doi.org/10.4236/apm.2019.94014

[8] Kienle Garrido, M., Breitenbach, T., Chudej, K. and Borzì, A. (2018) Modeling and Numerical Solution of a Cancer Therapy Optimal Control Problem. *Applied Mathematics*, **9**, 985-1004. https://doi.org/10.4236/am.2018.98067