

# Updating the Historical Perspective of the Interaction of Gravitational Field and Orbit in Sun-Planet-Moon System

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## Abstract

Studying the two famous old problems that why the moon can move around the Sun and why the orbit of the Moon around the Earth cannot be broken off by the Sun under the condition calculating with  $F = GMm/R^2$ , the attractive force of the Sun on the Moon is almost 2.2 times that of the Earth, we found that the planet and moon are unified as one single gravitational unit which results in that the Sun cannot have the force of  $F = GMm/R^2$  on the moon. The moon is moved by the gravitational unit orbiting around the Sun. It could indicate that the gravitational field of the moon is limited inside the unit and the gravitational fields of both the planet and moon are unified as one single field interacting with the Sun. The findings are further clarified by reestablishing Newton's repulsive gravity.

## Keywords

Interaction of Gravitational Field, Three-Body Problem, Orbital Perturbation Theory, Repulsive Gravity, Neutralization of Gravity

## 1. Introduction

The theory of gravity is developed from astronomy observation. It is the main achievement that the theory of gravity is valid to understand the celestial orbit and to design artificial orbit. In this work, we found, the current theory for the orbit in the Sun-planet-moon system need be re-explained and re-understood with the interaction of gravitational field. And, a theory for the interaction of gravitational field could be developed by further understanding the orbit in the Sun-planet-moon system.

In the time of the geocentric theory, through observation, the orbits of some celestial bodies can be accurately predicted and the accurate ephemeris and almanac are established. Observing the shape and orbit of the celestial bodies, Nicolaus Copernicus [1] established the heliocentric theory in 1507. Based on Kepler's law, [2] Sir Isaac Newton [3] established the theory of gravity in *The Mathematical Principles of Natural Philosophy* in 1687. And, Newton established the theory of orbit. After the Newtonian theory of gravity, the orbit is not only an observed result, but can be understood with celestial mechanics and can be designed artificially.

Factually, an orbit is not only determined with  $G \frac{Mm}{R^2} = \frac{mv^2}{R}$ , but is always perturbed by several celestial bodies. The orbital perturbation theory was initially formulated by Newton [3] [4]. Now, it is generally introduced in textbooks [5].

Almost in 1900, from the Newtonian theory of gravity, the Hill sphere was derived [6] [7]. The sphere of influence, including the Hill sphere and Laplace sphere, is generally used to study the extrasolar system and to design the interplanetary satellite orbiter [8]-[14]. The Hill sphere is valid to understand the orbital stability zone of a moon around a planet [10] [11] [12] [13] [14].

After 1900, Einstein's general relativity [15] and quantum mechanics were established. And, the quantum theory of gravity was presented. Generally, it is thought that Einstein's general relativity is a good theory for gravity while we are very far from having a complete quantum theory of gravity [16].

In this work, we noticed that there is a series of problems about the orbit in the solar system. 1) There is a famous old problem: [17] Why the orbit of the Moon around the Earth is stable under the condition that calculated with Newton's law of  $F = GMm/R^2$ , there is  $F_{sm}/F_{em} \approx 2.2$ , where  $F_{sm}$  and  $F_{em}$  are the force of the Sun and Earth on the Moon, respectively. 2) As the Earth is orbiting around the Sun, the Moon is also moving around the Sun. An old problem is: What is the mechanics that makes the Moon moved around the Sun? [18] 3) In 1860, Delaunay [17] [19] presented that the Earth-Moon system is binary planet. The binary planet was used to explain the Moon moving around the Sun by Turner [18] in 1912. And, now the Pluto-Charon system is thought binary [20] [21] [22] [23]. In 1700s it was reported that binary star was observed [24]. Now, the binary star/planet/blackhole is very prevailing. But, what is the mechanics that could make two planets/stars/blackholes orbit around the barycenter of them or orbit around each other? 4) Generally, in studying the orbit of the Moon, the Sun-Earth-Moon system was studied by Newton [3]. Before Poincaré, the Three-body problem had been studied generally [25]. Euler [26], Lagrange [27], Jacobi [28] and Hill [6] [7] had contribution to the restricted Three-body problem. Poincaré [29] [30] [31] published his study about the Three-body problem in 1892-99. Today, from the Poincaré's equation, it is generally believed that the orbits in the Three-body system are chaotic. But, why the orbits in the

typical three-body, such as the Sun-Earth-Moon system and Sun-Pluto-Charon system, are stable?

In the Pluto system, the mass of the Charon is almost 0.12 times that of the Pluto. This is a special case for the planet-moon system. The Charon is discovered in 1978 [32]. Therefore, some of the theories and concepts that presented before 1978 need be rechecked and re-understood with the Pluto system. Calculating with  $F = GMm/R^2$ , as the Pluto is at the perihelion of the orbit around the Sun, there is  $F_{cp}/F_{sp} \approx 40$ , where  $F_{sp}$  and  $F_{cp}$  are the gravitational force of the Sun and Charon on the Pluto, respectively. Therefore, from the Pluto-Charon system, we presented a new problem: Why the orbit of the Pluto around the Sun was not broken off by the Charon? (Now, the Pluto is excluded from the planet. But, in studying the orbits in the Pluto system, [20] [21] [22] [23] the mechanics and dynamics are just that for other planets. So, the Sun-Pluto-moon system can be treated as a Sun-planet-moon system.)

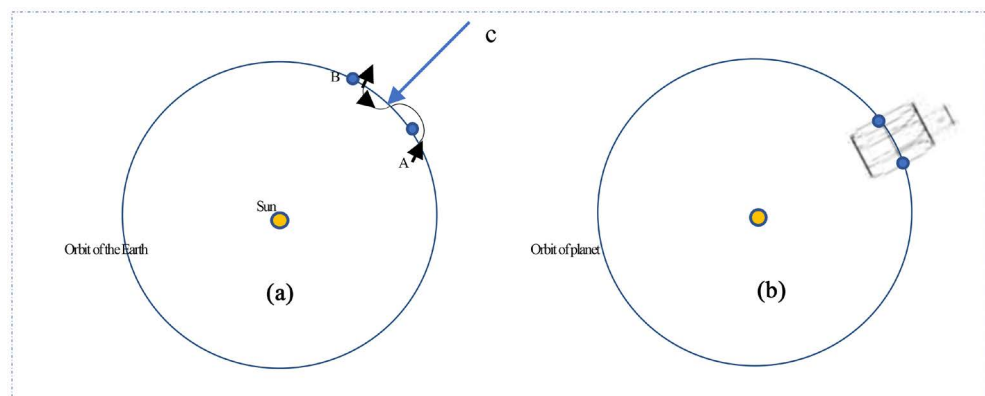
These problems are fundamental. Although having been studied by many scientists for long time, they are still open problems. It seems that a new line is needed to understand them. Here, we present, these problems could be understood with the interaction of gravitational field. And, the orbital perturbation theory is a valid theory for that it is well applied in designing artificial orbit. Therefore, here, we shall investigate other theories of orbit with the orbital perturbation theory. In Sec. 2, the line of the moon moving around the Sun is studied systematically. From the orbital perturbation theory, we found, a planet-moon system is unified as one single solid gravitational unit orbiting around the Sun. And, the gravitational fields of both the planet and moon are unified as one field interacting with that of the Sun while the field of the moon is limited in the unit. This is our main conclusion. In Sec. 3, the Poincaré's equation for Three-body problem is compared with the orbital perturbation equation. It is shown that, the gravitational unit and the interaction of gravitational field are implied in the orbital perturbation theory. And, it is wrong that a gravitational field could interact with any other ones with the force of  $F = GMm/R^2$ . Therefore, this comparison shows a clear outline about the problem in the current understanding about the gravitational field. In Sec. 4, it is shown that Newton's conclusion that the perturbation of the Sun to the Moon is always repulsive was well observed from the artificial orbit with modern technology. In Sec. 5, it was presented that, the Hill sphere and the orbital perturbation theory are complementary to each other. In Sec. 6, we presented that all of the gravitational fields are interacting with others, no gravitational field can be isolated from others. So, the gravitational field only can be understood from the interacting field. And, by analogy to the Maxwell equation, we establish a set of equations for the interaction of gravitational field. In Sec. 7, it is pointed out that our concept of gravitational unit is very analogous to the concept of binary planet which is currently used to explain the line of the moon moving around the Sun. But, observation showed that the Pluto and Charon is not binary system. The current

theory for binary system is questioned. In Sec. 8, we presented that our result could be valid to the orbit in the galaxy. The conclusion is Sec. 9. It is concluded that it is the time to remodel the theory of the gravity. And, the space for the interaction of gravitational field is much larger than that for other fields. It should be a good case to better observe and understand the interaction of all fields.

## 2. Orbital Perturbation Theory and the Lines of Moons Moving around the Sun

As a moon is orbiting around a planet, it is also moving around the Sun. But, till now, the line of the moon moving around the Sun has not been systematically studied. Here, it is shown that, there are two typical lines of the moon moving around the Sun as shown in **Figure 1**. In **Figure 1(a)**, the black line is that the Moon is moved around the Sun. As the Earth orbits around the Sun from point A to B, the Moon just orbited a period around the Earth. The direction of the orbit of the Moon moving around the Earth in AC is different from that in CB. In **Figure 1(b)**, the orbit of the moon around a planet is vertical to that of the planet around the Sun. The line of the moon moving around the Sun is helix. And, between the two kinds of typical lines there are many other kinds of the lines for the moon moving around the Sun. A crucial problem was presented [18]: What is the mechanics that makes the moon moved around the Sun?

**Figure 1** shows that relative to the direction of the moon moving around the Sun, the direction of the orbit of the moon around the planet can be varied in all directions. For example, in **Figure 1(a)**, as the Moon moves from point C to B, the direction of the Moon orbiting around the Earth is contrary to that of the Moon moving around the Sun. And, in **Figure 1(b)** the two directions are vertical to each other. And, the velocity of the Moon around the Sun is almost 30 km/s. It is almost equal to that of the Earth around the Sun while that of the Moon around the Earth is almost 1.023 km/s. No any force that is out of the planet-moon system could make the moon moved in such a case. The only reason is



**Figure 1.** The line of a moon moved around a star. Assuming that The orbit of the moon around the planet and that of the planet around the star are in a same plane, there are two typical kinds of lines as shown in (a) and (b). Between the two kinds of lines, there are many other different kinds of lines.

that a planet-moon system is one single gravitational unit that is moved by that the planet orbits around the Sun. A moon cannot independently move around the Sun. It is a part of a planet-moon gravitational unit that orbits around the Sun.

There are two features for a planet-moon unit orbiting the Sun. First, the orbit of a planet-moon unit around the Sun is only determined with the velocity  $v_p$  and mass  $m_p$  of the planet and the gravitational force of the Sun  $G \frac{M_s m_p}{R^2}$ , *i.e.*,  $G \frac{M_s m_p}{R^2} = m_p \frac{v_p^2}{R}$ , where  $s$  and  $p$  denote the Sun and planet respectively.

(For convenience, in this work, we assume all orbits are circular and on a same plane.) The mass and velocity of the moon cannot affect the orbit. The evidence is that the orbit of the Pluto around the Sun is not affected by the Charon although the calculated attractive force of the Charon on the Pluto is larger than 40 times that of the Sun. Second, the force out of a planet-moon unit cannot affect the orbit of the moon around the planet and the line of the moon around the Sun. The evidence is that the Moon is not moved to the Sun although the calculated gravitational force of the Sun on the Moon is almost 2.2 times that of the Earth on the Moon.

The planet-moon unit orbiting around the Sun could be well understood with the orbital perturbation theory and the Hill sphere.

The Hill sphere usually deals with the stability of the orbit of the moon around a planet. For the Sun-Earth-Moon system, it is approximately written as [8]

$$r_H \approx (1-e)a\sqrt[3]{m/3M} \quad (1)$$

where  $r_H$  is the Hill radius,  $M$  and  $m$  are the mass of the Sun and Earth,  $a$  and  $e$  are the semi-major axis and eccentricity of the orbit of the Earth, respectively.

The Hill sphere means that, inside the radius of  $r_H$ , the Earth dominates the gravity. And, the condition for that a moon orbits around a planet in a stable way is that only if the moon lies always within the Hill sphere.

The Pluto-Charon system shows another problem for the theory of gravity. Because of  $F_{cp}/F_{sp} \geq 40$ , if the formula  $F = GMm/R^2$  was valid for the gravitational force of the Charon acting on the Pluto, the orbit of the Pluto around the Sun should have been broken off long time ago. The orbit of the Pluto around the Sun is stable. It certainly shows that the Charon cannot act on the Pluto with the force  $F = GMm/R^2$ . It means that, out of the Hill sphere, the moon cannot have the force  $F = GMm/R^2$  on other bodies.

From the orbital perturbation theory, [5] the force of the Sun and Moon on an artificial satellite orbiting around the Earth was exactly known.

In the N-body system, the orbit of an artificial satellite around the Earth is determined with:

$$g_{total} = G \frac{M_E}{r^3} \mathbf{r} + \sum_{i=1}^n G m_i \left( \frac{\mathbf{r}_i}{r_i^3} - \frac{\mathbf{r}_j}{r_j^3} \right) \quad (2)$$

where  $M_E$  is the mass of the Earth,  $r$  is the distance between the Earth and the satellite.  $i$  is the  $i$ th body,  $m_i$  is the mass of  $i$ th body. For the Sun-Earth-Moon system,  $i = 2$ , *i.e.*, the Sun and Moon.  $r_i$  is the distance between the satellite and the  $i$ th body,  $r_j$  is the distance between the Earth and  $i$ th body,  $\mathbf{r}, \mathbf{r}_i$  and  $\mathbf{r}_j$  are vectors.

Equation (2) shows that the Earth is a central mass which determines the orbit of the Moon around the Earth, while the Sun only can have a perturbative force on the orbit.

For an artificial satellite, the distance between the satellite and the Sun is almost equal to that between the Sun and Earth, *i.e.*,  $r_i \approx r_j$ , the gravitational acceleration of the Sun on the satellite approximately is

$$g_{\text{perturb}} = 2G \frac{M_s}{r^3} \mathbf{R} \quad (3)$$

where  $M_s$  is the mass of the Sun,  $r$  is the distance between the Sun and Earth,  $R$  is the distance between the satellite and Earth. (The perturbation theory is discussed in detailed in Sec. 4.)

It is well known that, the perturbation force of the Sun on the satellite is very little. For a low orbit satellite, it is less than the force of the light pressure of the Sun on the same satellite. Usually,  $g_{\text{perturb}}$  of the Sun is on the level of  $10^{-7} \text{ ms}^{-2}$ .

Let's consider the Earth-Moon-spacecraft system. In **Figure 2**, as a spacecraft is out of the Hill sphere of the Moon, from the perturbation theory of the orbit we know, the total gravitational acceleration by the force of both the Earth and Moon on it is

$$g_{\text{total}} = G \frac{M_e}{r_{es}^2} + 2G \frac{M_m}{R^3} \mathbf{r}_{es} \quad (4)$$

where  $M_e$  and  $M_m$  are the mass of the Earth and Moon respectively,  $R$  is the distance between the Earth and Moon and  $r_{es}$  is the distance between the Earth and spacecraft.

It is emphasized, Equation (4) is well-confirmed. The theory of orbital perturbation is a well-understood and well-developed theory. It was used for the artificial orbit. The gravitational acceleration of the Earth and Moon on an artificial satellite around the Earth is just the Equation (4).

It is interesting, as this spacecraft is inside the Hill sphere of the Moon, the total acceleration by the Earth and the Moon on it become as:

$$g_{\text{total}} = G \frac{M_m}{r_{ms}^2} + 2G \frac{M_e}{R^3} \mathbf{r}_{ms} \quad (5)$$

where  $r_{ms}$  is the distance between the Moon and the spacecraft.

We also emphasize, Equation (5) is also well-confirmed. The total acceleration of the Sun and Earth on a satellite orbiting around the Earth just is

$$g_{\text{total}} = G \frac{M_e}{r_{es}^2} + 2G \frac{M_s}{R^3} \mathbf{r}_{es}, \text{ where s and e denote the Sun and Earth, respectively.}$$

The reason that the force of the Sun on a satellite orbiting around the Earth is

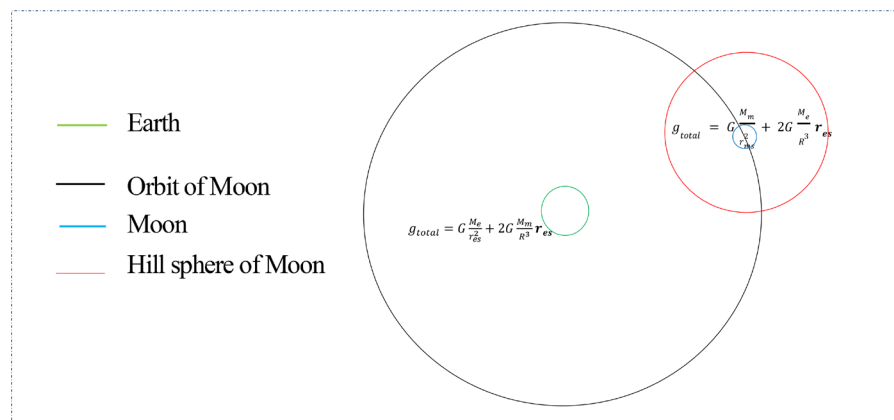
only the perturbation force is that this satellite is inside the Hill sphere of the Earth just as in Equation (5) the spacecraft is inside the Hill sphere of the Moon.

Comparing Equation (4) to Equation (5), it is shown that, for the same spacecraft, the central mass with the central force is corresponded with that the spacecraft is in which one Hill sphere.

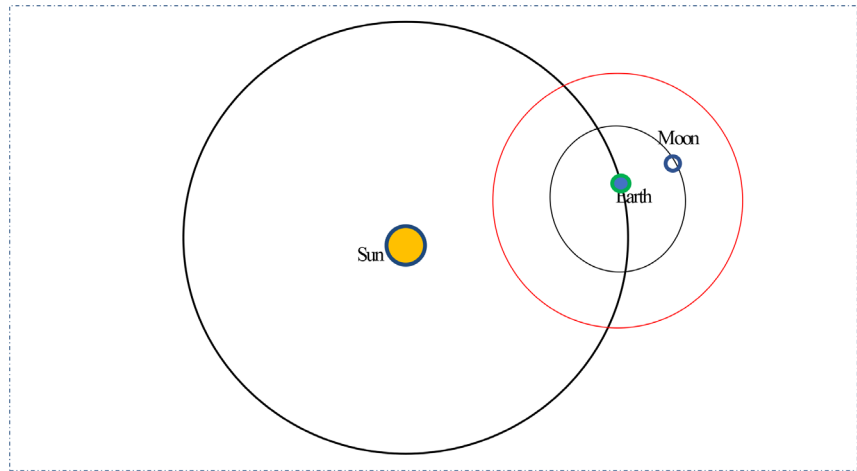
From the Hill sphere and Equations (4) and (5), understanding with the interaction of gravitational field, it could be concluded that, the gravitational field of the spacecraft could be limited inside the Hill sphere of the Moon as shown in **Figure 2**. As the field of the spacecraft is limited, the Earth cannot have the force of  $F = GMm/R^2$  on the spacecraft. Therefore, generally, as a field of a mass is limited or trapped, it means that it cannot extend freely. And, it cannot interact with any ones with  $F = GMm/R^2$  in infinite space.

Therefore, there are two observations for the limited field in the Earth-Moon system: 1) The field of the Moon is trapped or limited in the Hill radius of the Moon which is determined with Equation (1). It cannot extend infinitely. In another word, a gravitational field can be trapped into a limited zone by a large one. 2) Because it is trapped or limited inside the Hill radius zone, the force of the moon acting on the mass out of the Hill sphere is not  $F = GMm/R^2$ . It is also the perturbation force, just as the force of the Moon on an artificial satellite around the Earth is only the perturbation force for that this satellite is out of the Hill sphere of the Moon.

Therefore, there are the conclusions for the gravitational unit. 1) A primary gravitational unit is usually made up of a planet and a moon. It is inside the Hill sphere of the planet as shown in **Figure 3**. As the distance between the planet and the moon is less than  $\frac{1}{2}r_H$  ( $r_H$  is the Hill radius of the planet), the orbit of



**Figure 2.** The Hill sphere and the limited gravitational field. As a spacecraft is out of the Hill sphere of the Moon, the total gravitational acceleration by the force of both the Earth and Moon on it is  $g_{total} = G \frac{M_e}{r_{es}^2} + 2G \frac{M_m}{R^3} r_{es}$ ; As inside the Hill sphere of the Moon, it becomes  $g_{total} = G \frac{M_m}{r_{ms}^2} + 2G \frac{M_e}{R^3} r_{es}$ . It means that the field of the Moon is limited into the Hill sphere of the Moon.



**Figure 3.** The Hill sphere and gravitational union. The red circle is the Hill sphere. The Earth-Moon system orbits around the Sun on the orbit of the Earth as one single gravitational unit. The force of the Sun on the Earth or that of the Earth on the Moon is  $F = GMm/R^2$  while the Sun on the Moon is  $g_{\text{perturb}} = 2G \frac{M_s}{R^3} \mathbf{r}$ .

the moon is stable [10] [11]. 2) A gravitational unit has an orbit around the Sun and the orbit is only determined with the velocity and mass of the planet. 3) The fields of the planet and moon are unified as one single field to interact with the Sun. In this case, the direct action of the Sun on the planet is  $g_s = G \frac{M_s}{R^2}$  while on the moon only is  $g_{\text{perturb}} = 2G \frac{M_s}{R^3} \mathbf{r}$ . 4) The gravitational field of the moon is limited or trapped inside the Hill sphere. It cannot have the action of  $g = Gm/R^2$  on the planet. 5) The primary gravitational units can be combined to a larger unit, for example, the Sun-planets-moons unit contains several planet-moon units.

### 3. Two Different Understandings about Gravitational Field

Continuing a series of previous studies for the Three-body problem, [25] [26] [27] [28] Poincaré [29] [30] [31] published his equation for the Three-body problem in 1892-1899.

Denote the three masses by  $M_i$ , where  $i = 1, 2$  and  $3$ , the positions of them with respect to the origin of a Cartesian coordinate system by the vectors  $\mathbf{R}_i$ , and define the position of one body with respect to another by  $\mathbf{r}_{ij} = \mathbf{R}_j - \mathbf{R}_i$ , where  $\mathbf{r}_{ij} = -\mathbf{r}_{ji}$ ,  $j = 1, 2, 3$  and  $i \neq j$ , Poincaré's equation is [25]

$$M_i \frac{d^2 \mathbf{R}_i}{dt^2} = G \sum_{j=1}^3 \frac{M_i M_j}{r_{ij}^3} \mathbf{r}_{ij} \quad (6)$$

Comparing Equation (6) with Equation (2), it is easy to find that, in Equation (2) there is a central force from a central mass. Only the center mass  $M_E$  have the force of  $F = G \frac{M_E m}{R^2}$  on the satellite  $m$  that is orbiting around the center mass.



It is different from the perturbative force of  $F_{\text{perturb}} = \sum_{i=1}^n Gm_i m \left( \frac{\mathbf{r}_i}{r_i^3} - \frac{\mathbf{r}_j}{r_j^3} \right)$  by

other bodies  $m_i$ . While in (6), the force of anybody interacting with another one is always  $F = GMm/R^2$ . So, in Equation (6), no orbit in a Three-body system could be stable. (Even the orbit of artificial satellite around the Earth is unstable. The orbit of a real artificial satellite is only acted by the Sun and Moon with the perturbative force. So, Poincaré's restricted Three-body problem is questioned.) So, Equation (6) is invalid to understand why a real orbit, including the orbit of the Moon or an artificial satellite around the Earth, is stable.

We noticed, for Equation (2), the central mass with the central force for the Moon is an observed result. For example, for the Sun-Earth-Moon system, it only can be an observation that the central mass for the Moon orbiting around is the Earth. Conversely, if calculated with  $F = GMm/R^2$ , the central mass could be the Sun because the calculated force of the Sun on the Moon is 2.2 times that of the Earth. It means that, in the perturbation theory, the interaction of gravitational field was taken as a condition implied in Equation (2). It is defined that the Sun cannot have the force  $F = GMm/R^2$  on the Moon. But, in the Poincaré's equation, no observation about the real orbit was considered. It is a pure mathematics derivation based on  $F = GMm/R^2$ . The Sun-Earth-Moon system and Sun-Pluto-Charon system are typical Three-body problem. Their orbits are stable. And, as applied the Poincaré's equation on the orbits of the Sun-Earth-Moon and Sun-Pluto-Charon system, calculated with  $F = GMm/R^2$ , the force of the Sun on the Moon is 2.2 times that of the Earth on the Moon and the force of the Charon on the Pluto is larger than 40 times that of the Sun on the Pluto, the calculated orbits in the two systems are certainly unstable. It is emphasized that, this is the crucial evidence to show that the Poincaré's equation for Three-body problem is wrong.

The Poincaré's equation for Three-body problem is a strange presence. First, no orbit of the celestial body is chaotic. A broken orbit also is predictable. So, Poincaré's equation cannot be related with any real orbit. Second, the orbits of the typical Three-body system are stable. Poincaré's equation is invalid to understand these orbits. Third, Poincaré's equation is invalid to design an artificial orbit. It is very clear, the Poincaré's equation is nonsense in physics. But, there have been a big lot of works for the Poincaré's equation and now every year a lot of these kinds of works are being published. This strange presence needs to be understood and explained with the theory of scientific communication.

Comparing the Poincaré's equation with the orbital perturbation equation, it is clearly shown that, there are two different understandings about the gravitational field. It is noted that, the Poincaré's equation is based on the assumption that a gravitational field could interact with any other ones with the force  $F = GMm/R^2$ . This assumption is prevailing in current theory of gravity. With this understanding, in the Poincaré's equation the observation of the real orbit is omitted. It only is based on mathematics derivation. Now, it is presented that,

physics may be lost in mathematics which results in that the development of theoretical physics is stopped [33]. It seems that, the Poincaré's equation may be a case that physics is lost in mathematics.

In the orbital perturbation theory, Newton used mathematics to describe the observation of real orbit. But, in the age of Newton, the concept of field was not developed. Newton cannot know the interaction of the gravitational field clear. After Newton, it is misunderstood that a gravitational field can interact with any other ones with the force  $F = GMm/R^2$  has not been corrected. This misunderstanding is prevailing to nowadays.

#### 4. Newton's Repulsive Gravity

For convenience, in **Figure 4**, assuming that the Sun, Earth and Moon and the orbit of the Moon around the Earth are on a same plane.  $E$  is the Earth.  $M$  is the Moon which is orbiting around the Earth with a circle orbit. The Sun,  $M_1$ ,  $E$  and  $M_2$  are on a straight line.  $r$  and  $R$  are the distance between the Sun and Earth and between the Moon and Earth respectively. Under the condition of **Figure 4**, as the Moon is at point  $M_1$ , the orbit of the Moon is perturbed by the Sun with:

$$g = GM_s \left[ \frac{1}{(r-R)^2} - \frac{1}{r^2} \right] \quad (7)$$

where  $s$  and  $e$  denote the Sun and Earth,  $r$  and  $R$  are the distances between the Sun and Earth and between the Moon and Earth, respectively.

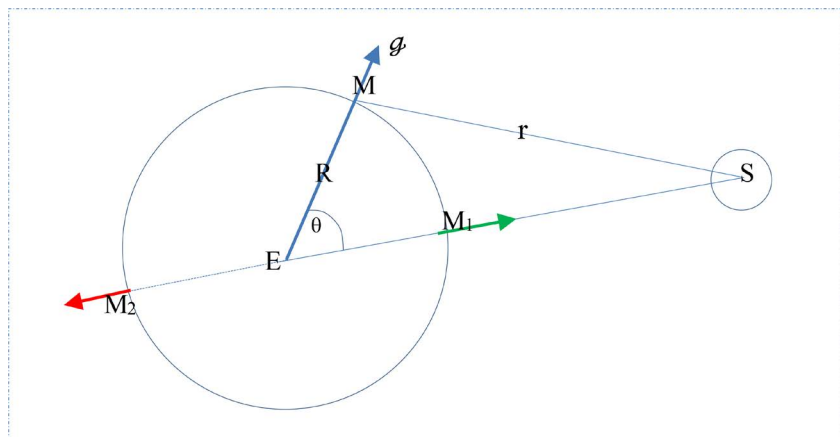
From Equation (7), there is

$$g = 2 \frac{GM_s}{r^3} R \quad (8)$$

As the Moon is moving on the orbit, from Equation (8) we have:

$$g = -\frac{GM_s}{r^3} R (3 \cos^2 \theta - 1) \quad (9)$$

It is stressed that Equation (9) is well applied in designing the orbit of artificial



**Figure 4.** The direction of the perturbative force.  $g$  is directed along line connected with the Earth and Moon.

satellite. It is well known that, the radius of an artificial orbit is varied by the transverse component  $g_{\perp} = \frac{3GM_s}{2r^3} R \sin 2\theta$  of Equation (9) [34]. The actual and predicted data of the radius of the GPS satellite at the precision of less than 3 cm usually can be provided by the GPS office [35] [36]. And, the force on an artificial satellite can be measured with the precision of  $10^{-8} \text{ m/s}^2$  [37]. So, Equation (9) was well measured in practice.

Here, it is emphasized that, in **Figure 4**, it is the transverse component  $g_{\perp} = \frac{3GM_s}{2r^3} R \sin 2\theta$  of Equation (9) that makes the radius of the orbit of the Moon varied [5]. It is clearly shown that, there is no force along the Sun-Moon line.

**The neutralization of gravity.** The orbital perturbation theory clearly and certainly shows that, the Sun cannot act on the Moon with the force of  $F = G \frac{M_s M_m}{r^2}$  along the line connected the Sun and Moon, where  $s$  and  $m$  denote the Sun and Moon, respectively. And, besides that as the Moon is at the point  $M_1$ , there is no attractive force along the Sun-Moon line. It means that, the Sun cannot act on the Moon with a direct force. Therefore, the field of the Moon is very analogous to such an electric field that is neutralized. As an electric field is neutralized, it also cannot be directly acted by another charge. So, we could conclude that, the gravitational field of the Moon could be neutralized by the Earth.

**The repulsive gravity.** It is noted that, in Equation (9), the direction of the force of the Sun on the Moon along the line  $EM_2$  (red arrow) is contrary to that along  $EM_1$  (green arrow) which is the direction that the Sun attracts the Moon. This is a clear observation.

Factually, Newton formulated that, the perturbation of the Sun on the Moon is always repulsive [3] [4]. Today, we can easily prove with measurement that Newton is right. At point  $M_2$ , the direction of the perturbative force is contrary to the attractive force of the Sun. And, besides along the green arrow, no other perturbative force is directed along the attractive force of the Sun. From Equation (9) we know, the force of the Sun perturbing to the Moon is along the blue line connected with the center of the Earth and the Moon. It is clear that the Moon is repulsed away from the center of the Earth along this direction by the perturbation of the Sun. In the artificial orbit, this force has been measured with the precision of  $10^{-8} \text{ m/s}^2$  [37]. It is certainly measured that the direction of the perturbative force of the Sun on an artificial satellite is just as that shown in **Figure 4**. But, Newtonian repulsive gravity has not been understood in current theory of gravity. Now, it is thought that the repulsive gravity is impossible. So, we think, now it is the time to reestablish Newtonian theory of repulsive gravity. Generally, the perturbative acceleration could be rewritten as

$$g = -2g \frac{R}{r} \quad (10)$$

where  $g = \frac{GM_s}{r^2}$  is the gravitational acceleration of the Sun,  $r$  and  $R$  are the

distance between the Earth and the Sun and between the Earth and Moon, respectively.  $\mathbf{R}$  is a vector. The sign “-” means that  $\mathbf{g}$  is a repulsive acceleration.

It is noted that, now, it is an observed result that the perturbative force is repulsive gravity. While the reason that results in the repulsive gravity has not been known in current theory. A new theory is needed to know the reason. But, it is clear, in the Newtonian orbital perturbation equation, the repulsive gravity is defined. It corresponds to the center gravity. Under the condition of **Figure 4**, as the Moon is at point  $M_2$ , the total acceleration of both the Sun and Earth on

the Moon is  $g_{total} = G \frac{M_e}{R^2} + GM_s \left[ \frac{1}{(r-R)^2} - \frac{1}{r^2} \right]$ . It is noted that, a general mi-

sunderstanding is that, as the Moon is at point  $M_2$ , the total acceleration of both

the Sun and Earth on the Moon is  $g'_{total} = G \frac{M_e}{R^2} + G \frac{M_s}{r^2}$ . And, as the Moon is at

point  $M_1$ , the total acceleration of both the Sun and Earth on the Moon is

$g''_{total} = G \frac{M_e}{R^2} - G \frac{M_s}{r^2}$ . But, in the orbital perturbation theory,  $g'_{total}$  and  $g''_{total}$

are radically excluded. It is clear, if  $g'_{total}$  and  $g''_{total}$  were true, the orbit of the Moon around the Earth should have been broken off long time ago. It is important, in any case, the acceleration of the Earth on the Moon is always

$g_e = G \frac{M_e}{R^2}$ . It determines that the orbit of the Moon around the Earth is stable.

So, in previous, we call the Earth the center mass and  $g_e$  the center acceleration. We repeat to emphasize, the center mass with the center force is an observed result. It cannot be determined with the calculation based on  $F = GMm/R^2$  for that, in this calculation, the force of the Sun on the Moon is 2.2 times that of the Earth. For the same reason, the perturbative force with perturbative mass also is an observed result. The perturbative force is a such kind of force that is produced from the interaction of gravitational field that is different from the interaction for the center force. Both of the two kinds of forces are defined by Newton with his orbital perturbation theory.

## 5. Hill Sphere and Gravitational Interaction

In history, the Hill sphere was derived from Newtonian theory of gravity through the formula analogous to that Poincaré used to derive the equation for the Three-body problem. Because Poincaré's equation is invalid, this derivation also is questioned. In this derivation, the condition for the Hill sphere is that, for mass  $M$  and  $m$ , it is needed that  $m/M$  is very little. But, in the Pluto-Charon system, there is  $m/M \approx 0.12$ . It is certain, the Hill sphere is valid to the system. So, the Hill sphere need be re-understood with this system. It is clear, the Hill sphere is a valid theory in understanding the celestial orbit and in designing the artificial interplanetary orbit. So, we prefer to believe that, the Hill sphere is an observed result.

Here, we present that, the Hill sphere could be better understood with the or-

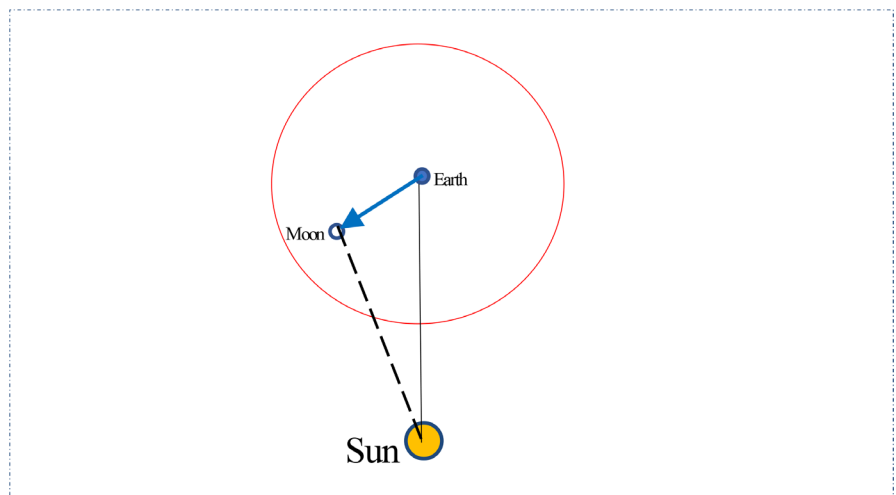
bital perturbation theory. We found, the Hill sphere and the orbital perturbation theory are complementary for each other. Both of them are needed to understand the orbits of the Three-body problem. The Hill sphere studies the stability zone of the orbit while the orbital perturbation theory studies the force of everybody on the orbit. In mechanics, the stability zone is determined with the force.

It is noted that, in current theory of gravity, the Hill sphere and the orbital perturbation theory are independent of each other. It is in this work, it is first presented that, the force inside and out of the Hill sphere is determined with orbital perturbation theory. Currently, it is known, the Hill sphere means that, inside the Hill radius of  $r_H$  of the Moon, the Moon dominates the gravity. But, it is not clear what is the force inside and out of the radius. The orbital perturbation theory determined that the total gravitational acceleration of the Earth and Moon on a satellite is Equations (4) or (5). But, it is not known what is the distance (radius) that Equations (4) or (5) can work. So, we think, the Hill sphere and the orbital perturbation theory are two sides of a coin. The orbital perturbation theory determined that the total acceleration is Equations (4) or (5). While the Hill radius determined the distance that Equations (4) or (5) can work. *i.e.*, inside the Hill radius Equation (5) works while out of the Hill radius Equation (4) works. For the reason, the Equations (1) and (10) could be combined as

$$\mathbf{g} = -2g_s \frac{\mathbf{R}}{r}, \quad R \leq r_H \quad (11)$$

where  $r_H$  is the Hill radius which is determined in Equation (1).

In Equation (11), because the perturbative force is measured with high precision, the force in a Hill sphere is clear. As shown in **Figure 5**, in the Hill sphere



**Figure 5.** Hill sphere and the force of the Sun on the Moon. In the Hill sphere, the Earth dominates the gravitational force on the Moon. The force of the Sun on the Moon is  $\mathbf{g} = -2g_s \frac{\mathbf{R}}{r}$  as shown in the blue arrow, where  $g_s = G \frac{M_s}{r^2}$ .  $\mathbf{g}$  means that the force of the Sun repulses the Moon away from the center of the Earth. There is no force along the line connected the Sun and Moon.

of the Earth, the Earth dominates the gravitational force. The force of the Sun in the Hill sphere is only the perturbative force with the gravitational acceleration of  $g = -2g \frac{R}{r}$ . It means that a repulsive force which repulses the Moon from away the center of the Earth along the Earth-Moon line. It is certain, there is not the force of  $F = G \frac{M_s m_m}{r^2}$  along the Sun-Moon line. As pointed out previously, calculated from this formula, the force of the Sun on the Moon is 2.2 times that of the Earth on the Moon. If there was such a force, it should make the Moon moved to the Sun. And, as shown in **Figure 4**, there is no attractive force along the Sun-Moon line.

In current theory of gravity, it is generally known that, in the Hill sphere of the Earth, the Earth dominates the gravity on the Moon. While it is unclear what is the force of the Sun on the Moon. However, this force need be known. Here, we may show that the force in the Hill sphere was observed with very high precision.

Now, the Hill sphere of some bodies in the solar system was probed [38] [39]. In technology, the Hill sphere for the Sun-planet-moon system can be accurately measured. This measurement should accurately show how the gravitational field of the moon is limited and how the gravitational fields of both the planet and moon are unified.

Artificial Hill sphere it is generally thought that a small mass, such as a spacecraft, above the surface of the Earth is with a Hill sphere. Therefore, according to Equation (1), an artificial Hill sphere can be obtained with the metal sphere with the density larger than  $16.65 \text{ g/cm}^3$ .

First, an artificial Hill sphere can be used as a probe to explore the gravitational field of the Earth. If a metal sphere on the surface of the Earth can have a Hill sphere, it means that the field of the Earth is homogenous. If the Hill sphere is produced at a height above the surface of the Earth, it means that this field has a structure with a layer. Second, it can be used to test the Hill sphere with high accurate and precision. Now, the formula for the Hill sphere is approximate. With the artificial Hill sphere, the precision radius about it can be measured. Third, it can be used to detect the interaction of gravitational field. We know, an orbit of an artificial satellite is determined with  $\frac{1}{2}mv^2 = G \frac{Mm}{R}$ . It means that there is an energy exchange between  $m$  and  $M$ . So, as an artificial Hill sphere in the field of the Earth, there also is an energy exchange between them. It could be detected through that the artificial Hill sphere may not be a right sphere.

## 6. Interaction of Gravitational Field

Now, we have known little about the interaction of gravitational field. Even we have not had a line to observe the interaction of the field.

Till now, only the one single gravitational field, which is not interacting with

other one, has been studied. It usually is believed that all the gravitational field can extend infinitely. In Newtonian theory of gravity, a gravitational field of a mass  $M$  is described with the equal potential surface:  $\psi = GM/R$ . In Einstein's general relativity, a gravitational field is described with Schwarzschild spherical symmetric solution of Einstein's field equation. But, it is the fact that none of the gravitational fields is isolating from other ones. All of them are always interacting with others. It is different from the electric field. There is the isolated electric field which is not interacting with other ones. So, the gravitational field only can be understood with the interacting fields. Thus, a theory for the interacting gravitational field is needed.

In the Newtonian theory of gravity, the gravitational force is  $F = GMm/R^2$ . It clearly showed that, the gravitational force is an interaction between two fields of  $M$  and  $m$ . Here, we presented that, if the gravitational force is propagated with the gravitational field, the variation of the field shall result in the variation of the force. On another hand, as the force is varied, the field also correspondently is varied. The orbit is determined with the gravitational force. Thus, the variation and interaction of the gravitational field can be observed through the orbit in the solar system.

In previous, from the orbit, we showed three observations for the interaction of field of N-body. First is the orbital perturbation equation. Second is the perturbative force. Third is the Hill sphere. From the three observations, the interaction of gravitational field of N-body could be described with a set of equations.

$$\begin{cases} g = g_{center} + \sum_{i=1}^n \mathbf{g} \\ g_{center} = G \frac{M_{center}}{R^2} \\ \mathbf{g} = -2g_i \frac{\mathbf{R}}{r_i}, R \leq r_H \\ r_H \approx (1-e)a \sqrt[3]{\frac{M_{center}}{3M}} \end{cases} \quad (12)$$

In Equation (12),  $g_{center}$  is the gravitational acceleration from the force of the center mass  $M_{center}$  on the mass  $m$  which is with a circle orbit around the  $M_{center}$  with the radius  $R$ .  $M_{center}$  and  $m$  is unified to a gravitational unit.  $r_i$  is the distance between  $M_{center}$  and  $m_i$ .  $g_i = G \frac{m_i}{r_i^2}$ .  $\mathbf{g}$  is the perturbation of

$m_i$  to the  $m$  which repulses the  $m$  away from the center of the  $M_{center}$  along the direction of  $\mathbf{R}$ .  $M$  is the mass that  $M_{center}$  orbits around. Just as that the force the Sun on the Moon is the perturbative force, the force of  $M$  on  $m$  also is the perturbative force.

In Equation (12), the four equations are related with each other as a whole. The first equation is general to describe the interaction of field of the  $M_{center} - m - m_i$  system. The second equation shows that the center mass  $M_{center}$  and the mass  $m$  unified as a gravitational unit. The third equation shows that the force of  $m_i$  on  $m$  is repulsive. The fourth equation determines the radius of the gravitational

unit and repulsive gravity.

Equation (12) is analogous to Maxwell equations for the electromagnetic interaction. The Maxwell equations are established from the well-developed electromagnetic theories, including the Coulomb law, the current law, the Biot-Savart law and the Faraday's law of induction. Our equations also are established from the well-developed theories of gravity. But, in physics, our equations are different from Maxwell equations. In the Maxwell equations, there are two different kinds of fields, *i.e.*, the electric and magnetic fields. In our equations, there is only one kind of field.

Our equations are different from Einstein's field equation. Till now, Einstein's field equation only has had the spherically symmetric solution for one field [40]. It cannot be used to understand the interaction of the fields of N-body. It was known by many people that the orbit perturbed by N-body cannot be studied with Einstein's field equation. So, the Newtonian theory of gravity is the unique theory to understand the celestial orbit and to design artificial orbit [41].

It is worth of emphasizing that Equation (12) is well confirmed. In another word, it is needed to design an artificial orbit. In current theory of gravity, these equations are separated independently. In our work, they are related with the interaction of gravitational field. It is very interesting, as we list the equations in current theory of gravity in one page, we can easily find some of new results. For example, as the orbital perturbation equation and the Poincaré's equation for Three-body problem are listed in one page, we can easily find the difference between them which clearly shows that the Poincaré's equation is invalid to the real orbit. And, as the orbital perturbation equation and the Hill sphere are listed in one page, we can easily find that the force of the Earth on the Moon is different from that of the Sun on the Moon. It means that it is the time to establish a set of equations for the theory of gravity to replace the current theory.

The limit of  $G \frac{M_1 M_2}{r^2} = M_1 \frac{v_1^2}{r}$ . For two mass  $M_1$  and  $M_2$  with the distance  $r$ , the orbital velocity for them could be determined respectively with  $v_1^2 = G \frac{M_2}{r}$  or  $v_2^2 = G \frac{M_1}{r}$ . But, Equation (12) shows that, the force of the Sun on the Moon is not  $F = G \frac{M_s M_m}{r^2}$ . Instead, it only is  $g = G \frac{M_s M_m}{r^3} R$ , where  $s$  and  $m$  denote the Sun and Moon,  $R$  is the distance between the Earth and Moon. It is clear,  $G \frac{M_1 M_2}{r^2} = M_1 \frac{v_1^2}{r}$  is not suitable to the Sun and Moon. As presented in the previous, if it was suitable for them, the Moon should have been moved to the Sun. This case was known with the Newtonian orbital perturbation equation. And,  $G \frac{M_1 M_2}{r^2} = M_1 \frac{v_1^2}{r}$  also is not suitable to the force of the Moon/Charon on the Earth/Pluto. As presented in the previous, calculated with  $F = G \frac{M_1 M_2}{r^2}$ , the force of the Charon on the Pluto is larger than 40 times that of the Sun on



the Pluto. If it was so, the orbit of the Pluto around the Sun should have been broken off. The limit of  $G \frac{M_1 M_2}{r^2} = M_1 \frac{v_1^2}{r}$  is first presented in this work.

The current observation shows that,  $G \frac{M_1 M_2}{r^2} = M_1 \frac{v_1^2}{r}$  is only suitable for that a moon orbits around a planet or a planet orbits around a star. From Equation (12), there is only  $v^2 = G \frac{M_{center}}{R}$ . That  $G \frac{M_1 M_2}{r^2} = M_1 \frac{v_1^2}{r}$  is used to the Sun on the Moon and the Charon on the Pluto is just the condition for the Poincaré's equation for Three-body problem. In such case, the orbits in the Sun-Earth-Moon and Sun-Pluto-Charon system should be chaotic.

## 7. Binary Planet or the Gravitational Unit

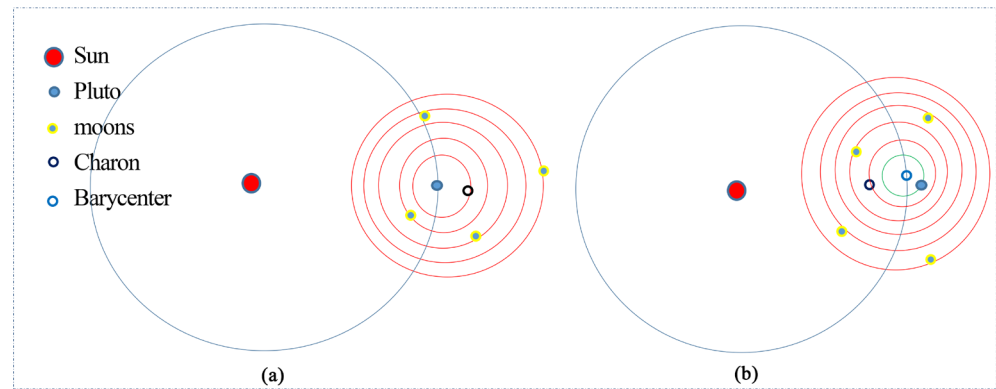
To explain the motion of the Moon around the Sun, it was presented that the Earth-Moon system is binary planet [18] [19]. It seems that, the binary planet is another kind of gravitational unit. The conclusions analogous to that deduced from our concept of gravitational unit, such as the gravitational field of the moon could be limited inside the unit, can be deduced from the binary planet. That the Pluto and its moons orbit around the barycenter of the Pluto-Charon system factually means that the Pluto cannot act on its moons with the force of  $F = GMm/R^2$ . So, in the binary planet, not only the field of the moons is limited, but that of the planet is done so. It seems that, the concept of binary planet is a strong evidence for our result. It shows that, to explain the orbit in the solar system, the planet-moon system need be considered as one single unit. But, we found, in mechanics and dynamics, the binary planet/star/blackhole cannot exist.

In 1870, it was claimed that the binary star was observed [24]. The binary system was introduced in current textbooks of celestial mechanics [5]. It was believed that the Pluto-Charon system is binary planet [20] [21] [22] [23]. It is different from other planet-moons system in which the moons orbit around a planet, in the Pluto system, the Pluto and its moons orbit around the barycenter of the system. The binary star is at a very distant place from us. Compared to the Pluto-Charon system, it is much more difficult to have accurate and precision observation. So, we think, the binary star/planet could be better understood from the Pluto-Charon system.

How to show the orbits of the moons of the Pluto in a figure? It should be easy to know whether or not the Pluto-Charon system is binary planet. As the orbits in the Sun-Pluto-moons are shown in a single figure, it should be clarified.

The orbit of the Sun-planet-moons system is well-known. For the Sun-Pluto-moons system, we can simply have **Figure 6(a)**. The Pluto orbits around the Sun while the moons, including the Charon, orbit around the Pluto. **Figure 6(a)** may be accepted easily by almost every people.

It is noted that, as the Earth-Moon system is believed a binary system, the barycenter of the system is within the Earth. So, the Earth is approximately on the



**Figure 6.** (a) The orbits of the Pluto and its moons. The Pluto orbits around the Sun while all of its moons (including the Charon) orbit around the Pluto. (b) The current thought of The orbits of the Pluto and its moons. The Pluto, Charon and four little moons orbit around the barycenter of the Pluto-Charon system. And, the Pluto is not on the orbit around the Sun.

orbit around the Sun. It is not apparently contradicted with observation. But, for the Pluto-Charon system, if it was true that, “Pluto’s motion is the result of the combination of its motion around the Sun, and its motion around the barycenter of its system” [20] and the other four little moons orbit around the barycenter, [20] [21] [22] [23] what a figure can we have?

In **Figure 6(a)**, the moons cannot orbit around the barycenter of the Pluto-Charon system. And, under the condition that the Pluto is moving on the orbit around the Sun, that the four little moons orbit around the barycenter of the Pluto-Charon system cannot be shown within a figure.

If as currently believed that “Pluto’s motion is the result of the combination of its motion around the Sun, and its motion around the barycenter of its system” [20], the figure of the Pluto system orbits around the Sun should be **Figure 6(b)**. If it was so, it should be the barycenter that is on the orbit of the Pluto-Charon system around the Sun. While the Pluto should not be on this orbit. And, the direction of the Pluto’s motion is varied and can be contrary to the direction of this orbit.

Besides **Figure 6(b)**, we cannot imagine another figure to show “Pluto’s motion is the result of the combination of its motion around the Sun, and its motion around the barycenter of its system” [20].

However, there is no evidence for **Figure 6(b)**. All observations showed that the Pluto is on the orbit around the Sun. The direction of the Pluto’s motion is just along the direction of the orbit as shown in **Figure 6(a)**.

It is important, it was observed that the orbit of the Charon is around the Pluto [21]. It means that, observation showed that the Pluto and Charon is not binary system.

Now, it is believed that the orbits of the four small moons are not around the Pluto. Instead, they are around the barycenter of the Pluto-Charon system [20] [21] [22] [23]. But, there are two problems: 1) What is the force of the Pluto on the moons? If the Pluto could not have the force of  $F = GMm/R^2$  on the

moons, it means that the Newtonian universe gravitation law should be invalid. 2) As shown in **Figure 6(a)**, as the orbit of the Charon is around the Pluto, the distances between the barycenter of the Pluto-Charon system and the moons are always varied. It certainly results in that the orbits of the moons are unstable if the orbits of the moons were around the barycenter. So, the belief is questioned.

It is noted that, the theory for the binary star/blackhole/neutron-star is that for the binary planet presented by Delaunay [17] [19]. The observation that the orbit of the Charon is around the Pluto shows that Delaunay's [19] theory for the binary planet is invalid. So, factually, till now, there has not been a theory for the binary star/blackhole/neutron-star.

It seems that the mechanics and dynamics for the binary star are questioned. Usually, the binary star/planet also is called as "double star/planet". In current textbooks, the binary star currently is described with a set of equations like the Equations (A)-(E) [5].

Assuming the distance between two stars  $M_1$  and  $M_2$  is  $R$ , the two stars orbit around the barycenter O of the  $M_1$ - $M_2$  system with the angle velocity  $\omega$ , the radius for the orbits of the two stars are respectively  $R_1$  and  $R_2$ ,  $R_1 + R_2 = R$ . In this case, for  $M_1$ , there is

$$F = G \frac{M_1 M_2}{R^2} = M_1 \omega^2 R_1 \quad (\text{A})$$

For  $M_2$ , there is

$$F = G \frac{M_1 M_2}{R^2} = M_2 \omega^2 R_2 \quad (\text{B})$$

and

$$M_1 \omega^2 R_1 = M_2 \omega^2 R_2 \quad (\text{C})$$

$$R_1 = \frac{M_2}{M_1 + M_2} R, \quad R_2 = \frac{M_1}{M_1 + M_2} R \quad (\text{D})$$

$$\omega = \sqrt{G(M_1 + M_2)/R^3} \quad \text{or} \quad v = \sqrt{G(M_1 + M_2)/R} \quad (\text{E})$$

Equations (A) and (B) means that  $M_1$  and  $M_2$  orbit around the barycenter O with the angle velocity  $\omega$  while the radius of the orbit for  $M_1$  is  $R_1$  and for  $M_2$  is  $R_2$ .

Here, the key problem is that, in the equations, the angle velocity  $\omega$  is not an observed result. Here, the angle velocity  $\omega$  only can mean that, if the binary star was true, such an angle velocity  $\omega$  is needed.

It is clear, Equations (A) and (B) violated both Newtonian constant gravitational law  $F = G \frac{M_1 M_2}{R^2}$  and orbital law  $G \frac{M_1 M_2}{R^2} = M_1 \frac{v_1^2}{R}$ . First, for (A), the Newtonian constant gravitational law  $F = G \frac{M_1 M_2}{R^2}$  only means that  $M_2$  has an acceleration on  $M_1$ . This acceleration is determined with  $M_1$ ,  $M_2$  and  $R$ . It cannot act on  $M_1$  with the distance  $R_1$  from  $M_1$ . Conversely, if  $F = G \frac{M_1 M_2}{R^2}$  could act

on  $M_1$  with the distance  $R_1$ , it means that the original meaning that  $F = G \frac{M_1 M_2}{R^2}$  acts on  $M_1$  with the distance  $R$  is broken off. And the formula  $G \frac{M_1 M_2}{R^2} = M_1 \frac{v_1^2}{R}$  is also broken off. This is clear wrong. So, there is no relationship between  $F = G \frac{M_1 M_2}{R^2}$  and  $R_1$ . This conclusion also is valid for Equation (B). Second, for Newtonian orbital law  $G \frac{M_1 M_2}{R^2} = M_1 \frac{v_1^2}{R}$ , we know, a centripetal force produced from the mass  $M_2$  is the necessary condition for  $M_1$  to move in a circle orbit around  $M_2$ . If there is no the centripetal force,  $M_1$  shall move freely which cannot form a circle orbit around  $M_2$ . It is clear, there is no mass at the barycenter O of  $M_1$ - $M_2$  system. Therefore, no centripetal force at O. So, no orbit around O can be formed.

The logic in Equations (A)-(E) seems unclear. It is unclear what is the center that  $M_1$  or  $M_2$  is orbiting around and what is the radius that the  $M_1$  or  $M_2$  is orbiting with. In Equations (A) and (B), the center is O and the radius is  $R_1$  or  $R_2$ . But, in Equation (E), the radius is  $R$  and the center can be  $M_1$  or  $M_2$ . Therefore, in (A)-(E), the center and radius are arbitrary. It is clear, the angle speed  $\omega$  for the orbit around O with radius of  $R_1$  and  $R_2$  in Equations (A) and (B) cannot be determined with the orbit around  $M_1$  or  $M_2$  with radius of  $R$  in (E). For example, as Equations (A)-(E) is applied on the Pluto-Charon system, if the angle speed  $\omega$  is that the Charon orbits around the barycenter O with the radius  $R_1$ , then, the same the angle speed  $\omega$  is invalid for that the Charon orbits around the Pluto with the radius  $R$ . But, Equations (A)-(E) means that the one single angle speed  $\omega$  can be valid for both of the cases. It is noted that, as Delaunay [19] presented that the Earth-Moon system is binary system, the barycenter of the system is within the surface of the Earth,  $R_m$  is almost equal to  $R$ , where  $R_m$  and  $R$  are the distance between the Moon and the barycenter and between the Moon and the Earth. Equations (A) and (E) can be approximately valid for the orbit of the Moon around the barycenter of the system. But, for the Pluto-Charon system, the barycenter is with a distance from the Pluto. It is clearly wrong to apply Equations (A)-(E) for the Pluto-Charon system.

In history, the equations like Equations (A)-(E) were presented for understanding the problems in the orbit of the Moon around the Earth, *i.e.*, Why the orbit of the Moon around the Earth is stable under the condition that calculated with Newton's law of  $F = GMm/R^2$ , there is  $F_{sm}/F_{em} \approx 2.2$  and accuracy about the perigee of the orbit of the Moon [4] [17]. In that time, several very famous scientists, including d'Alembert, Euler and Clairaut, tried to challenge Newton. They were unanimous in claiming that Newton's law of universal gravitation with the inverse-square-of-the-distance dependence does not account for the observed value. And some modified formulas were presented. From that time, many effort has been tried to modified Newtonian universal gravitational law. Now, this effort is still being continued. However, till now, none of the

modified formula is generally accepted. But, a concept that the orbit of the Moon is not only determined with the mass of the Earth but with both the masses of the Earth and Moon was remained to nowadays. In this concept, instead of

$$\frac{d^2 R}{dt^2} = G \frac{M_e}{R^3} R, \text{ the acceleration on the Moon is modified as}$$

$$\frac{d^2 R}{dt^2} = G(M_e + M_m) \frac{R}{R^3}, \text{ [4] [5] [17] where } e \text{ and } m \text{ denote the Earth and Moon}$$

respectively. However, from  $G \frac{M_e M_m}{R^2} = M_m \frac{v_m^2}{R}$ , we know, the velocity  $v_m$

must be determined with the centripetal force which is produced by the mass  $M_e$  of the Earth. But, the mass  $M_m$  of the Moon cannot have contribution to the centripetal force for the Moon orbits around the mass  $M_e$ . So, this concept is questioned.

It was argued that  $\frac{d^2 R}{dt^2} = G(M_e + M_m) \frac{R}{R^3}$  is suitable for the very big mass or

for such two masses that  $m/M$  is very large. But, it is noted that

$$\frac{d^2 R}{dt^2} = G(M_e + M_m) \frac{R}{R^3} \text{ was presented from the Earth-Moon system. So, it is}$$

not suitable for the big mass system. Especially, now, there has not been such a theory that can replace the Newtonian constant gravitational law  $F = G \frac{M_1 M_2}{R^2}$

and orbital law  $G \frac{M_1 M_2}{R^2} = M_1 \frac{v_1^2}{R}$  for the very big mass or the large ratio of

$m/M$ . Now, only the Newtonian theory of gravity is valid for the orbit of any

mass. And, it is known that  $\frac{d^2 R}{dt^2} = G(M_e + M_m) \frac{R}{R^3}$  is not helpful to under-

stand the problem in the orbit of the Moon for that  $\frac{d^2 R}{dt^2} = G(M_e + M_m) \frac{R}{R^3}$

was presented [4]. But, now, the formula as  $\frac{d^2 R}{dt^2} = G(M_e + M_m) \frac{R}{R^3}$  is being

generally used. Therefore, how to understand it is still a problem.

It is emphasized that, in a galaxy, almost every star is with a certain orbit around a bigger mass. The lone binary star has not been observed. So, as we study the binary system, **Figure 6** must be considered. But, in current theory about the binary star, only two stars isolated from other mass are considered. Therefore, no valid result can be obtained from this way. We think, this is the largest problem in current theory about the binary star.

It is noted, in visual, the Pluto and Charon appear as binary. Observed at the Charon, it appears that the Pluto was orbiting around the Charon just as that, observed at the Earth, the Sun was orbiting around the Earth. As we assume that the Earth is stationary, the line of the Sun moving around the Earth just is a cycle. It exactly appears as an orbit of the Sun around the Earth. In geometrics, as a body moves within a constant distance from another body, observed at any one of the two bodies, the line of another body moving is a circle. Therefore, ob-

served at an artificial satellite around the Earth, the line that the Earth moving just is a circle around the satellite. It is certain, no one should think that the Earth and the satellite are binary. For two stars in a very distance from us, the orbit of the main star orbit around a center is difficult to be observed. It also appears as that the two stars are orbiting around each other. But, after Copernicus heliocentric theory, we know it is unsuitable to say that the Sun is orbiting around the Earth. It is worth noting that, before Copernicus heliocentric theory, many celestial orbits and accurate ephemeris were well predicted with the geocentric theory. But, we cannot say that the geocentric theory is right. Therefore, although the binary star/planet is an observed result, it need be re-understood. So, we think, binary star/planet may be a misunderstanding by the geometric picture of the orbital motion.

We know, the four small moons were discovered in the predicted dynamical stability zoo as the Pluto-Charon system was treated as binary [23]. But, we do not think it could be an evidence that the binary system can be existed in mechanics and dynamics. It is noted, in mathematics, the difference between the dynamical stability zoo of the Pluto-Charon system and that of the Pluto is little.

Today, binary star is very fashioning. It was reported that the multi-star system was observed [42]. It is believed that there are the double black hole and double neutron star. And, the LIGO and VIRGO's detections of gravitational waves are based on the assumption of the double black hole [43] and double neutron star [44]. So, it is very important to clarify the binary system.

## 8. Orbits in a Galaxy and the Gravitational Unit

It is easy to observe that the Sun-planets-moons system is one gravitational unit. And, it is known that the orbit of the Sun is around the center of the Milky Way. Therefore, a star-planets-moons unit is just like a planet-moon unit, the field of the planets and the moons in a star-planets-moons unit are limited. And, other mass cannot have the action of  $g = Gm/r^2$  on the orbit of this star around the center of the Milky Way. If it was not so, the force of the Milky Way gravitating the stellar system should be determined with  $\sum \overline{g_i}$ , where  $\overline{g_i} = G \frac{m_i}{r_i^2} \overline{r_i}$ ,  $m_i$  is

the mass of star, planet, moon and other body and  $i$  is the number of the planet, star and moon in the Milky Way,  $r_i$  is the distance between this star and  $m_i$ ,  $\overline{r_i}$  is a vector. (Here, only the mass of the body with a volume is considered. Other object, such as disperse gas, was not considered. Maybe, the law for the force between a star and the disperse gas need be developed.) Approximately, the total mass of the Milky Way is  $5.8 \times 10^{11}$  times that of the Sun [45] and the mass of center of it is only  $4.5 \times 10^6$  times that of the Sun [46]. If the Star (or stellar system) was dominated by  $\sum \overline{g_i}$ , it should result in that the star (stellar systems) in different locations of the Milky Way could not be orbited around the center of the Milky Way for that the barycenter of mass of the Milky Way is determined with  $\sum r_i m_i / \sum m_i$ . For the stars in different locations of the Milky

Way, the barycenter of the mass for it is different. Therefore, the only condition for the stars orbiting around the center of the Milky Way is that the gravitational field of these stars can be limited. The star (stellar system) is not gravitated by  $\sum \overline{g_i}$ , but only by the center of the Milky Way with  $g = GM/R^2$ . In another words, the center of the Milky Way dominates the core of the stellar systems, this core dominates the stars, and the star dominates the planets. The mass of cores, stars and planets of a stellar system cannot affect the orbit of another stellar system.

We know, the dark matter was presented from the Galaxy rotation curves in which the observed orbital velocity is larger than the predicted one [47]. While a recent observation [48] reported that, the orbits of stars and other matter in the spiral galaxy are dominated by the Newtonian law of gravity. The two observations are contradicted with each other. Now, it is generally known that, till now, it is difficult to completely explain and understand the orbits in the Sun-Earth-Moon system [4]. The problem for the orbits in a galaxy is much more complicated than that in the Sun-Earth-Moon system. Therefore, more complete theory and much more accurate and precession observation are needed to better know the orbits in a galaxy. It is noted that, in current theory for the galaxy dynamics, the baryonic mass of a galaxy (the sum of its stars and gas) correlates with the amplitude of the flat rotation velocity [49]. But, we think, the orbital perturbation theory and the Hill sphere ought to be valid to the orbit in a galaxy. Therefore, it should be important to integrate the orbital perturbation theory and the Hill sphere with the current theory of orbit in the galaxy.

## 9. Conclusions

Newton formulated the orbital perturbation theory and the repulsive gravity [3] [4]. Therefore, he factually laid the foundation for the theory of interaction of gravitational field. But, in Newton's time, the theory of field had not been known. And, till now, it is believed that a mass can interact with any other ones with the force of  $F = GMm/R^2$ . It resulted in that the theory for the interaction of gravitational field cannot be developed. After Newton, the theory about gravity was developed in three aspects. First, in 1900s, the theory of field for the electricity and magnetism was established. The electromagnetic interaction was described with the Maxwell equation. It leads to that the gravitational interaction could be described with the theory of field by analogy to the electromagnetic field theory. Second, the Hill sphere was presented and generally applied [6]-[14]. It factually shows some of features of the interaction of gravitational field. Third, the artificial orbit becomes a general project which has been exploited by many nations. The gravitational force on an artificial satellite can be measured with high precision. Based on these developments, a new theory of gravity could be developed.

In our work, the current theories about orbit are investigated with the orbital perturbation theory. Our main conclusion is that the planet and moon are uni-

fied as one single gravitational unit and the field of the moon is limited in the unit which cannot interact with other ones out of the unit with  $F = GMm/R^2$ . We repeat to emphasize that this conclusion was implied in the Newtonian orbital perturbation equation. Therefore, our conclusion is based on the well-developed and well-applied theory. Now the gravitational force with the precision of  $10^{-8} \text{ m/s}^2$  perturbing to the artificial orbit can be measured [37] and the variation of the radius of the orbit of an artificial satellite can be measured with the precision of less than 3 cm [35] [36]. So, our conclusion is well confirmed experimentally and observationally. In another hand, from the contradiction between the orbital perturbation equation and the Poincaré's equation for Three-body problem, it is clearly shown that it is not right that a gravitational field can interact with any other ones with the force of  $F = GMm/R^2$ . Therefore, a new theory should be established for the interaction of gravitational field. It should be fundamental to the theory of gravity. Especially, that the interacting gravitational field could be limited and that the perturbation of the Sun to the Moon should be repulsive gravity which should lead to remodel the theory of gravity.

By analogy to the solar system, Ernest Rutherford [50] presented his atom model. Our observation shows that the planet and moon also unified as one single solid unit by gravitational field while the planet and moon are separated in space. It is analogous to that the nuclei and electrons are unified as an atom. The space for the interaction of gravitational field is much larger than that for the electromagnetic field, some of the new features of the interaction of field should be discovered from the interaction of gravitational field.

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<https://www.zhihu.com/question/447859297>.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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