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Modeling and Optimization of Interaction between Small Coastal Pelagic Fish, their Biological Predators and Fishermen

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Abstract

Small coastal pelagic fish are one of the fish families most affected by sea fishing. This man-made phenomenon leads to an imbalance in the marine and coastal ecosystem and is one of the main causes of migration north and offshore of the ranges. We used the ordinary differential equations to model the interactions existing between small pelagic resources and fishermen. Modelling follows the same of the Lotka-Volterra equations with a difference in the number of variables. This study confirmed the instability of the marine ecosystem. The objective is first of all to model a system of three interacting individuals composed of two distinct types of predators and two types of prey, and then optimise this interaction with the aim of conserving biodiversity in the ecosystem under study. Determining the Jacobian matrix made it possible to calculate the reproduction rate basic (R_0). The study of the strong connectedness has made it possible to reduce the number of variables without losing the objective of the study. A computer program implemented on the language computer python facilitated the visualisation of the results.

Keywords

Small Coastal Pelagics, Ordinary Differential Equations, Modelling, Optimisation, Basic Reproduction Rate

1. Introduction

Overfishing has a dual impact on the marine ecosystem. It devastates the fish families of a given area, but also destroys the corals that are places refuges and fish reproduction. This fact observed and multiplied by the changes climate leads to a displacement of fish shoals to other optimal areas by term of abiotic parameters and more stable in terms of exploitation. Understanding of the mechanism and the causes of the dynamics of small pelagic resources will allow the implementation of sustainable management policies. The reduction in the number of variables in the study facilitated the writing of interaction models. The strong connectedness in a graph allowed us to obtain strongly related subgraphs and thus to make variables out of them. It is important to know the (R_0) in this type of studies, because this is the objective to stabilise the basic reproduction system. Since the behaviour of the different species is modelled by a system of differential equations of population dynamics in interactions, the threshold R_0 of the variable representing small pelagics is determined from the stability study of the Jacobian matrix of the system. The mathematical model is an extension of the one of Lotka-volterra from two to three and then four variables. Numerical tests show a correlation with the research results published by a large number of biologists peaches. This decision-making tool is part of the contribution of science to the development of the fisheries sector, accurate in solving social and environmental problems. In one of their study, the authors in [1] have developed a three-variable differential equation system taking into account changes in the investment. This model has three variables: the biomass of the resource, the fishing effort and the market price of the resource. This model is based on the same principle as that of Lotka-Volterra, *i.e.* an interaction strong and natural exists between the species in the system. Biological scientists, mathematicians and other related profiles generally lean towards the pre-eminence of environmental factors and are increasingly directing their work in this direction. Thus for a better management of marine protected areas, researchers have compared two areas, one of which has artificial reefs and the other does not. They thus measured the attraction which exists between two areas with different nutritional potential [2]. The disappearance of some subfamilies of small pelagic fish such as sardines is indirectly imputed only to the action of fishermen and yet the latter protested against the development of intensive fishing which today threatens the balance of the marine ecosystem and economic development of coastal countries [3]. It is therefore important to consider in the models of management of fisheries the anthropic and economic aspects. In this study, the stock considered is that of small pelagic fish whose landings have increased slightly despite the very high increase in the number of fishermen in the seas [4]. These authors have proposed a solution for the spatialized management of small pelagic resources, in the Senegalese Exclusive Economic Zone. It is also a confirmation of the importance of having reliable statistical data in continuity, because the calculations of eight could disappear from the earth in the coming decades [5]. Biology has given rise to several computer disciplines, such as neural networks and genetic algorithms. She also represents the source of a new form of intelligence, which is intelligence collective; that of simple multicellular beings, the colonies of social insects and the human [6]. In their studies, they have used Multi-Agent systems to model population dynamics [6]. In [7], the authors modelled the dynamics of a population of groupers in a fishing area of a marine coast, taking into account to natural growth, fishing and migration, and to study the effect of poaching on this population. If one could at least describe the fluctuations in catchability by a dynamic model, then fishing mortality could be controlled in a way that would be more precise. By regulating fishing effort, catch per unit effort (CPUE) could be directly used as indicators of abundance [8]. Partial fishing mortality generated by a vessel or a fleet can be expressed in terms of catchability total (of all vessels/fleets exploiting this stock) and of the partial catch (of the vessel or fleet under consideration) [8].

The authors in [9] have developed a model to calculate the basic reproduction threshold. This knowledge of the threshold makes it possible to direct the study towards the search for this rate, which is important for the equilibrium of the system.

The predation equations of Lotka-Volterra, which are referred to as the also referred to as the "prey-predator model", are a pair of differential equations non-linear of the first order. They are commonly used to describe the dynamics biological systems in which a predator and its prey interact.

2. Materials and Methods

2.1. Choice of Variables

The environment studied is composed of: small pelagic fish (SP), predators (SP), artisanal fishermen (IP), industrial fishermen (IP), managers (M), decision-makers (D), women processors (WP), traders (T) and consumers (Cons). A mathematical model taking into account the interactions between all these variables is difficult to write and to solve. Modeling in the form of a problem-oriented graph allows us to study its strongly related components.

Figure 1 shows the interaction between the different variables of the problem under study.

The strongly related components are:

- Management := {G, D}
- Fishermen := {PA, PI, FT, C, Cons}
- Prey := {PP}
- Predator := {P}

The variables in the study are reduced to: Management, Fishermen, Prey and Predators.

2.2. Mathematical Model

In the logic of the Lotka-Volterra system, predators and their prey interact in a biological environment free of anthropic influence. The objective in this part is to introduce two new variables (fishermen and managers) into this system that

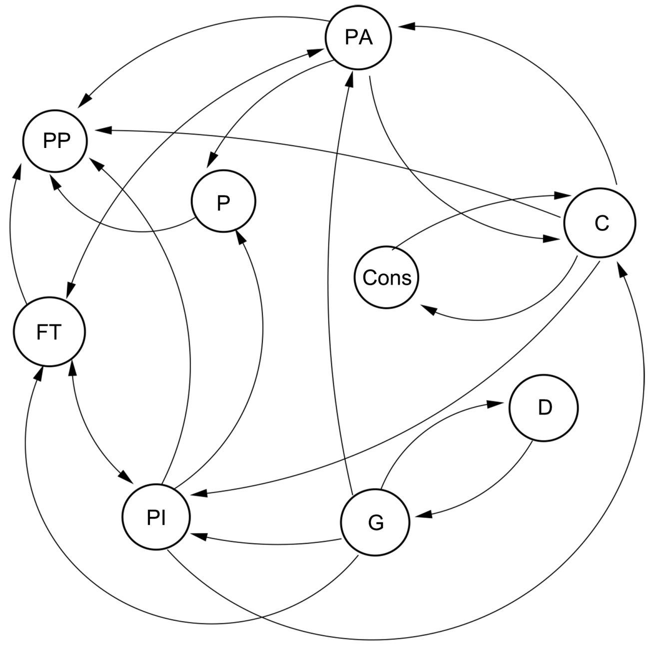

Figure 1. Interaction of ecosystem parameters.

act on predators as well as on prey. The human species is predatory of all other species living on the sea bed.

So what will the nature of the seabed be in a very long time and what is the solution to the problem? To the multiple assaults on this resource family. There is a double interaction between small pelagics and their ecosystem. The first interaction is that with predatory resources, the second is that with men. The three-variable model (men, predators and prey), allows:

- to know the bifurcation threshold which is the limit of disturbance of the system;
- to know the mutual behaviour of the three species;
- to know the level of disturbance of the system caused by the introduction of a new variable.

2.2.1. Model Assumptions

Based on the above, we make the following assumptions

(H1) We are in the presence of climate change.

(H2) the territory has favourable conditions for survival (a favourable sex ratio, a regime optimal food quality and a sufficiently high water temperature);

(H3) the ecosystem of our study is exclusively made up of small coastal pelagic fish. their natural predators and the fishermen's population

(H4) the initial level of the prey population is $\frac{\Theta}{\alpha}$[1].

2.2.2. Parameters and Variables

Table 1 groups together the terms used in the different parts.

[1]Result obtained with Schaefer's logistics function with infinite convergence.

Table 1. Parameters.

Parameters	Meanings
Θ	saturation level of the medium
α	natural growth rate of the small pelagic population
α_1'	natural mortality rate of prey
k_1	rate of increase due to management of small pelagic resource
k_2	regression rate due to small pelagic management
k_3	regression rate of predator management
β_1	mortality rate of small pelagics caused by predators
β_2	mortality rate of small pelagics caused by overfishing
ξ_1	proportion of decrease in sea trips
δ	growth rate of predators
r	intrinsic reproduction rate of the population
γ_1	mortality rate of fishing predators
γ_2	natural mortality rate of predators
δ	rate of increase in predators due to the abundance of small pelagics
η_1	Increase in the number of fishermen at sea caused by abundant prey
η_2	rate of change in the number of fishermen at sea caused by predators
μ_1	Rate of change in fishing effort caused by regulation.
μ_2	Rate of decline in fisheries management effort.

The decision variables used in the model are:

1) $X \in \mathbb{R}^+$: represents the population of small pelagics;
2) $Y \in \mathbb{R}^+$: represents the predator population;
3) $Z \in \mathbb{R}^+$: represents the population of fishermen.

Model formulation

$$\begin{cases} \dfrac{\mathrm{d}X}{\mathrm{d}t} = X\left(\alpha - \beta_1 Y - \beta_2 Z\right) & (1) \\ \dfrac{\mathrm{d}Y}{\mathrm{d}t} = Y\left(\delta X - \gamma_1 Z - \gamma_2\right) & (2) \\ \dfrac{\mathrm{d}Z}{\mathrm{d}t} = Z\left(X\eta_1 + Y\eta_2 - \xi\right) & (3) \end{cases} \tag{1}$$

$$\left(X(0), Y(0), Z(0)\right) = \left(X_0, Y_0, Z_0\right) = \left(X_0, 0, 0\right)$$

The objective of the model is to see the evolution of the three species interacting in a lotka-Volterra. The model (1) is a system of three differential equations unknown.

1) Equation (1) shows that the evolution of the prey population is controlled by pre organic date fishermen and fishermen.

2) Relationship (2) shows that the evolution of the predator population depends on the quantity of prey and fishermen.

3) Relationship (3) shows that the quantity of fishermen at sea depends on the quantity of prey and predators.

Existence and uniqueness of the solution

The system (1) can be written in the form of

$$\begin{cases} U'(t) = f(t, U(t)) \\ U(t_0) = U_0 \end{cases} \tag{2}$$

with $U'(t) = (X'(t), Y'(t), Z'(t))$

$f : I \times \mathbb{R}^d \to \mathbb{R}^d$

$(X'(t), Y'(t), Z'(t)) \mapsto (X(\alpha - \beta_1 Y - \beta_2 Z), Y(\delta X - \gamma_1 Z - \gamma_2), Z(X\eta_1 + Y\eta_2 - \xi))$

f is of class C^1 then according to the Chauchy-Lipschitz theorem the system (2) admits a single solution (2) admits a single solution $\Rightarrow$ (1) admits a single solution.

Bifurcation parameter and model stability

The bifurcation parameter ($R_0 \leq 1$) also called threshold, is the state of equilibrium of the system without exploitation. Knowledge of this parameter is important in biology and dynamics populations. It makes it possible to measure the real consequences of abusive exploitation of a resource family. The physical parameter at stake is the biomass of resource small pelagics. It intervenes when a small change in a physical parameter produces a major change in the organisation of the system under study. This threshold R0 represents the limit parameter for the exploitation of small pelagic resources by predators and fishermen.

In system (1) the solution space is $E = (X, Y, Z)$. The Jacobian matrix $J(E)$ gives:

$$J(X,Y,Z) = \begin{pmatrix} \alpha - \beta_1 Y - \beta_2 Z & -\beta_1 X & -\beta_2 X \\ Y\delta & \delta X - \gamma_1 Z - \gamma_2 & -\gamma_1 Y \\ \eta_1 Z & \eta_2 Z & X\eta_1 + Y\eta_2 - \xi \end{pmatrix}$$

The system is considered to be in equilibrium without any exploitation, without any attack from the predators on the prey population X as well as without the action of sea fishing. This equilibrium of the system (1) is denoted $E^0 = (X_0, 0, 0)$.

From this equilibrium $E^0 = (X_0, 0, 0)$ the Jacobian matrix is deduced:

$$J(E^0) = \begin{pmatrix} \alpha & -\beta_1 X_0 & -\beta_2 X_0 \\ 0 & \delta X_0 - \gamma_2 & 0 \\ 0 & 0 & X_0 \eta_1 - \xi \end{pmatrix}$$

The system (1) is stable if and only if the matrix (3) below is stable

$$\begin{pmatrix} \delta X_0 - \gamma_2 & 0 \\ 0 & X_0\eta_1 - \xi \end{pmatrix} \tag{3}$$

The matrix (3) is stable if and only if its determinant is positive. This results in:

$$(\delta X_0 - \gamma_2)(X_0\eta_1 - \xi) \geq 0$$

$$\delta X_0^2\eta_1 - \delta X_0\xi - \gamma_2 X_0\eta_1 + \gamma_2\xi \geq 0$$

$$-\delta X_0\xi - \gamma_2 X_0\eta_1 \geq -\delta X_0^2\eta_1 - \gamma_2\xi$$

$$\delta X_0\xi + \gamma_2 X_0\eta_1 \leq \delta X_0^2\eta_1 + \gamma_2\xi$$

$$\frac{\delta X_0\xi + \gamma_2 X_0\eta_1}{\delta X_0^2\eta_1 + \gamma_2\xi} \leq 1$$

then

$$R_0 = \frac{\delta X_0\xi + \gamma_2 X_0\eta_1}{\delta X_0^2\eta_1 + \gamma_2\xi}$$

R_0 represents in our case the predation value not to be exceeded, otherwise to disturb the balance of the system, avec $X_0 = \frac{\Theta}{\alpha}$.

2.2.3. Determination of System Equilibrium Points (1)

The point $O(0,0,0)$ is a point of equilibrium at the origin. Another point of equilibrium different from point O exists. To determine it, let us consider the system (1) at equilibrium:

$$\begin{cases} \alpha - \beta_1 Y - \beta_2 Z = 0 & (1) \\ \delta X - \gamma_1 Z - \gamma_2 = 0 & (2) \\ X\eta_1 + Y\eta_2 - \xi = 0 & (3) \end{cases} \tag{4}$$

with $(X(0), Y(0), Z(0)) = (X_0, Y_0, Z_0) = (X_0, 0, 0)$

To solve the model graphically we need to calculate the values of X, Y and Z in balance. Function (1) give us: $Z = \frac{\alpha - \beta_1 Y}{\beta_2}$. By replacing Z in the function (2) the following system is obtained:

$$\begin{cases} \delta X - \gamma_1\left(\frac{\alpha - \beta_1 Y}{\beta_2}\right) - \gamma_2 = 0 \\ X\eta_1 + Y\eta_2 - \xi = 0 \end{cases} \tag{5}$$

The resolution of the system (5) is:

$$Y_1 = \frac{\eta_1\gamma_1\alpha + \beta_2\gamma_2\eta_1 - \beta_2\xi\delta}{\beta_1\gamma_1\eta_1 - \beta_2\delta\eta_2}$$

$$X_1 = \frac{\xi}{\eta_1} - \frac{\eta_2}{\eta_1}(Y_1)$$

$$Z_1 = \frac{\alpha - \beta_1 Y_1}{\beta_2}$$

2.3. Stability of Critical Points

2.3.1. Critical Point Stability *O*(0, 0, 0)

As this model is non-linear, its stability is determined by the following theorem:

Theorem 1 (linarisation)

If f is differentiable en 0 and $f(0)=0$, then

- if $\forall\lambda$ belongs to the set of eigenvalues of the original matrix, $Re\lambda<0$, then 0 is asymptotically stable for (1).
- if $\exists\lambda$ belongs to the set of eigenvalues of the original matrix, $Re\lambda>0$, then 0 is unstable for (1).

We therefore know nothing about behaviour near a point of equilibrium if all the eigenvalues of the differential are of negative real part or zero, with at least one of the real part nul.

Let us study the stability of the point $O(0, 0, 0)$:

$$J(0,0,0)\begin{pmatrix} \alpha & 0 & 0 \\ 0 & -\gamma_2 & 0 \\ 0 & 0 & -\xi \end{pmatrix} \tag{6}$$

Let's calculate the eigenvalues of the Jacobian matrix *J*(0, 0, 0)

The eigenvalues are the solutions of the matrix determinant below:

$$\begin{pmatrix} \alpha-\lambda & 0 & 0 \\ 0 & -\gamma_2-\lambda & 0 \\ 0 & 0 & -\xi-\lambda \end{pmatrix}$$

its determinant gives: $(\alpha-\lambda)(-\gamma_2-\lambda)(-\xi-\lambda)$.

Let's ask

$$(\alpha-\lambda)(-\gamma_2-\lambda)(-\xi-\lambda)=0$$
$$\Rightarrow \alpha-\lambda=0 \quad \text{or} \quad -\gamma_2-\lambda=0 \quad \text{or} \quad -\xi-\lambda=0$$
$$\Rightarrow \lambda=\alpha \quad \text{or} \quad \lambda=-\gamma_2 \quad \text{or} \quad \lambda=-\xi$$
$$\text{or} \quad \alpha,\gamma,\xi\geq 0$$

The equilibrium $O(0, 0, 0)$ is unstable.

According to the theorem (1); if there is an eigenvalue λ with $Re\lambda>0$ the equilibrium (0, 0, 0) is unstable.

2.3.2. Stability of the Critical Point X_1, Y_1, Z_1

$$Y_1=\frac{\eta_1\gamma_1\alpha+\beta_2\gamma_2\eta_1-\beta_2\xi\delta}{\beta_1\gamma_1\eta_1-\beta_2\delta\eta_2}$$

$$X_1=\frac{\xi}{\eta_1}-\frac{\eta_2}{\eta_1}(Y_1)$$

$$Z_1=\frac{\alpha-\beta_1Y_1}{\beta_2}$$

Linearizing the system.

$$\begin{cases} X\left(\alpha-\beta_1 Y-\beta_2 Z\right) & (1) \\ Y\left(\delta X-\gamma_1 Z-\gamma_2\right) & (2) \\ Z\left(X\eta_1+Y\eta_2-\xi\right) & (3) \end{cases} \tag{7}$$

Let us determine the Jacobian matrix of the system (7) at the points $\left(X_1, Y_1, Z_1\right)$:

$$J=\begin{pmatrix} \alpha-\beta_1 Y_1-\beta_2 Z_1 & -\beta_1 X_1 & -\beta_2 X_1 \\ \delta Y_1 & \delta X_1-\gamma_1 Z_1-\gamma_2 & -\gamma_1 Y_1 \\ \eta_1 Z_1 & \eta_2 Z_1 & -X_1\eta_1+Y_1\eta_2-\xi \end{pmatrix}$$

Let's diagonalize the matrix J

$$\begin{aligned}\left(J-\lambda I_d\right)&=\begin{pmatrix} \alpha-\beta_1 Y_1-\beta_2 Z_1-\lambda & -\beta_1 X_1 & -\beta_2 X_1 \\ \delta Y_1 & \delta X_1-\gamma_1 Z_1-\gamma_2-\lambda & -\gamma_1 Y_1 \\ \eta_1 Z_1 & \eta_2 Z_1 & -X_1\eta_1+Y_1\eta_2-\xi-\lambda \end{pmatrix} \\ &=\begin{pmatrix} K_1-\lambda & m_1 & m_2 \\ m_3 & K_2-\lambda & m_4 \\ m_5 & m_6 & K_3-\lambda \end{pmatrix}\end{aligned}$$

with $K_1=\alpha-\beta_1 Y_1-\beta_2 Z_1$; $K_2=\delta X_1-\gamma_1 Z_1-\gamma_2$; $K_3=-X_1\eta_1+Y_1\eta_2-\xi$, $m_1=-\beta_1 X_1$, $m_2=-\beta_2 X_1$, $m_3=\delta Y_1$, $m_4=-\gamma_1 Y_1$, $m_5=\eta_1 Z_1$, $m_6=\eta_2 Z_1$ Determining eigenvalues from critical points $\left(X_1, Y_1, Z_1\right)$ by solving the equation: $\det\left(A-\lambda I_d\right)=0$).

$$\det\left(J-\lambda I_d\right)=0 \Rightarrow$$

$$\begin{aligned}&-\lambda^3+\lambda^2\left(K_1+K_3+K_1\right)+\lambda\left(-K_2K_3+m_1m_3+m_2m_5\right) \\ &+K_1K_2K_3-m_1m_3K_3+m_1^2m_5+m_3+m_3+m_6-m_2m_5K_2=0\end{aligned} \tag{8}$$

By posing: $\mathcal{K}_1=K_1+K_3+K_1$, $\mathcal{K}_2=-K_2K_3+m_1m_3+m_2m_5$ and $\mathcal{K}_3=K_1K_2K_3-m_1m_3K_3+m_1^2m_5+m_3+m_3+m_6-m_2m_5K_2$

The Equation (8) gives:

$$\lambda^3-\lambda^2\mathcal{K}_1-\lambda\mathcal{K}_2-\mathcal{K}_3=0 \tag{9}$$

The stability of critical point $\left(X_1, Y_1, Z_1\right)$ depends on the values and sign of $\mathcal{K}_1$, $\mathcal{K}_2$ and $\mathcal{K}_3$. This stability is almost impossible because the signs of $\mathcal{K}_1$, $\mathcal{K}_2$ and $\mathcal{K}_3$ depend on of variable parameters according to the seasons and climatic conditions.

3. Graphical Representation of the Numerical Test Results

Figure 2 represents the result of the simulation of the model(1) on the Python language:

Generally speaking, the interaction between three species in the same ecosystem, leads to the disappearance of one of the species. The other two species will continue to evolve periodically. Of the four simulations carried out after varying the different parameters of the model, it shows that the population of biological predators is in danger of extinction. In all simulations, this population either cancels out or tends towards 0.

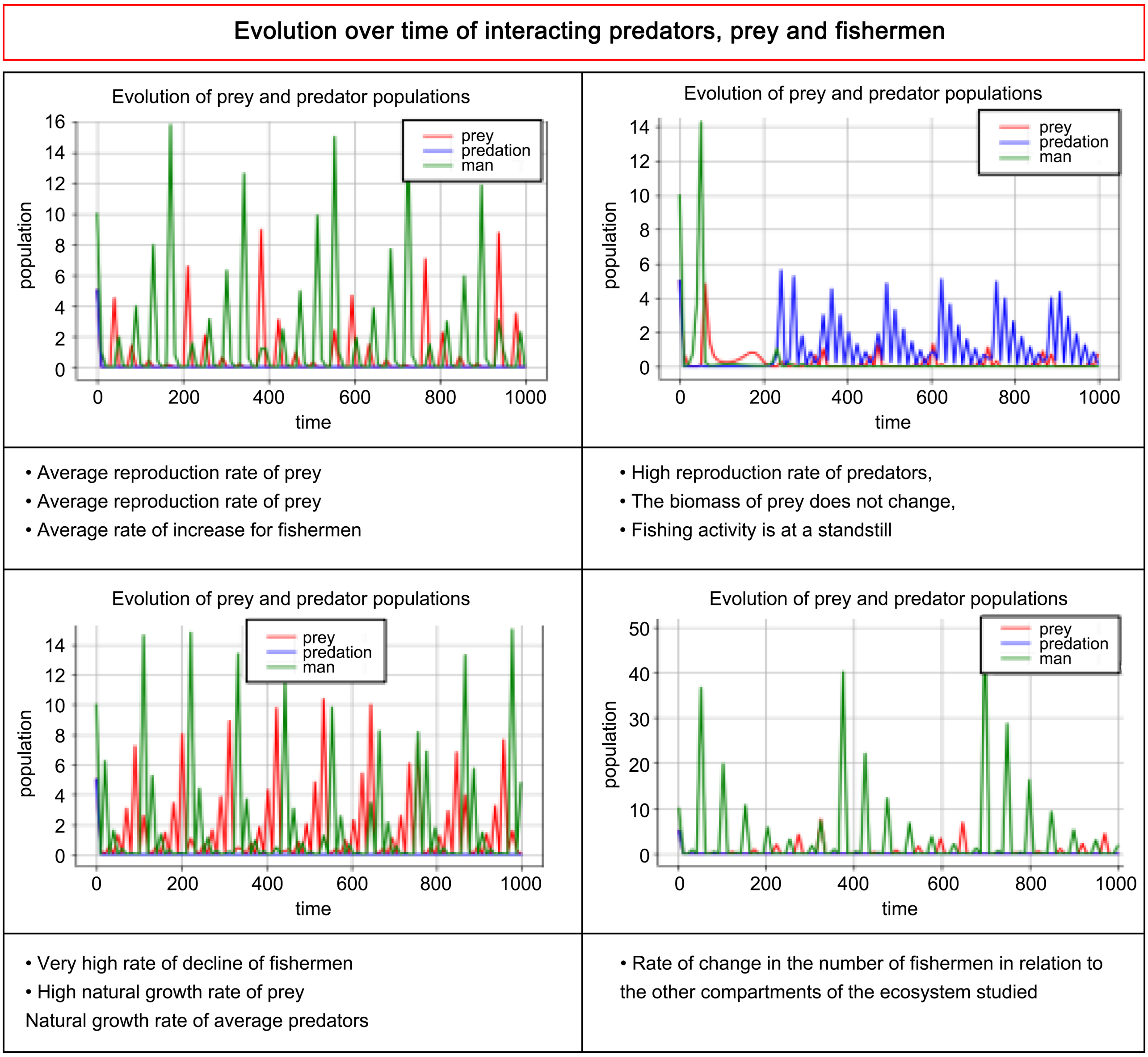

Figure 2. Temporal evolution of the interacting three-variable system.

These graphs show the instability of the points of origin. Policies for the protection of fishing resources could be oriented towards the protection of fish families predators, many of which are already extinct. These graphs show a perfect disturbance of the predator-prey biological system. This is due to the action of intensive and unregulated maritime fishing. Resilience mechanisms favouring the evolution of the natural rate of increase of this predatory fish stocks are a necessity.

The resolution is multiple but for an optimal solution we will consider an ordinary differential system by integrating a new variable (management).

Model Optimisation Solution (1)

In order to minimise the risk of extinction of the three species, we will introduce a new variable in the system. The management variable will interact with the other three system variables. It is noted W, the graph obtained becomes strongly related. The objective is to design a strongly related graph by adding to **Figure 2** a new variable named Management (W). The associated graph becomes $G'(S', A')$ with $S' = \{X, Y, Z, W\}$ et $A' = \{(Y, X), (Z, X), (Z, Y), (X, W), (W, Z), (W, Y)\}$ we have **Figure 3**.

Figure 3 shows us the interaction between the different variables in the model. Direct interactions are represented by solid arrows and indirect interactions by dotted arrows.

The mathematical model obtained from G' is:

$$\begin{cases} X' = X(\alpha - \beta_1 Y - \beta_2 Z + \alpha_1 W) \\ Y' = Y(\delta X - \gamma_1 Z - \gamma_2 + \alpha_2 W) \\ Z' = Z(X\eta_1 + Y\eta_2 - \xi - \alpha_3 W) \\ W' = W(-k_1 X - k_2 Y + k_3 Z) \end{cases} \tag{10}$$

$X(0) = X_0$, $Y(0) = Y_0$, $Z(0) = Z_0$ et $W(0) = W_0$.

Stability and calculation of the operating threshold R_0

At equilibrium without exploitation $E^0 = (X_0, 0, 0, 0)$.

$$J(E^0) = \begin{pmatrix} \alpha & -\beta_1 X_0 & -\beta_2 X_0 & \alpha_1 X_0 \\ 0 & \delta X_0 - \gamma_2 & 0 & 0 \\ 0 & 0 & X_0\eta_1 - \xi & 0 \\ 0 & 0 & 0 & -k_1 X_0 \end{pmatrix}$$

$J(E^0)$ is stable if the following matrix is stable:

$$\begin{pmatrix} X_0\eta_1 - \xi & 0 \\ 0 & -k_1 X_0 \end{pmatrix}$$

Threshold calculation: The threshold $R_1 \le 1$.

$$(X_0\eta_1 - \xi)(-k_1 X_0) \ge 0$$

$$X_0^2 \eta_1 k_1 \le \xi k_1 X_0$$

$$\frac{X_0^2 \eta_1 k_1}{\xi k_1 X_0} \le 1$$

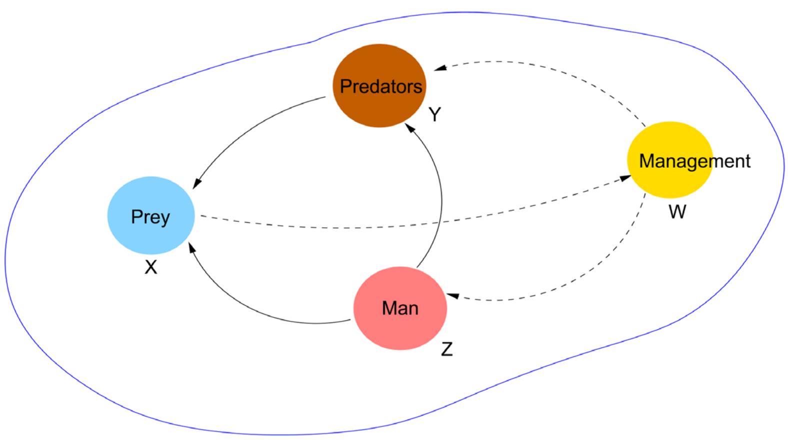

Figure 3. Graph of the strongly related after adding the variable W.

then

$$R_0 = \frac{X_0 \eta_1}{\xi_1}$$

This threshold, or basic reproduction rate, is the basic value for maintaining the biotic balance of the ecosystem under study [9]. The system (11) is stable if the threshold $R_1 = \frac{X_0 \eta}{\varepsilon_1} \le 1$.

One solution to solve the model is to propose three pairs of two variables each, to be solved independently of each other while remaining within the same system. For each couple, the variables that do not enter into a relationship will be considered as constant.

System 1: Small pelagics/Managers

1)

$$\begin{cases} X' = X\left(\alpha_1 W - \alpha_1'\right) & (1) \\ W' = W\left(k_1 - k_2 X\right) & (2) \\ \left(X(0), W(0)\right) = \left(X_0, W_0\right) & (3) \end{cases} \tag{11}$$

The system (11) is an interaction between small pelagic resources and the fisheries managers. Through better management of the resource, the resource has a normal growth.

2) **Numerical results**

These results of **Figure 4** show that management needs to be very active around fisheries. Effort management is determined by the fish biomass in exploitation state:

a) high biomass - passive management;

b) low biomass - active management.

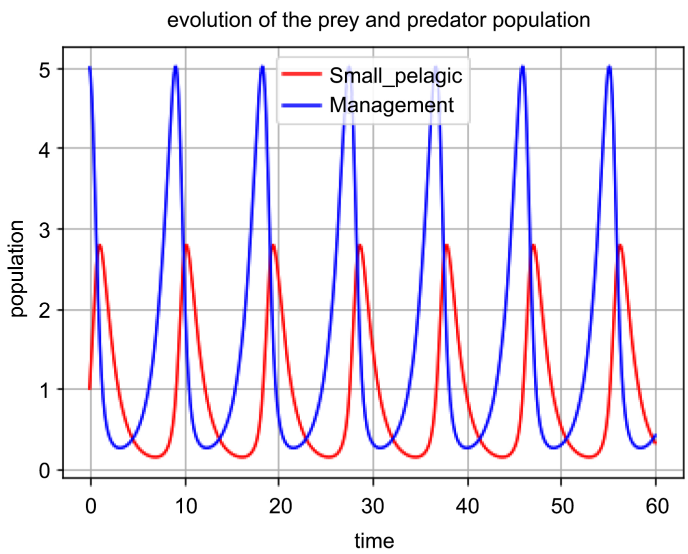

Figure 4. Interaction between fisheries managers and small pelagic resources.

System 2: predators/managers

1)

$$\begin{cases} Y' = Y(\delta W - \gamma) & (1) \\ W' = W(k_1 - k_3 Y) & (2) \\ (Y(0), W(0)) = (Y_0, W_0) & (3) \end{cases} \tag{12}$$

This system connects predatory fish with the fisheries manager. The growth of predatory resources (1) is controlled by the fisheries manager (2)

2) **Numerical results**

Figure 5 shows that the monitoring stocks of predatory resources is a necessity for a good management of fishing activity. This stock is the most important in this ecosystem sailor. The current dynamics of this family of resources is alarming. Fishery managers must monitor the behaviour of predatory resources at all times. by controlling their behaviour.

System 3: fishermen/managers

1)

$$\begin{cases} Z' = Z(\eta - \xi_1 W) & (1) \\ W' = W(\mu_1 Z - \mu_2) & (2) \\ (Z(0), W(0)) = (X_0, W_0) & (3) \end{cases} \tag{13}$$

This system connects fishermen and fisheries managers. This one allows for optimal maintenance of systems (11) and (12).

The manager has the possibility to control the activities of the fishermen. This control must be based on the control of the seabed and fish stocks. The function (1) models the evolution of fishermen under control.

Function (2) explains the behaviour of the manager in relation to that of the fishermen.

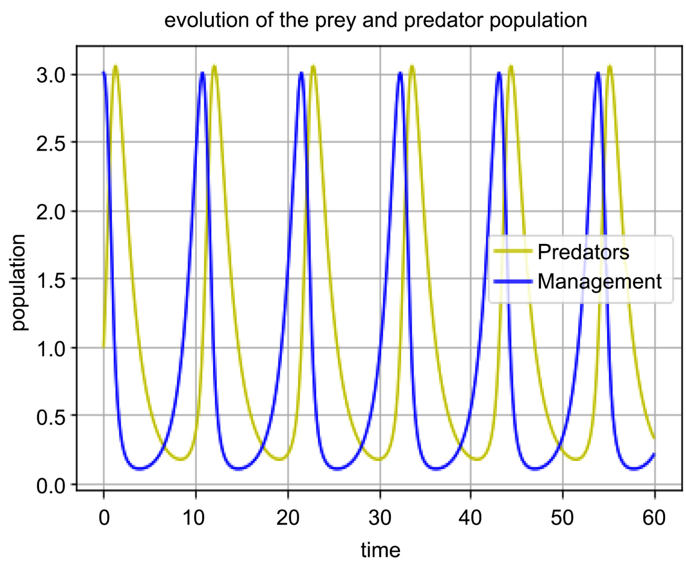

Figure 5. Interaction between manager and predatory.

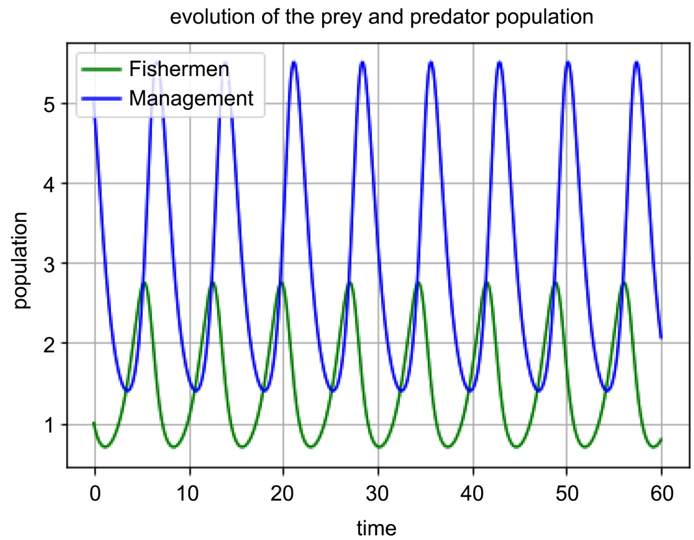

Figure 6. Interaction between managers and fishermen.

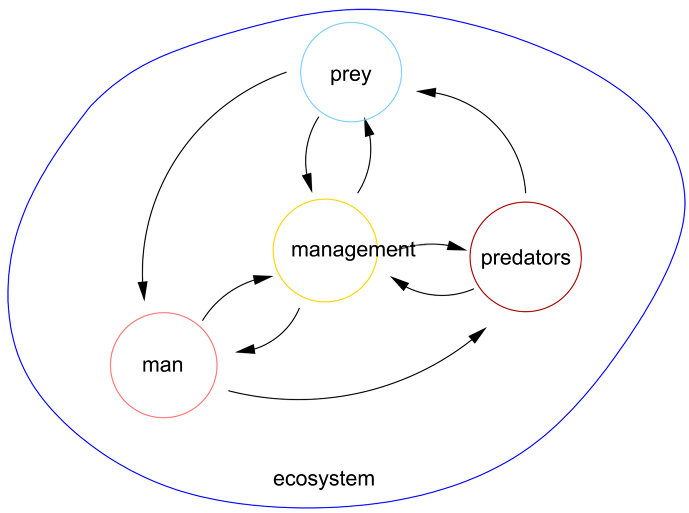

Figure 7. Graphical solution of the prey, predator, human and management system.

2) **Numerical results**

Figure 6 shows, as in logic, that the control must be at the level of the tool or the field controlled. Fisheries managers must be vigilant in order to effectively manage the problem of sea fishing. The graph shows that a fairly large number of managers allow optimal management of the activity. The behaviour of the two types of compartments in the system through this graph shows perfect stability and periodicity. Figure 7 shows graphically the solution to the problem.

4. Conclusion and Perspectives

In this section, we have developed two models. It is a system three-variable differential showing three different species interacting. This model has made it possible to describe the real consequences that threaten the human species if the same abusively exploitative behaviour continues. The second model (11) is an optimisation of the use of resources under the watchful eye of the peaches. The role of the manager is crucial for the balance of the marine ecosystem, because he can act at all levels separately. This study has made it possible to see the possible disturbances in a system of predators and of prey interacting and in the presence of the human hand. For better management of small pelagic resources, the solution lies in adopting the same behaviour as the natural biological interaction existing between predators and prey.

In other studies we will be interested:

- to the solution of the problem of fisheries management by graph theory;
- the optimisation of demersal resource landings in the Senegalese EEZ.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Auger, P. and Lett, C. (2019) Mathematical Models Can Help to Improve Managing Fisheries, Environmental Encyclopedia. https://www.encyclopedie-environnement.org/vivant/modeles-mathematiques-peche/

[2] Brochier, T., Auger, P., Thiam, N., Sow, M., Diouf, S., Sloterdijk, H. and Brehmer, P. (2015) Implementation of Artificial Habitats: Inside or Outside the Marine Protected Areas? Insights from a Mathematical Approach. *Ecological Modelling*, **297**, 98-106. https://doi.org/10.1016/j.ecolmodel.2014.10.034

[3] Cury, P. and Roy, C. (1991) West African Fisheries, Variability, Instability and Change. Scientific Editors, Paris.

[4] Diedhiou, I., Diagne, S.G., Thiam, N., Diop, C., Sow, F.N. and Ndiaye, G.B. (2018) Optimization of Sampling of Small Pelagic Fishes in the Exclusive Economic Zone of Senegal under the Climatic Impact. *Journal of Mathematics Researchers* (*JMR*), **10**, No. 4. https://doi.org/10.5539/jmr.v10n4p107

[5] Jacquot, G. (2019) Public Senat. Biodiversity: It Is Not Too Late. Biodiversit: The UN Shock Report. http://www.publicsenat.fr/article/societe/biodiversite-le-rapport-choc-de-l-onu-140868/

[6] Laroum, T. (2009) Multi-Agent Modeling in Population Ynamics, Application to the HIV-Memory Interface, Option Artitial Intelligence. Doctoral School of Computer Science in the East, 74 p.

[7] Miled, S.B. and Kebir, A. (2006) Modeling a Grouper Population, Poaching and migration. *Revue Africaine de la Recherche en Informatique et Mathématiques Appliquées*, **5**, 183-205. https://doi.org/10.46298/arima.1863

[8] Marchal, P. (2010) Dynamics of Fishing Mortality and Impact on the Management of Fisheries Resources. Authorization to Conduct Research, Research Framework at IFREMER. SIBAGHE Doctoral School, 43 p.

[9] Tewa, J.J. (2019) Modeling, Mathematical Analysis, Epidemiology. University of Yaound I, École Nationale Supérieure Polytechnique, Ummisco Centre of Yound.