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On the Controversy over the Logical Correctness of Einstein's First Paper on Mass-Energy Equivalence

Patrick Moylan, Lixian Yan, Michael Gironda

Department of Physics, The Pennsylvania State University, State College, USA
Email: pjm11@psu.edu, mtg5019@psu.edu, lpy5069@psu.edu

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Abstract

It is well-known that Einstein's first attempt to explain $E = mc^2$ which was published in *Annalen der Physik* in 1905, has been criticized as problematic. In particular, it has been shown by Ives and reiterated by Jammer that it suffers from the error of circular reasoning. Attempts have been made in the scientific literature to discount the circular reasoning objection of Ives, Jammer, Arzeliès and others. Fritz Rohrlich in 1990 gave a remarkably simple and concise derivation of $E = mc^2$ along lines similar to Einstein's but based on *both* momentum and energy conservation, in contrast to Einstein's which uses only energy considerations. Rohrlich's approach using momentum conservation is an alternative to Einstein's, which is free from objection in logical error, and we make it quite clear on the importance of the implicit assumption of momentum conservation in any attempt to refute the circular reasoning error in Einstein's paper. It is our contention that this point is overlooked or altogether avoided by those who have attempted to uproot the circular reasoning criticism of Einstein's paper.

Keywords

History of Science, Origins of Special Relativity

1. Introduction

In his first attempt at a derivation of mass-energy equivalence (Einstein, 1905b), which was published a few months after his first paper on special relativity (Einstein, 1905a), Einstein considers a body at rest in an inertial frame S that emits electromagnetic radiation of total energy L into two equal but oppositely directed amounts. He then considers the same emission process as seen from

another inertial frame S' moving with speed v relative to S. Applying his newly found discovery of the relativistic Doppler formula for light, which appeared in his first paper on relativity (Einstein, 1905a), along with the relativity of motion principle, Einstein was able to demonstrate that the mass of the body diminishes due to the emission of radiation, specifically by an amount which leads one to the conclusion that $E = mc^2$.

According to many scientists this first paper of Einstein on $E = mc^2$ suffers from the fallacy of circular reasoning. It involves a *petitio principii* or *circulus in probando*, in that the conclusions are embedded in the assumptions of what they are trying to prove. The most famous example of such *petitio principii* in mathematics is, beyond any doubt, the attempted proofs of Euclid's 5th axiom with Ptolemy, Omar Khayyám and Lambert among others falling victim to it.

The first person to criticize Einstein's 1905 paper on $E = mc^2$ was Planck (Planck, 1907) shortly after Einstein's paper was published. Years later, Herbert E. Ives, elaborating on Planck's remarks, interpreted Einstein's error as an error in logic: "*What Einstein did by setting down these equations* (our Equations (4) and (5)) (*as* "*clear*") *was to introduce*

$$L/\left(m-m'\right)c^2 = 1.$$

Now this is the very relation the derivation was supposed to yield (Ives, 1953)." This resulted in further criticism of the paper, most notably, by Max Jammer, the renowned physicist and philosopher of science (Jammer, 1961), and the well-known Einstein biographer, Arthur I. Miller (Miller, 1981). Additional investigations into the paper's shortcomings include those of reputable physicists such as H. Arzeliès (Arzeliès, 1966).

In more recent times, there have appeared papers, especially the paper of Stachel and Torretti (Stachel & Torretti, 1982) and of Fadner (Fadner, 1988), countering the claims of Planck (Planck, 1907), Ives (Ives, 1953), Jammer (Jammer, 1961), Miller (Miller, 1981) and Arzeliès (Arzeliès, 1966). According to them, Einstein's 1905 paper is not flawed, rather, it is their contention that "the paper (Einstein's) was basically sound" (Stachel & Torretti, 1982). Specifically, in the case of Stachel and Torretti: "we have to declare that Ives, Jammer, and Arzeliès—not Einstein—is guilty of a logical error" (Stachel & Torretti, 1982). The Stachel and Torretti paper is often referenced on Wikipedia and elsewhere on the internet as being the final judgement on the matter regarding the Planck-Ives criticism. Ohanian lends his support to Stachel and Torretti: "Stachel-Torretti was right in asserting that Ives, Jammer, and Arzeliès are wrong" (Ohanian, 2009). However, according to Ohanian, the point of view of Stachel and Torretti leads to even further difficulties, so that he still maintains that Einstein's paper is flawed (Ohanian, 2009). For more details on the history of $E = mc^2$ we refer the reader to the works of Sir E.T. Whittaker (Whittaker, 1973) and those of Max Jammer (Jammer, 1961; Jammer, 2000). For a recent article on the subject with a somewhat complete list of relevant references, we refer to the article by Hecht (Hecht, 2011).

An outline of the paper is as follows. In Section 2, we present a brief summary of Einstein's paper. Section 3 describes Rohrlich's derivation of $E = mc^2$. Section 4 is based on (Moylan et al., 2016) and it provides an explanation of Einstein's circular reasoning error in as simple as possible terms. In Section 5 we explain how Einstein's problematic assumptions (our Equations (4) and (5)) can be traced to momentum conservation. The results of this section make it clear how purported claims refuting the circular reasoning criticisms of Einstein's paper rest upon the assumption that Einstein implicitly made use of momentum conservation. In our conclusions, Section 5, we maintain that such purported counterclaims should be considered as valid only if one presupposes that Einstein made implicit use of momentum considerations and momentum conservation, something which is completely absent in his paper.

2. Summary of Einstein's 1905 Paper on $E = mc^2$

A brief summary of Einstein's derivation of $E = mc^2$ is as follows. We let E_0 be the energy of the body in the rest frame S of the body before the emission of electromagnetic energy in the form of two light rays in opposite directions, and we let E_1 be the energy of the body of mass m in S after the emission of light of total energy L. The results just stated, i.e. the various energies of the particle before and after emission of light and the total energy of the radiation, are listed in Table 1.

Similarly we let H_0 be the energy of the body in the frame S' before the emission of electromagnetic energy. We let H_1 be the energy of the body in S' after the emission of the radiation. We let K_0 be the kinetic energy of the body in the frame S' before the emission of electromagnetic energy and we let K_1 be the kinetic energy of the body in S' after the emission of the radiation. The energy of radiation in S' is L'.

Note that the speed of the body in S' both before and after the emission of light must be v, since, in the rest frame S of the body, the body is always at rest both before and after the emission of light. This fact is used by Einstein in obtaining his final conclusion (third to last sentence of his paper) on mass-energy equivalence out of his last equation.

Einstein in his first paper on special relativity (Einstein, 1905a) derived the relativistic Doppler formula for light from the Lorentz transformations; in particular, he established in that paper that if the radiation possesses in S a total energy L, then it possesses a total energy L' in S' with L and L' related as follows:

Table 1. Values of relevant quantities in rest frame S of particle.

S	Energy	Kinetic Energy
particle before emission of radiation	E_0	0
particle after emission of radiation	E_1	0
radiation	L	0

$$L' = L\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1}$$

where c is the velocity of light. Of course, in order for Equation (1) to make sense, v must be less than c, a fact which Einstein makes clear in his first paper on special relativity (Einstein, 1905a). The above stated results for energy considerations in S' are compiled in **Table 2**.

By making use of this equation and conservation of energy in the S and S' frames, respectively, Einstein obtains the following two equations:

$$E_0 = E_1 + \frac{1}{2}L + \frac{1}{2}L, \tag{2}$$

$$H_0 = H_1 + L\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{3}$$

Next Einstein comes to his assumptions criticized by Planck and Ives, which are:

$$H_0 - E_0 = K_0 + C, \tag{4}$$

$$H_1 - E_1 = K_1 + C. \tag{5}$$

where, according to Einstein, C is an additive constant which does not change during the emission of light.

From these assumptions together with the previous two equations, which were obtained from conservation of energy, Einstein obtains

$$K_0 - K_1 = L\left\{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right\}. \tag{6}$$

This is his second to last equation. His last equation is obtained from this by a Maclaurin series expansion of the r.h.s. which neglects terms of order higher than v^2/c^2. It is

$$K_0 - K_1 = \frac{1}{2}\frac{L}{c^2}v^2. \tag{7}$$

Table 2. Values of relevant quantities in the reference frame S' where the particle moves with speed v.

S'	Energy	Kinetic Energy
particle before emission of radiation	H_0	K_0
particle after emission of radiation	H_1	K_1
radiation	$L' = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$	0

To obtain his last statements (i.e. the four sentences after "From this equation it directly follows that:" one simply uses the non-relativistic formulae $K_0 = \frac{1}{2}mv^2$ and $K_1 = \frac{1}{2}m'v^2$ where m' is the mass of the body after emission of radiation.

3. Rohrlich's Derivation of $E = mc^2$

Fritz Rohrlich (1921-2018) was an outstanding theoretical physicist of Austrian-Jewish decent who contributed to the foundations of quantum and classical electrodynamics. He is perhaps most noted for his support of and further development of Fermi's viewpoint on classical electromagnetic models of the electron relating to their stability and Lorentz covariance (Fermi, 1922).

Rohrlich's derivation of $E = mc^2$, in so far as it is based on momentum conservation, can be traced back to Poincaré's treatment of mass-energy equivalence invoving a "Hertzian exciter emitting a pulse of radiation in a single direction" (Poincaré, 1900). For both Poincaré and Rohrlich the key ingredients are momentum of radiation, and momentum conservation. Apart from invoking Planck's quantum hypothesis and radiation momentum, the latter of which comes from Maxwell's electromagnetic theory, Rohrlich makes use only of non-relativistic considerations by assuming $v \ll c$. As such, it has at least as much validity as Einstein's does, since Einstein's derivation is also valid only to lowest order in v/c.

Rohrlich argument is based on the four following assumptions:[1]

1) The Newtonian fomulae for the kinetic energy and the linear momentum of a body of mass m and moving with speed $v \ll c$ are valid;

2) The laws of conservation of energy and momentum are valid;

3) Electromagnetic radiation of frequency f consists of photons of energy hf (Planck's quantum hypothesis (1900)) and momentum hf/c (momentum of radiation) where h is Planck's constant and c is the speed of light;

4) The Doppler effect, discovered by Christian Doppler in 1842 while a professor at the Polytechnic in Prague (Czech Technical University). In his most famous paper entitled, *On the Colored Light of the Double Stars and Certain Other Stars of the Heavens*, and presented to the Royal Bohemian Society of Science on the 25th of May, 1842 (Dopper, 1843), Doppler asserts that radiation of frequency f and speed c, as observed in a reference frame stationary relative to the rest frame of the source emitting the radiation, will be perceived to have that frequency altered by a factor $1+v/c$ $(1-v/c)$ when the observer moves relative to the source with speed v and in a direction toward it (away from it).

Rohrlich's argument runs as follows. A source of radiation emits two photons simultaneously while remaining at rest in some (Newtonian) inertial reference frame S. Conservation of momentum requires these two photons to have equal

[1]We would add to Rohrlich's list of assumptions the relativity of motion principle. He made use of it when he asserted that the body is at rest both before and after the radiation emission implies that the body in the moving reference frame "must therefore have the same speed v both before and after the emission."

and opposite momenta, and therefore equal frequencies *f*. Therefore, by Planck's quantum hypothesis (Rohrlich's assumption 3) they also have equal energies *hf*. Conservation of energy requires that the energy of the body diminishes by an amount

$$\Delta E = 2hf. \tag{8}$$

Next, Rohrlich considers, as does Einstein, the decay process from a reference frame S' moving with speed v relative to the rest frame S of the body. Conservation of momentum in S' leads him to the equation

$$p_i' = p_f' + \left(\frac{hf}{c}\right)\left(1+\frac{v}{c}\right) - \left(\frac{hf}{c}\right)\left(1-\frac{v}{c}\right) = p_f' + \Delta p' \tag{9}$$

where

$$\Delta p' = \left(2hf/c^2\right)v. \tag{10}$$

Since the body in reference frame S is at rest both before and after the emission of radiation, it must therefore have the same speed v in S' before and after the emission of radiation. Rohrlich's assumption (1) that the Newtonian formula for the momentum of the body hold true implies $p = mv$ and from Equation (10) this leads us to

$$\Delta m = 2hf/c^2. \tag{11}$$

Conservation of energy in S' gives

$$E_i' = E_f' + hf\left(1+v/c\right) + hf\left(1-v/c\right)$$

or

$$\Delta E' = E_f' - E_i' = 2hf = \Delta E. \tag{12}$$

Comparing Equation (8) and Equation (11) leads to

$$\Delta E = \Delta m c^2. \tag{13}$$

Rohrlich goes a step further and assumes that all of the mass of the body is used by the emitted photons. This means that the "mass then disappears, and its energy is present in the two photons that have total energy (Rohrlich, 1990) $E = m_i c^2$ where m_i the mass of the body.

4. The Simplest Possible Illustration of the Logical Fallacy in Einstein's Analysis

In order to better appreciate the logical fallacy in Einstein's argumentation leading to his last equation we take the special case considered by Rohrlich to which we just alluded, namely the case in which the particle ceases to exist after the emission of radiation. An example of such a process in particle physics is neutral pion decay:

$$\pi_0 \rightarrow \gamma + \gamma.$$

This decay is depicted in **Figure 1** for the case where the two photons fly off at angles φ and $\varphi + \pi$ with respect to the positive *x* axis. We consider the case

where $\varphi = 0$. In this case aberration does not enter into our discussion, since

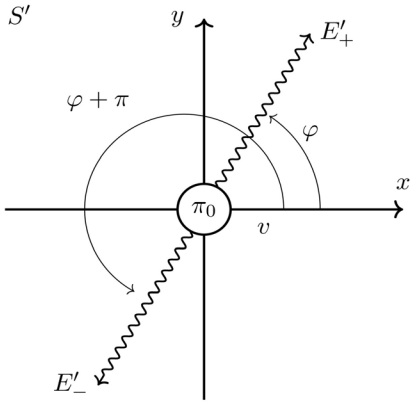

Figure 1. Pion decay as viewed in the reference frame S' where the pion is moving with speed v to the right. Note that $E'_+ + E'_- = L$ by conservation of energy.

the decay into radiation occurs along a line and the aberration effect is absent in one dimension. Let E_+ be the energy of the photon travelling in the right direction and E_- be the energy of the photon in the left direction as observed in the rest frame of the pion.

For the energies of the two photons in the frame S' we have the following situation. We let E'_+ be the energy of the photon in the "fore" (right) direction and E'_- is in the "back" (left) direction. Then, according to the relativistic Doppler formula for light (Einstein, 1905a; Moylan et al., 2016) we have for the energies $E'_\pm$ of the emitted photons in the reference frame S' in which the pion is moving to the right with speed v:

$$E'_\pm = E_\pm \frac{1 \pm \frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{14}$$

Now we come to Einstein's problematic assumptions, Equations (4) and (5), which, for our special situation, are easily seen to reduce to (cf. **Table 1** and **Table 2**):

$$\begin{gathered} H_0 - E_0 = K_0 + C \quad (4') \\ 0 - 0 = 0 + C \quad (5') \\ \Rightarrow C = 0 \\ \Rightarrow H_0 = K_0 + E_0. \end{gathered} \tag{15}$$

Conservation of energy in the S frame implies that

$$E_0 = E_+ + E_- = L. \tag{16}$$

Conservation of energy in the S' frame implies

$$H_0 = E'_+ + E'_- = E_+ \frac{1+\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} + E_- \frac{1-\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} = L \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{17}$$

Using Equations (15) and (16) together with this equation we obtain:

$$K_0 = H_0 - E_0 = L\left\{\frac{1}{\sqrt{1-v^2/c^2}} - 1\right\} \tag{18}$$

which is a special case of Einstein's second to last equation

$$K_0 - K_1 = L\left\{\frac{1}{\sqrt{1-v^2/c^2}} - 1\right\} \tag{19}$$

adapted to our situation of π_0 annihilation. (For us $K_1 = 0$, since there is no particle after the decay and thus no K_1 (kinetic energy of the particle after the decay)).

Einstein obtains energy-mass equivalence out of his second to last equation by using the non-relativistic approximation for kinetic energy and equating it to the lowest order term in a Maclaurin expansion of the right hand side this equation in powers of v^2/c^2. Adapted to our case for which $K_1 = 0$, this gives out of Equation (19)

$$\frac{1}{2}mv^2 + \cdots = L\left\{\frac{1}{\sqrt{1-v^2/c^2}} - 1\right\} = L\frac{1}{2}\frac{v^2}{c^2} + \cdots \tag{20}$$

where the dots mean terms of higher order in v/c than v^2/c^2. By comparing the right and left hand sides of this equation and neglecting the higher order terms, it leads us to

$$m = L/c^2. \tag{21}$$

Einstein in 1905 did not write down Equation (20). However, it is implicit in Einstein's statement in the 3rd to last sentence of his paper.[2] Since $E_0 = L$, we equivalently get

$$E_0 = mc^2. \tag{22}$$

To see the error of circular reasoning hidden in Einstein's analysis we reiterate his final analysis following from his second to last equation which is our Equation (19): "Neglecting magnitudes of fourth and higher orders we may place $K_0 - K_1 = \frac{1}{2}\frac{L}{c^2}v^2$."

This for our case is simply the statement that

$K_0 = H_0 - E_0$ to order v^2/c^2 plus conservation of energy to order v^2/c^2 implies $E_0 = mc^2$.

Since conservation of energy is always a true statement we are permitted to add this to the antecedent of the italicized statement to obtain the equivalent statement:

$K_0 = H_0 - E_0$ to order v^2/c^2 plus conservation of energy to order v^2/c^2 implies $E_0 = mc^2$ plus conservation of energy to order v^2/c^2.

If we can demonstrate that the converse statement is also true, namely that

[2]"The mass of a body is a measure of its energy-content; if the energy changes by *L*, the mass changes in the same sense by $L/9 \times 10^{20}$, the energy being measured in ergs, and the mass in grammes."

$E_0 = mc^2$ *plus conservation of energy to order* v^2/c^2 *implies* $K_0 = H_0 - E_0$ *to order* v^2/c^2 *plus conservation of energy to order* v^2/c^2, then we'll have shown that Einstein's statement suffers from the logical fallacy of circular reasoning. Following (Moylan et al., 2016) the argument runs as follows: conservation of energy in S' gives

$$H_0 = \frac{L}{\sqrt{1-v^2/c^2}}. \tag{23}$$

This to order v^2/c^2 reads

$$H_0 = L + \frac{1}{2}\frac{L}{c^2}v^2. \tag{24}$$

Now use conservation of energy in S which is $E_0 = L$ to get

$$H_0 = E_0 + \frac{1}{2}\frac{E_0}{c^2}v^2. \tag{25}$$

Finally use $E_0 = mc^2$ in this equation to get

$$H_0 = E_0 + \frac{1}{2}mv^2, \tag{26}$$

which is exactly what we need to show in order to demonstrate circular reasoning, since this equation is just a rewriting of the antecedent in the above converse statement, namely: $K_0 = H_0 - E_0$ *to order* v^2/c^2 where K_0 is the kinetic energy of the pion in S'.

5. The Role of Momentum Conservation in Eliminating Einstein's Circular Reasoning Error

We now show how assuming momentum conservation in S' together with momentum of radiation[3] which is (Halliday, Resnick, & Walker, 2011)

$$p_\gamma = \frac{E_\gamma}{c} \tag{27}$$

where E_γ is the energy of a photon of momentum p_γ remedies the above difficulties in Einstein's paper. Let p_+ and p_- be the magnitudes of the momenta of the the two photons of the previous section in "fore" and "back" directions, respectively. According to Equation (27) we have $p_+ = \frac{1}{2}\frac{L}{c}$ and $p_- = \frac{1}{2}\frac{L}{c}$, since $\frac{1}{2}L$ is the energy of each photon in the frame S. Since the pion is at rest in the reference frame S, momentum conservation in S reduces to:

$$p_+ = p_- \tag{28}$$

which implies from Equation (27) that $E_+ = E_-$, an assumption which Einstein made at the outset of his analysis, but did not provide any justification for it.

Without using momentum conservation, we are not allowed to assert

[3]Max Abraham, one of the most important figures in creation of classical electromagnetic theory and a very influential physicist at the time Einstein wrote his paper, attributes momentum of radiation, Equation (27), to the paper (Poincaré, 1900) i.e. to Poincaré. (Abraham, 1902; Abraham, 1903) Janssen and Mecklenburg, in Ref. (Janssen & Mecklenburg, 2006) also maintain that the concept originated with Poincaré.

$E_+ = E_-$. It violates the relativity principle: just as either of the two twins in the twin paradox (Schild, 1959) has the right to insist that he is at rest and it is the other twin who is in motion, so also here, an observer in S' has just as much right to assert that $E'_+ = E'_-$ as does an observer in S has the right to assert that $E_+ = E_-$. Momentum conservation, however, eliminates the second possibility, since, in S the total momentum before the pion decay is zero and so it must also be zero afterwards, which implies $p_+ = p_-$. Then, as before, using Equation (27) to relate momentum and energy, it follows that $E_+ = E_-$.

Conservation of momentum in the S' frame gives out of Equation (14) and momentum of radiation, i.e. Equation (27):

$$p = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v}{c^2}, \tag{29}$$

where p is the momentum of the pion in the reference frame S'. Using the non-relativistic formula for momentum of the pion i.e. $p = mv$ and a Maclaurin expansion of the right hand side of this equation to order v/c gives:

$$L = mc^2. \tag{30}$$

Finally conservation of energy in S which is $E_0 = L$ gives the desired result, namely:

$$E_0 = mc^2. \tag{31}$$

In fact, Equation (29) together with conservation of energy is also equivalent to $E_0 = mc^2$ plus conservation of energy, the argumentation being essentially identical to the above arguments establishing that "$H_0 = E_0 + K_0$ together with conservation of energy" is equivalent to "$E_0 = mc^2$ plus conservation of energy." The difference between the two is that $H_0 = E_0 + K_0$ is an unjustified "ad hoc" assumption made by Einstein. In our case Equation (29) is not an "ad hoc" assumption subject to dispute; rather it comes directly from momentum conservation, a law of physics, unambiguous as to its meaning and interpretation. This means that the arguments presented here in this section do not fall victim to circular reasoning. Momentum conservation falls into the category of one of n postulates of Fadner's theorem (Fadner, 1988), which theorem we do not dispute. Einstein's assumptions (our Equations (4) and (5)) do not. They are not laws of physics and cannot be viewed as such, since, as we have argued, *their meaning is ambiguous and open to different interpretations*.

Without using conservation of momentum, Einstein has no right to assert that K_0 in his formula $H_0 - E_0 = K_0$ is the kinetic energy of the pion. He does have the right to assert that it is the *kinetic energy of a particle of some mass, but not necessarily of mass m*. Without using momentum conservation, one can assert that K_0 in $H_0 - E_0 = K_0$ is the kinetic energy of the pion if and only if $E_0 = mc^2$ which is circular reasoning.

Finally we note that the peculiar objection to the Ives criticism raised by Fadner in (Fadner, 1988) is irrelevant from our way of looking at things. The n

postulates of Fadner's theorem are assumed to be true and unambiguous as regards to interpretation. According to what we have just explained in the preceding paragraph, this is not the case for Einstein's assumptions. Rather it is a matter of confusion in interpretation of K_0 in $H_0 - E_0 = K_0$. *Einstein's has no right to interpret* K_0 *as the relativistic kinetic energy of a particle of inertial mass* m, *but rather only the relativistic kinetic energy of a particle associated with some mass*, *possibly different than* m, unless, of course, he assumes that which he wants to prove, or invokes additional considerations involving momentum and conservation of momentum.

6. Conclusion

Rohrlich's analysis, which is based on momentum conservation, shows Einstein's assumptions, in addition to being problematic, are unnecessary. Rohrlich uses only the relativity principle, the conservation laws of energy and momentum, the Doppler effect and Poincaré's momentum of radiation to obtain $E = mc^2$.

Without the presupposition of momentum conservation, Einstein's argument, using only the relativity principle, conservation of energy and properties of radiation, to get at mass-energy equivalence is flawed. It suffers from the error of circular reasoning. It is clearly not the way to proceed. Rather, one should also make use of the other conservation law which we typically make use of for studying collision and emission processes, i.e. that of momentum conservation. There is no need for Einstein's problematic assumptions and there is no reason why Einstein could not have made use of momentum conservation in his paper, given that Poincaré had introduced momentum of radiation already in 1900, and that Einstein was most probably aware of it at the time he wrote his first papers on relativity (Martinez, 2009).

Stachel and Torretti (Stachel & Torretti, 1982) (and also Fadner (Fadner, 1988)) take it for granted that Einstein implicitly used momentum conservation; for they write in their paper that the body in its rest frame "loses energy but not momentum" (Stachel & Torretti, 1982). Using momentum conservation one is led to a unique interpretation of K_0 as the kinetic energy of a particle of mass m, in which case the basis on which the Ives objection rests is no longer true. This is why they can contend that "the paper (Einstein's) was basically sound" (Stachel & Torretti, 1982). In other words, they corrected Einstein's oversight by sneaking the premise of momentum conservation into Einstein's argument, so that they could get around the Ives criticism.

However, all three authors are silent as to what might have led Einstein to implicitly assume momentum conservation, nor do they explain how he could have made implicit use of momentum conservation even though Einstein never made any mention whatsoever of momentum considerations in his paper. Such would mean that he would have had to attributed momentum to radiation, a concept explained by Poincaré five years earlier (Poincaré 1900), (Abraham, 1902), (Abraham, 1903), (Janssen & Mecklenburg, 2006) and far from trivial at the begin-

ning of the twentieth century.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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