An Analytic Model for Representing Required Capacity of an Urban Bus Route

R. Jiang, Yu Zhou, Min Mi

Faculty of Automotive and Mechanical Engineering, Changsha University of Science and Technology, Changsha, Hunan 410114, China

jiang@csust.edu.cn

Abstract: To provide a desired service quality and cost effectiveness of urban bus transport service, it needs to solve a series of planning problems including various scheduling problems (e.g., vehicle scheduling, driver scheduling, maintenance scheduling, and so on). An important input of these planning problems is the required capacity to deliver passengers. In this paper, we give its definition and propose an analytic model to represent it. The model is a superposition of three parts. One part deals with the required capacity in off-peak hours and the other two parts represents the required capacities in morning and evening peak hours. A case study is included to illustrate how the model parameters are determined. A comparative analysis between the required and actual capacities is carried out to show the usefulness of the proposed model.

Keywords: Bus transport; service quality; operational planning; demand; capacity

1 Introduction

In China, most of the people in cities use urban bus transport systems to go to and get back from workplaces and schools. Though more and more people own themselves cars, it is crucially important to reduce the use of cars in urban environments by providing reliable and convenient public transit service so as to improve the environment, road safety and traffic congestion in urban areas, particularly in peak hours.

To maintain and enhance the public transit systems, a critical issue is to balance the service quality and cost effectiveness of the system. According to References [1] and [2], the present urban bus transport systems leave much to be desired in terms of outcomes for users. The most common bus service problems may be overcrowding and poor service reliability (in terms of on-time performance, headway between buses, running time, waiting time and so on). To improve the situation, many works can be done. For a detailed discussions on economics, policy and planning of bus transport, see Reference [3]; for a comprehensive review of transit economics, see Reference [4]; and for a detained discussion and review on planning problems of public transit, see Reference [5].

This paper deals with the planning problem of urban bus transport. We focus on modeling the required capacity derived from the passenger demand. The passenger demand for a given bus route can be represented by the total number of passengers in all the stops in the route in a unit time at time t, and the required capacity is defined as the one that the bus service should carry the passengers per unit time at t. It depends on the bus capacity and the road traffic condition (which impacts the bus speed). To model the demand, a detailed statistical analysis based on a large amount of field survey is needed. To guide the survey, reduce the survey burdens and represent the survey results, we propose an analytic model to represent the required capacity. The model is a superposition of three parts. The first part represents the required capacity in the off-peak hours, which is assumed to be proportional to a beta density function; the other two parts represent the required capacities in the morning and evening peak hours, respectively, which are assumed to be proportional to a normal density function. We present a method to determine the model parameters based on some field surveys. A case study is carried out to illustrate the modeling approach and its usefulness.

The paper is organized as follows. Section 2 deals with relevant concepts and definition. The proposed model and a case study are presented in Sections 3 and 4, respectively. The paper is concluded with a brief summary in Section 5.

2 Relevant concepts and definition

Urban bus transport is a service system. Passenger arrival behavior varies with the travel direction in a route

This research was supported by the Key Project of Scientific Research Fund of Hunan Provincial Education Department (No. 06A004).

978-1-935068-05-1 © 2010 SciRes.

and time (different time in a day, different day in a week, and different season in a year). In this section, we look at the passengers' arrival and transition behavior and their influence on the bus capacity requirement.

2.1 Route representation

A bus route can be represented by the following elements:

- Two terminals (which are sometimes the same) and two travel directions. We let A and B denote the terminals and AB [BA] denote the travel direction from Point A [B] to Point B [A].
- In the AB direction, there are $n-1$ stops: $\{S_i, 0 \le i \le n\}$ with $S_0 = A$ and $S_n = B$; and in the BA direction, there are $m-1$ stops: $\{s_i, 0 \le i \le m\}$ with $s_0 = B$ and $s_m = A$. Let K_i [k_i] denote the distance (in kilometer) between S_{i-1} [s_{i-1}] and S_i [s_i], $1 \le i \le n$ [$1 \le i \le m$]. The total length of the route in AB [BA] direction equals $K_{AB} = \sum_{i=1}^{n} K_i$ [$k_{BA} = \sum_{i=1}^{m} k_i$]. When we do not need to differentiate the travel direction, we adopt the notations of the AB direction.
- The first [last] bus leaves Point A at t_S [$t_E^{'}$]. The time from Point A to Point B and back to Point A in off-peak hours is τ. The service finishes at about $t_E = t_E^{'} + \tau$.
- N buses (monotypic or mixed) are used to provide the service. Each bus has a nominal seats number (seating and standing) n_b.
- The morning [evening] peak hours are known, in the period (t_{s1}, t_{e1}) [(t_{s2}, t_{e2})]. The other time periods are off-peak hours.
- A bus leaves Point A per Δt_p [Δt_o] minutes (i.e., theoretic headway) in peak [off-peak] hours. Let Δt denote the actual headway, which is often different from the theoretic headway, particularly in peak hours.

We define the actual capacity of the bus service as below:

$$y_a(t) = n_b / \Delta t . \tag{1}$$

Three factors that impact $y_a(t)$ are (a) the nominal seats number of the bus, (b) the theoretic headway (determined by the operational scheduling), and (c) the road traffic condition (congestion or not).

2.2 Arrival and transition behavior of passengers

For a given travel direction, let t and $t+\Delta t$ denote the instant when the first and second buses just leave Stop i. Let $n_i(t)$ denote the number of passengers who arrive at Stop i during ($t, t+\Delta t$). For a given time period, $n_i(t)$ is a random variable and usually follows a Poisson distribution or the like. In the following discussion, we consider its average. The total number of passengers on the line is given by:

$$N(t) = \sum_{i=0}^{n-1} n_i(t), \tag{2}$$

And the total number of passengers on the line per unit time is given by:

$$D(t) = \frac{N(t)}{\Delta t}, \tag{3}$$

The passengers can have different origins and destinations. The number of passengers in the vehicle depends on the distributions of these origins and destinations. This information can be represented by an origin-destination (or O-D) matrix

$$M_{OD} = [p_{ij}, j > i, 0 \le i \le n-1, p_{ij} \in (0,1), p_{n-1,n} = 1], \sum_{j=i+1}^{n} p_{ij} = 1, \tag{4}$$

where p_{ij} is a probability or proportion. It is noted that the O-D matrix has $n(n-1)/2$ independent parameters.

Let n_{di} denote the numbers of passengers getting-down at Stop i. If the arrival rate of passengers is approximately a constant, the bus capacity is sufficiently large so that all the passengers can board on the bus and their transitions follow (4), we have

$$n_{di} = \sum_{l=0}^{i-1} n_l p_{li}, \tag{5}$$

Discussions: The "sufficiently large capacity" assumption is plausible for off-peak hours but not true for peak hours. To consider the finite capacity case, we divide $n_i(t)$ into two parts: $n_i^{(b)}$ and $n_i^{(d)}$, where $n_i^{(b)}$ is the number of the passengers who get on board and $n_i^{(d)}$ is the number of the passengers who have not gotten on board. To get the actual demand information, we should include the information of $n_i^{(d)}$ and assume that their transitions would follow (4).

Let N_i and N_{di} denote the cumulative numbers of passengers boarding-on and getting-down at Stop i, respectively. We have

$$N_i = \sum_{l=0}^{i-1} n_l, \quad N_{di} = \sum_{l=1}^{i} n_{dl}. \tag{6}$$

The total number of passengers on the bus between Stops i and $i+1$ is given by

$$n_p(i) = N_i - N_{di} \geq 0 . \qquad (7)$$

Clearly, for the purpose of operational scheduling, $n_p(i)$ is more meaningful than $N(t)$. The passenger transition behavior in a run is illustrated in Figure 1.

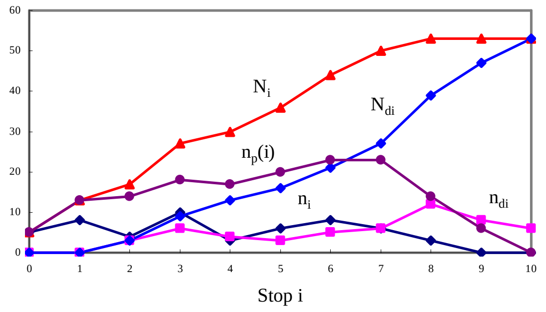

Figure 1. Passengers distribution in a route

2.3 Required capacity

Let

$$n_{p,\max} = \max[n_p(i), 0 \leq i \leq n-1] , \qquad (8)$$

The required capacity of the route at time t can be represented by

$$y(t) = n_{p,\max} / \Delta t , \qquad (9)$$

For the case shown in Figure 1, if $\Delta t = 5$ minutes, then we have:

$n_{p,\max} = 23$, $y(t) = 4.6$ (passengers per minute).

2.4 Some important measures

Service quality (or social benefit) relevant measures include:

1) the total number of passengers delivered is N_n, which also represents the fare box revenues of the run,

2) the total passenger-kilometers of the run given by

$$K_p = \sum_{i=1}^{n} n_p(i) K_i , \qquad (10)$$

3) the unmet demand, which represents service unreliability or social loss, given by

$$L(t) = \sum_{i=1}^{n} n_i^{(d)}(i) \qquad (11)$$

4) the total operational time given by $\tau = \sum_{i=1}^{n} \tau_i$,

where τ_i denote the time (in minute) from the instant when the bus leaves Stop $i-1$ to the instant when the bus leaves Stop i (or arrives at Stop n).

The capacity utilization measures include:

1) relative load ratio is given by $\overline{n}_p / n_b$, where $\overline{n}_p$ is the average number of passengers given by

$$\overline{n}_p = \frac{1}{n} \sum_{i=0}^{n-1} n_p(i) , \qquad (12)$$

2) load unevenness given by $\delta = n_{p,\max} / \overline{n}_p$.

3 Model for representing required capacity

We use (13) to represent $y(t)$:

$$y(t) = \delta_0 (t - t_S)^p (t_E - t)^q + \delta_1 \exp[-(\frac{t-\mu_1}{\sqrt{2}\sigma_1})^2] + \delta_2 \exp[-(\frac{t-\mu_2}{\sqrt{2}\sigma_2})^2] \qquad (13)$$

The first term of RHS of (13) is proportional to a beta density function and describes the required capacity in off-peak hours, the second term is proportional to a normal density function and describes the additional capacity requirement in the morning peak hours, and the third term is also proportional to a normal density function and describes the additional capacity requirement in the evening peak hours.

The model contains nine parameters ($\delta_j, 1 \leq j \leq 3, p, q, \mu_l, \sigma_l, l = 1, 2$). The procedure to determine these parameters are as follows.

Suppose that we have completed field survey and obtained a set of observations of $\{t, n_{p,\max}, \Delta t\}$. Step 1 is to use those observations in the off-peak hours to determine the values of δ_0, p and q using a least squared error approach. Generally, p and q are larger than but very close to zero since a constant demand requires $p = q = 0$ and the demand is close to zero at t_S and t_E.

The next step is to use those observations in the morning peak hours to determine the values of δ_1, μ_1 and σ_1. For this case, we have:

$$y(t) - \delta_0 (t - t_S)^p (t_E - t)^q \approx \delta_1 \exp[-(\frac{t-\mu_1}{\sigma_1})^2 / 2] . \qquad (14)$$

The parameters can be determined using the least squared error method.

The final step is to use those observations in the evening peak hours to determine the values of δ_2, μ_2 and σ_2. For this case, we have:

$$y(t) - \delta_0 (t - t_S)^p (t_E - t)^q \approx \delta_2 \exp[-(\frac{t-\mu_2}{\sigma_2})^2 / 2] . \qquad (14)$$

The parameters can be determined in a similar way.

978-1-935068-05-1 © 2010 SciRes.

4 Case studies

4.1 Route characteristics

The proposed model is illustrated using a case study in this section we focus on a certain route in the Changsha city. The characteristics of the route are as follows.

- Two travel directions are not overlapped with $n=35$ and $m=29$.
- $t_S = 6:20$, $t_E^{'} = 21:30$, $\tau = 80$ minutes. As a result, $t_E \approx 22:50$.
- 35 monotypic buses with $n_b = 86$ run in the route to provide the service.
- The morning and evening peak hours are in the period (7:00, 9:00) and (16:30, 18:30), respectively.
- The theoretic headway is 3 minutes for the peak hours and 5 minutes for the off-peak hours.

4.2 Data and estimated parameters

The field survey was conducted in two stages. The first stage aimed to seek the stop where $n_p(i)$ achieves its maximum. This was done by several travels on the board. Once the stop was identified, the survey was conducted in the identified stop. When a bus arrives at the stop, the on-boarding passengers (and the passengers that failed to catch the bus if applicable) were quickly counted. We assume that the counting error follows a normal distribution with a zero mean (i.e., no systematic errors). In such a way, we obtained 115 observations during the period from 7:08 to 19:38 of a Monday. Among them, 32 observations are used to fit the additional morning-peak demand, 26 to fit the additional evening-peak demand, and 57 to fit the off-peak demand.

Using the data obtained from the field survey and the approach outlined in Section 3, we have the following estimated parameters:

$$(\delta_0,\ p,\ q) = (7.8007, 0.0000, 0.0712),$$
$$(\delta_1,\ \mu_1,\ \sigma_1) = (12.2478, 8.2552, 0.5984),$$
$$(\delta_2,\ \mu_2,\ \sigma_2) = (18.0865, 18.3589, 0.5742).$$

Namely, the maximum demand in the morning occurs at 8:15, and the maximum demand in the evening occurs at 18:22. If we take $\mu \pm 2\sigma$ as the peak hours, from the fitted model, the morning peak-hours are in the period (7:03, 9:27), and the evening peak-hours are in the period (17:12, 19:30). These are consistent with our intuition.

4.3 Analysis and discussion

The actual headway can be derived from the observed data. It varies with time as shown in Table 1. As a result, we can calculate the actual capacity from (1) and the required capacity from (13), and they are shown in Figure 2.

Table 1. Observed headway at a fixed stop

7:08-9:00	9:05-10:28	14:56-16:27	17:19-19:38	Other
4.44 min	3.67 min	3.50 min	4.48 min	5 min

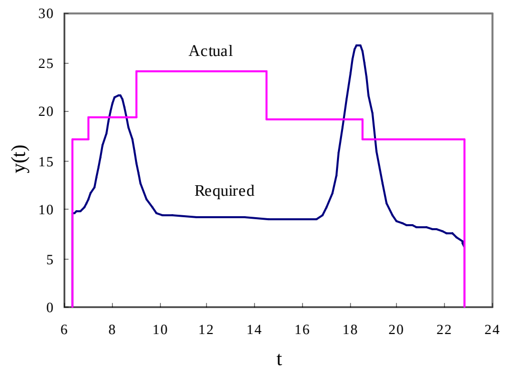

Figure 2. Actual and required capacities of the route

From Table 1 and Figure 2, we have the following two important observations:

1) The actual headway in the peak hours is longer than the theoretic headway in peak hours and shorter than the theoretic headway in off-peak hours. This implies that the actual headway is heavily dependent on the traffic condition rather than only dependent on the operational scheduling.

2) The actual capacity is much larger than the required capacity in off-peak hours and much smaller than the required capacity in peak hours.

To improve the situation, both the bus operator and government need to make some efforts. For the operator, more practical strategies can be offered to monitor and improve the quality of bus service, including the following:

1) To meet the demand in peak hours, buses with a larger nominal seats number should be used since it is difficult to increase the capacity by shortening the theoretic headway.

2) There are two ways to reduce the supplied capacity in off-peak hours. One is to adjust the service

frequency but this will reduce the service quality and probably fare box revenues. The other is to use smaller buses (i.e., with fewer nominal seat number). This will reduce the operational cost (e.g., fuel consumption) and maintain an appropriate service quality but probably increase the capital costs. As a result, a detailed analysis is needed to optimize a mixed-type bus fleet to achieve a good tradeoff between the service quality and cost effectiveness (e.g., see References [4] and [6]).

For the government, a successful practice is to use the exclusive bus lanes to reduce the actual headway in the peak hours.

5 Conclusions and discussion

In this paper, we have defined the concept of a required capacity for a bus route and proposed a three-fold mixture model to represent the required capacity. A case study has been presented to illustrate its usefulness in evaluating the current practices and future directions for improvement.

Two important findings have been:

- The actual service frequency is heavily dependent on the traffic condition in the peak hours.
- The actually supplied capacity does not match the required capacity in both off-peak and peak hours using a monotypic bus fleet.

To improve the situation, the operator needs to look at various planning problems to optimize its service networks, routes and stops, fleet size and operational schedule so as to minimize its operational cost without loss of service quality. Similarly, the government can play an important rule, e.g., designing appropriate public transport policy, giving special rights-of-way for buses, giving operators financial incentives for passenger growth and service quality, setting down the service quality relevant requirements and monitoring the actual performances of service quality, and so on.

References

[1] Xumei Chen, Lei Yu, Yushi Zhang, Jifu Guo. Analyzing urban bus service reliability at the stop, route, and network levels [J]. *Transportation Research Part A: Policy and Practice*, 2009, 43(8), P722-734.

[2] Madhav G. Badami, Murtaza Haider. An analysis of public bus transit performance in Indian cities [J]. *Transportation Research Part A: Policy and Practice*, 2007, 41(10), P961-981.

[3] David A. Hensher. Bus transport: Economics, policy and planning [M]. Research in Transportation Economics, 2007. 1-507.

[4] Ken Gwilliam. A review of issues in transit economics [J]. *Research in Transportation Economics*, 2008, 23(1), P4–22.

[5] Guy Desaulniers, Mark D. Hickman. Handbooks in Operations Research and Management Science, 2007. 69-127.

[6] Oldfield R. H, Bly P. H. An analytic investigation of optimal bus size [M]. *Transportation Research Part B: Methodological*, 1988, 22(5), P319-337.