

Hydropower Production Optimization from Inflow: Case Study of Songloulou Hydroplant

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Abstract

The model of nonlinear power generation function is developed to generate optimal operational policies for Songloulou inflow in Cameroon and test these policies in real time conditions. Our model is used to adjust operational regimes for the Songloulou reservoir under varying flows (turbined and de-versed) using a dynamic program. A more interesting approach, proposed in this article, consists of combining both the principle of decomposition by resources (or quantities) and the technique of dynamic programming. Dynamic programming is an appropriating optimization algorithm that is used for complex non-linear inflow operational policies and strategies. In this case study, our optimization model is used and confirmed maximizing large scale of hydropower in a period of time step by the integration of several. The high non linearity of our study object is the first stage of difficulty which brought us to combined least squared and Time Varying Acceleration Coefficients Particle Swarm (TVACPSO) to obtain appropriate production function which generated optimal operational policies for the Songloulou hydropower in sub-Saharan region and after we tested it in the company policies operational at real time conditions. The model could be successfully applied to other hydropower dams in the region.

Keywords

Hydropower, Reservoir, Optimization, Simulation, Dynamic Programming

1. Introduction

Furthermore, Cameroon is the second highest hydro potential in Central Africa after the Democratic Republic of Congo [1]. With its vast watershed representing more than 25% of the total area of the country and its numerous rapids, the Sanaga, the largest river in the Republic of Cameroon, constitutes a first-rate hydroelectric energy reservoir. The Songloulou hydroelectric dam is the largest hydroelectric plant in Cameroon. Located on the Sanaga in the Massock Songloulou commune, about 55 km to the northwest and upstream of Edea, the Songloulou hydroelectric power station has a capacity of 394 MW as run-of the-river hydroplant. Its retention dimension is 528 meters. The Songloulou power station has 8 Francis groups with a capacity of 49.5 MW. Songloulou is now the biggest power plant which makes him the center piece in production planning. Cameroon faces a deficit in electrical energy despite government efforts to resolve this through the construction of new power plants currently underway. There is an urgent need to start economic and social development towards an emergence planned for 2035. In view of the above, it would be important to have a tool that corresponds to the environmental, technological and economic realities to match the power demand. Many research themes relate to improving energy quality and increasing electricity production capacity for both renewable and fossil sources [2]-[11].

Hydro power is represented by a nonlinear function of the turbined flow and of the storage in the reservoirs, and sometimes also of the spillage. In literature, many works are implemented on the modeling and scheduling of the hydro-power in the long, mid and short terms planning by taking into account hill curves, input/output relationship data collection, forbidden operating zones [12] [13]. Usual procedures of mathematic modeling are to represent the power as a family of nonlinear concave functions of the discharge, one at each given head [14] [15] [16], or as a unique function of both discharge and storage [17]. Formulations as seen in literature depend on the data sets of the hydro plant, as shown in [18] [19] [20]. In this paper, a simulation of the using of the best alternatives in the hourly dispatch of Songloulou hydro plants is performed, and the deviations from the gap operation obtained are presented under statistical correlation factor. The objective of this paper is to build a combined optimization-simulation dynamic model with the following aims:

- Construct a mathematical model for each hydro unit power factor by least squared with Time Varying Acceleration Coefficients Particle Swarm (TVACPSO) regression technique from values obtained in company data during chosen highly non linearity operational time.
- Generate an optimal policy planning, which consists to maximize the production of each unit at the level of the upstream by developing mathematical model of downstream as well as the possible flow and the spillage took account
- Compare the generate policies with the currently existed historical input/output data.

The design model uses the dynamic structure due to the stochasticity of the reservoir inflow with his spillage for respect the maximum level of upstream. The Songloulou hydroplant is used here as a case study. Eneo is responsible for the management, operation and maintenance of the hydraulic layout of the power plant. Currently, the reservoir releases are done to match the energy demands.

This paper is organized as follows: Section 2 describes the hydro power generation function and makes review of mathematical formulations. Section 3 presents the dynamic programming used here in our optimal planification. We show how solving the generalized problem with physical and hydraulic constraints can be done effectively inside a scheme which combine both PD's technic and decomposition's method with resources. Section 4 is the phase of development of the model in detail. Numerical results are shown then discuss and the conclusion are given in Section 6.

2. Production Function of Hydro Generation

The goal of the production function is to quantify the generation power of a hydropower plant. The calculus procedure takes account the difference between forebay and tailrace levels and the water discharge by applying the losses of the turbine-generator or group set and penstock [14] [21] [22] [23]:

$$P = k \cdot \eta_t \cdot \eta_g \cdot [h_{fb}(V) - h_r(u) - h_{pl}] \quad (1)$$

For individual generating unit of the plant by taking into account the losses of the turbine-generator or group set and the penstock, Equation (1) can be rewritten as follows:

$$P = k \cdot \eta^G \cdot h_g \cdot q \quad (2)$$

This transformation of net head to gross head does not compromise the record of plant operations because, in general, values of turbine efficiency, generator efficiency, and penstock head loss are not recorded separately in the database for each hour (or half-hour) of operation [16].

Usually, almost head values are calculated by computational models developed for the hydroelectric system's operation.

2.1. Data Collection and Area of Modeling

Development of hydroelectricity power plant model required the following records data use during one year since June 2017 to May 2018 around 8760 hours:

- Hill chart of typical turbine unit of the plant,
- Hourly power generation, upstream elevation, downstream elevation, turbined flow, deversing flow, gross head of each unit.

These data serve to study the schedule of hydro plant with:

- Turbo-generator model,
- Tailrace elevation model.

2.2. Songloulou Hydroelectric Plant Model

Songloulou is a run-of-the river plant which produces the high generation electricity in the country. Scheduling of hydro power plants needs an expression output power in terms of head and discharges, *i.e.*, hydro turbine model, reservoir elevation model & tailrace elevation model. Mathematical model required to explain the proper scheduling to help maximize the power generation.

Modeling of the hydroelectric plant is a very complex task and as such there is no uniform modeling as each one is unique to its location and requirement. The diversity of these designs makes it necessary to model each one individually. The parameters of modeling are nonlinear and highly dependent on the control variables. In case of the power station study, the coefficients have to be determined by collating precedent announcing plant data.

1) Turbo-generator model

Under uncertain raining conditions, maximizing of hydropower becomes a challenging task which is the scope of all producer. Of course, the mathematical formulation took into account the head variation over the operation periods as the net head changes from hour to hour and affected power generation. Additionally, the formulation considered the operation of 8 heterogeneous generating units and the nonlinear power generation function of each unit. A three-dimensional interpolation technique is used to accurately represent the nonlinear power generation function of each unit, taking into account the time-varying head as well as the nonsmooth limitations for power output and power release. It is very important to model the turbo-generator as it is a part of the objective function in the hydroelectric scheduling problem. This model relates the magnitude of power generated in terms of head and discharge as shown previously. As in this case, power is a dependent variable, whereas head and discharge are the independent variables, so it can be modeled using multiple regression analysis. The mathematical model of the individual hydro plant is obtained by minimizing the average square error of the power values provided by the model as compared with the collecting data as in [15] with TVACPSO [24].

$$P_i = a_0 H b * Q_i + a_1 Q_i * H b^3 + a_2 H b^2 * Q_i^2 + a_3 Q_i^3 + a_4 H b^2 * Q_i^4 + a_5 Q_i^5 \quad (3)$$

In order to ensure that we have an appreciable mathematical model, we could calculate the factor of correlation. By convention, we will say that the relation between single X and Y is:

- perfect if $r^2 = 1$.
- very strong if $r^2 > 0.8$.
- strong if r^2 is between 0.5 and 0.8.
- medium intensity if r^2 is between 0.2 and 0.5.
- weak if r^2 is between 0 and 0.2.
- Null if $r^2 = 0$.

We evaluate quality of our model by calculating coefficient of correlation.

Table 1 presents each individual turbo-generator model below.

Table 1. Individual unit model of Songloulou hydro plant.

No. of turbo-generator	Turbo-generator unit model	Coefficients of each unit	Coefficient of correlation r^2
1		$a_0 = 1.316679217634740e-02$ $a_1 = -2.715864995727833e-06$ $a_2 = -3.733512519404666e-08$ $a_3 = 1.074220489716463e-06$ $a_4 = -4.641641738924982e-12$ $a_5 = 1.697020112231463e-11$	9.976130591100405e-01
2		$a_0 = 1.322042487911197e-02$ $a_1 = -2.807677454934436e-06$ $a_2 = 2.168789884485183e-08$ $a_3 = 5.467947653238211e-07$ $a_4 = -5.995505182364090e-12$ $a_5 = 3.477122715583562e-11$	9.971746324792786e-01
3		$a_0 = 1.313878893709308e-02$ $a_1 = -2.729078969976513e-06$ $a_2 = -7.486153024467089e-09$ $a_3 = 8.667242388630805e-07$ $a_4 = -5.903715276125817e-12$ $a_5 = 2.983455647910659e-11$	9.971567005397561e-01
4		$a_0 = 1.316774373781319e-02$ $a_1 = -2.749230895725366e-06$ $a_2 = -5.499993769277984e-09$ $a_3 = 8.273787397806270e-07$ $a_4 = -5.706247574557738e-12$ $a_5 = 2.869290899153645e-11$	9.969636630527057e-01
	$p_i = a_0 Hb * Q_i + a_1 Q_i * Hb^3 + a_2 Hb^2 * Q_i^2$ $+ a_3 Q_i^3 + a_4 Hb^2 * Q_i^4 + a_5 Q_i^5$ <p style="text-align: center;">Avec $i = 1, \dots, 8$</p>		
5		$a_0 = 1.315808878162106e-02$ $a_1 = -2.725205361635298e-06$ $a_2 = -2.150986014924703e-08$ $a_3 = 9.588104330438762e-07$ $a_4 = -5.377147197223959e-12$ $a_5 = 2.457111076654736e-11$	9.975286024741136e-01
6		$a_0 = 1.318297544416945e-02$ $a_1 = -2.773909771353523e-06$ $a_2 = 9.156785037522066e-09$ $a_3 = 7.008256127892839e-07$ $a_4 = -5.939328289569685e-12$ $a_5 = 3.185215586671757e-11$	9.983840447826141e-01
7		$a_0 = 1.311543722428082e-02$ $a_1 = -2.715389147495682e-06$ $a_2 = -9.287696266877517e-09$ $a_3 = 9.426327676216690e-07$ $a_4 = -6.181351649510719e-12$ $a_5 = 3.027123318831389e-11$	9.977870255672673e-01
8		$a_0 = 1.319119946943368e-02$ $a_1 = -2.764287356826369e-06$ $a_2 = -2.450985943737339e-09$ $a_3 = 7.530299368429817e-07$ $a_4 = -5.503092320525912e-12$ $a_5 = 2.851255228666815e-11$	9.980396489376582e-01

2) Tailrace elevation model

As per design of the hydroelectric plant, the station discharge either through turbine or spillway can raise the tailrace elevation, which decreases the effective head. This model related the water level in the tailrace channel with total discharge (discharge through turbine & spillway) through hydro power plant. Tailrace elevation is the independent variable and total discharge is a dependent variable. The model can be obtained by the same development as turbo-generator model by using non linear least squared.

$$h_{tl} = -1.441091209563667 \times 10^{-5} Q_{tot}^{1.68178} + 4.569880405719042 \times 10^2 Q_{tot}^{0.00969} + 3.001647346575221 \times 10^{-11} Q_{tot}^{3.21} \quad (4)$$

with correlation coefficient $r^2 = 0.9999999087038359$.

Given the slight variation in the upstream elevation, we had prohibited the determination of its model according to the volume of the tank. In our optimization, the hydro plant manager introduces the level of the upstream elevation and the specific total flow before computing the planing program.

3. Optimal Planification of Songloulou Hydro Plant

3.1. Concepts of Dynamic Programming Dispatching

Short-term planning (horizon of a few days with an hourly step) takes into account the global decisions made by medium-term planning and provides a plant operation plan with all the considerations linked to the choice of turbine generator sets to be engaged or remove for maintenance, prohibited areas of operation from groups, etc. This work took account decision variables as turbined flow and deversing flow to expect maximizing the power generation of Songloulou plant. One of the best method of dispatching of this kind of problem is dynamic programming which took account all sub-problems during the process.

Dynamic programming has stages and states. Each sub-problem is a step whereas at each step, a state of the system is defined. The solution is built by back induction, that is to say that the resolution of the problem begins at the last step and the sub-problems are solved by going back one step each time, until all the sub -problems be resolved. One of the major problems with this method is that quickly the problem becomes difficult to solve given the number of variables. In fact, there is a need to discretize state space. For example, in hydroelectric optimization, we must discretize the volume, the unit flow, the total flow, without forgetting that we must consider the number of turbines as well as the periods. Obviously, this enumeration depends on the modeling of the problem. One of the advantages of dynamic programming is that non-linearities are easily handled, since all combinations of variables are evaluated. The principal inconvenient of dynamic programming is when the number of state variable become high [25] [26]. In this paper, modified dynamic programming is proposed by avoiding high dimension of the variable state. So we have divided the main problem into many subproblems with equal constraints.

Literature shows that some authors of [27] [28] [29] use dynamic programming to determine the number of turbines in operation on an hourly basis. To do this, the steps are the hours, the states are the number of turbines running at each step and the decision variables are the number of starts and stops of turbines at each step. A compromise between the efficiency of the power produced and the starts is minimized in the objective function of the dynamic programming problem. The authors do not consider the water constraints, because the upstream elevation does not vary much over hourly periods. In our modelization, we consider the variations of upstream elevation over hourly periods but not took account starts up and start down of turbines cost.

3.2. Formulation of Songloulou Problem of Maximization

The conventional formulation problem (C) of dynamic programming is:

$$\max \sum_{j=1}^8 P_j(Q_j) \quad (5)$$

$$\sum_{i=1}^8 q_i = Q_T \quad (6)$$

$$q_i \in D_i \quad (7)$$

The conventional formulation has a flaw that is open to criticism. Indeed, the use of the equality constraint (6) to model the total flow to be discharged by the power station can generate a certain ineffectiveness due to the presence of the prohibited zones or global constraints on a subset of groups (like the constraints of terminals on the power to generate).

The second ineffective situation of the model (C) is obtained by combining the equality constraint (6) with an overall power constraint. To do this, we consider two identical groups whose power-flow function is illustrated in **Figure 1**. The global constraint on the two groups is of type:

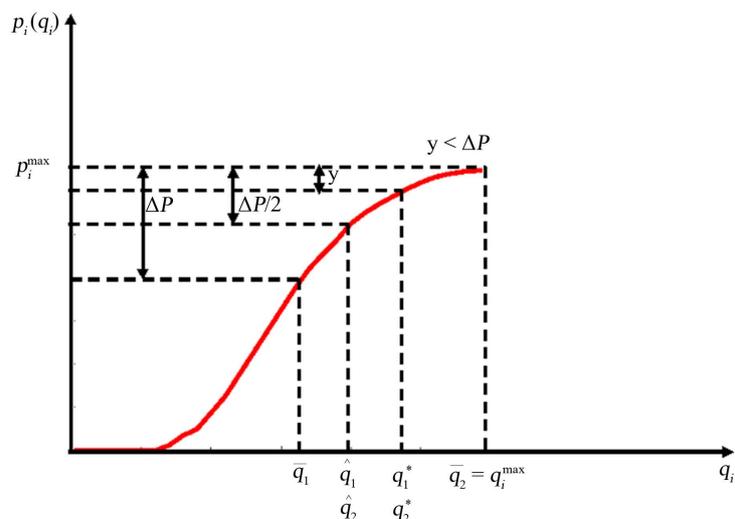


Figure 1. Example with global power constraint.

$$p_1(q_1) + p_2(q_2) \leq 2p_1^{\max} - \Delta P \quad (8)$$

With p_1, p_2 the power of the two groups. $\Delta P = p_2 - p_1$.

The solution of the problem, neglecting the global constraint (8), consists in sharing the production between the two groups, that is to say $q_1^* = q_2^* = \frac{Q_T}{2}$.

The solution of the problem (C) to which we add the constraint [8] is (\bar{q}_1, \bar{q}_2) thus totaling a power of $(2p_1^{\max} - \Delta P)$. An alternative solution (\hat{q}_1, \hat{q}_2) (satisfying the constraint (8)) generates the same power by discharging less flow. This solution is also obtained by substituting the equality constraint (6) with an inequality.

The two examples show that the formulation of the constraint (6) in the form of equality can generate a certain ineffectiveness in the loading of the groups. For this reason, in what follows and along the chapters of this thesis, we opt for the formulation of this constraint in the form of inferior or equal inequality (\leq).

Our proposed formulation optimization problem (G) of Songloulou hydro plant is:

$$\max \sum_{j=1}^8 P_j(Q_j) \quad (9)$$

With Q_j the total allocated flow of each turbine and P_j the production function of each unit.

The constrained use in our problem is:

$$\sum_{j=1}^8 Q_j \leq Q_{tot} \quad (10)$$

$$Q_j \geq 0 \quad (11)$$

$$Q_i \in D_i \quad (12)$$

where D_i is admissible interval of turbine, Q_i unit discharge, Q_{tot} total discharge.

Note that the sum of the flows can be lower than the total flow allocated, because in some cases, it is necessary to pour water from the tank directly if you do not want to lose power. Unlike the model (C), the set point constraint (10) is an inequality.

Then, after having mathematically modeled our problem, we were able to reflect on the dynamic programming of it. We therefore determined:

- Steps
- The states
- Decision variables

Our goal is to determine the flow to allocate per turbine from the total flow (Q_{tot}).

So we have the decision variable Q_j , which is the amount of flow allocated to the turbine. Each turbine is a step. The states are Q_r the flow remaining to be allocated.

For the programming language, we chose to use python 3 because it is an ob-

ject-oriented language that is easy to use and very efficient for mathematical calculations.

A more interesting approach, proposed in this article, consists of combining both the principle of decomposition by resources (or quantities) and the technique of dynamic programming. We show how solving the generalizes problem, with physical and hydraulic constraints can be done effectively inside a scheme which combine both PD's technic and dkomposition's method with resources.

A direct extension of the PD algorithm (to this generalized model) consists in associating with each global constraint of the problem a state variable. This represents, at each stage of the PD, the quantity of water used by the groups supplied by a given channel or the power generated by the groups of a given network. So, we are left with a state vector of dimension equal to the number of coupling constraints of the problem.

Although this approach is relatively simple to implement, its drawback is that the computation time increases rapidly with the dimension of the state vector. A more interesting approach, proposed in this paper, consists of combining both the principle of decomposition by resources (or quantities) and the dynamic programming technique.

At the upper level, the total available Q_r speed is shared between the different channels; then, the flow rate of each channel is distributed between the subsets of groups connected to the same electrical network; finally, each bit rate relating to this last partition is allocated between the corresponding groups. individually, these successive allocations must be carried out optimally.

Formally, we can announce the principle of decomposition as follows: the initial problem G is subdivided into sub-problems SC so that each of them involves only the decision variables $(q_i)_{i \in I}$ representative the groups of the water supply of channel I . In the same way, the sub-problem SC is decomposed into sub-problems SR such that each restricted sub-problem contains only the decision variables linked to the network r of the channel I .

Thus, let $P_r^I(Q_r)$ be the optimal production of the network r of the channel I to which a flow resource Q has been allocated. This production is calculated by solving the following sub-problem:

$$\left\{ \begin{array}{l} \max \sum_{i \in J_r^I} p_i(q_i) \\ \sum_{i \in J_r^I} q_i = Q_r \\ q_i \in D_i, \forall i \in J_r^I \end{array} \right\} \text{SR} \quad (13)$$

At the level of channel I , we denote V_r^I the possible values of the bit rate to be allocated to the groups of the network r of channel I . This set is defined by the following domain:

$$V_r^I = \left\{ Q_r \in \left[\sum_{i \in J_r^I} p^{\min}, \sum_{i \in J_r^I} p^{\max} \right] : P_r^I \in DP \right\} \quad (14)$$

Thus, the channel $l \in L$ works optimally by solving the sub-problem:

$$\left\{ \begin{array}{l} \max \sum_{i \in R_l} p_i(q_i) \\ \sum_{i \in R_r^l} q_i = Q_r \\ q_i \in V_r^l, \forall i \in R_l \end{array} \right\} \text{SC} \tag{15}$$

Our procedure took into account the combination of decomposition and dynamic programming to provide a solution to the planning of Songloulou groups as described above.

4. Results and Discussion

After the implementation of our proposed problem with each production unit and the tailrace elevation using here to calculate individual output power, We implement an GUI interface, as shown in **Figure 2**.

Our GUI interface allows adjusting in input the current value of tailrace elevation and total inflow of the hydro plant and to obtain in the output the current unit inflow, total inflow, unit power, and total power according to the right discretization as shown in **Figure 3**.

To improve our proposed method, we compare our results with the company's results in the following **Table 2**.

Serie 1 is the tailrace elevation of Songloulou plant.

Serie 2 is the output power of the Songloulou hydro power.

Serie 3 is the output power result of our proposed optimization.

Table 2 shows the higher quality of production with our proposed method. From **Table 2**, we can see that the results are very good, because in the tests that

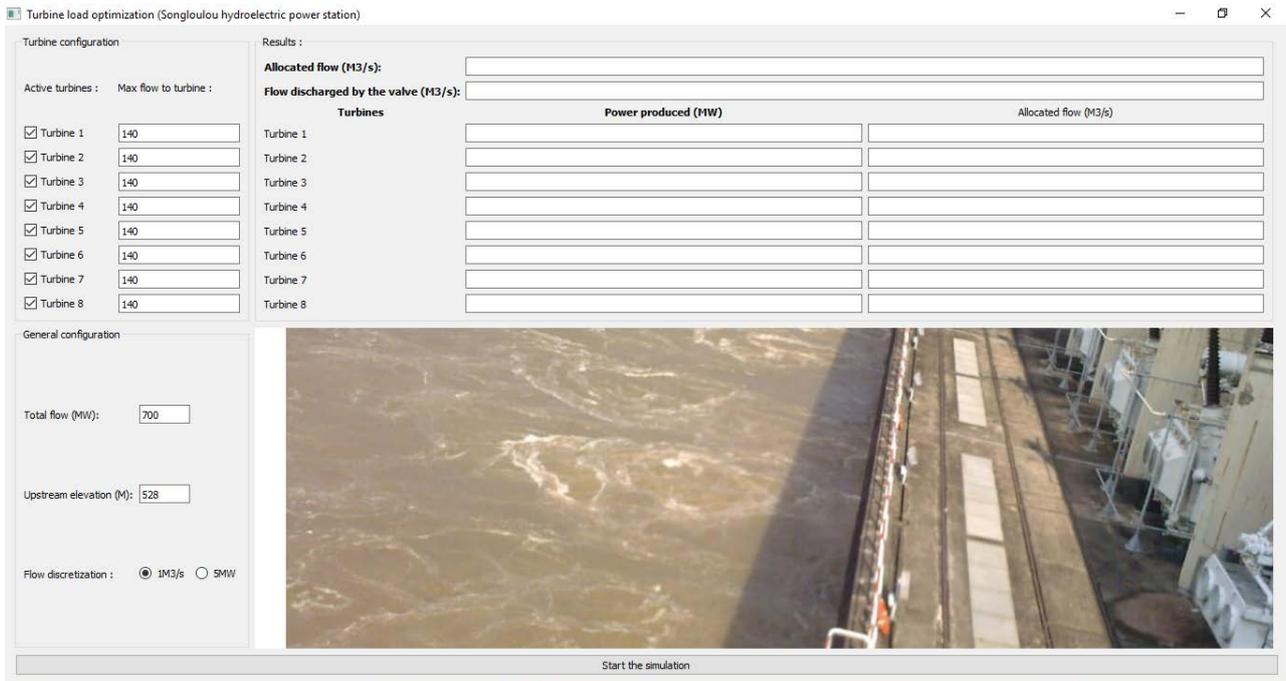


Figure 2. Gui of our dynamic programming dispatching.

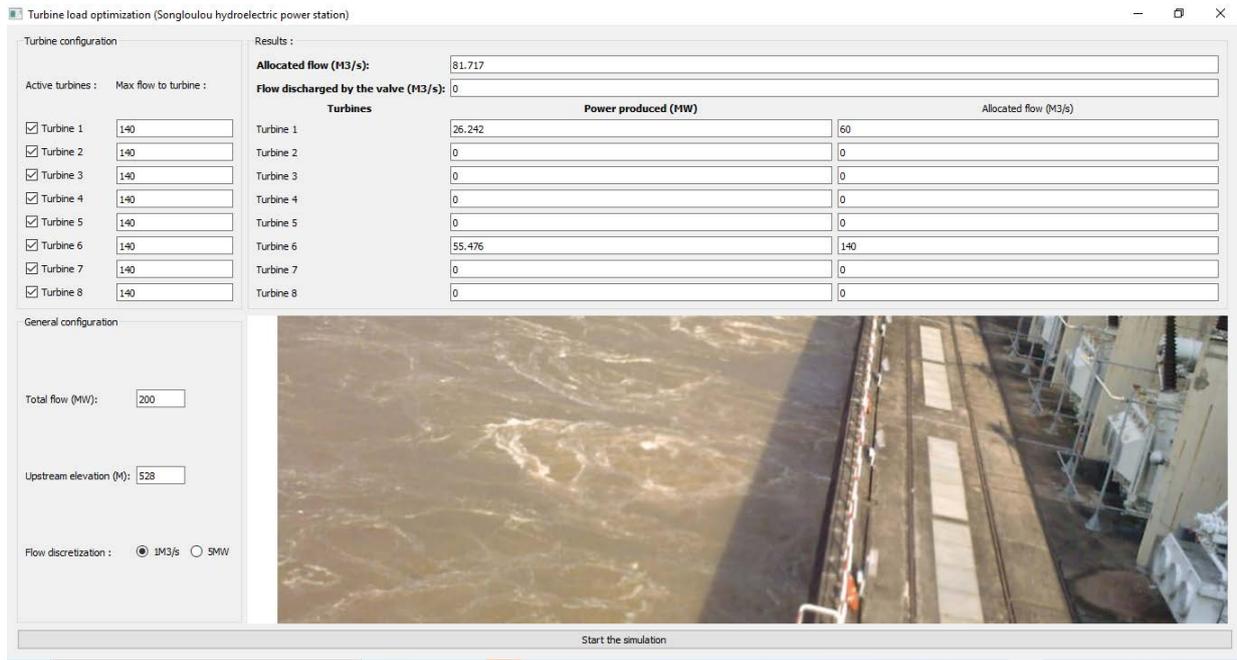


Figure 3. Sample of dynamic programming dispatching simulation.

Table 2. Comparison of results between our proposed algorithm and company implementation.

Allocated coast	Owner method	Qtot	P1	P2	P3	P4	P5	P6	P7	P8	Qdev	Ptot
527	SLL	1679	31	34	34	42	38	29	44	44	837	296
	Proposed		49	49	49	49	49	49	49	49	559	392
527.5	SLL	1545	48	46	49	44	46	47	48	45	463	373
	Proposed		49	49	49	49	49	49	49	49	425	392
528	SLL	1364	47	47	48	46	48	48	46	47	287	377
	Proposed		49	49	49	49	49	49	49	49	244	392
528	SLL	1105	49	46	48	47	47	46	47	48	20	378
	Proposed		49	49	49	49	42	49	49	49	0	385
527.5	SLL	1006	45	41	42	46	48	43	47	42	0	354
	Proposed		48.9	38.7	48.9	48.9	18.9	48.9	48.9	48.9	0	351
528	SLL	817	32	39	34	33	35	38	40	35	823	286
	Proposed		36.776	35.733	35.391	35.733	0	47.83	45.419	48.865	0	286.235
528	SLL	1003	44	46	42	45	44	42	45	43	0	351
	Proposed		44.164	44.156	44.16	43.809	80	49	49	49	0	351.539
527.5	SLL	679	19	0	40	47	35	48	0	48	0	237
	Proposed		28.745	30.491	29.802	29.794	0	39.538	39.195	40.228	0	238.027
527.80	SLL	3662	43	47	44	47	43	47	45	48	2622	364
	Proposed		45.659	45.597	45.628	45.61	0	45.612	45.642	45.636	2542	364.953
528.20	SLL	1560	40	0	35	37	39	34	26	31	869	242
	Proposed		49	49	49	49	49	49	49	49	440	392
528.27	SLL	1548	34	33	28	30	28	34	30	30	842	247
	Proposed		49	49	49	49	49	49	49	49	428	392
527.94	SLL	1460	41	43	42	46	43	47	43	42	474	347
	Proposed		49	49	49	49	49	49	49	49	340	392

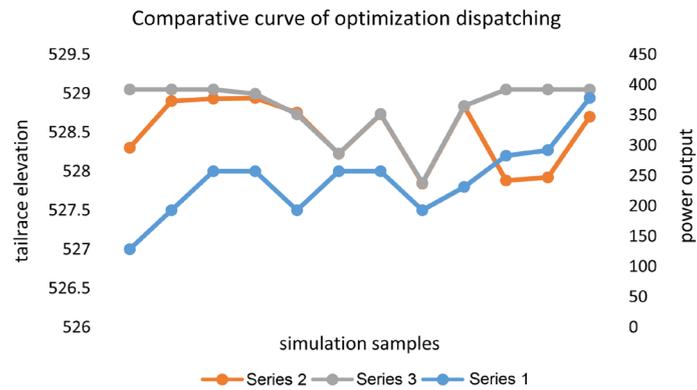


Figure 4. Comparative curve of optimization dispatching of Songloulou hydro plant.

we carried out, we find ourselves roughly in the same range of values as on the Songloulou file. In the moments when the eight turbines work in the file, on our algorithm, the eight turbines also work. If a turbine is not working in the file, in our algorithm a turbine does not rotate either. Besides, when we have to open the discharge valve, our algorithm also proposes to open the discharge valve at the same time as in the file provided by Songloulou. **Figure 4** presents the gap between our proposal algorithm against results of Songloulou hydro plant. Our program always maximizes production by avoiding deversing flow without producing the maximum power.

5. Conclusion

The increase in electricity production requires taking into account the operating conditions. The latter makes it possible to promote the longevity of the plant by avoiding untimely stops of one or more during the operation following a breakdown. In this work, the main objective was the maximizing of the production of the 8 groups of the Songloulou power station by taking into account the level of water availability in the upstream basin, the admissible area of each turbine, the stop/start of the system while respecting the operating conditions and the quantity of water evacuation not participating in the production. From these parameters used, we obtained convincing results with an increase in Songloulou production. This work did not take into account the criteria for group maintenance and loss of transmission of the power produced in the electrical network which will be the subject of our next study.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Notation:

The following notation is used throughout the paper:

- $i = 1, 2, \dots, n$: Index of turbine
- $l = 1, 2, \dots, L$: Index of network channel
- r Index of electrical network
- V Volume of water in upstream reservoir
- u Volume of water in downstream reservoir
- P power obtained in the conversion process of hydraulic potential energy into electric energy (MW)
- k gravity constant, multiplied by the water specific weight and divided by 106 {its value is 0.00981 [MW/(m³/s)/m]}
- η_t turbine efficiency in the conversion process of kinetic energy into mechanical energy
- η_g generator efficiency in the conversion process of mechanical energy into electrical energy
- h_{fb} forebay elevation which is function of the water storage (m); V = water storage in the reservoir of plant (hm³)
- h_r tailrace elevation which is function of the water release (m); u =water release by the turbines and the spillway of the plant
- h_{pl} penstock head loss which is function of the water discharge (m); and q = water discharge by the turbines of the powerhouse (m³/s)
- η^G unit efficiency considering the losses of the turbine generator set ($\eta_t \cdot \eta_g$)
- h_g difference between forebay elevation and tailrace elevation, that is $h_{fb}(V) - h_r(u)$
- Q Flow discharge by unit
- Q_T Turbine flow discharged in the hydro plant
- Q_r Flow remaining after allocation
- D Unit interval of operation
- G Initial optimization problem
- SC Optimization sub-problem 1
- SR Optimization sub-problem 2
- DP required interval of production power
- J_r^t Active turbine during operation