A New Theory and Technique on Measurement Axial Symmetric and Static Electric Field

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Abstract: This paper studied the problems on solving passive electric field. In the circumstances of the electric field which is limited and differentiable on a symmetric axis, it provides a new theory and technique on measurement axial symmetric and static electric field by making use of Maxwell equations. Out-of-axis electric field can be transformed into series by means of the method. The series include the field on the symmetric axis and its different order derivatives. Applying the method is very easy, quick and exact to perform numerical calculation. Taking the electric field emitted by an electric dipole as an example, approximate calculations were performed. When choosing only the first fifteen terms, on the curved surface that r and z is in a ratio of two to five, the relative errors of approximate results and their exact values of z and r components of the electric field are 0.000000008% and 0.000000005% respectively.

Keywords: new efficient measurement method; electric dipole; static electric field; axial symmetry

1 Introduction

The phenomena and problems of the nature are extremely complex. To describe these phenomena quantitatively and solve them, scientific researchers always try to abstract them into equations of mathematical physics and solve them. Because the problems are very complex and diverse, it is quite difficult to give generally analytic solutions. Therefore, they have given and developed many special solving methods. For instance, to nonlinear problems, scientists have already established mapping deformation method [1], sine-cosine method [2], generalized hirota transformation method [3], wavelet function and quasi-wavelet method [4] and so on.

Alike, the propagation and detection, or calculation of electromagnetic field has also been attended widely. In the process, scientists gave fast multipole method [5], implicit marching-on-in-time method [6], Green function method [7], etc.

Similarly, the phenomenon of axial symmetry is also common in the nature. Solving axial symmetric electric field is a problem that we often meet in the electromagnetic field theory and the practical application. If it is highly axial symmetric, that is, the electric field is only a function of cylindrical coordinates r, it can be solved directly by using Gauss's theorem, loop theorem and axial symmetry. Otherwise, if it is normally axial symmetric, that is, the electric field is relative to both cylindrical coordinates r and z, it is very difficult to get its generally analytic solutions. In this case, we can perform approximate calculation by using computers. Yet, owing to the complexity of problems and improper handling of them at times, the approximate solutions are not easy to approach exact solutions, or it takes a long time to get more ideal approximate solutions, or even it is quite difficult to perform approximate calculation. Along with the high performance of electronic computers in recent years, numerical simulation and analytical calculations of electromagnetic fields have been prosperous. In the process of researches and calculations people have presented many methods, such as sparse-matrix canonical-grid method [8], two-grid method [9], multilevel sparse-matrix canonical-grid method [10], time domain finite difference method [11], impedance approximate method [12], etc. To optimize algorithm and improve computational efficiency, this paper built a precise theoretical model and gave a computational method with high performance. In the circumstances of the axial symmetric, static and passive electric field which is limited and differentiable on the symmetric axis, this paper gives a new kind of computing method and its results in the series form by using Maxwell equations and calculus. That is, the out-of-axis electric field can be determined by the electric field on the symmetric axis and its different order derivatives. We take the electric field emitted by an electric dipole as an example and apply the method to computers for performing approximate calculation. The procedure and results of approximate calculation state that the results in the series form are the most ideal results for computer to perform approximate calculation with extremely high degree of accuracy and quick speed.

The new computing method given by this paper is very important in theory and application since signal detection and calculation is an important part in scientific research and engineering. Along with the progress of science and technology, scientific researchers invented many measuring or detecting devices [13]. They can directly measure or detect signals by using them [14]. On the other

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hand, the researchers did in-depth theoretical studies in the analysis and presumption of the signal components [15]. In fact, many signals can be described as electromagnetic wave. So the problems become the calculation or detection of electromagnetic wave. As known, in some cases, measuring devices can not reach the regions of some signals because of limitation. So this paper gives a new kind of computing method and results to calculate or detect signals. That is, the electric field on one point can be calculated, detected or presumed with another point. As you know, an electric field can be regarded as a static state approximately if the signal changes slowly. Then, to the static and passive field, there is certain theoretical and applied value for the study how to solve the electric field at one point by calculating or detecting another point. To optimize algorithm and improve computational efficiency, this writer built a precise theoretical model and gave a computational method with high performance. Not long ago, the authors studied the electric field generated by a point charge, the fifth and sixth power of charge distribution in sphere symmetry using the method [16-20]. This paper is its further development. Under the circumstances of axial symmetric and passive electromagnetic wave that is limited and differentiable on the symmetric axis, using Maxwell equations and calculus gives a kind of evolutionary computing method and its results in the series form. Applying this method, out-of-axis electromagnetic wave can be described by the electromagnetic wave on the symmetric axis and its different order derivatives. We have tested the evolutionary computing method in a monochromatic electromagnetic wave emitted by a magnetic dipole and given the relative errors.

2 The Alternating Iteration of Axial Symmetric and Static Electric Field

We choose cylindrical coordinates r, φ, z, and the z axis is the symmetric axis of the electric field. In the circumstances of vacuum and static state, applying Maxwell equations and axial symmetry, we can obtain

$$-\vec{e}_r \,\partial E_\varphi/\partial z + \vec{e}_\varphi(\partial E_r/\partial z - \partial E_z/\partial r) + \vec{e}_z\,\partial(rE_\varphi)/\partial r/r = 0 \quad (1)$$

$$\partial(rE_r)/\partial r/r + \partial E_z/\partial z = 0 \quad (2)$$

From the (1) and (2), we have

$$E_r(r,z) = -(1/r)\int_0^r [\partial E_z(r,z)/\partial z] r dr \quad (3)$$

$$E_z(r,z) = E(z) + \int_0^r [\partial E_r(r,z)/\partial z] dr \quad (4)$$

$$E_\varphi(r,z) = 0 \quad (5)$$

$E(z) = E_z(0,z)$ in the (4).

The equations above show that the axial symmetric electric field $E_\phi(r,z)$ is identically vanishing. However, because of the nestification of $E_r(r,z)$ and $E_z(r,z)$, none of them can be solved directly. In order to solve $E_r(r,z)$ and $E_z(r,z)$, we suggest the new computing method (alternating iteration method). That is, one should choose a proper initial value and make use of the (3) and (4) to perform alternating iteration. Obviously, approximate solutions are given by the alternating iteration of finite degrees and times. With increasing the degrees and times of alternating iteration, the solutions are more exact. After the alternating iteration of infinite degrees and times, the exact solutions of the out-of-axis electric field can be given. Here, we choose $E_z(r,z)$'s value on the symmetric axis as the initial value, that is

$$E_{0z}(r,z) = E_z(0,z) = E(z) \quad (6)$$

Substituting the (6) into the (3), we can get

$$E_{0r}(r,z) = (-1)^{0+1} E^{(2\times 0+1)}(z)(r/2)^{2\times 0+1}/(0+1)!0! \quad (7)$$

$E^{(n)}(z)$ is the n th derivative of $E(z)$ with respect to z in the (7). Substituting the (7) into the (4), we get

$$E_{1z}(r,z) = E(z) + (-1)^1 E^{(2\times 1)}(z)(r/2)^{2\times 1}/(1!)^2 \quad (8)$$

Substituting the (8) into the (3), we obtain

$$E_{1r}(r,z) = (-1)^{0+1} E^{(2\times 0+1)}(z)(r/2)^{2\times 0+1}/(0+1)!0! + (-1)^{1+1} E^{(2\times 1+1)}(z)(r/2)^{2\times 1+1}/(1+1)!1! \quad (9)$$

The rest may be deduced by analogy. The exact solutions of $E_z(r,z)$ and $E_r(r,z)$ can be given by the alternating iteration of infinite degree. That is

$$E_z(r,z) = \sum_{n=0}^{\infty}(-1)^n E^{(2n)}(z)(r/2)^{2n}/(n!)^2 \quad (10)$$

$$E_r(r,z) = \sum_{n=0}^{\infty}(-1)^{n+1} E^{(2n+1)}(z)(r/2)^{2n+1}/(n+1)!n! \quad (11)$$

3 The Test of the Efficient Measurement Methods

In this paper, the alternating iterative method, not only in theory has a certain significance, but also in the application of great value. In the electromagnetic field, there are some scopes in which the electromagnetic field to be measured difficult (or unable). To resolve this kind of questions (difficulties), this method is better and more useful. We can speculate the electromagnetic field of some scopes which is out the symmetry axis with this method by measuring these quantities of the symmetry axis. This method is convenient to use, and the results measured are very accurate. In this article, the method is used for measuring the electric field of the electric dipole, and giving out the accuracy of measurement. Now let's test the applicability to perform approximate calculation with the theory, method and results given by this paper. To compare with the exact values easily, we suppose that in cylindrical coordinates the electric field is

$$\vec{E}(r,z) = A[3rz\vec{e}_r + (2z^2 - r^2)\vec{e}_z]/(r^2 + z^2)^{5/2} \qquad (12)$$

where A is constant. It is necessary to emphasize that the theory, method and results not only may be applied to such situation as shown in the (12), but also can be suited to a more complex one. The reason why the paper chooses such a situation is that the exact values of the electric field can be solved easily and it will be easy to compare it with the approximate results got by performing numerical calculation using the (10) and (11). Through the comparison we can test the applicability to perform approximate calculation with the theory, method and results. On the z axis, the electric field is

$$E(z) = 2!A/z^3 \qquad (13)$$

On the curved surface $z = 5r/2$, the electric field will be discussed. From the (13), we get

$$E(z) = (2\times 0+2)!A/z^{(2\times 0+3)} \qquad (14)$$

$$E^{(1)}(z) = (-1)(2\times 0+3)!A/z^{(2\times 0+4)} \qquad (15)$$

$$E^{(2)}(z) = (2\times 1+2)!A/z^{(2\times 1+3)} \qquad (16)$$

And so on and so forth, we have

$$E^{(2n)}(z) = (2n+2)!A/z^{(2n+3)} \qquad (17)$$

$$E^{(2n+1)}(z) = (-1)(2n+3)!A/z^{(2n+4)} \qquad (18)$$

Substituting the (17) and (18) into the (10) and (11), we obtain

$$E_z(r,z) = \sum_{n=0}^{\infty}(-1)^n\,(2n+2)!E_0/(n!)^2 5^{2n} \qquad (19)$$

$$E_r(r,z) = \sum_{n=0}^{\infty}(-1)^n\,(2n+3)!E_0/(n+1)!n!5^{2n+1} \qquad (20)$$

$E_0 = A/z^3$ in the (19) and (20). On the curved surface $z = 5r/2$, the exact solutions of E_z and E_r got from the (12) are

$$E_z = 1.269617354E_0 \qquad (21)$$

$$E_r = 0.828011318E_0 \qquad (22)$$

respectively.

Using (19) and (20), we can perform approximate calculation with computer. The approximate results are listed in the Table 1 and Table 2, which also list the relative errors of the approximate results in different order numerical calculations and their exact values the (21) and (22).

Table 1. The different order approximate results and the relative errors Δ of E_z

n	$E_{nz}(E_0)$	$\Delta nz(\%)$
0	2.000000000	57.52777749
1	1.040000000	18.08555571
2	1.328000000	4.598444251
3	1.256320000	1.047351294
4	1.272448000	0.222952703
5	1.269041766	0.045335501
6	1.269730583	0.008918335
7	1.269595631	0.001710988
8	1.269621440	0.000321869
9	1.269616597	0.000059605
10	1.269617492	0.000010891
11	1.269617329	0.000001972
12	1.269617358	0.000000350
13	1.269617353	0.000000065
14	1.269617354	0.000000008

Table 2. The different order approximate results and the relative errors Δ of $E_r \cdot n$

	$E_{nr}(E_0)$	$\Delta nr(\%)$
0	1.200000000	44.92555529
1	0.720000000	13.04466683
2	0.854400000	3.186995365
3	0.822144000	0.708603561
4	0.829240320	0.148428203
5	0.827764285	0.029834404
6	0.828059492	0.005818117
7	0.828002138	0.001108658
8	0.828013035	0.000207430
9	0.828011001	0.000038240
10	0.828011375	0.000006962
11	0.828011307	0.000001256
12	0.828011319	0.000000222
13	0.828011317	0.000000041
14	0.828011318	0.000000005

4 Conclusions

In the circumstances of the axial symmetric, static and passive electric field which is limited and differentiable on the symmetric axis, this paper provides a new kind of computing method (alternating iteration method) and its results in the series form by making use of Maxwell equations and calculus. That is, the out-of-axis electric field can be determined by the electric field on the symmetric axis and its different order derivatives. In order to compare with the exact values easily and achieve the goal of testing the applicability to perform approximate calculation with the theory, method and results, we take the electric field emitted by an electric dipole as an example to perform approximate calculation. The research process and results of numerical calculation show that the results in the series form are the most ideal results for

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computers to perform approximate calculation. When we choose only the first fifteen terms, on the curved surface $z = 5r/2$, the relative error of E_{14z}'s approximate result and its exact value is 0.000000008%, and the relative error of E_{14r} is 0.000000005%. It has been exact enough, and it is very easy to perform approximate calculation with quite high speed. Therefore, the theory, method and results given by this paper not only are of certain theoretical meaning, but also are of quite great-applied value. Especially to those problems of some out-of-axis areas that are not easy to measure or get exact solutions from theory, it is easy, quick and exact to solve them by using the method and results given by this paper.

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