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Circular Scale of Time Applied in Classifying the Quantum-Mechanical Energy Terms Entering the Framework of the Schrödinger Perturbation Theory

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Abstract

The paper applies a one-to-one correspondence which exists between individual Schrödinger perturbation terms and the diagrams obtained on a circular scale of time to whole sets of the Schrödinger terms belonging to a definite perturbation order. In effect the diagram properties allowed us to derive the recurrence formulae giving the number of higher perturbative terms from the number of lower order terms. This recurrence formalism is based on a complementary property that any perturbation order N can be composed of two positive integer components N_a, N_b combined into N in all possible ways. Another result concerns the degeneracy of the perturbative terms. This degeneracy is shown to be only twofold and the terms having it are easily detectable on the basis of a circular scale. An analysis of this type demonstrates that the degeneracy of the perturbative terms does not exist for very low perturbative orders. But when the perturbative order exceeds five, the number of degenerate terms predominates heavily over that of nondegenerate terms.

Keywords: Circular Scale of Time, Quantum-Mechanical Energy Terms, Complementary Relations, Schrödinger's Perturbation Theory

1. Introduction

As soon as quantum mechanics in its wave-mechanical form was developed, the perturbation problem of the eigenenergies and eigenstates came into consideration [1]. A necessity of the perturbation theory was dictated by the fact that only very few systems on the atomic level could be examined in an exact quantum-mechanical way. In an overwhelming part of physical problems, the approximate methods had to be developed, and one of them was the Rayleigh-Schrödinger (RS) perturbation framework [2]. Rayleigh's name was involved because a similar perturbation method was applied by that author in a treatment of the acoustic waves; see [2]. More developed approaches to the perturbation theory than [2] are in [3-8].

In fact, perturbation theory is a complicated formalism already at the level of a one-particle (one-electron) system that occupies a non-degenerate unperturbed state [1, 2]. Essentially the method is based on the calculations of the matrix elements between the unperturbed wave functions

$$|p\rangle,\ |q\rangle, \tag{1}$$

and the potential function V which represents a perturbation of an originally unperturbed energy Hamiltonian. If any matrix element is considered as a result of a single particle scattering on the potential V, the number of the RS terms required to calculate the perturbation energy increases dramatically, especially when many different scatterings on V are taken into account. In principle, a full RS series is obtained when the number of the different types of scatterings on V is allowed to go to infinity. In practice, however, we consider only the perturbations of some finite order N, where N defines the number of factors of the type

$$U_{pq} = \langle p|V|q\rangle \tag{2}$$

entering a product of terms (2) forming a single perturbation term. In general, any of these factors is based on the eigenstates of the kind of (1) of an unperturbed Hamiltonian.

The RS formalism of the perturbation of a non-degenerate state strictly limits the number S_N of kinds of perturbative terms characteristic for a given N. This number is represented by

$$S_N = \frac{(2N-2)!}{N!(N-1)!}. \quad (3)$$

Different values of S_N for different N numbers are listed in **Table 1**.

The derivation of (3) is based on a complicated combinatorial formalism [9,10]. But, in many cases, we look for a recurrence formulae that allows us to calculate S_N from the lower-order terms

$$S_1, S_2, S_3, \cdots, S_{N-2}, S_{N-1}. \quad (4)$$

The aim of the present paper is to provide such formulae. This treatment is based on a circular time scale.

The idea of introducing the time variable as a tool to place the perturbative terms in order according to the size of N came from Feynman [4,11]. The scattering events with the perturbation V are arranged along a straight-linear time scale and all diagrams connecting a given number of events should be taken into account. However, the number

$$P_N = (N-1)! \quad (5)$$

of Feynman diagrams necessary to calculate the RS terms for a given N exceeds S_N; see **Table 1**.

Consequently, the energy contributions from the Feynman diagrams should be appropriately combined to give the proper contributions coming from the individual RS terms. The necessity of such a combination of diagrams makes the Feynman formalism especially cumbersome when N is large. For example, for $N = 20$ we have

$$S_{20} = 1\,767\,263\,190 \quad (6)$$

Table 1. The number S_N of the RS perturbative terms [see (3)] and the number P_N of the Feynman diagrams [see (5)] of a given perturbative order N.

N	S_N	P_N
1	1	1
2	1	1
3	2	2
4	5	6
5	14	24
6	42	120
7	132	720
8	429	5040
9	1430	40320
10	4862	362880

and

$$P_{20} = (10-1)! \cong 1.216 \times 10^{17}. \quad (7)$$

Consequently, a combination of P_N energy terms into S_N terms becomes an uneasy task.

But the difficulties of the Feynman formalism can be avoided when its straight-linear time scale is replaced by a circular time scale. In the latter case, a one-to-one correspondence exists between the diagrams obtained on the circular scale and the RS perturbation terms [12-16]. In effect, any component term entering the set of S_N terms corresponds to a separate diagram contributing a definite formula of the RS perturbation energy of order N. This result is attained by applying an appropriate contraction rule for the scattering events on the circular scale. Any contraction prescribed by this rule is different, and the whole number of diagrams obtained in this way becomes exactly equal to S_N. Moreover, an analysis of all contractions for a given N gives precisely the same energy terms, as they are provided by the RS theory.

In Section 3 we present the recurrence formulae for S_N attained on the basis of a graphical analysis applied in [12-16]. In fact, these formulae represent the complementary relations for S_N obtained on the basis of S_{N_a} and S_{N_b} having N_a, $N_b < N$. The property of complementarity becomes evident if we note that any component of S_N is a product of S_{N_a} and S_{N_b} for which there is satisfied the relation:

$$N_a + N_b = N \quad (8)$$

for any pair of integer numbers N_a and N_b. The changes of S_N due to the change of N are reported in Section 3.

Another advantage of the circular scale is its use in detecting the degeneracy of the Schrödinger perturbation terms. In fact, any diagram representing the perturbation term is either symmetrical for itself, or asymmetrical with respect to another diagram; see Section 4. This property provides us with a simple rule that there is no degeneracy of energy for a symmetrical diagram, but a twofold degeneracy is connected with any pair of asymmetrical diagrams. No other kind of degeneracy is obtained on the basis of the symmetry analysis of the diagrams. Sections 4 and 5 demonstrate that the twofold degeneracy due to a circular character of the time scale holds for the most part of the perturbative energy terms belonging to a given N on condition $N > 5$.

2. Recurrence Formulae for S_N and Their Complementary Character

In the first step, we point out that the recurrence formalism for S_N can be obtained without an analysis of all

contractions occurring on a circular scale for a given N. If all scattering events are arranged on a line—see e.g. **Figure 1** for $N=6$—we can separate successively $1,2,3,4,\cdots,N-2,N-1$ points on that line.

Figure 2 presents an example of such separations for $N=6$ performed in each case with the aid of a single vertical line.

According to the results from previous studies [12-16], any set of separated points can be considered as lying on some special loop of time of the circular scale characteristic for a given N. In the next step, such a loop has its characteristic number of contractions leading to a corresponding number of diagrams for that loop. In effect, the number of the RS terms corresponding to any separation of the kind represented in **Figure 2** labeled by s_i is dictated by the size of the component integer numbers that satisfy the relation

$$N_a, N_b < N . \tag{9}$$

Any pair of N_a, N_b entering (9) should be different. Moreover, any term forming the set should satisfy the two-component sum rule (8).

Here N is the same constant number for all terms in (8) and (9). A full set of (N_a, N_b) for $N=6$ is given in **Figure 2**. The number of terms s_i in the set is evidently equal to $N-1$.

Because the loops,with their possible further contractions, behave independently each of other, any s_i term of the set characterized by (9) provides us with the number of perturbative terms equal to

$$s_i = S^{(i)}_{N_a,N_b} = S_{N_a} S_{N_b} . \tag{10}$$

The total number of S_N is therefore equal to

$$S_N = \sum_{i=1}^{i=N-1} S^{(i)}_{N_a,N_b} . \tag{11}$$

For $N=6$ this gives the formula

$$\begin{aligned} S_6 &= S_1S_5 + S_2S_4 + S_3S_3 + S_4S_2 + S_5S_1 \\ &= 1\times 14 + 1\times 5 + 2\times 2 + 5\times 1 + 14\times 1 = 42. \end{aligned} \tag{12}$$

The Formula (11) is a very simple result that is checked in **Table 2** up to the order $N=10$.

3. A Change ΔS of S_N

In many cases we like to calculate the number S_N of some perturbative order N from S_{N-1} of order $N-1$. This calculation can easily be performed following the diagram on **Figure 3** with the case $N=6$ as an example. From **Figure 4**, we obtain:

$$\begin{aligned} \Delta S = S_6 - S_5 &= S_1(S_5 - S_4) + S_2(S_4 - S_3) + S_3(S_3 - S_2) \\ &+ S_4(S_2 - S_1) + S_5 \end{aligned}$$

Figure 1. Fundamental pattern of scattering points for calculating the number S_N in the RS perturbation theory; see (3). The pattern for the perturbative order $N=6$ is taken as an example.

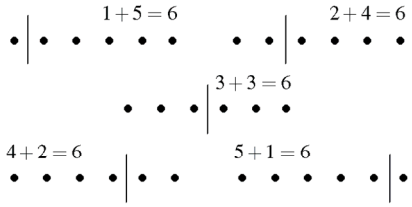


Figure 2. Separations of a fundamental pattern of scattering points in Figure 1 useful in calculating S_N. The perturbative order $N=6$ is taken as an example. The effect of separations is presented in (12).

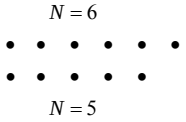


Figure 3. The fundamental pattern of scattering points for calculating the increment $\Delta S = S_N - S_{N-1}$ in the RS perturbation theory; see (13) and (15). The perturbative order $N=6$ is taken as an example.

Table 2. The S_N numbers calculated for $2 \le N \le 10$ from the recurrence formulae of (10) and (11).

$$S_2 = S_1S_1 = 1\times 1 = 1$$

$$S_3 = S_1S_2 + S_2S_1 = 1 + 1 = 2$$

$$S_4 = S_1S_3 + S_2S_2 + S_3S_1 = 2 + 1 + 2 = 5$$

$$S_5 = S_1S_4 + S_2S_3 + S_3S_2 + S_4S_1 = 5 + 2 + 2 + 5 = 14$$

$$S_6 = S_1S_5 + S_2S_4 + S_3S_3 + S_4S_2 + S_5S_1 = 14 + 5 + 4 + 5 + 14 = 42$$

$$S_7 = S_1S_6 + S_2S_5 + S_3S_4 + S_4S_3 + S_5S_2 + S_6S_1 = 42 + 14 + 10 + 10 + 14 + 42 = 132$$

$$S_8 = S_1S_7 + S_2S_6 + S_3S_5 + S_4S_4 + S_5S_3 + S_6S_2 + S_7S_1 = 132 + 42 + 28 + 25 + 28 + 42 + 132 = 429$$

$$S_9 = S_1S_8 + S_2S_7 + S_3S_6 + S_4S_5 + S_5S_4 + S_6S_3 + S_7S_2 + S_8S_1 = 429 + 132 + 84 + 70 + 70 + 84 + 132 + 429 = 1430$$

$$S_{10} = S_1S_9 + S_2S_8 + S_3S_7 + S_4S_6 + S_5S_5 + S_6S_4 + S_7S_3 + S_8S_2 + S_9S_1 = 1430 + 429 + 264 + 210 + 196 + 210 + 264 + 429 + 1430 = 4862$$

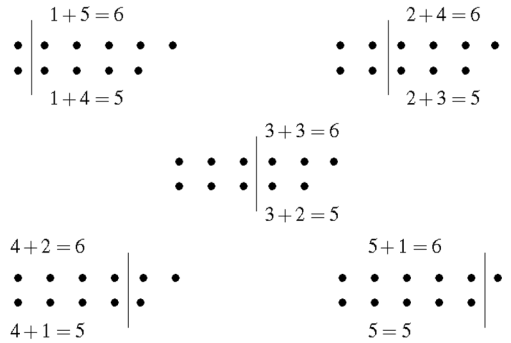


Figure 4. Separations of a fundamental pattern of scattering points in Figure 3 useful in calculation of $\Delta S = S_N - S_{N-1}$ for $N = 6$; see (13).

$$= 1\times(14-5)+1\times(5-2)+2\times(2-1)+5\times(1-1)+14 = 28. \quad (13)$$

The difference (13) added to S_5 gives:

$$S_6 = S_5 + \Delta S = 14 + 28 = 42. \quad (14)$$

Calculations similar to those presented in (13) and (14) performed for N not exceeding 10 are presented in **Table 3**. The differences other than (15), such as those between S_N and S_{N-2}, or S_N and S_{N-3}, can be calculated on the same footing as the difference between S_N and S_{N-1}.

The result of (14) agrees with the Huby-Tong formula of (3); cf. here the data in **Table 1**. A general formula for ΔS is

$$\Delta S = S_N - S_{N-1} = S_1(S_{N-1} - S_{N-2}) + S_2(S_{N-2} - S_{N-3}) + \cdots + S_{N-2}(S_2 - S_1) + S_{N-1}. \quad (15)$$

4. Symmetry of the Time Scale and Degeneracy of the Perturbation Terms

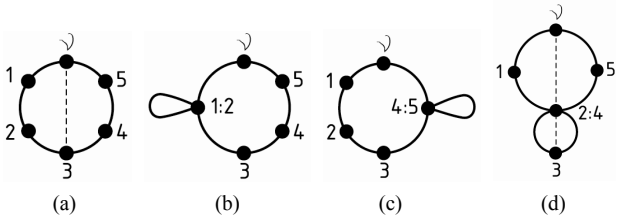
A topological symmetry of energy diagrams on a circular scale can be easily demonstrated by an example.

Suppose we have the energy terms of the perturbative order $N = 6$. In **Figure 5** we present four diagrams of $N = 6$; a full set of the $S_6 = 42$ diagrams is given in [12]. Diagram (a), also called the main time loop, has no contractions; it is symmetrical with respect to the line joining the beginning-end point of the loop with point 3, which is the most distant point from ν.

The line divides the loop into halves. Diagram (d) has a similar symmetry that is characteristic by the time contraction 2:4. On the other hand, diagrams (b) and (c) represent a different kind of symmetry: a mirror reflection with respect to the line joining points ν and 3 gives diagram (c) from (b), and similar reflection of (c) gives

Table 3. The $S^{(i)}_{N_a,N_b}$ numbers and S_N terms calculated from the recurrence Formula (13) for $3 \le N \le 10$.

$N=3$	$\Delta S = S_1(S_2 - S_1) + S_2 = 0 + 1 = 1$ $S_3 = S_2 + 1 = 1 + 1 = 2$
$N=4$	$\Delta S = S_1(S_3 - S_2) + S_2(S_2 - S_1) + S_3 = 1 + 2 = 3$ $S_4 = S_3 + 3 = 2 + 3 = 5$
$N=5$	$\Delta S = S_1(S_4 - S_3) + S_2(S_3 - S_2) + S_3(S_2 - S_1) + S_4 = 3 + 1 + 5 = 9$ $S_5 = S_4 + 9 = 14$
$N=6$	$\Delta S = S_1(S_5 - S_4) + S_2(S_4 - S_3) + S_3(S_3 - S_2) + S_4(S_2 - S_1) + S_5 = 9 + 3 + 2 + 14 = 28$ $S_6 = S_5 + 28 = 42$
$N=7$	$\Delta S = S_1(S_6 - S_5) + S_2(S_5 - S_4) + S_3(S_4 - S_3) + S_4(S_3 - S_2) + S_5(S_2 - S_1) + S_6 = 28 + 9 + 6 + 5 + 42 = 90$ $S_7 = S_6 + 90 = 132$
$N=8$	$\Delta S = S_1(S_7 - S_6) + S_2(S_6 - S_5) + S_3(S_5 - S_4) + S_4(S_4 - S_3) + S_5(S_3 - S_2) + S_6(S_2 - S_1) + S_7 = 90 + 28 + 18 + 15 + 14 + 132 = 297$ $S_8 = S_7 + 297 = 429$
$N=9$	$\Delta S = S_1(S_8 - S_7) + S_2(S_7 - S_6) + S_3(S_6 - S_5) + S_4(S_5 - S_4) + S_5(S_4 - S_3) + S_6(S_3 - S_2) + S_7(S_2 - S_1) + S_8$ $= 297 + 90 + 56 + 45 + 42 + 42 + 429 = 1001$ $S_9 = S_8 + 1001 = 1430$
$N=10$	$\Delta S = S_1(S_9 - S_8) + S_2(S_8 - S_7) + S_3(S_7 - S_6) + S_4(S_6 - S_5) + S_5(S_5 - S_4) + S_6(S_4 - S_3) + S_7(S_3 - S_2) + S_8(S_2 - S_1) + S_9$ $= 1001 + 297 + 180 + 56 + 140 + 126 + 126 + 132 + 1430 = 3432$ $S_{10} = S_9 + 3432 = 4862$



(a) (b) (c) (d)

Figure 5. Two symmetric and two asymmetric diagrams for energy terms belonging to $N=6$. Symmetric diagrams (a) and (d) represent non-degenerate energy terms, asymmetric diagrams (b) and (c) lead to two degenerate components of the perturbation energy.

diagram (b). Either of symmetry mentioned above is characteristic for any diagram of order N, so the diagrams can be classified as either symmetric, or asymmetric with respect to the line joining point ν on the main loop of time with a point that is the most distant from ν. For even N, the second point is labeled by a point $N/2$ on the loop. However, for odd N, the most distant point from ν does not coincide with a scattering event on the time scale, but is half of the time interval between the scattering points, labeled by $(1/2)(N-1)$ and $(1/2)(N+1)$. For example, for $N=1$, this is no point on the loop beyond the beginning-end point ν and there are no contractions on that loop; see **Figure 6**.

The symmetry behavior of the diagrams affects the perturbation energy: the symmetric diagrams are non-degenerate, which means that the perurbative term corresponding to such diagram is different from all other terms. But the asymmetric diagrams give pairs of degenerate energy terms. This degeneracy is only twofold because it is due solely to the property of asymmetry of diagrams entering given pair. According to the rules applied earlier [12-16] the diagrams presented in **Figure 5** provide us with the following perturbation terms:

$$\Delta E_{(a)} = \frac{U_{np}U_{pq}U_{qr}U_{rs}U_{st}U_{tn}}{(E_n-E_p)(E_n-E_q)(E_n-E_r)(E_n-E_s)(E_n-E_t)}; \tag{16}$$

$$\Delta E_{(b)} = -U_{nn}\frac{U_{np}U_{pq}U_{qr}U_{rs}U_{sn}}{(E_n-E_p)^2(E_n-E_q)(E_n-E_r)(E_n-E_s)}$$

$$= \Delta E_{(c)} = -U_{nn}\frac{U_{np}U_{pq}U_{qr}U_{rs}U_{sn}}{(E_n-E_p)(E_n-E_q)(E_n-E_r)(E_n-E_s)^2}; \tag{17}$$

$$\Delta E_{(d)} = -\frac{U_{np}U_{pq}U_{qr}U_{rn}}{(E_n-E_p)(E_n-E_q)^2(E_n-E_r)}\frac{U_{np}U_{pn}}{(E_n-E_p)^2}. \tag{18}$$

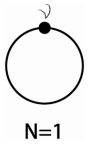


Figure 6. Diagram of the perturbative order $N=1$ representing the side loops of diagrams (b) and (c) given in Figure 5. There is no scattering point beyond the beginning-end point ν on the loop.

In cases (b) and (c) [see (17)], the side loop provides us with a factor U_{nn} that represents the first-order perturbation energy of $N=1$. Another factor is entering the energy (18):

$$\frac{U_{np}U_{pn}}{(E_n-E_p)^2}. \tag{18a}$$

This factor corresponds (apart of its sign) to the second-order perturbation energy $(N=2)$ because the side loop of diagram (d) has two points on it. In the Formulae (16)-(18a) E_n is a non-degenerate energy of an original (unperturbed) quantum state $|n\rangle$.

The symbols

$$E_p, E_q, E_r \cdots \tag{19}$$

label unperturbed energies of states $|p\rangle, |q\rangle, |r\rangle \cdots$ The formula for the matrix element U_{pq} between states $|p\rangle$ and $|q\rangle$ is given in (2). The repetition of the indices $p,q,r,\cdots$ in the numerator of the energy expressions given in (16)-(18a) implies a summation over p, q, r, $\cdots$ In Section 5, we give a selection rule for the degenerate perturbation terms of a given order N.

5. Algebra of the Time Loops in Diagrams and Selection of Degenerate Perturbation Terms

Graphical presentation of the perturbation terms is a rather tedious way to select the degenerate terms, but an algebraic expression of the diagrams can be developed to simplify the selection problem. As an example, we apply this kind of algebraic expression for diagrams of order $N=6$.

Any diagram can be represented as a product referring to the number of points lying on the loops in that diagram. The side loops are considered in chronological order, which means that the loops containing earlier scattering points are represented before those containing later points. An exception is the loop having the beginning-end point ν: this loop is presented regularly as the end factor of any product. For example, the loops in **Figure 5** have the following notation:

L_6 represents diagram(a),

$$\begin{aligned} &L_1L_5 \quad \text{represents diagram}(b),\\ &L_1L_5 \quad \text{represents diagram}(c),\\ &L_2L_4 \quad \text{represents diagram}(d). \end{aligned} \tag{20}$$

As a rule, the sum of the loop indices of L in a given product is equal to the perturbative order N. A full set of 42 diagrams with $N=6$ is given in [12]; a list of their algebraic representations is presented in **Table 4**. When one of two equal side loops is nearer to the point v than another loop, the first loop is labeled by a superscript (n) and the farther loop by a superscript (f). When more than one of equal loops begin in a given contraction point, their number is indicated by a power exponent of L.

Every pair of degenerate terms is represented by the same product of L, for example, the second and the third row in (20). Nevertheless, a sequence of L in degenerate products may be different, with the exception of the last term of the product, which should be the same for a given degenerate pair. The degenerate pairs are identified in **Table 5**, and the non-degenerate energy terms are listed in **Table 6**. A characteristic point is that no kind of degeneracy but a two-fold one due to an arrangement of scattering points on the time scale is detected for the perturbation terms.

Table 4. Algebraic representation of 42 energy diagrams of perturbative order $N=6$. For the diagrams see [12].

$0_6 = L_6$	$\mathrm{XXI}_6 = L_1^3L_3$
$\mathrm{I}_6 = L_1^{(n)}L_6$	$\mathrm{XXII}_6 = L_1^2L_2L_2$
$\mathrm{II}_6 = L_2^{(n)}L_4$	$\mathrm{XXIII}_6 = L_1L_2L_1L_2$
$\mathrm{III}_6 = L_3L_3$	$\mathrm{XXIV}_6 = L_2L_1^2L_2$
$\mathrm{IV}_6 = L_4L_2$	$\mathrm{XXV}_6 = L_1^3L_3$
$\mathrm{V}_6 = L_1^{(f)}L_5$	$\mathrm{XXVI}_6 = L_1^4L_2$
$\mathrm{VI}_6 = L_2L_4$	$\mathrm{XXVII}_6 = L_1L_1L_4$
$\mathrm{VII}_6 = L_3L_3$	$\mathrm{XXVIII}_6 = L_1L_2L_3$
$\mathrm{VIII}_6 = L_1^{(f)}L_5$	$\mathrm{XXIX}_6 = L_1L_1L_4$
$\mathrm{IX}_6 = L_2^{(n)}L_4$	$\mathrm{XXX}_6 = L_1L_1^2L_3$
$\mathrm{X}_6 = L_1^{(n)}L_5$	$\mathrm{XXXI}_6 = L_2L_1L_3$
$\mathrm{XI}_6 = L_1^2L_4$	$\mathrm{XXXII}_6 = L_1L_1L_4$
$\mathrm{XII}_6 = L_1L_2L_3$	$\mathrm{XXXIII}_6 = L_1^2L_1L_3$
$\mathrm{XIII}_6 = L_1L_3L_2$	$\mathrm{XXXIV}_6 = L_1L_2L_3$
$\mathrm{XIV}_6 = L_2L_1L_3$	$\mathrm{XXXV}_6 = L_1L_3L_2$
$\mathrm{XV}_6 = L_2^2L_2$	$\mathrm{XXXVI}_6 = L_2L_2L_2$
$\mathrm{XVI}_6 = L_3L_1L_2$	$\mathrm{XXXVII}_6 = L_1L_3L_2$
$\mathrm{XVII}_6 = L_1^2L_4$	$\mathrm{XXXVIII}_6 = L_1^2L_2L_2$
$\mathrm{XVIII}_6 = L_1L_2L_3$	$\mathrm{XXXIX}_6 = L_1L_2L_3$
$\mathrm{XIX}_6 = L_2L_1L_3$	$\mathrm{XL}_6 = L_1L_2L_1L_2$
$\mathrm{XX}_6 = L_1^2L_4$	$\mathrm{XLI}_6 = L_1L_1L_2L_2$

Table 5. A list of twofold degenerate contributions to the perturbation energy of order $N=6$. Their algebraic representations are given in Table 4. The non-degenerate energy terms are listed in Table 6. The diagrams on a circular scale corresponding to the listed terms are presented in [12].

$\mathrm{I}_6 = \mathrm{X}_6$;	$\mathrm{XXI}_6 = \mathrm{XXV}_6$;
$\mathrm{II}_6 = \mathrm{IX}_6$;	$\mathrm{XXII}_6 = \mathrm{XXIV}_6$;
$\mathrm{III}_6 = \mathrm{VII}_6$;	$\mathrm{XXVII}_6 = \mathrm{XXXII}_6$;
$\mathrm{V}_6 = \mathrm{VIII}_6$;	$\mathrm{XXVIII}_6 = \mathrm{XXXI}_6$;
$\mathrm{XI}_6 = \mathrm{XX}_6$;	$\mathrm{XXX}_6 = \mathrm{XXXIII}_6$;
$\mathrm{XII}_6 = \mathrm{XIV}_6$;	$\mathrm{XXXIV}_6 = \mathrm{XXXIX}_6$;
$\mathrm{XIII}_6 = \mathrm{XVI}_6$;	$\mathrm{XXXV}_6 = \mathrm{XXXVII}_6$;
$\mathrm{XVIII}_6 = \mathrm{XIX}_6$;	$\mathrm{XL}_6 = \mathrm{XLI}_6$;

Table 6. Non-degenerate terms contributing to the perturbation energy of order $N=6$. Their algebraic representations are given in Table 4, for the diagrams see [12].

0_6; IV_6; VI_6; XV_6; XVII_6; XIII_6;
XXVI_6; XXIX_6; XXXVI_6; $\mathrm{XXXVIII}_6$.

The number of degenerate terms is equal to 2 multiplied by

$$0,0,1,4,16,56\cdots \tag{21}$$

for $N=2,3,4,5,6,7\cdots$, respectively. Beginning with $N=5$ this number is much larger than the number of non-degenerate terms, which is

$$1,2,3,6,10,20\cdots \tag{22}$$

for the same N as they are listed below (21).

It can be noted that an equality of the algebraic representations of the two diagrams is a necessary but not sufficient condition for the degeneracy of the diagram energies: some equal algebraic representations can also give non-degenerate terms. For example, XXVII_6 and XXXII_6 in **Table 4** give degenerate terms, but XXIX_6 is a non-degenerate contribution.

6. Summary

In the first step, we presented a graphical and algebraic derivation of the number S_N of the RS perturbative components of the energy for some perturbative order N from the values S_{N_a}, S_{N_b} for the lower perturbative orders N_a, N_b, where N_a and N_b satisfy the complementarity rule (8). The perturbed quantum state is assumed to be a non-degenerate one. A similar recurrence formula is obtained for an increment ΔS which should be added to S_{N-1} to obtain S_N, so

$$S_N = S_{N-1} + \Delta S \ . \qquad (23)$$

The calculations make reference to the properties of the time contractions characteristic for a circular scale of time along which the perturbative effect of a quantum-mechanical system is developed.

The property of asymmetry of the time loops on a circular scale is applied in examining the degeneracy of the perturbation terms. An absence of degeneracy for the energy terms belonging to very low perturbative orders is found. However, a twofold degeneracy of most of perturbative terms can be detected when the perturbative order N becomes larger than 5.

7. References

[1] E. Schrödinger, "Quantization as an Eigenvalue Problem III," *Annalen der Physik*, Vol. 80, 1926, pp. 437-490.

[2] P. M. Morse and H. Feshbach, "Methods of Theoretical Physics, Part 2," McGraw-Hill, New York, 1953.

[3] N. H. March, W. H. Young and S. Sampanthar, "The Many-Body Problem in Quantum Mechanics," Cambridge University Press, Cambridge, 1967.

[4] R. D. Mattuck, "A Guide to Feynman Diagrams in the Many-Body Problem," 2nd Edition, McGraw-Hill, New York, 1976.

[5] J. Killingbeck, "Quantum-Mechanical Perturbation Theory," *Reports on Progress in Physics*, Vol. 40, No. 9, 1977, pp. 963-1031. doi:10.1088/0034-4885/40/9/001

[6] P. O. Löwdin, "Proceedings of the International Workshop on Perturbation Theory of Large Order," *International Journal of Quantum Chemistry*, Vol. 21, 1982, pp. 1-353.

[7] G. A. Arteca, F. M. Fernandez and E. A. Castro, "Large Order Perturbation Theory and Summation Methods in Quantum Mechanics," Springer, Berlin, 1990.

[8] J. C. Le Guillou and J. Zinn-Justin, "Large-Order Behaviour of Perturbation Theory," North-Holland, Amsterdam, 1990.

[9] R. P. Feynman, "The Theory of Positrons," *Physical Review*, Vol. 76, No. 6, 1949, pp. 749-759. doi:10.1103/PhysRev.76.749

[10] R. Huby, "Formulae for Rayleigh-Schrödinger Perturbation Theory in Any Order," *Proceedings of the Physical Societ*, London, Vol. 78, 1961, pp. 529-536.

[11] B. Y. Tong, "On Huby's Rules for Non-Degenerate Rayleigh-Schrodinger Perturbation Theory in Any Order," *Proceedings of the Physical Society*, London, Vol. 80, 1962, pp. 1101-1104.

[12] S. Olszewski, "Time Scale and Its Application in the Perturbation Theory," *Zeitschrift fur Naturforschung A*, Vol. 46, 1991, pp. 313-320.

[13] S. Olszewski, "Time Topology for Some Classical and Quantum Non-Relativistic Systems," *Studia Philosophiae Christianae*, Vol. 28, 1992, pp. 119-135.

[14] S. Olszewski and T. Kwiatkowski, "A Topological Approach to Evaluation of Non-Degenerate Schrodinger Perturbation Energy Based on a Circular Scale of Time," *Computation Chemistry*, Vol. 22, No. 6, 1998, pp. 445-461. doi:10.1016/S0097-8485(98)00023-0

[15] S. Olszewski, "Two Pathways of the Time Parameter Characteristic for the Perturbation Problem in Quantum Chemistry," *Trends in Physical Chemistry*, Vol. 9, 2003, pp. 69-101.

[16] S. Olszewski, "Combinatorial Analysis of the Rayleigh-Schrodinger Perturbation Theory Based on a Circular Scale of Time," *International Journal of Quantum Chemistry*, Vol. 97, No. 3, 2004, pp. 784-801. doi:10.1002/qua.10776

