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# There Is a Way to Comprise Half-Integer Eigenvalues for Photon Spin

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**Abstract**

In this article, an attempt based on **Spin Topological Space**, **STS**, to give a reasonable detailed account of the cause of **photonic fermionization phenomena** of light photon is made.

**STS** is an unconventional spin space in quantum mechanics, which can be used to account for where the unconventional half-integer spin eigenvalues phenomenon of light photon comes from.

We suggest to dectect the possible existence of photonic one-third-spinization phenomenon of light photon, by using three beams of light photon in interference experiment.

**Keywords**

Spin Topological Space, STS, Non-Hermitian matrix, Casimir operator, photonic fermionization phenomena, half-integer spin eigenvalues, one third, one fourth spin eigenvalues of photon spin

## 1 Introduction

Kyle E. Ballantine, John F. Donegan and Paul R. Eastham [1] measured the total angular momentum of the beam of light with their interferometer, and observed some curious optical phenomena. They found: the eigenvalues of angular momentum of light photon obviously shifted away from the normal physical values that are ruled by the general axioms accepted in today's quantum mechanics world.

Normal angular momentum quantum numbers of the photon must be integers, in units of the Planck constant $\hbar$: eigenvalues of spin are $-1\hbar, 0\hbar, +1\hbar$ and eigenvalues of orbital are $0\hbar, 1\hbar, 2\hbar, 3\hbar, \ldots$

However, as the title of their paper, " There are many ways to spin a photon: Half-quantization of a total optical angular momentum " [1] shows: the experimental data in [1] were half-integer, $+\hbar/2$ and $-\hbar/2$, or even may be $+1.5\hbar$ and $-1.5\hbar$...! It is an important physicial experiment result, and indeed, light photon is boson, however possesses fermionic, spectrum ! curious phenomena...

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This present article, " There is a way to comprise half-integer eigenvalues for photon spin ", is in the frame of **Spin Topological Space**, **STS** [2] to consider the contributions of spin effects of light photon, and tries to clear up the cause of the photonic fermionization phenomena, which emerged from the experiment [1].

The contributions of orbital effects of light photon, which show half-integer eigenvalues, could appeal to the mechanism of Non-Hermitian orbital angular momentum $\mathbb{L}_3, \mathbb{L}^2$ [3].

Normally, in quantum mechanics, different kinds of spin particles possess different dimensional spaces, which are expressed by finite dimensional matrices, and these finite dimensional matrices are all Hermiticity.

According to **STS** , spin angular momentum $\vec{\pi}(l)$ of particles is expressed by infinite dimensional matrices in three-physical space. The first component $\pi_1(l)$ and the second component $\pi_2(l)$ are Non-Hermitian matrices; the third component $\pi_3(l)$ is Hermitian diagonal matrix. Here, mark " $l$ " indicates the $l$th generation spin particles, $l = 1, 2, 3, \ldots$

**2 Three groups of matrices** $\vec{\pi}_{+3,-3}(2), \vec{\pi}_{+2,-1}(1), \vec{\pi}_{+3/2,-3/2}(1)$ of light photon particle $\vec{\pi}(l)$, which satisfy spin angular momentum commutation relus, play the major role **in elaborating the machanism of photonic fermionization phenomena**.

$$\vec{\pi}_{+3,-3}(2) \times \vec{\pi}_{+3,-3}(2) = i\hbar\vec{\pi}_{+3,-3}(2) \tag{1}$$

$$\vec{\pi}_{+2,-1}(1) \times \vec{\pi}_{+2,-1}(1) = i\hbar\vec{\pi}_{+2,-1}(1) \tag{2}$$

$$\vec{\pi}_{+3/2,-3/2}(1) \times \vec{\pi}_{+3/2,-3/2}(1) = i\hbar\vec{\pi}_{+3/2,-3/2}(1) \tag{3}$$

Where

$$\vec{\pi}_{+3,-3}(2) = \{ \pi_{1;+3,-3}(2), \pi_{2;+3,-3}(2), \pi_{3;+3,-3}(2) \} \tag{4}$$

$$\vec{\pi}_{+2,-1}(1) = \{ \pi_{1;+2,-1}(1), \pi_{2;+2,-1}(1), \pi_{3;+2,-1}(1) \} \tag{5}$$

$$\vec{\pi}_{+3/2,-3/2}(1) = \{ \pi_{1;+3/2,-3/2}(1), \pi_{2;+3/2,-3/2}(1), \pi_{3;+3/2,-3/2}(1) \} \tag{6}$$

Or instead of (1), (2), (3), in terms of raising matrix operator $\pi_j^+$, lowering matrix operator $\pi_k^-$ and $\pi_{3;j,k}$, *i. e.* (7) below, to represent commutation rules (8), (9), (10) of light photon with three different kinds of spin state ($\hbar = 1$):

$$\{ \pi_j^+(l), \pi_k^-(l), \pi_{3;j,k}(l) \} \tag{7}$$

$$\pi_{+3}^+(2)\pi_{-3}^-(2) - \pi_{-3}^-(2)\pi_{+3}^+(2) = 2\pi_{3;+3,-3}(2) \tag{8.1}$$

$$\pi_{3;+3,-3}(2)\pi_{+3}^+(2) - \pi_{+3}^+(2)\pi_{3;+3,-3}(2) = +\pi_{+3}^+(2) \tag{8.2}$$

$$\pi_{3;+3,-3}(2)\pi_{-3}^-(2) - \pi_{-3}^-(2)\pi_{3;+3,-3}(2) = -\pi_{-3}^-(2) \tag{8.3}$$

$$\pi_{+2}^+(1)\pi_{-1}^-(1) - \pi_{-1}^-(1)\pi_{+2}^+(1) = 2\pi_{3;+2,-1}(1) \tag{9.1}$$

$$\pi_{3;+2,-1}(1)\pi_{+2}^+(1) - \pi_{+2}^+(1)\pi_{3;+2,-1}(1) = +\pi_{+2}^+(1) \tag{9.2}$$

$$\pi_{3;+2,-1}(1)\pi_{-1}^-(1) - \pi_{-1}^-(1)\pi_{3;+2,-1}(1) = -\pi_{-1}^-(1) \tag{9.3}$$

$$\pi_{+3/2}^+(1)\pi_{-3/2}^-(1) - \pi_{-3/2}^-(1)\pi_{+3/2}^+(1) = 2\pi_{3;+3/2,-3/2}(1) \tag{10.1}$$

$$\pi_{3;+3/2,-3/2}(1)\pi_{+3/2}^+(1) - \pi_{+3/2}^+(1)\pi_{3;+3/2,-3/2}(1) = +\pi_{+3/2}^+(1) \tag{10.2}$$

$$\pi_{3;+3/2,-3/2}(1)\pi_{-3/2}^-(1) - \pi_{-3/2}^-(1)\pi_{3;+3/2,-3/2}(1) = -\pi_{-3/2}^-(1) \tag{10.3}$$

Write down the explicit representations of raising matrix operators and lowering matrix operators that appear in the above three formulas (8), (9), (10):

$$\pi^{+}_{+3}(2) = \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ 8,\ 7,\ 6,\ 5,\ 4,\ 3,\ 2,\ 1,\ 0,\ -1,\ -2,\ ,\ \}_{+2} \qquad (11)$$
$$\pi^{-}_{-3}(2) = \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ -7,\ -8,\ ,\ \}_{-2} \qquad (12)$$

$$\pi^{+}_{+2}(1) = \mathrm{diag}\{\ ,\ 7,\ 6,\ 5,\ 4,\ 3,\ 2,\ 1,\ 0,\ -1,\ -2,\ -3,\ ,\ \}_{+1} \qquad (13)$$
$$\pi^{-}_{-1}(1) = \mathrm{diag}\{\ ,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ ,\ \}_{-1} \qquad (14)$$

$$\pi^{+}_{+3/2}(1) = \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ 13,\ 11,\ 9,\ 7,\ 5,\ 3,\ 1,\ -1,\ -3,\ -5,\ -7,\ ,\ \}_{+1} \qquad (15)$$
$$\pi^{-}_{-3/2}(1) = \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ -7,\ -5,\ -3,\ -1,\ 1,\ 3,\ 5,\ 7,\ 9,\ 11,\ -13,\ ,\ \}_{-1} \qquad (16)$$

Subscripts " +1 " and " -1 " represent the first minor top-right diagonal and the first minor down-left diagonal.

Subscripts " +2 " and " -2 " represent the second minor top-right diagonal and the second minor down-left diagonal.

Subscripts " 0 " indicates major diagonal, sometimes for convenience be omitted.

In condition for keeping photon’s Casimir operator invariant, that is, keeping

$$\pi^{2}_{+3,-3}(2) = \pi^{2}_{+2\,,\,-1}(1) = \pi^{2}_{+3/2,-3/2}(1) = 1(1+1)I_0\hbar^2 = 2I_0\hbar^2 \qquad (17)$$
$$I_0 = \mathrm{diag}\{\ ,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ ,\ \}_0 \qquad (18)$$

Further, next three groups of math series forms of the spin third component $\pi_3(l)$ of light photon are obtained as below

$$\pi_{3;\ +3,-3}(2) = \mathrm{diag}\{,\ 3\hbar,\ 2.5\hbar,\ 2\hbar,\ \boxed{1.5\hbar,}\ 1\hbar,\ \boxed{0.5\hbar,}\ 0\hbar,\ \boxed{-0.5\hbar,}\ -1\hbar,\ \boxed{-1.5\hbar,}\ -2\hbar,\ ,\}_0 \qquad (19)$$

$$\pi_{3;\ +2\,,\,-1}(1) = \mathrm{diag}\{,\ 6\hbar,\ 5\hbar,\ 4\hbar,\ 3\hbar,\ \boxed{2\hbar,}\ \boxed{1\hbar,}\ \boxed{0\hbar,}\ \boxed{-1\hbar,}\ \boxed{-2\hbar,}\ -3\hbar,\ -4\hbar,\ ,\}_0 \qquad (20)$$

$$\pi_{3;\ +3/2,-3/2}(1) = \mathrm{diag}\{,\ 5.5\hbar,\ 4.5\hbar,\ 3.5\hbar,\ 2.5\hbar,\ \boxed{1.5\hbar,}\ \boxed{0.5\hbar,}\ \boxed{-0.5\hbar,}\ \boxed{-1.5\hbar,}\ -2.5\hbar,\ -3.5\hbar,\ -4.5\hbar,\ ,\}_0 \qquad (21)$$

(19): Alternating series form of Integer eigenvalues and Half-integer eigenvalues

(20): Integer eigenvalues series form

(21): Half-integer eigenvalues series form

(19), (20), (21) are just seperately the figures of what happening in Kyle E. Ballantine’s and his colleagues’ experiment:

Integer eigenvalues series form (20) and half-integer eigenvalues series form (21) give the accounts of " One family includes have the expected bosonic spectrum with integer eigenvalues, and other family, has a fermionic spectrum, comprising half-integer eigenvalues. " (quoted passage from the paper [1]).

By the way, (20) $\pi_{3;\ +2,-1}(1)$ and (21) $\pi_{3;\ +3/2,-3/2}(1)$, both of them are together involved in (19) $\pi_{3;\ +3,-3}(2)$. So it seems that there should exist the third family, alternating series form of Integer eigenvalues and Half-integer eigenvalues (19).

**3 Physical behavior mechanism of photonic fermionization of light photon experiment**

Now, matrices (8), (9), (10) can be used to describe the experiment rusults (17) and (19), (20), (21) of photonic fermionization phenomena of light photon, but from what kind of experimental procedure of physical behavior mechanism, these experimental results arise ?

For this reason, deeper research is given. Be concise, the sign of "(1)", is omitted in follows.

Because $\vec{\pi}_{+2,-1}$ and $\vec{\pi}_{m+2,m-1}$ are spin angular momentums in **STS**, it means：

$$\vec{\pi}_{+2,-1} \times \vec{\pi}_{+2,-1} = i\vec{\pi}_{+2,-1} \tag{2}$$

$$\vec{\pi}_{m+2,m-1} \times \vec{\pi}_{m+2,m-1} = i\vec{\pi}_{m+2,m-1} \tag{22}$$

Using the linear combinanation of (2) with (22), a new spin angular momentum $\vec{\pi}_{m/2+2,\ m/2-1}$ (23) is composed, and it obeys commutation rule (24)

$$\vec{\pi}_{m/2+2,\ m/2-1} = \tfrac{1}{2}\{\vec{\pi}_{m+2,m-1} + \vec{\pi}_{+2,-1}\}\ \ldots \tag{23}$$

$$\vec{\pi}_{m/2+2,\ m/2-1} \times \vec{\pi}_{m/2+2,\ m/2-1} = i\ \vec{\pi}_{m/2+2,\ m/2-1} \tag{24}$$

$\vec{\pi}_{+2,-1}$, $\vec{\pi}_{m+2,m-1}$ and $\vec{\pi}_{m/2+2,\ m/2-1}$ all are light photon, since their Carsimir operators equal to $2\hbar^2$, *i. e.*

$$\pi^2_{+2,-1} = \pi^2_{m+2,m-1} = \pi^2_{m/2+2,\ m/2-1} = 1(1+1)I_0\hbar^2 = 2I_0\hbar^2 \tag{25}$$

Write down the third component of (23), and its exlicit formulation (26. *m*) as below:

$$\pi_{3;\ m/2+2,\ m/2-1} = \tfrac{1}{2}\{\pi_{3;\ m+2,m-1} + \underline{\pi_{3;+2,-1}}\}\ ;\ m = 0, \pm1, \pm2, \pm3, \pm4, \ldots \quad (26.\,m)$$

$$♦\pi_{3;+4,+1} = \tfrac{1}{2}\{\pi_{3;+6,+3} + \underline{\pi_{3;+2,-1}}\}$$

$$= \mathrm{diag}\{\ ,\ 8,\ 7,\ 6,\ 5,\ 4,\ \underline{3},\ 2,\ 1,\ 0,\ -1,\ -2,\ ,\} \quad (26.4)$$

$$◇\pi_{3;+7/2,+1/2} = \tfrac{1}{2}\{\pi_{3;+5,+2} + \underline{\pi_{3;+2,-1}}\}$$

$$= \tfrac{1}{2}\mathrm{diag}\{\ ,\ 15,\ 13,\ 11,\ 9,\ 7,\ \underline{5},\ 3,\ 1,\ -1,\ -3,\ -5,\ ,\} \quad (26.3)$$

$$♦\pi_{3;+3,0} = \tfrac{1}{2}\{\pi_{3;+4,+1} + \underline{\pi_{3;+2,-1}}\}$$

$$= \mathrm{diag}\{\ ,\ 7,\ 6,\ 5,\ 4,\ 3,\ \underline{2},\ 1,\ 0,\ -1,\ -2,\ -3,\ ,\} \quad (26.2)$$

$$◇\pi_{3;+5/2,-1/2} = \tfrac{1}{2}\{\pi_{3;+3,0} + \underline{\pi_{3;+2,-1}}\}$$

$$= \tfrac{1}{2}\mathrm{diag}\{\ ,\ 13,\ 11,\ 9,\ 7,\ 5,\ \underline{3},\ 1,\ -1,\ -3,\ -5,\ -7,\ ,\} \quad (26.1)$$

$$♦♦\boxed{\pi_{3;+2,-1}} = \tfrac{1}{2}\{\pi_{3;+2,-1} + \underline{\pi_{3;+2,-1}}\}$$

$$= \mathrm{diag}\{\ ,\ 6,\ 5,\ 4,\ 3,\ 2,\ \underline{1},\ 0,\ -1,\ -2,\ -3,\ -4,\ ,\} \quad (26.0)$$

$$◇\boxed{\pi_{3;+3/2,-3/2}} = \tfrac{1}{2}\{\pi_{3;+1,-2} + \underline{\pi_{3;+2,-1}}\}$$

$$= \tfrac{1}{2}\mathrm{diag}\{\ ,\ 11,\ 9,\ 7,\ 5,\ 3,\ \underline{1},\ -1,\ -3,\ -5,\ -7,\ -9,\ ,\} \quad (26.\text{-}1)$$

$$\blacklozenge\pi_{3;\,+1,-2} = \tfrac{1}{2}\{\ \pi_{3;\,0\,,-3} + \underline{\pi_{3;\,+2,-1}}\ \}$$
$$= \mathrm{diag}\{\ ,\ 5,\ 4,\ 3,\ 2,\ 1,\ \underline{0},\ -1,\ -2,\ -3,\ -4,\ -5,\ ,\} \quad (26.\text{-}2)$$

$$\lozenge\pi_{3;\,+1/2,-5/2} = \tfrac{1}{2}\{\ \pi_{3;\,-1\,,-4} + \underline{\pi_{3;\,+2,-1}}\ \}$$
$$= \tfrac{1}{2}\,\mathrm{diag}\{\ ,\ 9,\ 7,\ 5,\ 3,\ 1,\ \underline{-1},\ -3,\ -5,\ -7,\ -9,\ -11,\ ,\} \quad (26.\text{-}3)$$

$$\blacklozenge\pi_{3;\,0\,,-3} = \tfrac{1}{2}\{\ \pi_{3;\,-2\,,-5} + \underline{\pi_{3;\,+2,-1}}\ \}$$
$$= \mathrm{diag}\{\ ,4,\ 3,\ 2,\ 1,\ 0,\ \underline{-1},\ -2,\ -3,\ -4,\ -5,\ -6\ ,\} \quad (26.\text{-}4)$$

**From (26), two important conclusions are given**

**1)** There exist two diffrent families of the third component of light photon:

family **BP: B**osonization of **P**hoton, labelled by " ♦ ",

family **FP: F**ermionization of **P**hoton, labelled by " ◇ "

For light photon, the angular momentum addition of two angular momentums, one angular momentum $\mathbf{BP}_1$ with other angular momentum $\mathbf{FP}_2$, may generate two diffrent families of the third component of light photon.

$$\mathbf{BP}\blacklozenge(m) = \tfrac{1}{2}\{\ \mathbf{BP}_1(m) + \mathbf{BP}_2(0)\ \}\ ;\ m = 0, \pm2, \pm4, \pm6, \ldots \quad (27)$$

$$\mathbf{FP}\lozenge(m) = \tfrac{1}{2}\{\ \mathbf{BP}_1(m) + \mathbf{BP}_2(0)\ \}\ ;\ m = \pm1, \pm3, \pm5, \pm7, \ldots \quad (28)$$

**BP♦** and **FP◇** alternately appear with *m*.

**2)** For a fixed term of the new spin angular momentum $\pi_{3;\ m/2+2\,,\ m/2-1}$, there are many options to choose from the general expression (29).

$$\mathbf{BP}\blacklozenge(m,m'),\ \mathbf{FP}\lozenge(m,m') = \tfrac{1}{2}\{\ \mathbf{BP}_1(m) + \mathbf{BP}_2(m')\ \} \quad (29)$$
$$m,\ m' = 0, \pm1, \pm2, \pm3, \ldots$$

Family **BP♦** (27) and Family **FP◇** (28) are the simplest couple, in which one spin angular momentum $\mathbf{BP}_2(0)$, $\underline{\pi_{3;\,+2,-1}}$ is keeping invariant, as other spin angular momentum $\mathbf{BP}_1(m)$, $\vec{\pi}_{m+2\,,\,m-1}$ varies with $m$ in expression $\pi_{3;\ m/2+2\,,\ m/2-1}$ (26. *m*).

(20) $\pi_{3;\ +2\,,-1}(1)$ and (21) $\pi_{3;\ +3/2,-3/2}(1)$, which are the part of expression $\pi_{3;\ m/2+2\,,\ m/2-1}$ (26. *m*). When $m$ equals to 0 and -1, (20) and (21) are just (26.0) family **BP♦**(0) and (26.-1) family **FP◇**(-1).

The relationship between (20) and (21), or equivalent to that between matrices (9) and (10), could refer to the math statements (26.0) and (26.-1). They are the results of the additions of spin angular momentum photon $\pi_{3;\,+2,-1}$ with photon $\underline{\pi_{3;\,+2,-1}}$, and photo $\pi_{3;\,+1,\,-2}$ with photon $\underline{\pi_{3;\,+2,-1}}$ in Ballantine's and his colleagues' experiment.

By the way, the intervals between two adjoining **BP♦** and **FP◇** is $\frac{\hbar}{2}$

$$\Delta\pi_{3;\ m/2+2\,,\ m/2-1} = \mathbf{FP}\lozenge(m \pm 1) - \mathbf{BP}\blacklozenge(m) = \pm\tfrac{\hbar}{2}\, I_0 \quad (30)$$

**4 Prediction about one-third-spinization phenomenon of light photon**

Proceeding in above way, parallel to the math structure of three groups of matrices (11), (12) and (13), (14), (15), (16) for photonic fermionization, we guess at the existent of so-call photonic one-third-spinization phenomenon of light photon, and four groups of matrices (31), (32) and (33), (34), (35), (36), (37), (38) are given, below labelled by " ♣ ".

$$\pi^{+}_{+4}(3) = \tfrac{1}{2}\,\text{diag}\{\ ,\ 9,\ 8,\ 7,\ 6,\ 5,\ 4,\ 3,\ 2,\ 1,\ 0,\ -1,\ ,\ \}_{+3} \qquad (31)$$

$$\pi^{-}_{-5}(3) = \tfrac{1}{2}\,\text{diag}\{\ ,\ 0,\ -1,\ -2,\ -3,\ -4,\ -5,\ -6,\ -7,\ -8,\ -9,\ -10,\ ,\ \}_{-3} \qquad (32)$$

$$\pi^{+}_{+2}(1) = \text{diag}\{\ ,\ 7,\ 6,\ 5,\ 4,\ 3,\ 2,\ 1,\ 0,\ -1,\ -2,\ -3,\ ,\ \}_{+1} \qquad (33)$$

$$\pi^{-}_{-1}(1) = \text{diag}\{\ ,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ ,\ \}_{-1} \qquad (34)$$

$$\pi^{+}_{+5/3}(1) = \tfrac{1}{3}\,\text{diag}\{\ ,20,\ 17,\ 14,\ 11,\ 8,\ \underline{5},\ 2,\ -1,\ -4,\ -7,\ -10,\ ,\ \}_{+1} \qquad (35)$$

$$\pi^{-}_{-4/3}(1) = \tfrac{1}{3}\,\text{diag}\{\ ,11,\ 8,\ 5,\ 2,\ -1,\ \underline{-4},\ -7,\ -10,\ -13,\ -16,\ -19,\ ,\ \}_{-1} \qquad (36)$$

$$\pi^{+}_{+4/3}(1) = \tfrac{1}{3}\,\text{diag}\{\ ,19,\ 16,\ 13,\ 10,\ 7,\ \underline{4},\ 1,\ -2,\ -5,\ -8,\ -11,\ ,\ \}_{+1} \qquad (37)$$

$$\pi^{-}_{-5/3}(1) = \tfrac{1}{3}\,\text{diag}\{\ ,10,\ 7,\ 4,\ 1,\ -2,\ \underline{-5},\ -8,\ -11,\ -14,\ -17,\ -20,\ ,\ \}_{-1} \qquad (38)$$

As well as photon's Casimir operator

$$\pi^{2}_{+4,-5}(3) = \pi^{2}_{+2\,,\,-1}(1) = \pi^{2}_{+5/3,-4/3}(1) = \pi^{2}_{+4/3,-5/3}(1) = 1(1 + 1)I_0\hbar^2 = 2I_0\hbar^2 \qquad (39)$$

Accordingly, next four groups of math series forms of the spin third component $\pi_3(l)$ of light photon are obtained as below

$$\pi_{3;\ +4,\,-5}(3) =$$
$$\text{diag}\{,\ 2,\ 5/3,\ 4/3,\ 1,\ 2/3,\ \boxed{\underline{1/3},}\ \ 0,\ \boxed{-1/3,}\ -2/3,\ -1,\ -4/3,\ ,\}_0 \qquad (40)$$

$$\pi_{3;\ +2\,,\,-1}(1) =$$
$$\text{diag}\{,\ 6,\ 5,\ 4,\ 3,\ 2,\ \boxed{\underline{1},}\ \ 0,\ -1,\ -2,\ -3,\ -4,\ ,\}_0 \qquad (41)$$
$$\text{diag}\{,\ 18/3,\ 15/3,\ 12/3,\ 9/3,\ 6/3,\ \boxed{\underline{3/3},}\ \boxed{0\,/3,}\ -3/3,\ -6/3,\ -9/3,\ -12/3,\ ,\}_0$$

$$\pi_{3;\ +5/3,-4/3}(1) =$$
$$\text{diag}\{,\ 17/3,\ 14/3,\ 11/3,\ 8/3,\ 5/3,\ \boxed{\underline{2/3},}\ \boxed{-1/3,}\ -4/3,\ -7/3,\ -10/3,\ -13/3,\ ,\}_0 \qquad (42)$$

$$\pi_{3;\ +4/3,-5/3}(1) =$$
$$\text{diag}\{,\ 16/3,\ 13/3,\ 10/3,\ 7/3,\ 4/3,\ \boxed{\underline{1/3},}\ \boxed{-2/3,}\ -5/3,\ -8/3,\ -11/3,\ -14/3,\ ,\}_0 \qquad (43)$$

(41), (42), (43) combine to form (40). All of them imply that the third component eigenvalues of light photon can be integer, one-third-integer series.

Let us have some acquaintance with the relationship among (41), (42), (43), by the general formula of addition of spin angular momentum of light photon (44.*m*).

It is shown that (41), (42), (43) are (44.0), (44.-1), (44.-2), which are parts of general formular (44) below

General formula of the addition of light photon are given by (44.*m*)

$$\pi_{3;\ m/3+2\ ,\ m/3-1} = \tfrac{1}{3}\{\ \pi_{3;\ m+2\ ,\ m-1} + \underline{2\pi_{3;\ +2,-1}}\ \}\ ;\quad m = 0, \pm1, \pm2, \pm3, \ldots \qquad (44.m)$$

$$\blacklozenge\pi_{3;\ +4,\ +1} = \tfrac{1}{3}\{\ \pi_{3;\ +8,\ +5} + \underline{2\pi_{3;\ +2,-1}}\ \}$$
$$= \mathrm{diag}\{\ ,\ 8,\ 7,\ 6,\ 5,\ 4,\ \underline{3},\ 2,\ 1,\ 0,\ -1,\ -2,\ ,\ \} \qquad (44.6)$$

$$\clubsuit\pi_{3;\ +11/3,\ +2/3} = \tfrac{1}{3}\{\ \pi_{3;\ +7,\ +4} + \underline{2\pi_{3;\ +2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 23,\ 20,\ 17,\ 14,\ 11,\ \underline{8},\ 5,\ 2,\ -1,\ -4,\ -7,\ ,\ \} \qquad (44.5)$$

$$\clubsuit\pi_{3;\ +10/3,\ +1/3} = \tfrac{1}{3}\{\ \pi_{3;\ +6,\ +3} + \underline{2\pi_{3;\ +2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 22,\ 19,\ 16,\ 13,\ 10,\ \underline{7},\ 4,\ 1,\ -2,\ -5,\ -8,\ ,\ \} \qquad (44.4)$$

$$\blacklozenge\pi_{3;\ +3,\ 0} = \tfrac{1}{3}\{\ \pi_{3;\ +5,\ +2} + \underline{2\pi_{3;\ +2,-1}}\ \}$$
$$= \mathrm{diag}\{\ ,\ 7,\ 6,\ 5,\ 4,\ 3,\ \underline{2},\ 1,\ 0,\ -1,\ -2,\ -3,\ ,\ \} \qquad (44.3)$$

$$\clubsuit\pi_{3;\ +8/3,-1/3} = \tfrac{1}{3}\{\ \pi_{3;\ +4,\ +1} + \underline{2\pi_{3;\ +2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 20,\ 17,\ 14,\ 11,\ 8,\ \underline{5},\ 2,\ -1,\ -4,\ -7,\ -10,\ ,\ \} \qquad (44.2)$$

$$\clubsuit\pi_{3;\ +7/3,-2/3} = \tfrac{1}{3}\{\ \pi_{3;\ +3,\ 0} + \underline{2\pi_{3;\ +2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 19,\ 16,\ 13,\ 10,\ 7,\ \underline{4},\ 1,\ -2,\ -5,\ -8,\ -11,\ ,\ \} \qquad (44.1)$$

$$\blacklozenge\blacklozenge\underline{\pi_{3;\ +2,\ -1}} = \tfrac{1}{3}\{\ \pi_{3;\ +2,-1} + \underline{2\pi_{3;\ +2,-1}}\ \}$$
$$= \mathrm{diag}\{\ ,\ 6,\ 5,\ 4,\ 3,\ 2,\ \underline{1},\ 0,\ -1,\ -2,\ -3,\ -4,\ ,\ \} \qquad (44.0)$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 18,\ 15,\ 12,\ 9,\ 6,\ \boxed{\underline{3}},\ ,\ 0\ \ -3,\ -6,\ -9,\ -12,\ ,\ \}$$

$$\clubsuit\pi_{3;\ +5/3,-4/3} = \tfrac{1}{3}\{\ \pi_{3;\ +1,-2} + \underline{2\pi_{3;\ +2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 17,\ 14,\ 11,\ 8,\ 5,\ \boxed{\underline{2}},\ -1,\ -4,\ -7,\ -10,\ -13,\ ,\ \} \qquad (44.-1)$$

$$\clubsuit\pi_{3;\ +4/3,-5/3} = \tfrac{1}{3}\{\ \pi_{3;\ 0\ ,-3} + \underline{2\pi_{3;\ +2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 16,\ 13,\ 10,\ 7,\ 4,\ \boxed{\underline{1}},\ -2,\ -5,\ -8,\ -11,\ -14,\ ,\ \} \qquad (44.-2)$$

$$\blacklozenge\pi_{3;\ +1,-2} = \tfrac{1}{3}\{\ \pi_{3;\ -1,-4} + \underline{2\pi_{3;\ +2,-1}}\ \}$$
$$= \mathrm{diag}\{\ ,\ 5,\ 4,\ 3,\ 2,\ 1,\ \underline{0},\ -1,\ -2,\ -3,\ -4,\ -5,\ ,\ \} \qquad (44.-3)$$

$$\clubsuit\pi_{3;\ +2/3,-7/3} = \tfrac{1}{3}\{\ \pi_{3;\ -2,-5} + \underline{2\pi_{3;\ +2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 14,\ 11,\ 8,\ 5,\ 2,\ \underline{-1},\ -4,\ -7,\ -10,\ -13,\ -16,\ ,\ \} \qquad (44.-4)$$

$$\clubsuit\pi_{3;\ +1/3,-8/3}(1) = \tfrac{1}{3}\{\ \pi_{3;\ -3,-6} + \underline{2\pi_{3;\ +2,-1}}\ \}$$
$$= \tfrac{1}{3}\,\mathrm{diag}\{\ ,\ 13,\ 10,\ 7,\ 4,\ 1,\ \underline{-2},\ -5,\ -8,\ -11,\ -14,\ -17,\ ,\ \} \qquad (44.-5)$$

$$\blacklozenge\pi_{3;\ 0\ ,-3} = \tfrac{1}{3}\{\ \pi_{3;\ -4,-7} + \underline{2\pi_{3;\ +2,-1}}\ \}$$
$$= \mathrm{diag}\{\ ,\ 4,\ 3,\ 2,\ 1,\ 0,\ \underline{-1},\ -2,\ -3,\ -4,\ -5,\ -6,\ ,\ \} \qquad (44.-6)$$

$$\mathbf{BP}\blacklozenge(m) = \tfrac{1}{3}\{\ \mathbf{BP}_1(m) + 2\mathbf{BP}_2(0)\ \}\ ;\quad m = 0, \pm3, \pm6.\ ,\ldots \qquad (45)$$
$$\mathbf{DP}\clubsuit(m) = \tfrac{1}{3}\{\ \mathbf{BP}_1(m) + 2\mathbf{BP}_2(0)\ \}\ ;\quad m = \pm1, \pm2, \pm4, \pm5, \ldots \qquad (46)$$

$$\Delta\pi_{3;\ m/3+2\ ,\ m/3-1} = \mathbf{DP}\clubsuit(m \pm 1) - \mathbf{BP}\blacklozenge(m) = \pm\tfrac{\hbar}{3}I_0 \qquad (47)$$

And $\pi^2_{+2,-1} = \pi^2_{m+2\ ,\ m-1} = \pi^2_{m/3+2\ ,\ m/3-1} = 1(1 + 1)I_0\hbar^2 = 2I_0\hbar^2$ (48)

Combine (44), (26), obtian:

$$\pi_{3;\ m/6+2,\ m/6-1} = \frac{1}{3}\{\ \pi_{3;\ m+2,\ m-1} + \underline{2\pi_{3;+2,-1}}\ \}\ ;\ m = 0, \pm1, \pm2, \pm3, \ldots \qquad (49.m)$$

♦$\pi_{3;+4,+1} = \frac{1}{3}\{\ \pi_{3;+8,+5} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \text{diag}\{\ ,\ 8,\ 7,\ 6,\ 5,\ 4,\ \underline{3},\ 2,\ 1,\ 0,\ -1,\ -2,\ ,\}$ (49.12)

$\pi_{3;+23/6,+5/6} = \frac{1}{3}\{\ \pi_{3;+15/2,+9/2} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \frac{1}{6}\,\text{diag}\{\ ,\ 47,\ 41,\ 35,\ 29,\ 23,\ \underline{17},\ 11,\ 5,\ -1,\ -7,\ -13,\ \}$ (49.11)

♣$\pi_{3;+11/3,+2/3} = \frac{1}{3}\{\ \pi_{3;+7,+4} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \frac{1}{3}\,\text{diag}\{\ ,\ 23,\ 20,\ 17,\ 14,\ 11,\ \underline{8},\ 5,\ 2,\ -1,\ -4,\ -7,\ \}$ (49.10)

◇$\pi_{3;+7/2,+1/2} = \frac{1}{3}\{\ \pi_{3;+13/2,+7/2} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \frac{1}{2}\,\text{diag}\{\ ,\ 15,\ 13,\ 11,\ 9,\ 7,\ \underline{5},\ 3,\ 1,\ -1,\ -3,\ -5,\ ,\}$ (49.9)

♣$\pi_{3;+10/3,+1/3}(1) = \frac{1}{3}\{\ \pi_{3;+6,+3} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \frac{1}{3}\,\text{diag}\{\ ,\ 22,\ 19,\ 16,\ 13,\ 10,\ \underline{7},\ 4,\ 1,\ -2,\ -5,\ -8,\ \}$ (49.8)

$\pi_{3;+19/6,+1/6} = \frac{1}{3}\{\ \pi_{3;+11/2,+5/2} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \frac{1}{6}\,\text{diag}\{\ ,\ 43,\ 37,\ 31,\ 25,\ 19,\ \underline{13},\ 7,\ 1,\ -5,\ -11,\ -17,\ \}$ (49.7)

♦$\pi_{3;+3,0} = \frac{1}{3}\{\ \pi_{3;+5,+2} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \text{diag}\{\ ,\ 7,\ 6,\ 5,\ 4,\ 3,\ \underline{2},\ 1,\ 0,\ -1,\ -2,\ -3,\ ,\}$ (49.6)

$\pi_{3;+17/6,-1/6} = \frac{1}{3}\{\ \pi_{3;+9/2,+3/2} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \frac{1}{6}\,\text{diag}\{\ ,\ 41,\ 35,\ 29,\ 23,\ 17,\ \underline{11},\ 5,\ -1,\ -7,\ -13,\ -19,\ \}$ (49.5)

♣$\pi_{3;+8/3,-1/3} = \frac{1}{3}\{\ \pi_{3;+4,+1} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \frac{1}{3}\,\text{diag}\{\ ,\ 20,\ 17,\ 14,\ 11,\ 8,\ \underline{5},\ 2,\ -1,\ -4,\ -7,\ -10,\ \}$ (49.4)

◇$\pi_{3;+5/2,-1/2} = \frac{1}{3}\{\ \pi_{3;+7/2,+1/2} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \frac{1}{2}\,\text{diag}\{\ ,\ 13,\ 11,\ 9,\ 7,\ 5,\ \underline{3},\ 1,\ -1,\ -3,\ -5,\ -7,\ ,\}$ (49.3)

♣$\pi_{3;+7/3,-2/3} = \frac{1}{3}\{\ \pi_{3;+3,0} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \frac{1}{3}\,\text{diag}\{\ ,\ 19,\ 16,\ 13,\ 10,\ 7,\ \underline{4},\ 1,\ -2,\ -5,\ -8,\ -11,\ ,\}$ (49.2)

$\pi_{3;+13/6,-5/6} = \frac{1}{3}\{\ \pi_{3;+5/2,-1/2} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \frac{1}{6}\,\text{diag}\{\ ,\ 37,\ 31,\ 25,\ 19,\ 13,\ \underline{7},\ 1,\ -5,\ -11,\ -17,\ -23,\ ,\}$ (49.1)

♦♦$\underline{\pi_{3;+2,-1}} = \frac{1}{3}\{\ \pi_{3;+2,-1} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \text{diag}\{\ ,\ 6,\ 5,\ 4,\ 3,\ 2,\ \underline{1},\ 0,\ -1,\ -2,\ -3,\ -4,\ ,\}$ (49.0)

$\pi_{3;+11/6,-7/6} = \frac{1}{3}\{\ \pi_{3;+3/2,-3/2} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \frac{1}{6}\,\text{diag}\{\ ,\ 35,\ 29,\ 23,\ 17,\ 11,\ \underline{5},\ -1,\ -7,\ -13,\ -19,\ -25,\ ,\}$ (49.-1)

♣$\pi_{3;+5/3,-4/3} = \frac{1}{3}\{\ \pi_{3;+1,-2} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \frac{1}{3}\,\text{diag}\{\ ,\ 17,\ 14,\ 11,\ 8,\ 5,\ \underline{2},\ -1,\ -4,\ -7,\ -10,\ -13,\ ,\}$ (49.-2)

◇$\pi_{3;+3/2,-3/2} = \frac{1}{3}\{\ \pi_{3;+1/2,-5/2} + \underline{2\pi_{3;+2,-1}}\ \}$

$= \frac{1}{2}\,\text{diag}\{\ ,\ 11,\ 9,\ 7,\ 5,\ 3,\ \underline{1},\ -1,\ -3,\ -5,\ -7,\ -9,\ ,\}$ (49.-3)

♣$\pi_{3;\,+4/3,-5/3} \;=\; \frac{1}{3}\{\,\pi_{3;\,0,-3} \;+\; \underline{2\pi_{3;\,+2,-1}}\,\}$

$= \frac{1}{3}\,\mathrm{diag}\{\ ,\ 16,\ 13,\ 10,\ 7,\ 4,\ \underline{1},\ -2,\ -5,\ -8,\ -11,\ -14,\ ,\}$ (49.-4)

$\pi_{3;\,+7/6,-11/6} \;=\; \frac{1}{3}\{\,\pi_{3;\,-1/2,\,-7/2} \;+\; 2\pi_{3;\,+2,-1}\,\}$

$= \frac{1}{6}\,\mathrm{diag}\{\ ,\ 31,\ 25,\ 19,\ 13,\ 7,\ \underline{1},\ -5,\ -11,\ -17,\ -23,\ -29,\ \}$ (49.-5)

♦$\pi_{3;\,+1,-2} \;=\; \frac{1}{3}\{\,\pi_{3;\,-1,-4} \;+\; \underline{2\pi_{3;\,+2,-1}}\,\}$

$= \quad \mathrm{diag}\{\ ,5,\ 4,\ 3,\ 2,\ 1,\ \underline{0},\ -1,\ -2,\ -3,\ -4,\ -5,\ ,\}$ (49.-6)

$\pi_{3;\,+5/6,-1/2} \;=\; \frac{1}{3}\{\,\pi_{3;\,-3/2,\,-9/2} \;+\; \underline{2\pi_{3;\,+2,-1}}\,\}$

$= \frac{1}{6}\,\mathrm{diag}\{\ ,\ 29,\ 23,\ 17,\ 11,\ 5,\ \underline{-1},\ -7,\ -13,\ -19,\ -25,\ -31,\ ,\}$ (49.-7)

♣$\pi_{3;\,+2/3,-7/3} \;=\; \frac{1}{3}\{\,\pi_{3;\,-2,-5} \;+\; \underline{2\pi_{3;\,+2,-1}}\,\}$

$= \frac{1}{3}\,\mathrm{diag}\{\ ,\ 14,\ 11,\ 8,\ 5,\ 2,\ \underline{-1},\ -4,\ -7,\ -10,\ -13,\ -16,\ ,\}$ (49.-8)

◇$\pi_{3;\,+1/2,-5/2} \;=\; \frac{1}{3}\{\,\pi_{3;\,-5/2,\,-11/2} \;+\; 2\pi_{3;\,+2,-1}\,\}$

$= \frac{1}{2}\,\mathrm{diag}\{\ ,\ 9,\ 7,\ 5,\ 3,\ 1,\ \underline{-1},\ -3,\ -5,\ -7,\ -9,\ -11,\ ,\}$ (49.-9)

♣$\pi_{3;\,+1/3,-8/3}(1) \;=\; \frac{1}{3}\{\,\pi_{3;\,-3,-6} \;+\; \underline{2\pi_{3;\,+2,-1}}\,\}$

$\frac{1}{3}\,\mathrm{diag}\{\ ,\ 13,\ 10,\ 7,\ 4,\ 1,\ \underline{-2},\ -5,\ -8,\ -11,\ -14,\ -17,\ ,\}$ (49.-10)

$\pi_{3;\,+1/6,-17/6} \;=\; \frac{1}{3}\{\,\pi_{3;\,-7/2,\,-13/2} \;+\; \underline{2\pi_{3;\,+2,-1}}\,\}$

$= \frac{1}{6}\,\mathrm{diag}\{\ ,\ 25,\ 19,\ 13,\ 7,\ 1,\ \underline{-5},\ -11,\ -17,\ -23,\ -29,\ -35,\ ,\}$ (49.-11)

♦$\pi_{3;\,0,-3} \;=\; \frac{1}{3}\{\,\pi_{3;\,-4,-7} \;+\; \underline{2\pi_{3;\,+2,-1}}\,\}$

$= \quad \mathrm{diag}\{\ ,4,\ 3,\ 2,\ 1,\ 0,\ \underline{-1},\ -2,\ -3,\ -4,\ -5,\ -6,\ ,\}$ (49.-12)

By the way, the intervals between above two adjoining $\pi_3$ is $\frac{\hbar}{6}$

$$\Delta\pi_{3;\ m/6+2,\ m/6-1} \;=\; \pi_{3;\ (m+1)/6+2,\ (m+1)/6-1} \;-\; \pi_{3;\ m/6+2,\ m/6-1} \;=\; \frac{\hbar}{6}I_0 \qquad (50)$$

Reducing (30), (47), (50) to following limitation

$$\lim_{n\to\infty} \Delta\pi_{3;\ m/2n+2,\ m/2n-1} \;=\; \pi_{3;\ (m+1)/2n+2,\ (m+1)/2n-1} \;-\; \pi_{3;\ m/2n+2,\ m/2n-1} \;=\; \frac{\hbar}{2n}I_0 \qquad (51)$$

And $\pi^2_{+4,-5}(3) \;=\; \pi^2_{m+2,\,m-1}(1) \;=\; \pi^2_{m/6+2,\ m/6-1}(1) \;=\; 1(1\;+\;1)I_0\hbar^2 \;=\; 2I_0\hbar^2 \qquad (52)$

## 5 Conclusions

This paper bases on the principle of the addition of spin angular momentums in **STS** frame, trying to explain the Non-boson-spinization phenomenon of light photon that occurred in [1]. Particle’s spin angular momentums itself, which are influencing on the light photon interference, maybe, rather than the physical quantity phase of propagating light wave causing alone, from previous experiences.

**By Table A.** Explanation for what happening in photonic fermionization of light photon experiment [1].

**By Table B.** Suggestion for dectecting the possible existence of photonic one-third-spinization phenomenon of light photon, by using three beams of light photon in interference experiment.

**By** (51) When the numbers of beams of light photon increase, the intervals between two adjoining $\pi_3$ become narrower, and the interference patterns approach to continuous spectrum.

**Table A.** **<u>Interference by two beams of light photon</u>** [1]

**fermionization phenomenon of light photon**

| | |
|---|---|
| **boson-spinization** | **fermionization** |
| $\vec{\pi}_{+2,-1}$ , $\vec{\pi}_{+2,-1}$ | $\vec{\pi}_{+1,-2}$ , $\vec{\pi}_{+2,-1}$ |
| $\frac{1}{2}\{\pi_{3;+2,-1} + \underline{\pi_{3;+2,-1}}\}$ (26.0) | $\frac{1}{2}\{\pi_{3;+1,-2} + \underline{\pi_{3;+2,-1}}\}$ (26.-1) |
| $\pi_{3;+2,-1}(1)$ (20) | $\pi_{3;+3/2,-3/2}(1)$ (21) |
| ..., $2\hbar$, $1\hbar$, $0\hbar$, $-1\hbar$, $-2\hbar$, ... | ..., $1.5\hbar$, $0.5\hbar$, $-0.5\hbar$, $-1.5\hbar$, ... |

**Table B.** **<u>Interference by three beams of light photon</u>**

**one-third-spinization phenomenon of light photon**

| | | |
|---|---|---|
| **boson-spinization** | **one-third-spinization** | **one-third-spinization** |
| $\vec{\pi}_{+2,-1}$, $\vec{\pi}_{+2,-1}$, $\vec{\pi}_{+2,-1}$ | $\vec{\pi}_{+1,-2}$, $\vec{\pi}_{+2,-1}$, $\vec{\pi}_{+2,-1}$ | $\vec{\pi}_{0,-3}$, $\vec{\pi}_{+2,-1}$, $\vec{\pi}_{+2,-1}$ |
| $\frac{1}{3}\{\pi_{3;+2,-1} + \underline{2\pi_{3;+2,-1}}\}$ (44.0) | $\frac{1}{3}\{\pi_{3;+1,-2} + \underline{2\pi_{3;+2,-1}}\}$ (44.-1) | $\frac{1}{3}\{\pi_{3;0,-3} + \underline{2\pi_{3;+2,-1}}\}$ (44.-2) |
| $\pi_{3;+2,-1}(1)$ (41) | $\pi_{3;+5/3,-4/3}(1)$ (42) | $\pi_{3;+4/3,-5/3}(1)$ (43) |
| ..., $\underline{3\hbar/3}$, $0\hbar/3$, ... | ..., $\underline{2\hbar/3}$, $-1\hbar/3$, ... | ..., $\underline{1\hbar/3}$, $-2\hbar/3$, ... |

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