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A Multi-Objective Obnoxious Facility Location Model on a Plane

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Abstract

In this paper a Vertex Covering Obnoxious Facility Location model on a Plane has been designed with a combination of three interacting criteria as follows: 1) Minimize the overall importance of the various existing facility points; 2) Maximize the minimum distance from the facility to be located to the existing facility points; 3) Maximize the number of existing facility points covered. Area restriction concept has been incorporated so that the facility to be located should be within certain restricted area. The model developed here is a class of maximal covering problem, that is covering maximum number of points where the facility is within the upper bounds of the corresponding mth feasible region. Two types of compromise solution methods have been designed to get a satisfactory solution of the multi-objective problem. A transformed non-linear programming algorithm has been designed for the proposed non-linear model. Rectilinear distance norm has been considered as the distance measure as it is more appropriate to various realistic situations. A numerical example has been presented to illustrate the solution algorithm.

1. Introduction

For service facility location problems when the costs are the increasing function of distance, it is reasonable to consider either minimum of the sum of distances or the weighted distances. On the other hand, for some vital facilities it may be desired to minimize the maximum distances. However, there are types of location situations where cost decreases as distance increases and is considered the name in the literature as the obnoxious or undesirable facility location problems.

Erkut and Neuman [1] have given an excellent survey on undesirable facility location problems. Models containing maximization of some function of distances as one of the objectives were considered for analysis. Examples appropriate for the above cases are garbage dump, chemical plant or a nuclear reactor. Another type of problems called semi desirable, have been found in the literature. Examples appropriate for these models are baseball stadium, incineration plants etc.

In the location literature many people have worked on MAXIMIN criterion with Euclidean distances. Shamos [2] defines the unweighted MAXIMIN problem as the largest empty circle problem in R^2 and provides an algorithm

for solving that problem. Dasarathy and White [3] extended the unweighted maximin problem to a higher dimensional space and a convex feasible region. They provide an algorithm for a three dimensional space. Drezner and Wesolowsky [4] present a solution to a maximin problem assuming a feasible region which is the intersection of the circles of prescribed radii whose centers are existing facility points. Melachrinoudis and Cullinane [5] solved maximin problem for the case of non convex feasible region S in the presence of forbidden circles.

Drezner and Wesolowsky [6] first introduced the rectilinear maximin problem for locating an obnoxious facility. They developed a solution procedure by dividing the feasible region into rectilinear sub regions and solving a linear programming problem for each of this sub regions. Melachrinoudis [7] proved several properties of the optimal solution, developed elimination strategies for the sub regions and solved the duals of the LPs for the remaining sub regions. Mehrez *et al*. [8] suggested an improvement of Drezner and Wesolowsky's algorithm, based on bounds, which reduces the size as well as the number of sub problems to be solved. Arie Tamir [9] has presented a subquadratic algorithm for location two obnoxious facilities using the weighted maximin criterion.

Banez *et. al.* [10] have considered a problem of locating an obnoxious plane and solved in O(n^3) time and O(n^2) space. Plastria and Carrizosa [11] have considered of locating an undesirable facility within some feasible region of any shape in the plane or on a planar network by considering two criteria: a radius of influence to be maximized and the total covered population to be minimized. Low complexity polynomial algorithms are derived to determine all non dominated solutions.

A bibliography for some fundamental problem categories such covering models are given by C.S. Revelle, H.A. Eiselt, M.S. Daskin [12].

In this present investigation a vertex covering obnoxious facility location model has been designed with multiple objectives. In this model weights as importance has been assigned to the various demand points and considers as a separate objective. Because of undesirable facility, the facility points which has to be kept more distance away from the undesirable facility location should be given less weitages compared to the facility points which may be kept comparatively closer. The problem has been modeled as a pure planar location problem. Area restriction concept has been incorporated so that the facility to be located should be within certain restricted area. Incorporation of the area restriction has been implemented by inducting a convex polygon in the feasible region. Another advantage of introduction of a convex polygon in the constraint set is that it might reduce the number of transformed non-linear programming problems to be solved. Two types of compromise solution methods have been designed to get the satisfactory solution of the original multi-objective non linear model. The proposed model has another advantage to give different weightages to the different criteria separately to reach out to a desired compromise solution. Rectilinear distance norm has been considered as a distance measure to design the model. A numerical example has been presented to illustrate the solution algorithm.

Preliminaries are mentioned in Section 2. Model formulation for the vertex covering obnoxious facility problem has been given in Section 3. An algorithm has been designed in Section 4. A numerical example has been presented in Section 5. Concluding remarks are made in Section 6.

2. Preliminaries

1-Maxi-min Problem: Single facility location problem with maximum objective can be broadly classified into maximin and maxi sum objectives. The problem 1-maxi min which is under study in this paper is described below.

Let (a_i, b_i) for $i = 1, 2, \cdots, n$ be the location of n demand points. (x, y) be the co-ordinates of the facility to be located.

Then $F(x, y) = \min\limits_{i} \{|x - a_i| + |y - b_i|\}$ be the minimum rectangular distances from the facility to the demand point *i*. Then the mathematical model is

Max $\qquad\qquad F(x, y)$

subject to

$$AX \leq b, x \geq 0.$$

3. Mathematical Formulation

Let (a_i, b_i) be the location of the *i*th existing facility and (x, y) is the coordinates of the point to be located. The objective corresponding to the Minimize the overall importance of the various demand points is formulated as

P1.

Min $\qquad\qquad \sum\limits_{i=1}^{n} w_i x_i$

where $\qquad x_i = \begin{cases} 1, & |x - a_i| + |y - b_i| \geq Z_i \\ 0 & Otherwise \end{cases}$.

and w_i is the weights as importance assigned to the ith demand points for every *i*.

The objective corresponding to maximize the minimum distance from the facility to the demand points objective can be written as follows.

P2.

Maximize $\qquad \{Minimize \quad d_i(x, y)\}$

$\qquad\qquad (x, y) \in S \qquad i$

where $\qquad d_i(x, y) = |x - a_i| + |y - b_i|$

The third objective maximize the demand units covered can be formulated as

P3.

Maximize $\qquad\qquad \sum\limits_{i=1}^{n} x_i$

where x_i is as defined earlier.

Thus the multi-objective formulation of the vertex covering obnoxious facility location problem may be written as follows

P4.

Minimize $\qquad\qquad \sum\limits_{i=1}^{n} w_i x_i$

Maximize $\qquad\qquad Z_1$

Maximize $\qquad\qquad \sum\limits_{i=1}^{n} x_i$

subject to,

$$|x - a_i| + |y - b_i| > Z_1, \qquad i = 1, 2, \cdots, n$$
$$c_{j_1} x + c_{j_2} y \leq c_{j_3}, \qquad j = 1, 2, \cdots, k$$
$$(x, y) \in S$$

where $x_i = \begin{cases} 1, & if\ |x - a_i| + |y - b_i| \geq Z_1 \\ 0 & Otherwise \end{cases}$

4. Algorithm for Vertex Covering Obnoxious Facility Location Problem

Let us consider the grid of lines formed by drawing horizontal and vertical lines through every demand point (a_i, b_i). This will form at most $(n+1)^2$ rectangular regions, some of them bounded by infinity. Consider now the mathematical formulation which may be written as

P5.

Maximize $\quad\quad\quad\quad\quad\quad Z_1$

Minimize $\quad\quad\quad\quad\quad\quad \sum_{i=1}^{n} w_i x_i$

Maximize $\quad\quad\quad\quad\quad\quad \sum_{i=1}^{n} x_i$

subject to,

$$|x - a_i| + |y - b_i| \geq Z_1 X_i, \quad\quad i = 1, 2, \cdots, n$$

$$Z_1 \leq UB_m$$
$$c_{j_1} x + c_{j_2} y \leq c_{j_3}, \quad\quad\quad j = 1, 2, \cdots, k$$

where $\quad\quad x_i = \begin{cases} 1, & if\ |x - a_i| + |y - b_i| \geq Z_1 \\ 0 & Otherwise \end{cases}$

and UB_m is the upper bound corresponding to the rectangle m.

Next to find the upper bound inside rectangle m. The maximum value of Z_1 on m is as given below:

$$Z_m = Max\left\{Min\left(|x - a_i| + |y - b_i|\right)\right\}$$

$$(x, y) \in m \quad\quad i$$

$$\leq Min\left\{Max\left(|x - a_i| + |y - b_i|\right)\right\}$$

$$i \quad\quad (x, y) \in m$$

The maximum inside rectangle m must occur on *V*, where *V* is the set of four vertices of the rectangle (some of this vertices may be at infinity). Hence, an upper bound on Z_1 is

$$UB_m = Min\left\{Max\left(|x - a_i| + |y - b_i|\right)\right\}$$

$$i \quad\quad (x, y) \in m$$

For an open rectangle UB in infinite.

The single objective formulation of the above multi-objective problem is as given below.

The non-linear constraints of Problem (5) can be broken down into four alternative sub problems by using the inequality relations. The four sub problems are as follows.

P6.

Maximize $\quad\quad\quad\quad\quad\quad Z_1$

Minimize $\quad\quad\quad\quad\quad\quad \sum_{i=1}^{n} w_i x_i$

Maximize $\quad\quad\quad\quad\quad\quad \sum_{i=1}^{n} x_i$

subject to,

$$x + y - a_i - b_i + Z_1 x_i \geq 0$$

$$Z_1 \leq UB_m$$

$$c_{j_1} x + c_{j_2} y \leq c_{j_3}, \quad\quad j = 1, 2, \cdots, k .$$

or
P7.

Maximize $\quad\quad\quad\quad\quad\quad Z_1$

Minimize $\quad\quad\quad\quad\quad\quad \sum_{i=1}^{n} w_i x_i$

Maximize $\quad\quad\quad\quad\quad\quad \sum_{i=1}^{n} x_i$

subject to,

$$x - y - a_i + b_i - Z_1 x_i \geq 0$$

$$Z_1 \leq UB_m$$

$$c_{j_1} x + c_{j_2} y \leq c_{j_3}, \quad\quad j = 1, 2, \cdots, k .$$

or **P8.**

Maximize $\quad\quad\quad\quad\quad\quad Z_1$

Minimize $\quad\quad\quad\quad\quad\quad \sum_{i=1}^{n} w_i x_i$

Maximize $\quad\quad\quad\quad\quad\quad \sum_{i=1}^{n} x_i$

subject to,

$$x - y - a_i + b_i + Z_1 x_i \geq 0$$

$$Z_1 \leq UB_m$$

$$c_{j_1} x + c_{j_2} y \leq c_{j_3}, \quad\quad j = 1, 2, \cdots, k .$$

or **P9.**

Maximize $\quad\quad\quad\quad\quad\quad Z_1$

Minimize $\quad\quad\quad\quad\quad\quad \sum_{i=1}^{n} w_i x_i$

Maximize $\quad\quad\quad\quad\quad\quad \sum_{i=1}^{n} x_i$

subject to,

$$x - y + a_i + b_i - Z_1 x_i \geq 0$$

$$Z_1 \leq UB_m$$

$$c_{j_1}x + c_{j_2}y \le c_{j_3} \qquad j = 1, 2, \cdots, k.$$

For all the four problems P6 - P9

$$x_i = \begin{cases} 1, & if \ |x - a_i| + |y - b_i| \ge Z_1 \\ 0 & Otherwise \end{cases}.$$

And UB_m is the upper bound corresponding to the rectangle m.

For solving the above multi-objective problem two approaches are suggested.

Type Ⅰ Compromise solution:

Define the variable U_K in such a way that

$$U_K = \left[\frac{Z_k(X)}{Z_k^*} \right] \qquad X \in S$$

For getting the compromise solution following objective function is defined.

P10.

$$Maximize = V = \theta_1 U_1 + \theta_2 U_2 - \theta_3 U_3$$

such that $X \in S$

Where Z_K^* is the maximum value of Z_K over the constraint set. Z_K^* can be obtained by solving the individual single objective problem. θ_K is the weight attached to the ith objective function.

The problem thus transformed to four sets as the constraints given in P6, P7, P8, P9 with objective function for all the problems as given in P10.

Type Ⅱ compromise solution

The second type of compromise solution is obtained by defining the objective function

P11.

Minimize $\qquad W = \sum_{k=1}^{3} \theta_K U_K^2$

Where U_K is defined as follows:

$$U_1 = \left(1 - \frac{f_1(X)}{Z_1^\bullet}\right), U_2 = \left(1 - \frac{f_2(X)}{Z_2^\bullet}\right), U_3 = \left(1 - \left[\frac{-f_3(X)}{Z_3^\bullet}\right]\right)$$

Where Z_K^* is the maximum value of Z_K over the constraint set. Z_K^* can be obtained by solving the individual single objective problem. θ_K is the weight attached to the ith objective function. Adjustment in U_3 to avoid zero in the denominator.

The Algorithm for solving multi objective formulation of the Obnoxious Facility Location Problem is as given below.

Step-**1**. Formulate the multi-objective problem as given in P10 for **Type Ⅰ** compromise solution and P11 for **Type Ⅱ** compromise solution. Choose the weightages to be assigned to the different objectives.

Step-**2.** Find the enclosing feasible rectangle by the

procedure-1.

Step-**3**. Restrict the grid and rectangles to be considered to those that lie within the enclosing feasible rectangle.

Step-**4**. Eliminate all infeasible rectangles by Result 1.

Step-**5.** Find the upper bound on Z_1 for all the remaining rectangles by using Procedure 1. Sort these upper bounds of Z_1 from the largest to the smallest.

Step-**6**. Solve the multi-objective non-linear programming problem by breaking the problem into four alternative sub problems.

Step-**7**. Solve the sub problems starting with the rectangle with the highest upper bound on Z_1. Stop when the upper bound for the next rectangle is not greater than the best Z_1 value solution found so far.

Step-**8**. Take the solution of the sub problem which gives maximum objective function value for **type Ⅰ** compromise solution and the solution of the sub problem which gives minimum objective function value.

Procedure **1**. Find the enclosing feasible rectangle (Drezner & Wesolowsky (1983)).

To find $x = X_L$, the left vertical line defining the rectangle, solve a linear programming problem with x as the objective to be minimized and the constraints ( ).

Maximizing x will give X_U, which defines the right vertical line. The lower and upper horizontal lines $y = Y_L$ and $y = Y_U$ are found similarly.

Let us consider the rectangle m defined by the lines $x = a_{i_1}$ $\quad x = a_{i_2}$ $\quad y = b_{i_3}$ $\quad y = b_{i_4}$, where $a_{i_1} < a_{i_2}$ and $b_{i_3} < b_{i_4}$. Also we define the segments $\left[x_L^{(i)} \le x_U^{(i)} \right]$ and $\left[y_L^{(i)} \le y_U^{(i)} \right]$ where $x_L^{(i)} \le x_U^{(i)}, y_L^{(i)} \le y_U^{(i)}$; these are the parts of the line respectively $y = b_i$ and $x = a_i$.

Result 1. If all the following conditions hold, then there is no feasible point inside rectangle m (Drezner & Wesolowsky (1983)).

$$Y_L^{(i_1)} > b_{i_4} \ or \ y_U^{(i_1)} < b_{i_3} \qquad (1)$$

$$y_L^{(i_2)} > b_{i_4} \ or \ y_U^{(i_2)} < b_{i_3} \qquad (2)$$

$$x_L^{i_3} > a_{i_4} \ or \ x_U^{(i_3)} < a_{i_1} \qquad (3)$$

$$x_L^{i_4} > a_{i_2} \ or \ x_U^{(i_4)} < a_{i_1} \qquad (4)$$

5. Numerical Example

Let A(2,1), B(3,5), C(6,9), D(4,1), E(9,7) be the five demand points on a plane, and $3x + 5y \le 45$, $5x + 4y \le 42$, $x \ge 0, y \ge 0$ are the boundary of the convex feasible region. Let the weights attached to the five demand points are 0.30, 0.15, 0.20, 0.10, 0.25 respectively.

The upper bound corresponding to the twenty feasible rectangles (see **Figure 1**) are obtained by procedure 1

and theorem 1 and are given in the sorted form, starting from largest to the smallest in **Table 1**.

For Type Ⅰ compromise solution

In the first iteration the mathematical programming formulation of the first sub problem corresponding to the highest upper bound of Z_1 as 9 is given in the following

Maximize $V = 0.4Z_1(0.11) + 0.06Z_2 - 0.3Z_3$

Subject to,

$$x + y - 2 - 1 + Z_1 x_1 \geq 0;$$
$$x + y - 8 + Z_1 x_2 \geq 0;$$
$$x + y - 15 + Z_1 x_3 \geq 0;$$
$$x + y - 5 + Z_1 x_4 \geq 0;$$
$$x + y - 16 + Z_1 x_5 \geq 0;$$
$$x_1 + x_2 + x_3 + x_4 + x_5 - z_2 = 0;$$
$$0.3x_1 + 0.15x_2 + 0.2x_3 + 0.1x_4 + 0.25x_5 5 - z_3 = 0;$$
$$3x + 5y \leq 45;$$
$$5x + 4y \leq 40;$$
$$x \geq 0; y \geq 0;$$
$$x_1, x_2, x_3, x_4, x_5 = (0,1);$$

$$Z_1 \leq 9;$$

Where 9,5,1 are the ideal solutions obtained for the first, second and the third objectives respectively. The weights attached to the three objectives are 0.4,0.3,0.3 respectively.

Solution obtained for is as given below:

SP-1

$V = 0.426,$

$Z_1 = 9, Z_2 = 4, Z_3 = 0.7,$

$x = 1.53, y = 8.07,$

$x_1 = 0, x_2 = x_3 = x_4 = x_5 = 1$

Similarly the solutions for the other sub problems are as given below.

SP-2

$V = 0.411,$

$Z_1 = 9, Z_2 = 1, Z_3 = 0.15,$

$x = 8, y = 0,$

$x_1 = x_3 = x_4 = x_5 = 0, x_2 = 1$

SP-3

$V = 0.426,$

$Z_1 = 9, Z_2 = 1, Z_3 = 0.10,$

$x = 0, y = 9,$

$x_1 = x_2 = x_3 = x_5 = 0, x_4 = 1$

SP-4

$V = 0.396$

$Z_1 = 9, Z_2 = 0, Z_3 = 0,$

$x = 0, y = 0,$

$x_1 = x_2 = x_3 = x_4 = x_5 = 0$

Table 1. Upper bound for various rectangles.

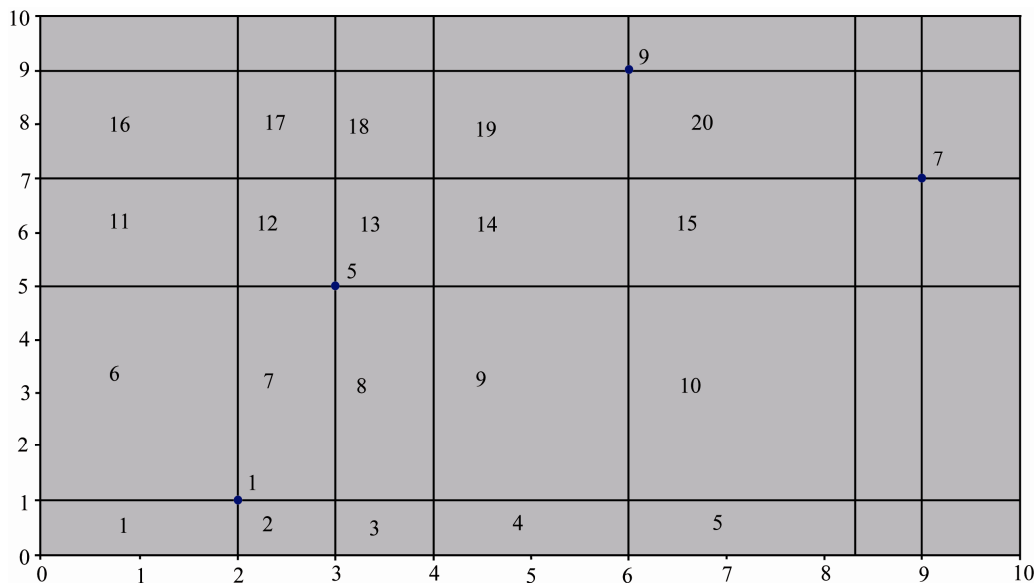| Rectangle Number	10	6	9	5	7	8	11	14	15	16	17	18	19	20	1	4	12	13	2	3
UB_m	9	6	6	6	5	5	5	5	5	5	5	5	4	5	3	3	3	3	2	2

Figure 1. Rectangles.

If we go to the next iteration with the next Z_1 value, the solution obtained can not dominate the Z_1 value satisfactory solution for the multi-objective problem are the solutions of **SP-1** and **SP-3**. So in this case we are getting the alternative optimal compromise solutions.

For Type Ⅱ compromise solution

In the first iteration the mathematical programming formulation of the first sub problem corresponding to the highest upper bound of Z_1 as 9 is given in the following.

Minimize

$$W = 0.4\big[1 - Z_1(0.11)\big]^2 + 0.3\big[1 - Z_2(0.2)\big]^2 + 0.3\big(1 - Z_3\big)^2$$

Subject to,

$$x + y - 2 - 1 + Z_1 x_1 \geq 0;$$
$$x + y - 8 + Z_1 x_2 \geq 0;$$
$$x + y - 15 + Z_1 x_3 \geq 0;$$
$$x + y - 5 + Z_1 x_4 \geq 0;$$
$$x + y - 16 + Z_1 x_5 \geq 0;$$
$$x_1 + x_2 + x_3 + x_4 + x_5 - z_2 = 0;$$
$$0.3x_1 + 0.15x_2 + 0.2x_3 + 0.1x_4 + 0.25x_5 - z_3 = 0;$$
$$3x + 5y \leq 45;$$
$$5x + 4y \leq 40;$$
$$x \geq 0; y \geq 0;$$
$$x_1, x_2, x_3, x_4, x_5 = (0,1);$$
$$Z_1 \leq 9;$$

The solution obtained by using software LINGO for the four sub problems are as given below. The weights attached to the three objectives are 0.4,0.3,0.3 respectively.

SP-1
$W = 0.4\text{E-}04$
$Z_1 = 9, Z_2 = 5, Z_3 = 1,$
$x = 7, y = 0,$
$x_1 = x_2 = x_3 = x_4 = x_5 = 1$

Similarly the solution obtained for the other three sub problems are as given below.

SP-2
$W = 0.5887$
$Z_1 = 9, Z_2 = 1, Z_3 = 0.15,$
$x = 8, y = 0,$
$x_1 = x_3 = x_4 = x_5 = 0, x_2 = 1$

SP-3
$W = 0.555$
$Z_1 = 9, Z_2 = 1, Z_3 = 0.10,$
$x = 0, y = 9,$
$x_1 = x_2 = x_3 = x_5 = 0, x_4 = 1$

SP-4

$W = 0.594$
$Z_1 = 8, Z_2 = 1, Z_3 = 0.10,$
$x = 0, y = 0,$
$x_1 = x_3 = x_4 = x_5 = 0, x_2 = 1$

The solution obtained with the next Z_1 value can not dominate the solution obtained so far. Thus we stop the process here and the satisfactory solution for the multi-objective problem by using **Type Ⅱ** compromise solution methods are the solution of **SP-1**.

In the **Type Ⅰ** compromise solution method we have got the alternative optimal solutions where as for the **Type Ⅱ** compromise solution method we have got first sub problem as the optimal compromise solution.

6. Concluding Remarks

- In this paper a vertex covering obnoxious facility location problem has been modeled as a multi-objective planar location model.
- A modified non linear programming algorithm has been designed to solve the proposed model.
- Two types of compromise solution methods have been designed. In the first method each individual objectives are divided by the ideal solutions and the sum of them are maximized. Where as in the second method each objective deviations have been divided by the ideal solution and square of them has been minimized.
- Other distance norm such as Euclidean, Geodesic etc. may be considered to model various other situations.
- Models with general feasible regions (Union of disjoint and non-convex sets) may be considered to model various geographic regions.
- The model developed here is a class of maximal covering problem, that is covering maximum number of points where the facility is within the upper bounds of the corresponding mth feasible region.

7. References

[1] E. Erkut and S. Neuman, "Analytical Models for Locating Undesirable Facilities," *European Journal of Operational Research*, Vol. 40, No. 3, 1989, pp. 275-291. doi:10.1016/0377-2217(89)90420-7

[2] M. I. Shamos, "Computational Geometry," Ph.D. Dissertation, Department of Computer Science, Yale University, New Haven, 1977.

[3] B. Dasarathy and L. White, "A Maximin Location Problem," *Operations Research*, Vol. 28, No. 6, 1980, pp. 1385-1401. doi:10.1016/0377-2217(89)90420-7

[4] Z. Drezner and G. O. Wesolowsky, "A Maximin Location Problem with Maximum Distance Constraints," *AIIE Transaction*, Vol. 12, No. 3, 1980, pp. 249-252.

doi:10.1080/05695558008974513

[5] E. Melachrinoudis and T. P. Cullinane, "Locating an Undesirable facility within a Geographical Region Using the Maximin Criterion," *Journal of Regional Science*, Vol. 25, No. 1, 1985, pp. 115-127. doi:10.1111/j.1467-9787.1985.tb00297.x

[6] Z. Drezner and G. O. Wesolowsky, "The Location of an Obnoxious Facility with Rectangular Distance," *Journal of Regional Science*, Vol. 23, No. 2, 1983, pp. 241-248. doi:10.1111/j.1467-9787.1983.tb00800.x

[7] E. Melachrindis, "An Efficient Computational Procedure for the Rectilinear Maximin Location Problem," *Transportation Science*, Vol. 22, No. 3, 1988, pp. 217-223. doi:10.1287/trsc.22.3.217

[8] A. Mehrez, Z. Sinuany-Stern and A. Stulman, "An Enhancement of the Drezner-Wesolowsky Algorithm for Single Facility Location with Maximin of Rectangular Distance," *Journal of Operations Research Society*, Vol.

37, No. 10, 1986, pp. 971-977.

[9] A. Tamir, "Locating Two Obnoxious Facilities using the Weighted Maximin Criterion," *Operations Research Letter*, Vol. 34, No. 1, 2006, pp. 97-105. doi:10.1016/j.orl.2005.02.004

[10] J. M. Diaz-Banez, M. A. Lopez and J. A. Sellaves, "Locating an Obnoxious Plane," *European Journal of Operations Research*, Vol. 173, No. 2, 2006, pp. 556-564. doi:10.1016/j.ejor.2005.02.048

[11] F. Plastria, and E. Carrizosa, "Undesirable Facility Location with Minimal Covering Objectives," *European Journal of Operational Research*, Vol. 119, No. 1, 1999, pp. 158-180. doi:10.1016/S0377-2217(98)00335-X

[12] C. S. ReVelle, H. A. Eiselt, M. S. Daskin, "A Bibliography for Some Fundamental Problem Categories in Discrete Location Science," *European Journal of Operational Rsearch*, Vol. 184, No. 3, 2008, pp. 817-848. doi:10.1016/j.ejor.2006.12.044