

A Monte Carlo Based Robustness Optimization Method in New Product Design Process: A Case Study

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Abstract

Monte Carlo method can analyze, solve and optimize many mathematical or physical problems through generating a large number of statistical random samples to simulating stochastic events. It also can be used to remarkably improve design quality of new product. In new product design process, setting distribution characteristics of the design variables is vital to product quality and production robustness. Firstly, response surface model between output characteristics and design variables in new product design is proposed, and the distribution characteristics of design variables and response output are analyzed; then position error model of response output and standard value and allowed error maximum is presented; and then the differences of position error model and allowed error maximum are counted, and reliability ratio is built and calculated, and design robustness of the new product is increased by adjusting the precision value of random design variables in Monte Carlo experiments. Finally, a case is brought forward to verify the validity of the method.

Keywords

Monte Carlo Simulation, Robustness Optimization, New Product Design, Process Improvement

1. Monte Carlo Method

Monte Carlo method is a computer-based simulation or experiment method. It approximately simulates and solves mathematical or physical stochastic problem with statistical random sampling. Compared with traditional algebraic method, due to their reliance on repeated computation of random or pseudo-random numbers, Monte Carlo method can apply Normal distribution, Exponential distribution, Weibull distribution etc. to model phenomena with significant uncertainty in inputs when it is unfeasible or impossible to compute an exact result with a deterministic algo-

rithm, and does not need to know parameter's distribution type and probability parameter [1]-[5]. In new product design, usually design parameter is a random variable that follows a probability distribution. Design parameter is a key impact factor to design robustness, and distribution of these design parameters determines greatly robustness of output response. It is not reliable to optimize this product design problem using traditional deterministic method. How to find the optimum value of these design parameters is an important problem to improve the design robustness of new product. Here, we propose a robustness optimization method of new product based on Monte Carlo simulation.

Procedures are as followed [6]-[10]:

1) To analyze the problem existing in new product design, and define the relationship between design variables X and response output as a response surface model $f(x) = f(x_1, x_2, \dots, x_n)$;

2) To analyze design variables' distribution types and define design variables' distribution characteristics, such as mean and standard deviation, etc.;

3) To sample from populations of random design variables X , and get the sample $x(j) = \{x_{j1}, x_{j2}, \dots, x_{jn}\}^T$;

4) To bring sample $x(j)$ into position error model between response surface model and standard value, $f(x)$, then get a r position error sample and compose an experiment

$$\Delta_{i\max} = g(x_j, H_i) = g\{x_{j1}, x_{j2}, \dots, x_{jn}, B_i\}^T, (B_i \text{ is response output, } i = 1, 2, \dots, n);$$

5) To check if it meets $Y_j = f(x) = \Delta_{i\max} - [\Delta] < 0$, $[\Delta]$ is allowed error maximum.

6) To repeat step 2) to 5) by k times, and calculate the number of $Y_j < 0$ in all independent samples Y_j ,

$$K_T = \sum_{j=1}^k U_j, \text{ here } K_T \text{ is the number of } Y_j < 0, U_j = \begin{cases} 0 & Y_j > 0 \\ 1 & Y_j < 0 \end{cases};$$

7) To calculate reliability ratio $P = K_T/K$. If $P > D$ (D is acceptable reliability), then process or product is robust, otherwise we need to modify the precision value of random design variables to enhance reliability ratio and robustness of the manufacture process, then the manufacture process is optimized.

2. Case Study

Considering design of a pressure container, according to mechanics of materials, the pressure container's axial stress is $\sigma_1 = \frac{pR}{2\delta}$, hoop stress is $\sigma_2 = \frac{pR}{\delta} \left(1 - \frac{R^2}{2H^2}\right)$, here p is internal pressure of container; δ is wall

thickness of container; R is internal radius of container; H is half height of container. Material of container is 15 MnV. Through observation of experiment, we defined the design variables and their distribution characteristics as followed: internal work pressure of container p follows lognormal distribution $p \sim LN(14.495, 1.4495^2)$, the tolerance of p is (10, 20), the material strength limit σ_s follows lognormal distribution $\sigma_s \sim LN(392, 19^2)$, the tolerance of σ_s is (340, 420), wall thickness of container δ follows normal distribution $\delta \sim N(3.0, 0.1133^2)$, the tolerance of δ is (1.5, 4.5), internal radius of container R follows normal distribution $R \sim N(88, 0.88^2)$, the tolerance of R is (50, 100), half height of container H follows normal distribution $H \sim N(220, 11^2)$, the tolerance of H is (130, 210). Our objective is to maximize container's volume under 95% failure probability (confidence level) of strength and container size falling Interval $2 \leq H/R \leq 2.5$.

As known, our objective function is volume maximum of the pressure container, that is

$$\max f(p, \sigma_s, \delta, R, H) = \frac{4}{3} \pi (R - \delta)^3 + 2\pi (R - \delta)^2 (H - R)$$

Constraints are

$$g_1: \frac{pR}{2\delta} - \sigma_s \leq 0$$

$$g_2: \frac{pR}{\delta} \left(1 - \frac{R^2}{2H^2}\right) - \sigma_s \leq 0$$

$$g_3: 2 \leq H/R \leq 2.5$$

Firstly we established response surface model of response variables f, g_1, g_2, g_3 concerned pressure con-

tainer and constraints condition, and run Monte Carlo experiment 3000 times using Crystal Ball 7. According to Anderson-Darling testing, Chi-Square testing and K-S testing [11]–[13], we can fit the probability distribution type of all four response variables as Normal distribution, Gamma distribution, Gamma distribution and Beta distribution, see **Table 1**.

Then we get probability distribution and cumulative probability distribution of response variable and constraints as showed in **Figure 1**.

1) Robust analysis

We can know from **Figure 2** that probability which product design met constraint g_1 is $\int_{-\infty}^0 g_1 dy = 1 > 95\%$, and probability which product design met constraint g_2 is $\int_{-\infty}^0 g_2 dy = 0.5037 < 95\%$, and probability which product design met constraint g_3 is $\int_2^{2.5} g_3 dy = 0.5052 < 95\%$ under current design variables value. That is, wave range of constraints both g_1 and g_2 go beyond their allowed range. According to requirements, current response output value is not robust and need to improve product design level.

2) Sensitivity analysis

We can know from Sensitivity Analysis of Response Variables and Constraints in **Figure 3** that the significant impact factors of volume function f of pressure container are H (positive correlation) and R (positive correlation) in order; the significant impact factors of constraint g_1 are p (positive correlation), σ_s (negative correlation) and δ (negative correlation) in order; the significant impact factors of constraint g_2 are p (positive correlation), σ_s (negative correlation) and δ (negative correlation) in order; the significant impact factors of constraint g_3 are H (positive correlation) and R (negative correlation) in order.

3) Modification of design variables precision

According to relationships between design variables and response output variable, and contributions to variance view from **Figure 3**, we modified parameter values of the design variables $p, \sigma_s, \delta, R, H$ to $p \sim LN(16, 1.4495^2)$, $\sigma_s \sim LN(392, 19^2)$, $\delta \sim N(3.0, 0.1133^2)$, $R \sim N(74, 1^2)$, $H \sim N(180, 4^2)$, and get the corresponding probability distribution and cumulative probability distribution as followed in **Figure 4**.

After Modification of design variables precision, we analyze robustness of response variable and constraints

Table 1. Analysis of response variables' distribution characteristics.

Variables	Mean	Median	Standard deviation	Skewness	Kurtosis	Ceff. of variability	Distribution type
f	8568233.77	8568233.77	534933.47	0.00	2.97	0.0624	Normal
g_1	1522.92	1511.21	208.31	0.3379	3.17	0.1368	Gamma
g_2	-0.22	-3.93	47.08	0.4748	3.34	-216.04	Gamma
g_3	2.50	2.50	0.13	0.00	2.97	0.0519	Beta

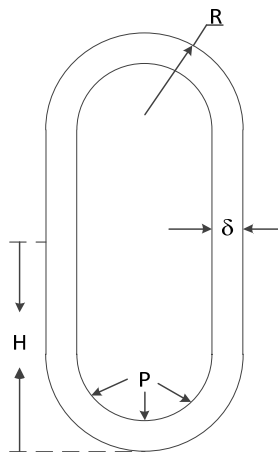


Figure 1. Pressure container chart.

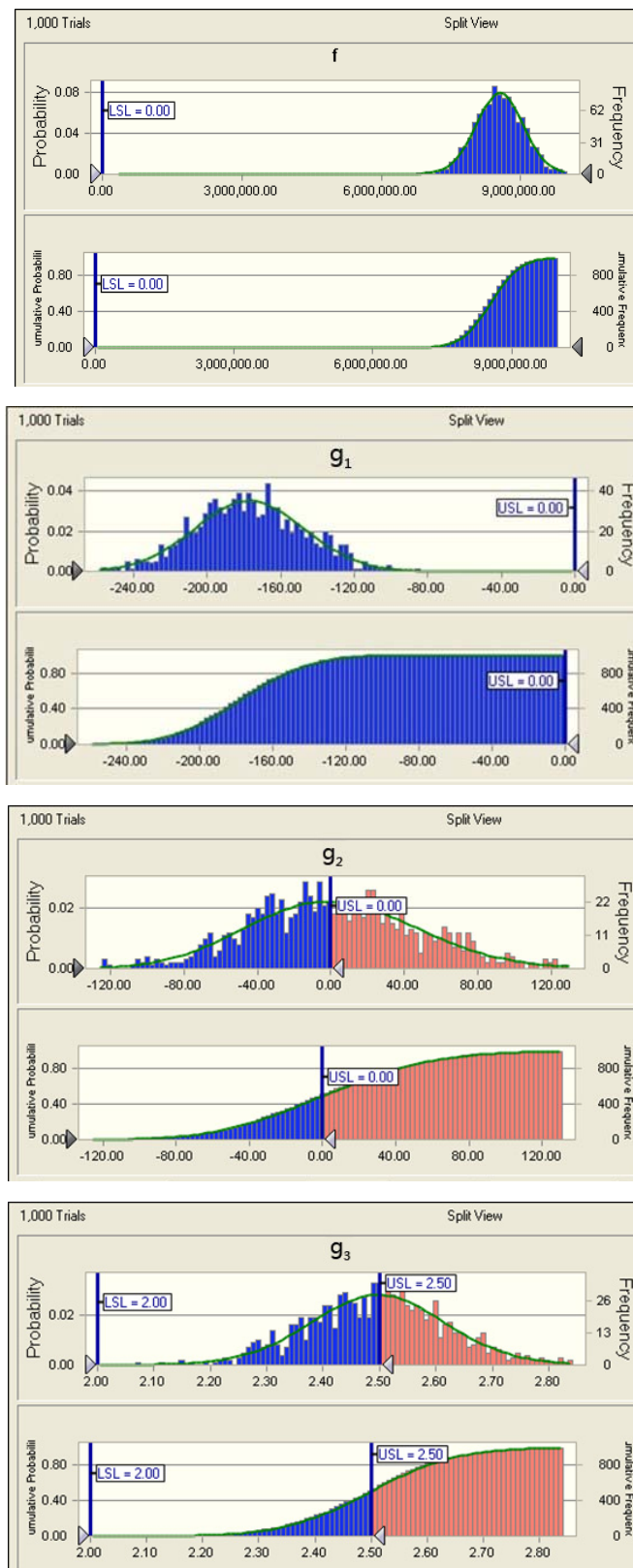


Figure 2. Probability distribution and cumulative probability distribution of response variable and constraints.

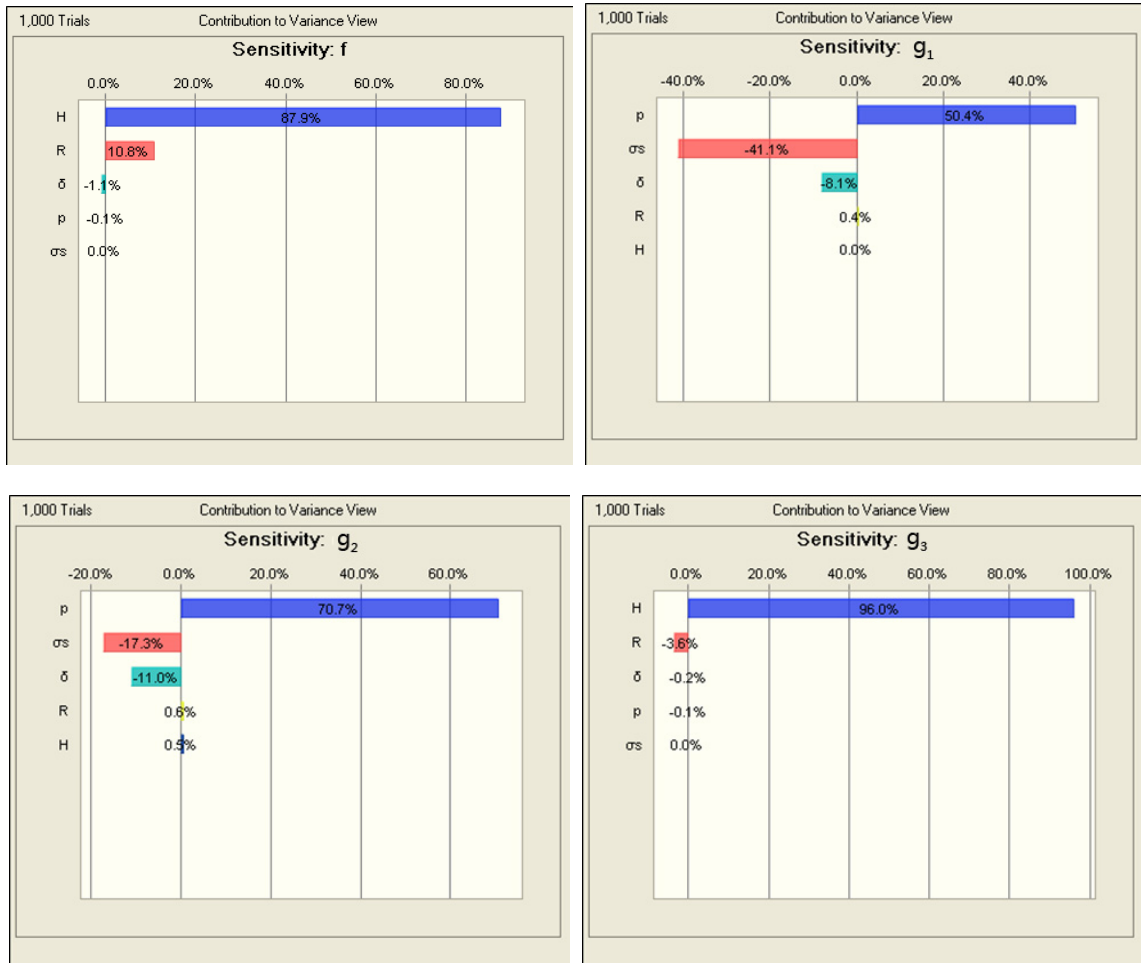


Figure 3. Sensitivity analysis of response variables and constraints.

again, and get the probability that product design met constraints g_1 is $\int_{-\infty}^0 g_1 dy = 1 > 95\%$, and the probability that product design met constraints g_2 is $\int_{-\infty}^0 g_2 dy = 0.9922 > 95\%$, and the probability that product design met constraints g_3 is $\int_2^{2.5} g_3 dy = 0.9993 > 95\%$, which reached product design requirements, that is, pressure container is robust enough, and pass percentage of pressure container has been enhanced greatly. And the maximum volume of container is $f = 4,525,100$, product robustness has been greatly improved. Meanwhile, we knew from sensitivity analysis of response variables in **Figure 3** that we should keep monitoring the fluctuation of the significant impact factors of response variables and constraints to hold robustness of pressure container.

4) Design optimization

Now we further optimize the pressure container to enhance the design robustness using OptQuest optimizer. Here we create the OptQuest model and run simulation experiments 1500 times according to distribution characteristics of design variables and constraints condition [4]–[6], and get the optimum of design variables $p, \sigma_s, \delta, R, H$ are $p, \sigma_s, \delta, R, H = [17.9259, 395.177, 3.49114, 69.6651, 173.633]$ in **Table 2**, then the probability that product design met constraints g_1 are $\int_{-\infty}^0 g_1 dy = 1 > 95\%$, and the probability that product design met constraints g_2 was $\int_{-\infty}^0 g_2 dy = 0.9918 > 95\%$, and the probability that product design met constraints g_3 is $\int_2^{2.5} g_3 dy = 1 > 95\%$ from **Figure 5**, and the maximum volume of container is $f = 4,553,200$ as **Figure 6** showed, so design robustness of pressure container has been further improved than that of last time.

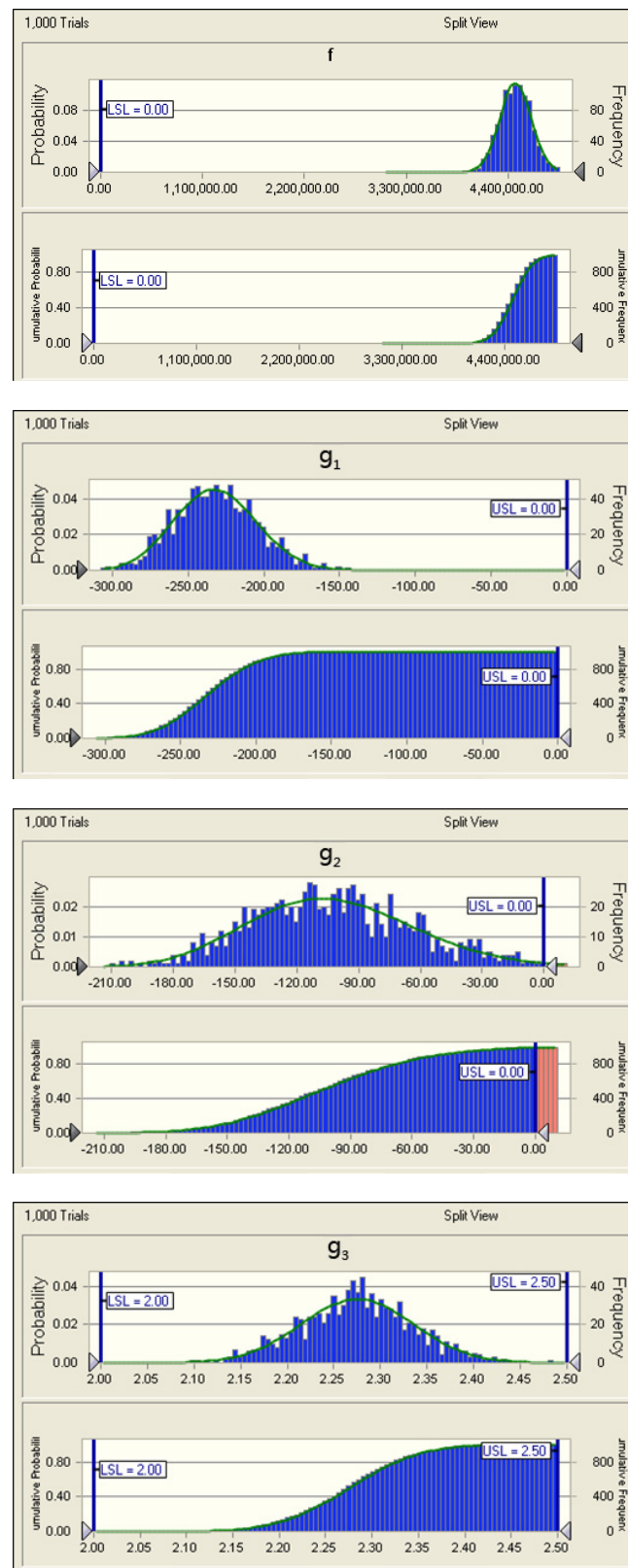
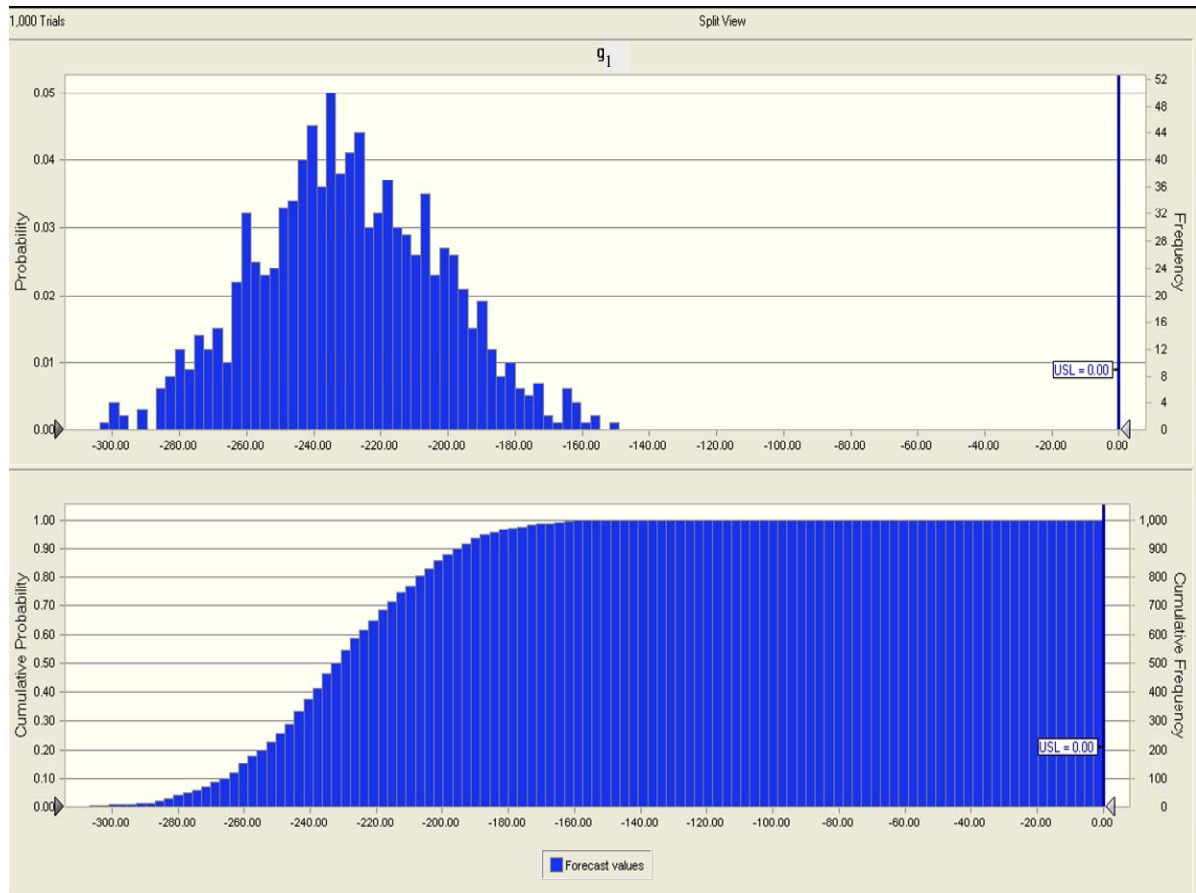
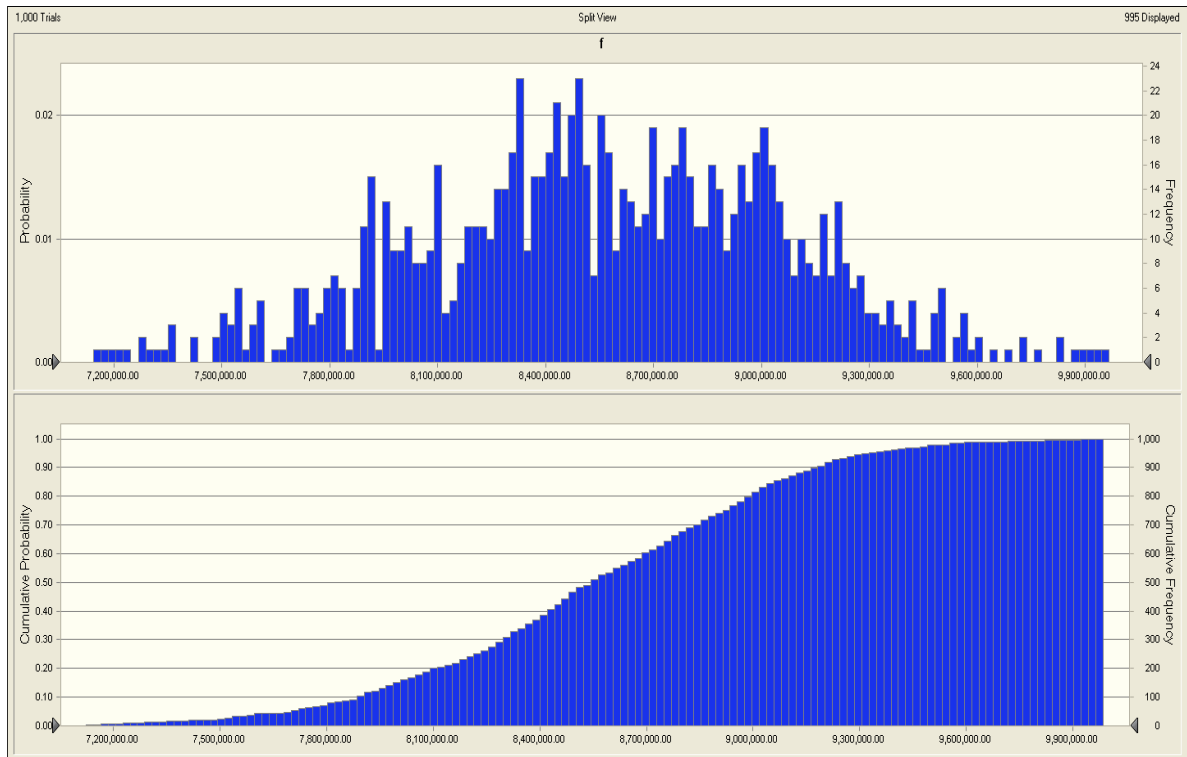


Figure 4. Probability distribution and cumulative probability distribution of response variable and constraints after modification of parameter precision.



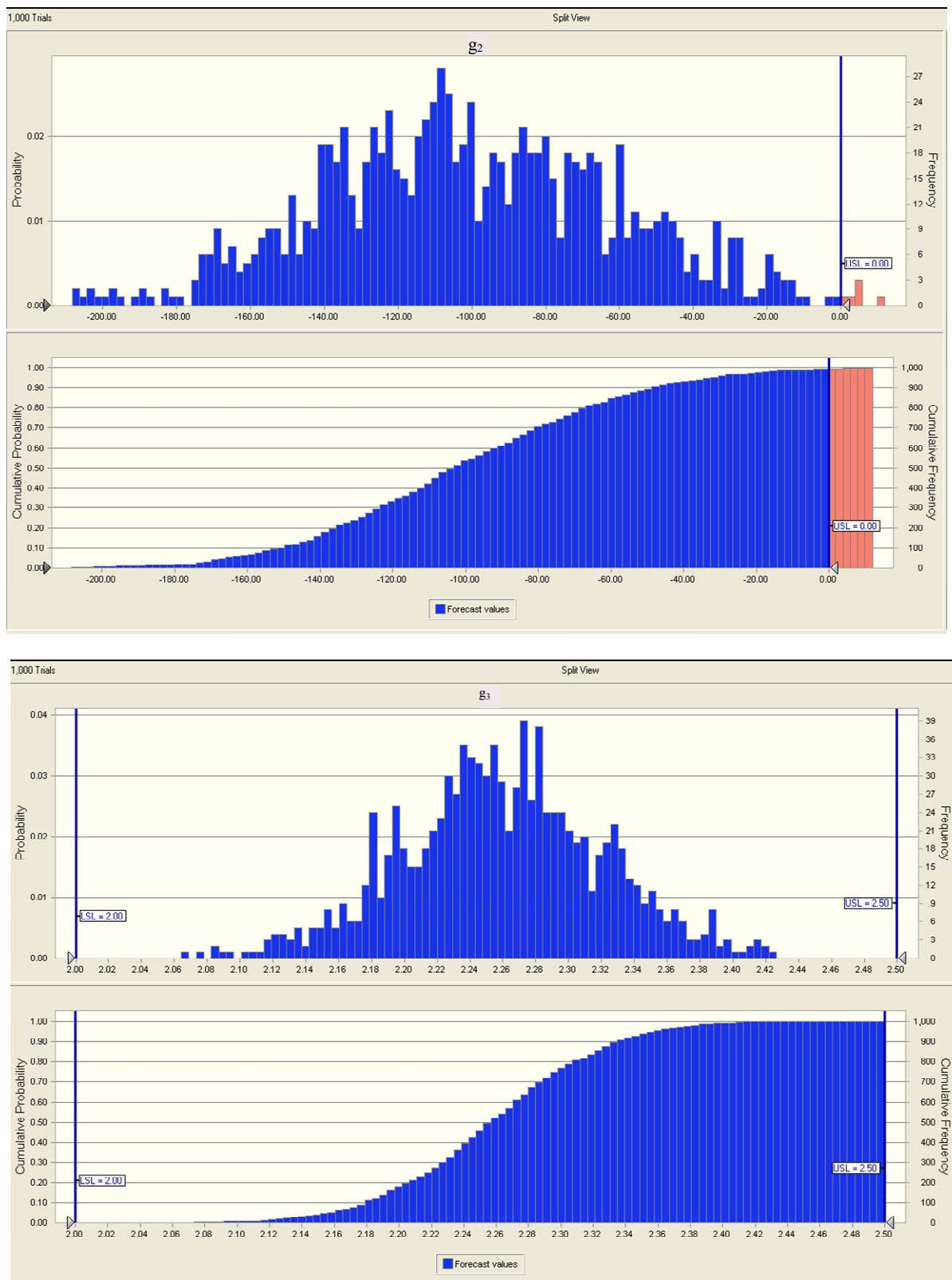
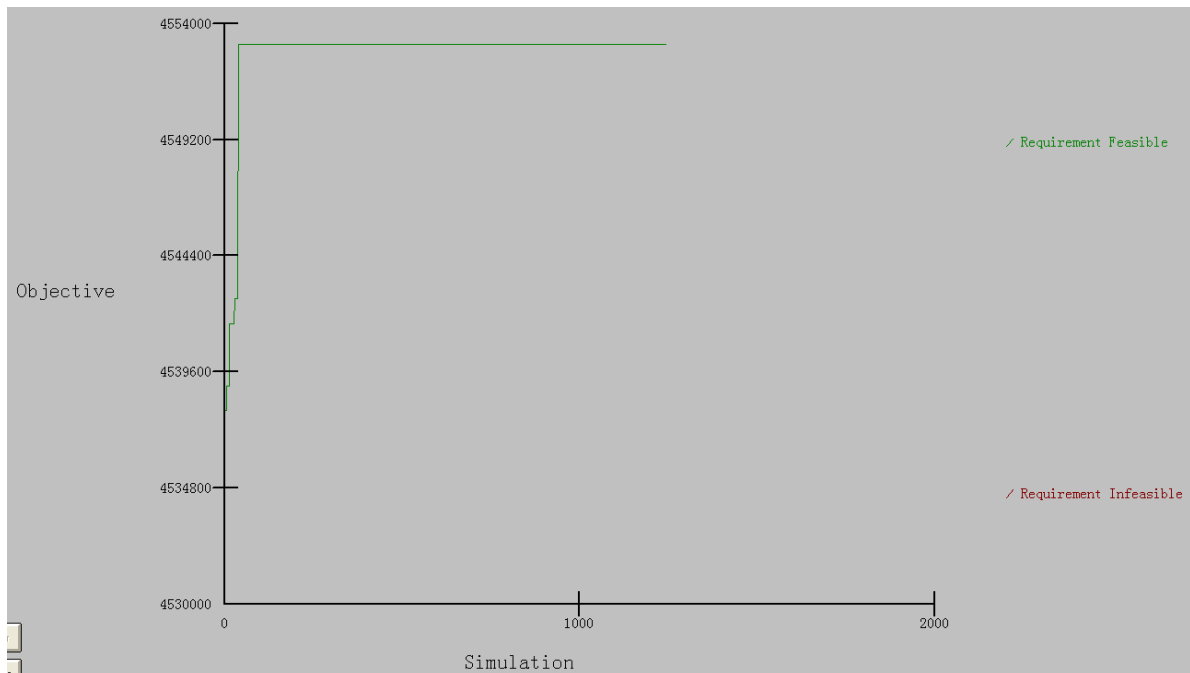


Figure 5. Probability distribution and cumulative probability distribution of response variable and constraints after optimization after OptQuest optimization.

Table 2. Optimum design settings of pressure container with OptQuest.

Status	Time Remaining: 00 Simulation:1245								
Simulation	Maximize objective mean	Requirement g_1	Requirement g_2	Requirement g_3	p	σ_s	δ	R	H
1	4.5380E+06	100.00	98.8506	100.00	15	385	3	75	170
4	4.5390E+06	100.00	98.7106	100.00	19.6990	414.026	1.65588	79.4806	131.772
12	4.5416E+06	100.00	99.4676	100.00	16.6921	410.204	1.64555	78.5969	143.661
27	4.5426E+06	100.00	99.4423	100.00	19.6574	412.131	1.50000	73.8756	130
Best: 38	4.5532E+06	100.00	99.1782	100.00	17.9259	395.177	3.49114	69.6651	173.633

**Figure 6.** Optimum design performance graph of pressure container with OptQuest.

3. Conclusion

When random changes exist in design variables, traditional determined optimization method cannot guarantee robustness of product and process design to the extent. While Monte Carlo method can be used to precision control and optimization in product and process design, which can avoid increasing cost due to duplicate experiments and excessive design precision, as well as low pass percentage due to deficient design precision. In robust analysis and design, Monte Carlo method also can study on change of response model of product and process brought by modification of design variables, get probability distribution and statistical parameters' values of response variables, further improve design robustness of product and process and realize robustness design and optimization of product and process. It proved that analysis and optimization of response surface model based on Monte Carlo method was a good robustness design method, and can markedly improve robustness, precision and pass percentage of product and process.

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