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Surface Wave Propagation in a Generalized Thermoelastic Material with Voids

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Abstract

In the present paper, the propagation of surface wave in a generalized thermoelastic solid with voids is considered. The governing equations are solved to obtain the general solution in *x-z* plane. The appropriate boundary conditions at an interface between two dissimilar half-spaces are satisfied by appropriate particular solutions to obtain the frequency equation of the surface wave in the medium. Some special cases are also discussed.

Keywords: Thermoelasticity, Surface Waves, Boundary Conditions, Voids

1. Introduction

Theory of linear elastic materials with voids is an important generalization of the classical theory of elasticity. The theory is used for investigating various types of geological and biological materials for which classical theory of elasticity is not adequate. The theory of linear elastic materials with voids deals the materials with a distribution of small pores or voids, where the volume of void is included among the kinematics variables. The theory reduces to the classical theory in the limiting case of volume of void tending to zero. Non-linear theory of elastic materials with voids was developed by Nunziato and Cowin [1]. Cowin and Nunziato [2] developed a theory of linear elastic materials with voids to study mathematically the mechanical behavior of porous solids. Puri and Cowin [3] studied the behavior of plane waves in a linear elastic material with voids. Iesan [4] developed the linear theory of thermoelastic materials with voids.

Dhaliwal and Wang [5] formulated the heat-flux dependent thermoelasticity theory for an elastic material with voids. This theory includes the heat-flux among the constitutive variables and assumes an evolution equation for the heat-flux. Ciarletta and Scalia [6] developed a nonlinear theory of non-simple thermoelastic materials with voids. Ciarletta and Scarpetta [7] studied some results on thermoelasticity for dielectric materials with voids. Marin [8-9] studied uniqueness and domain of influence results in thermoelastic bodies with voids. Chirita and Scalia [10] studied the spatial and temporal behavior in linear thermoelasticity of materials with voids. A theory of thermoelastic materials with voids and without energy dissipation is developed by Cicco and Diaco [11]. Ciarletta *et al.* [12] presented a model for acoustic wave propagation in a porous material which also allows for propagation of a thermal displacement wave. Singh [13] studied the wave propagation in a homogeneous, isotropic generalized thermoelastic half space with voids in context of Lord and Shulman theory. Ciarletta *et al.* [14] studied the linear theory of micropolar thermoelasticity for materials with voids. Recently, Aoudai [15] derived the equations of the linear theory of thermoelastic diffusion in porous media based on the concept of volume fraction.

Lord Rayleigh [16] investigated the surface wave along the plane surface of an elastic solid. Chandrasekharaiah [17] discussed the effect of voids on Rayleigh waves in an elastic solid with voids and on Rayleigh-lamb waves in homogeneous elastic plate with voids. Many researchers have studied the surface waves in various theories of thermoelasticity. For example, Chadwick and Windle [18], Agarwal [19], Sharma and Singh [20], Mayer [21], Semerak [22], Chandrasekharaiah [23], Sharma *et al.* [24], Sharma and Kaur [25] and many others.

The present paper is motivated by the linear theory of thermoelasticity with voids developed by Iesan [4]. In Section 2, the governing equations are generalized with the help of Lord and Shulman [26] theory. In Section 3, these equations are solved for general solutions. In Sections 4 and 5, the particular solutions are obtained and applied at required boundary conditions to obtain the frequency equation of surface waves in thermoelastic material with voids. In Section 6, some limiting cases of the problem are discussed. In last section, some concluding remarks are given.

2. Governing Equations

Following, Iesan [4] and Lord and Shulman [26], the constitutive equations and field equations in terms of the displacement, volume fraction and temperature, for homogeneous isotropic generalized thermo-elastic material with voids in the absence of the body forces, heat sources and extrinsic equilibrated body forces are

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}\left[\lambda e_{kk} - \beta\Theta + b\Phi\right] \tag{1}$$

$$q_i + \tau_0 \ddot{q}_i = K\Theta_{,i} \tag{2}$$

$$h_i = \alpha\Phi_{,i} \tag{3}$$

$$\rho\eta = \beta e_{kk} + a\Theta + m\Phi \tag{4}$$

$$g = -be_{kk} - \xi\Phi + m\Theta \tag{5}$$

$$\rho T_0 \dot{\eta} = q_{i,i} \tag{6}$$

$$\mu u_{i,jj} + (\lambda+\mu) u_{j,ij} - \beta\Theta_{,i} + b\Phi_{,i} = \rho\ddot{u}_i \tag{7}$$

$$\rho c_E\left(\dot{\Theta} + \tau_0\ddot{\Theta}\right) + \beta T_0\left(\dot{u}_{k,k} + \tau_0\ddot{u}_{k,k}\right) + mT_0\left(\dot{\Phi} + \tau_0\ddot{\Phi}\right) = K\Theta_{,ii} \tag{8}$$

$$\alpha\Phi_{,ii} - bu_{k,k} - \xi\Phi + m\Theta = \rho\chi\ddot{\Phi} \tag{9}$$

where λ, μ are Lame's constants. $\Theta = T - T_0$, T_0 is the temperature of the medium in its natural state assumed to be such that $\left|\Theta/T_0\right| \ll 1$, T is the absolute temperature, σ_{ij} are the components of the stress tensor, $e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$, u_i are the components of the displacement vector, η is the entropy per unit mass, K is the coefficient of thermal conductivity, τ_0 is the thermal relaxation time. α, b, ξ are void material parameters, m is thermo-void coefficient, $\beta = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion, δ_{ij} is Kronecker delta, q_i are the components of heat flux vector, h_i are the components of equilibrated stress tensor, Φ is change in volume fraction field, g is the intrinsic equilibrated body force and a is thermal constant.

The Equations (7) to (9) are written in x-z plane as

$$(\lambda+2\mu)\frac{\partial^2 u_1}{\partial x^2} + (\lambda+\mu)\frac{\partial^2 u_3}{\partial x\partial z} + \mu\frac{\partial^2 u_1}{\partial z^2} - \beta\frac{\partial\Theta}{\partial x} + b\frac{\partial\Phi}{\partial x} = \rho\frac{\partial^2 u_1}{\partial t^2}, \tag{10}$$

$$(\lambda+2\mu)\frac{\partial^2 u_3}{\partial z^2} + (\lambda+\mu)\frac{\partial^2 u_1}{\partial z\partial x} + \mu\frac{\partial^2 u_3}{\partial x^2} - \beta\frac{\partial\Theta}{\partial z} + b\frac{\partial\Phi}{\partial z} = \rho\frac{\partial^2 u_3}{\partial t^2} \tag{11}$$

$$\rho c_E\tau\frac{\partial\Theta}{\partial t} + \beta T_0\tau\left(\frac{\partial^2 u_1}{\partial x\partial t} + \frac{\partial^2 u_3}{\partial z\partial t}\right) + mT_0\tau\frac{\partial\Phi}{\partial t} = K\left(\frac{\partial^2\Theta}{\partial x^2} + \frac{\partial^2\Theta}{\partial z^2}\right), \tag{12}$$

$$\alpha\left(\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2}\right) - b\left(\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z}\right) - \xi\Phi + m\Theta = \rho\chi\frac{\partial^2\Phi}{\partial t^2}, \tag{13}$$

where $\tau = 1 + \tau_0\frac{\partial}{\partial t}$.

Now the displacement components u_1 and u_3 are written in terms of potential function ϕ and ψ as

$$u_1 = \frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial z},\ u_3 = \frac{\partial\phi}{\partial z} + \frac{\partial\psi}{\partial x} \tag{14}$$

Using Equation (14) into Equations (10) to (13), we have

$$\mu\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial z^2}\right) = \rho\frac{\partial^2\psi}{\partial t^2}, \tag{15}$$

$$(\lambda+2\mu)\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2}\right) + b\Phi - \beta\Theta = \rho\frac{\partial^2\phi}{\partial t^2}, \tag{16}$$

$$\omega^*\nabla^2\Theta = \tau\left[\frac{\partial\Theta}{\partial t} + \varepsilon\frac{\partial}{\partial t}\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2}\right) + \varepsilon_1\frac{\partial\Phi}{\partial t}\right], \tag{17}$$

$$\alpha\left(\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2}\right) - b\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2}\right) - \xi\Phi + m\Theta = \rho\chi\frac{\partial^2\Phi}{\partial t^2}, \tag{18}$$

where,

$$\omega^* = \frac{K}{\rho c_E},\ \varepsilon = \frac{\beta T_0}{\rho c_E},\ \varepsilon_1 = \frac{mT_0}{\rho c_E}.$$

Here Equation (15) is uncoupled, whereas Equations (16), (17) and (18) are coupled in ϕ, Θ and Φ.

3. Solutions

To solve the Equations (16) to (18), we consider

$$(\phi, \Theta, \Phi) = \left\{\bar{\phi}(z), \bar{\Theta}(z), \bar{\Phi}(z)\right\}e^{ik(x-ct)}, \tag{19}$$

Substituting (19) in Equations (16)-(18), we obtain

$$\left[D^2+k^2\left(\Omega^2-1\right)\right]\bar{\phi}(z)-\beta_1\bar{\Theta}(z)+b_1\bar{\Phi}(z)=0, \tag{20}$$

$$\begin{aligned}&i\varepsilon\omega\tau\left(-k^2+D^2\right)\bar{\phi}(z)+\left[\omega^*\left(-k^2+D^2\right)+i\tau\omega\right]\bar{\Theta}(z)\\&+i\varepsilon_1\omega\tau\bar{\Phi}(z)=0,\end{aligned} \tag{21}$$

$$\begin{aligned}&-b\left(-k^2+D^2\right)\bar{\phi}(z)+m\bar{\Theta}(z)\\&+\left[\alpha\left(-k^2+D^2\right)-\xi+\rho\chi\omega^2\right]\bar{\Phi}(z)=0\end{aligned} \tag{22}$$

where,

$$D^2=\frac{\mathrm{d}^2}{\mathrm{d}z^2},\ \Omega^2=\frac{\rho c^2}{\lambda+2\mu},\ \beta_1=\frac{\beta}{\lambda+2\mu}\ \text{and}$$
$$b_1=\frac{b}{\lambda+2\mu},\ \omega=kc$$

The non-trivial solutions of Equations (20) to (22) exist if

$$L_0D^6+L_1D^4+L_2D^2+L_3=0 \tag{23}$$

where

$$\begin{aligned}
L_0&=\omega^*\alpha\\
L_1&=\alpha\omega^*k^2\left(\Omega^2-1\right)+i\beta_1\alpha\varepsilon\tau\omega-bb_1\omega^2-2\omega^*\alpha k^2\\
&\quad-\omega^*\xi+\omega^*\rho\omega\chi+i\omega\alpha\\
L_2&=k^2\left(\Omega^2-1\right)\left(-2\omega^*\alpha k^2-\omega^*\xi+\omega^*\rho\omega\chi+i\omega\alpha\right)\\
&\quad+\omega^*\alpha k^4+\omega^*b^2\xi-\omega^*k^2\rho\chi\omega^2-i\omega\tau k^2\alpha-i\omega\tau\xi\\
&\quad+i\omega^3\rho\tau\chi+im\xi\tau\omega-2\alpha\beta_1\varepsilon\tau\omega k^2\\
&\quad-i\beta_1\varepsilon\tau\omega\xi+i\beta_1\xi\tau\omega^3\rho\chi+i\beta_1\xi\tau\omega b+ib_1\varepsilon m\tau\omega\\
&\quad+bb_1\omega^2k^2+bb_1\omega^*k^2-ib_1\omega b\tau\\
L_3&=k^2\left(\Omega^2-1\right)\left(\omega^*\alpha k^4+\omega^*b^2\xi-\omega^*b^2\rho\chi\omega^2\right.\\
&\quad\left.-i\omega\tau k^2\alpha-i\omega\tau\xi+i\omega^3\rho\tau\chi-im\xi\tau\omega\right)+i\beta_1\alpha\varepsilon\tau\omega k^4\\
&\quad+i\beta_1\tau\xi\omega k^2-i\beta_1\varepsilon\tau\omega^3\rho k^2\chi-i\beta_1\xi\rho\omega bk^2\\
&\quad-ib_1m\varepsilon\tau\omega k^2+b^2b_1k^4\omega^*+ibb_1\omega\tau k^2
\end{aligned}$$

Let m_1, m_2, m_3 be the roots of the auxiliary Equation (23), then the general solutions ϕ, Θ and Φ are written as

$$\begin{aligned}\phi=&\left(A_1\mathrm{e}^{-m_1z}+A_2\mathrm{e}^{-m_2z}+A_3\mathrm{e}^{-m_3z}+A_4\mathrm{e}^{m_1z}\right.\\&\left.+A_5\mathrm{e}^{m_2z}+A_6\mathrm{e}^{m_3z}\right)\mathrm{e}^{ik(x-ct)}\end{aligned} \tag{24}$$

$$\begin{aligned}\Theta=&\left(\zeta_1A_1\mathrm{e}^{-m_1z}+\zeta_2A_2\mathrm{e}^{-m_2z}+\zeta_3A_3\mathrm{e}^{-m_3z}+\zeta_1A_4\mathrm{e}^{m_1z}\right.\\&\left.+\zeta_2A_5\mathrm{e}^{m_2z}+\zeta_3A_6\mathrm{e}^{m_3z}\right)\mathrm{e}^{ik(x-ct)}\end{aligned} \tag{25}$$

$$\begin{aligned}\Phi=&\left(\eta_1A_1\mathrm{e}^{-m_1z}+\eta_2A_2\mathrm{e}^{-m_2z}+\eta_3A_3\mathrm{e}^{-m_3z}+\eta_1A_4\mathrm{e}^{m_1z}\right.\\&\left.+\eta_2A_5\mathrm{e}^{m_2z}+\eta_3A_6\mathrm{e}^{m_3z}\right)\mathrm{e}^{ik(x-ct)}\end{aligned} \tag{26}$$

where

$$\zeta_i=\frac{\left(m_i^2-k^2\right)\left[b^2-G_i\left(\lambda+2\mu\right)\right]+G_i\rho\omega^2}{\rho G_i+mb},i=\left(1,2,3\right)$$

$$\eta_i=\frac{1}{b}\left[\left(k^2-m_i^2\right)\left(\lambda+2\mu\right)-\rho\omega^2\right]+\frac{\beta}{b}\zeta_i,i=\left(1,2,3\right)$$

$$G_i=\chi\rho\omega^2-\alpha\left(k^2-m_i^2\right)-\xi$$

The general solution ψ of Equation (15) is written as

$$\psi=\left(B_0^*\mathrm{e}^{-m_4z}+B_1^*\mathrm{e}^{m_4z}\right)\mathrm{e}^{ik(x-ct)}, \tag{27}$$

where, $m_4^2=k^2\left(1-\frac{\rho}{\mu}c^2\right)$.

4. Formulation of the Problem

Let us consider two semi-infinite half-spaces of thermoelastic solid with voids in welded contact as shown in **Figure 1**. The particular solutions in half-spaces M and M' are as follows:

For medium M,

$$\phi=\left(A_1\mathrm{e}^{-m_1z}+A_2\mathrm{e}^{-m_2z}+A_3\mathrm{e}^{-m_3z}\right)\mathrm{e}^{ik(x-ct)} \tag{28}$$

$$\Theta=\left(\zeta_1A_1\mathrm{e}^{-m_1z}+\zeta_2A_2\mathrm{e}^{-m_2z}+\zeta_3A_3\mathrm{e}^{-m_3z}\right)\mathrm{e}^{ik(x-ct)} \tag{29}$$

$$\Phi=\left(\eta_1A_1\mathrm{e}^{-m_1z}+\eta_2A_2\mathrm{e}^{-m_2z}+\eta_3A_3\mathrm{e}^{-m_3z}\right)\mathrm{e}^{ik(x-ct)} \tag{30}$$

$$\psi=B_1\mathrm{e}^{-m_4z+ik(x-ct)} \tag{31}$$

where,

$${m_1}^2+{m_2}^2+{m_3}^2=-\frac{L_1}{L_0},$$

$${m_1}^2{m_2}^2+{m_2}^2{m_3}^2+{m_3}^2{m_1}^2=\frac{L_2}{L_0},{m_1}^2{m_2}^2{m_3}^2=-\frac{L_3}{L_0}$$

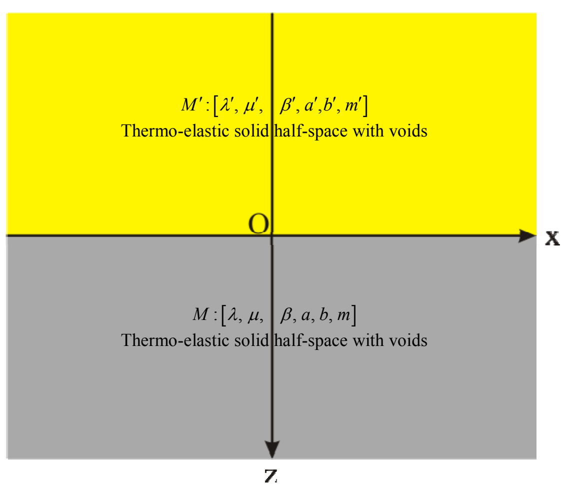

Figure 1. Geometry of the problem.

Similarly, for medium M'

$$\phi' = \left(A_1' e^{m_1' z} + A_2' e^{m_2' z} + A_3' e^{m_3' z} \right) e^{ik(x-ct)} \tag{32}$$

$$\Theta' = \left(\zeta_1' A_1' e^{m_1' z} + \zeta_2' A_2' e^{m_2' z} + \zeta_3' A_3' e^{m_3' z} \right) e^{ik(x-ct)} \tag{33}$$

$$\Phi' = \left(\eta_1' A_1' e^{m_1' z} + \eta_2' A_2' e^{m_2' z} + \eta_3' A_3' e^{m_3' z} \right) e^{ik(x-ct)} \tag{34}$$

$$\psi' = B_1' e^{m_4' z + ik(x-ct)} \tag{35}$$

Here, the symbols with primes in the following sections correspond to medium M'.

5. Boundary Conditions

The boundary conditions at the interface $z = 0$ are the continuity of force stress components, displacement components, heat flux, temperature and volume fractional field, *i.e.*

$$\begin{aligned} &\sigma_{zz} = \sigma_{zz}', \ \sigma_{zx} = \sigma_{zx}', \\ &\frac{\partial \Theta}{\partial z} = \chi_1 \frac{\partial \Theta'}{\partial z}, \ \frac{\partial \Phi}{\partial z} = \chi_2 \frac{\partial \Phi'}{\partial z}, \\ &u_3 = u_3', \ u_1 = u_1', \ \Theta = \Theta', \ \Phi = \Phi'. \end{aligned} \tag{36}$$

where,

$$\begin{aligned} &\sigma_{zz} = 2\mu e_{zz} + \lambda e_{kk} - \beta\Theta + b\Phi, \\ &\sigma_{zz}' = 2\mu' e_{zz}' + \lambda' e_{kk}' - \beta'\Theta' + b'\Phi' \\ &\sigma_{zx} = 2\mu e_{zx}, \sigma_{zx}' = 2\mu' e'_{zx}, \\ &u_3 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}, u_3' = \frac{\partial \phi'}{\partial z} + \frac{\partial \psi'}{\partial x} \\ &u_1 = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, u_1' = \frac{\partial \phi'}{\partial x} - \frac{\partial \psi'}{\partial z}, \\ &\chi_1 = \frac{K'}{K} \frac{1 - ikc\tau_0}{1 - ikc\tau_0'}, \chi_2 = \frac{\alpha'}{\alpha} \end{aligned}$$

The particular solutions (29) to (35) satisfy the above boundary conditions if

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & a_{28} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} & a_{38} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} & a_{58} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} & a_{68} \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} & a_{78} \\ a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & a_{88} \end{vmatrix} = 0 \tag{37}$$

where

$$a_{11} = -\lambda k^2 + (\lambda + 2\mu) m_1^2 - \beta\zeta_1 + b\eta_1$$

$$a_{12} = -\lambda k^2 + (\lambda + 2\mu) m_2^2 - \beta\zeta_2 + b\eta_2$$

$$a_{13} = -\lambda k^2 + (\lambda + 2\mu) m_3^2 - \beta\zeta_3 + b\eta_3,$$

$$a_{14} = -2i\mu k m_4,$$

$$a_{15} = \lambda' k^2 - (\lambda' + 2\mu') m_1'^2 + \beta'\zeta_1' - b'\eta_1',$$

$$a_{16} = \lambda' k^2 - (\lambda' + 2\mu') m_2'^2 + \beta'\zeta_2' - b'\eta_2',$$

$$a_{17} = \lambda' k^2 - (\lambda' + 2\mu') m_3'^2 + \beta'\zeta_3' - b'\eta_3',$$

$$a_{18} = -2i\mu' k m_4', a_{21} = -2i\mu k m_1,$$

$$a_{22} = -2i\mu k m_2, a_{23} = -2i\mu k m_3,$$

$$a_{24} = -\mu\left(k^2 + m_4^2\right), a_{25} = -2i\mu' k m_1', a_{26} = -2i\mu' k m_2',$$

$$a_{27} = -2i\mu' k m_3', a_{28} = \mu'\left(k^2 + m_4'^2\right).$$

$$a_{31} = m_1\zeta_1, a_{32} = m_2\zeta_2, a_{33} = m_3\zeta_3, a_{34} = 0,$$

$$a_{35} = m_1'\chi_1\zeta_1', a_{36} = m_2'\chi_1\zeta_2', a_{37} = m_3'\chi_1\zeta_3',$$

$$a_{38} = 0.$$

$$a_{41} = m_1\eta_1, a_{42} = m_2\eta_2, a_{43} = m_3\eta_3, a_{44} = 0,$$

$$a_{45} = m_1'\eta_1'\chi_2, a_{46} = m_2'\eta_2'\chi_2, a_{47} = m_3'\eta_3'\chi_2,$$

$$a_{48} = 0.$$

$$a_{51} = m_1, a_{52} = m_2, a_{53} = m_3, a_{54} = -ik,$$

$$a_{55} = m_1', a_{56} = m_2', a_{57} = m_3', a_{58} = ik.$$

$$a_{61} = ik, a_{62} = ik, a_{63} = ik, a_{64} = m_4,$$

$$a_{65} = -ik, a_{66} = -ik, a_{67} = -ik, a_{68} = m_4'.$$

$$a_{71} = \zeta_1, a_{72} = \zeta_2, a_{73} = \zeta_3, a_{74} = 0,$$

$$a_{75} = -\zeta_1', a_{76} = -\zeta_2', a_{77} = -\zeta_3', a_{78} = 0.$$

$$a_{81} = \eta_1, a_{82} = \eta_2, a_{83} = \eta_3, a_{84} = 0,$$

$$a_{85} = -\eta_1', a_{86} = -\eta_2', a_{87} = -\eta_3', a_{88} = 0.$$

The Equation (37) gives the frequency equation for surface wave in a generalized thermo-elastic medium with voids.

6. Limiting Cases

1) If we neglect the void parameters, then the Equation (37) reduces to

$$\begin{vmatrix} a_{11}' & a_{12}' & a_{13}' & a_{14}' & a_{15}' & a_{16}' \\ a_{21}' & a_{22}' & a_{23}' & a_{24}' & a_{25}' & a_{26}' \\ a_{31}' & a_{32}' & a_{33}' & a_{34}' & a_{35}' & a_{36}' \\ a_{41}' & a_{42}' & a_{43}' & a_{44}' & a_{45}' & a_{46}' \\ a_{51}' & a_{52}' & a_{53}' & a_{54}' & a_{55}' & a_{56}' \\ a_{61}' & a_{62}' & a_{63}' & a_{64}' & a_{65}' & a_{66}' \end{vmatrix} \tag{38}$$

where,

$$a_{11}' = -\lambda k^2 + (\lambda + 2\mu) m_1^2 - \beta\zeta_1,$$

$$a_{12}' = -\lambda k^2 + (\lambda + 2\mu) m_2^2 - \beta\zeta_2, a_{13}' = -2i\mu k m_4,$$

$$a_{14}' = \lambda' k^2 - (\lambda' + 2\mu') m_1'^2 + \beta'\zeta_1',$$

$$a'_{15} = \lambda' k^2 - (\lambda' + 2\mu') m_2'^2 + \beta' \zeta'_2,\ a'_{16} = -2i\mu' k m'_4,$$
$$a'_{21} = -2im_1\mu k,\ a'_{22} = -2im_2\mu k,\ a'_{23} = -\mu\left(k^2 + m_4^2\right),$$
$$a'_{24} = -2im'_1\mu' k,\ a'_{25} = -2im'_2\mu' k,\ a'_{26} = \mu'\left(k^2 + m_4'^2\right),$$
$$a'_{31} = m_1\zeta_1, a'_{32} = m_2\zeta_2, a'_{33} = 0,$$
$$a'_{34} = \chi_1 m'_1\zeta'_1, a'_{35} = \chi_1 m'_2\zeta'_2, a'_{36} = 0,$$
$$a'_{41} = m_1, a'_{42} = m_2, a'_{43} = -ik,$$
$$a'_{44} = m'_1, a'_{45} = m'_2, a'_{46} = ik.$$
$$a'_{51} = ik, a'_{52} = ik, a'_{53} = m_{4,}$$
$$a'_{54} = -ik, a'_{55} = -ik, a'_{56} = m'_{4,}$$
$$a'_{61} = \zeta_1, a'_{62} = \zeta_2, a'_{63} = 0,$$
$$a'_{64} = -\zeta'_1, a'_{65} = -\zeta'_2, a'_{66} = 0.$$

The Equation (38) gives the frequency equation for surface waves in a generalized thermo-elastic medium.

2) If we neglect thermal parameters, then the Equation (37) reduces to

$$\begin{vmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} & b_{46} \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} & b_{56} \\ b_{61} & b_{62} & b_{63} & b_{64} & b_{65} & b_{66} \end{vmatrix} = 0 \qquad (39)$$

where

$$b_{11} = -\lambda k^2 + (\lambda + 2\mu) m_1^2 + b\eta_1,$$
$$b_{12} = -\lambda k^2 + (\lambda + 2\mu) m_3^2 + b\eta_3,\ b_{13} = -2i\mu k m_4,$$
$$b_{14} = \lambda' k^2 - (\lambda' + 2\mu') m_1'^2 - b'\eta'_1,$$
$$b_{15} = \lambda' k^2 - (\lambda' + 2\mu') m_3'^2 - b'\eta'_3,\ b_{16} = -2i\mu' k m'_4,$$
$$b_{21} = -2im_1\mu k, b_{22} = -2im_3\mu k, b_{23} = -\mu\left(k^2 + m_4^2\right),$$
$$b_{24} = -2im'_1\mu' k, b_{25} = -2im'_3\mu' k, b_{26} = \mu'\left(k^2 + m_4'^2\right).$$
$$b_{31} = m_1\eta_1, b_{32} = m_3\eta_3, b_{33} = 0,$$
$$b_{34} = \chi_2 m'_1\eta'_1, b_{35} = \chi_2 m'_3\eta'_3, b_{36} = 0.$$
$$b_{41} = m_1, b_{42} = m_3, b_{43} = -ik,$$
$$b_{44} = m'_1, b_{45} = m'_3, b_{46} = ik.$$
$$b_{51} = ik, b_{52} = ik, b_{53} = m_{4,}$$
$$b_{54} = -ik, b_{55} = -ik, b_{56} = m'_4$$
$$b_{61} = \eta_1, b_{62} = \eta_3, b_{63} = 0,$$
$$b_{64} = -\eta'_1, b_{65} = -\eta'_3, b_{66} = 0.$$

The Equation (39) gives the frequency equation for surface wave in an elastic medium with voids.

3) If we neglect void and thermal parameters, then the Equation (37) reduces to

$$\begin{vmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{vmatrix} = 0 \qquad (40)$$

where

$$c_{11} = -\lambda k^2 + (\lambda + 2\mu) m_1^2,\ c_{12} = -2i\mu k m_4,$$
$$c_{13} = \lambda' k^2 - (\lambda' + 2\mu') m_1'^2,\ c_{14} = 2i\mu' k m'_4,$$
$$c_{21} = -2im_1\mu k, c_{22} = -\mu\left(k^2 + m_4^2\right),$$
$$c_{23} = -2im'_1\mu' k, c_{24} = \mu'\left(k^2 + m_4'^2\right).$$
$$c_{31} = m_1, c_{32} = -ik,$$
$$c_{33} = m'_1, c_{34} = ik$$
$$c_{41} = ik, c_{42} = m_4,$$
$$c_{43} = -ik, c_{44} = m'_4.$$

The Equation (40) gives the frequency equation for surface wave in an elastic medium

4) If we remove the upper half-space, then the Equation (37) reduces to

$$\begin{vmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{vmatrix} = 0 \qquad (41)$$

where

$$d_{11} = -\lambda k^2 + (\lambda + 2\mu) m_1^2 - \beta\zeta_1 + b\eta_1,$$
$$d_{12} = -\lambda k^2 + (\lambda + 2\mu) m_2^2 - \beta\zeta_2 + b\eta_2,$$
$$d_{13} = -\lambda k^2 + (\lambda + 2\mu) m_3^2 - \beta\zeta_3 + b\eta_3,$$
$$d_{14} = -2i\mu k m_4 d_{21} = -2i\mu k m_1,\ d_{22} = -2i\mu k m_2,$$
$$d_{23} = -2i\mu k m_3,\ d_{24} = -\mu\left(k^2 + m_4^2\right).$$
$$d_{31} = m_1\zeta_1,\ d_{32} = m_2\zeta_2,\ d_{33} = m_3\zeta_3,\ d_{34} = 0.$$
$$d_{41} = m_1\eta_1,\ d_{42} = m_2\eta_2,\ d_{43} = m_3\eta_3,\ d_{44} = 0.$$

The Equation (41) gives the frequency equation of a Rayleigh surface wave in a half-space of a generalized thermo-elastic material with void.

7. Conclusions

The frequency equation of surface waves in generalized thermoelastic material with voids is obtained. The frequency equation of Rayleigh surface wave is obtained as limiting case. The theoretical results indicate that the speed of surface wave depends on various material parameters. Present analytical solutions can be used to find numerically the speed of surface wave for a particular material modeled as thermoelastic material with voids.

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