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Generalization of Uniqueness Theorems for Entire and Meromorphic Functions

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Abstract

In this paper, we deal with the uniqueness problems on entire and meromorphic functions concerning differential polynomials that share fixed-points. Moreover, we generalise and improve some results of Weichuan Lin, Hongxun Yi, Meng Chao, C. Y. Fang, M. L. Fang and Junfeng xu.

Keywords

Nevanlinna Theory, Uniqueness, Entire Functions, Meromorphic Functions, Differential Polynomials, Fixed Points

1. Introduction

In this paper, the term "meromorphic" will always mean meromorphic in the complex plane C. Let a be a complex number and $\alpha(z)$ be a meromorphic function such that $T(r,\alpha)=o\{T(r,f)\}$. We say f and g share the value a CM, if $f-a$ and $g-a$ assume the same zeros with the same multiplicities; if $f(z)-\alpha(z)$ and $g(z)-\alpha(z)$ assume the same zeros with the same multiplicities, then we say $f(z)$ and $g(z)$ share $\alpha(z)$ CM, especially we say that $f(z)$ and $g(z)$ have the same fixed-points when $\alpha(z)=z$. It is assumed that the reader is familiar with the notations of the Nevanlinna theory that can be found, for instance, in [1]. We denote by $S(r,f)$ any function satisfying

$$S(r,f)=o\{T(r,f)\},$$

as $r\to\infty$, possibly outside of finite measure.

Set

$$N_k\left(r,\frac{1}{f-a}\right)=\bar{N}\left(r,\frac{1}{f-a}\right)+\bar{N}_{(2}\left(r,\frac{1}{f-a}\right)+\cdots+\bar{N}_{(k}\left(r,\frac{1}{f-a}\right).$$

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It is well known that if *f* and *g* share four distinct values CM, then *f* is a fractional transformation of *g*. In 1997, corresponding to one famous question of Hayman, C. C. Yang and X. H. Hua showed the similar conclusions hold for certain types of differential polynomials when they share only one value. They proved the following result.

Theorem A ([2]). *Let* f *and* g *be two non-constant meromorphic functions,* $n \geq 11$ *be an integer and* $a \in C - \{0\}$. *If* $f^n f'$ *and* $g^n g'$ *share the value a CM, then either* $f = dg$ *for some* $(n+1)^{th}$ *root of unity d or* $g(z) = c_1 e^{cz}$ *and* $f(z) = c_2 e^{-cz}$, *where* c, c_1 *and* c_2 *are constants and satisfy* $(c_1 c_2)^{n+1} c^2 = -a^2$.

In 2001, M. L. Fang and W. Hong obtained the following result.

Theorem B ([3]). *Let f and g be two transcendental entire functions,* $n \geq 11$ *an integer. If* $f^n(f-1)f'$ *and* $g^n(g-1)g'$ *share the value* 1 *CM, then* $f \equiv g$.

Recently, W. C. Lin and H. X. Yi extended the above theorem with respect to fixed point. They proved the following results.

Theorem C ([4]). *Let* f *and* g *be two transcendental meromorphic functions,* $n \geq 12$ *an integer. If* $f^n(f-1)f'$ *and* $g^n(g-1)g'$ *share z CM, then either* $f(z) \equiv g(z)$ *or*

$$f = \frac{(n+2)h(1-h^{n+1})}{(n+1)(1-h^{n+2})}, \quad g = \frac{(n+2)(1-h^{n+1})}{(n+1)(1-h^{n+2})}$$

where *h* is a nonconstant meromorphic function.

Theorem D ([4]). *Let* f *and* g *be two transcendental meromorphic functions,* $n \geq 13$ *an integer. If* $f^n(f-1)^2 f'$ *and* $g^n(g-1)^2 g'$ *share z CM, then* $f \equiv g$.

We generalise the above results and prove the following Theorem.

Theorem 1.1 *Let* f *and* g *be two transcendental meromorphic functions,* $n \geq m+11$ *an integer. If* $f^n(f-1)^m f'$ *and* $g^n(g-1)^m g'$ *share* z *CM then* $f \equiv g$.

For $m = 1$, we get Theorem C.

For $m = 2$, we get Theorem D.

One may ask the following question, can the nature of the fixed point *z* be relaxed to *IM* in the above theorems?

In 2008, Meng Chao answered to the above question and proved the following theorems.

Theorem E ([5]). *Let* f *and* g *be two transcendental meromorphic functions,* $n \geq 27$ *an integer. If* $f^n(f-1)f'$ *and* $g^n(g-1)g'$ *share z IM, then either* $f(z) \equiv g(z)$ *or*

$$f = \frac{(n+2)h(1-h^{n+1})}{(n+1)(1-h^{n+2})}, \quad g = \frac{(n+2)(1-h^{n+1})}{(n+1)(1-h^{n+2})}$$

where *h* is a nonconstant meromorphic function.

Theorem F([5]). *Let* f *and* g *be two transcendental meromorphic functions,* $n \geq 28$ *an integer. If* $f^n(f-1)^2 f'$ *and* $g^n(g-1)^2 g'$ *share z IM, then* $f \equiv g$.

We generalise the above results and prove the following Theorem.

Theorem 1.2 *Let* f *and* g *be two transcendental meromorphic functions,* $n \geq m+26$ *an integer. If* $f^n(f-1)^m f'$ *and* $g^n(g-1)^m g'$ *share* z *IM then* $f \equiv g$.

For $m = 1$, we get $n \geq 24$ which improves Theorem E.

For $m = 2$, we get $n \geq 28$, we get Theorem F.

In 2002, Fang and Fang [6] proved that there exists a differential polynomial *d* such that for any pair of nonconstant entire functions *f* and *g* we can get $f \equiv g$, if $d(f)$ and $d(g)$ share one value *CM*.

Theorem G ([6]). *Let* f *and* g *be two nonconstant entire functions,* $n \geq 8$ *be a positive integer. If* $f^n(f-1)f'$ *and* $g^n(g-1)g'$ *share* 1 *CM, then* $f \equiv g$.

In 2004, Lin-Yi [7] and Qiu-Fang [8] proved that Theorem G remains valid for $n \geq 7$.

Theorem H ([7] [8]). *Let* f *and* g *be two nonconstant entire functions,* $n \geq 7$ *be a positive integer. If* $f^n(f-1)f'$ *and* $g^n(g-1)g'$ *share* 1 *CM, then* $f \equiv g$.

We generalise the above results and prove the following theorem.

Theorem 1.3 *Let f and g be two transcendental entire functions,* $n \geq m+6$ *an integer. If* $f^n(f-1)^m f'$ *and* $g^n(g-1)^m g'$ *share* z *CM then* $f \equiv g$.

For $m=1$, $n\geq 7$ we get Theorem H.

For $m=2$, $n\geq 8$, we get new result.

Fang-Fang discussed Theorem H by replacing CM with IM and proved the following Theorem.

Theorem I ([6]). *Let f and g be two nonconstant entire functions, n be a positive integer. If* $f^n(f-1)f'$ *and* $g^n(g-1)g'$ *share* 1 *IM and* $n\geq 17$, *then* $f\equiv g$.

We generalise the above results and prove the following Theorem.

Theorem 1.4 *Let f and g be two transcendental entire functions,* $n\geq m+15$ *an integer. If* $f^n(f-1)^m f'$ *and* $g^n(g-1)^m g'$ *share z IM then* $f\equiv g$.

For $m=1$, $n\geq 16$ which improves Theorem I.

For $m=2$, $n\geq 17$, we get new result.

2. Some Lemmas

Lemma 2.1 ([9]) *Let f be a nonconstant meromorphic function, n be a positive integer.* $P(f)=a_n f^n+a_{n-1}f^{n-1}+\cdots+a_1 f$ *where* a_i *is a meromorphic function satisfying* $T(r,a_i)=S(r,f)$ $(i=1,2,3,\cdots,n)$. *Then*

$$T(r,P(f))=nT(r,f)+S(r,f).$$

Lemma 2.2 ([10]) *Let f be a non-constant meromorphic function k be a positive integer, then*

$$N_p\left(r,\frac{1}{f^{(k)}}\right)\leq N_{p+k}\left(r,\frac{1}{f}\right)+k\bar{N}(r,f)+S(r,f),$$

where $N_p\left(r,\frac{1}{f^{(k)}}\right)$ denotes the counting function of the zero's of $\frac{1}{f^{(k)}}$ where a zero of multiplicity m is counted m times if $m\leq p$ and p times if $m>p$. Clearly $\bar{N}\left(r,\frac{1}{f^{(k)}}\right)=N_1\left(r,\frac{1}{f^{(k)}}\right)$.

Lemma 2.3 ([11] [12]) *Let F and G be two nonconstant meromorphic functions sharing the value* 1 *IM. Let*

$$H=\left(\frac{F''}{F'}-2\frac{F'}{F-1}\right)-\left(\frac{G''}{G'}-2\frac{G'}{G-1}\right).$$

If $H\neq 0$, *then*

$$T(r,F)+T(r,G)\leq 2\left[N_2(r,F)+N_2(r,G)+N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)\right]$$
$$+3\left[\bar{N}(r,F)+\bar{N}(r,G)+\bar{N}\left(r,\frac{1}{F}\right)+\bar{N}\left(r,\frac{1}{G}\right)\right]+S(r,F)+S(r,G).$$

Lemma 2.4 ([5]) *Let f and g be two nonconstant meromorphic functions,* $m<7$, $n>7-m$ *positive integers,* $\alpha(z)$ *denotes as in section* 1 *and* $\alpha\neq 0,\infty$, *and let*

$$F=f^n(f-1)^m f',\quad G=g^n(g-1)^m g'$$

if F and G share $\alpha(z)$ *IM, then* $S(r,f)=S(r,g)$.

Lemma 2.5 ([13]) *Let H be defined as above. If* $H\equiv 0$ *and*

$$\limsup_{r\to\infty}\frac{\bar{N}\left(r,\frac{1}{F}\right)+\bar{N}(r,F)+\bar{N}\left(r,\frac{1}{G}\right)+\bar{N}(r,G)}{T(r)}<1,\quad r\in I,$$

where $T(r)=\max\{T(r,F),T(r,G)\}$ *and I is a set with infinite linear measure, then* $F\equiv G$ *or* $FG\equiv 1$.

Lemma 2.6 ([14]) *Let* $Q(w)=(n-1)^2(w^n-1)(w^{n-2}-1)-n(n-2)(w^{n-1}-1)^2$, *then*

$$Q(w)=(w-1)^4(w-\beta_1)(w-\beta_2)\cdots(w-\beta_{2n-6})$$

where $\beta_j \in C\setminus\{0,1\}$ $(j=1,2,3,\cdots,2n-6)$, *which are distinct respectively.*

3. Proofs of the Theorems

In this section, we present the proofs of the main results.

Proof of Theorem 1.2.

Lemma 2.4 implies that $S(r,f)=S(r,g)$.

Let

$$F=\frac{f^n(f-1)^m f'}{z}, \tag{1}$$

$$G=\frac{g^n(g-1)^m g'}{z}. \tag{2}$$

and

$$F^*=\frac{1}{n+m+1}f^{n+m+1}-\frac{{}^mC_1}{n+m}f^{n+m}+\frac{{}^mC_2}{n+m-1}f^{n+m-1}+\cdots+(-1)^p\frac{1}{n+1}f^{n+1}, \tag{3}$$

$$G^*=\frac{1}{n+m+1}g^{n+m+1}-\frac{{}^mC_1}{n+m}g^{n+m}+\frac{{}^mC_2}{n+m-1}g^{n+m-1}+\cdots+(-1)^p\frac{1}{n+1}g^{n+1}, \tag{4}$$

where $p=0,1,2,\cdots$

Thus we obtain that F and G share the value 1 IM. Moreover, by Lemma 2.1, we have

$$T(r,F^*)=(n+m+1)T(r,f)+S(r,f), \tag{5}$$

$$T(r,G^*)=(n+m+1)T(r,g)+S(r,g). \tag{6}$$

Noting that $(F^*)'=Fz$, we deduce

$$m\left(r,\frac{1}{F^*}\right)\le m\left(r,\frac{1}{zF}\right)+S(r,f)\le m\left(r,\frac{1}{F}\right)+\log r+S(r,f), \tag{7}$$

and by the First Fundamental Theorem,

$$T(r,F^*)\le T(r,F)+N\left(r,\frac{1}{F^*}\right)-N\left(r,\frac{1}{F}\right)+\log r+S(r,f). \tag{8}$$

Note that,

$$N\left(r,\frac{1}{F^*}\right)=(n+1)N\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{f-a_1}\right)+N\left(r,\frac{1}{f-a_2}\right)+\cdots+N\left(r,\frac{1}{f-a_m}\right), \tag{9}$$

where $a_1,a_2,\cdots,a_m$ are distinct roots of the algebraic equation

$$\frac{{}^mC_0}{n+m+1}Z^m-\frac{{}^mC_1}{n+m}Z^{m-1}+\frac{{}^mC_2}{n+m-1}Z^{m-2}+\cdots+(-1)^p\frac{1}{n+1}=0,$$

and

$$N\left(r,\frac{1}{F}\right)=nN\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{(f-1)^m}\right)+N\left(r,\frac{1}{f'}\right). \tag{10}$$

Since F and G share 1 IM, by Lemma 2.3, we have

$$\begin{aligned}T(r,F)+T(r,G)\le{}&2\left[N_2(r,F)+N_2(r,G)+N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right)\right]\\&+3\left[\overline{N}(r,F)+\overline{N}(r,G)+\overline{N}\left(r,\frac{1}{F}\right)+\overline{N}\left(r,\frac{1}{G}\right)\right]+S(r,F)+S(r,G).\end{aligned} \tag{11}$$

Obviously, we have

$$N_2(r,F)+N_2\left(r,\frac{1}{F}\right)\le 2\bar{N}(r,f)+2N\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{(f-1)^m}\right)+N\left(r,\frac{1}{f'}\right)+\log r, \tag{12}$$

$$N_2(r,G)+N_2\left(r,\frac{1}{G}\right)\le 2\bar{N}(r,g)+2N\left(r,\frac{1}{g}\right)+N\left(r,\frac{1}{(g-1)^m}\right)+N\left(r,\frac{1}{g'}\right)+\log r. \tag{13}$$

So, we have

$$\begin{aligned}T(r,F^*)+T(r,G^*)\le{}& T(r,F)+T(r,G)+N\left(r,\frac{1}{F^*}\right)+N\left(r,\frac{1}{G^*}\right)-N\left(r,\frac{1}{F}\right)\\&-N\left(r,\frac{1}{G}\right)+2\log r+S(r,f)+S(r,g).\end{aligned} \tag{14}$$

From (5) to (14), we have

$$(n-m-25)T(r,f)+(n-m-25)T(r,g)\le 6\log r+S(r,f)+S(r,g). \tag{15}$$

We obtain that $n\le m+25$ which contradicts $n>m+26$.
Therefore $H\equiv 0$, that is

$$\frac{F''}{F'}-2\frac{F'}{F-1}\equiv\frac{G''}{G'}-2\frac{G'}{G-1}. \tag{16}$$

By integration, we have

$$\frac{A}{F-1}+B=\frac{1}{G-1}, \tag{17}$$

where $A(\ne 0)$ and B are constants. Thus

$$T(r,F)=T(r,G)+S(r,f). \tag{18}$$

Since,

$$\bar{N}\left(r,\frac{1}{f'}\right)\le T(r,f')-m\left(r,\frac{1}{f'}\right)\le 2T(r,f)-m\left(r,\frac{1}{f'}\right)+S(r,f), \tag{19}$$

we note that,

$$\begin{aligned}&\bar{N}\left(r,\frac{1}{F}\right)+\bar{N}\left(r,\frac{1}{G}\right)+\bar{N}(r,F)+\bar{N}(r,G)\\&\le\bar{N}\left(r,\frac{1}{f}\right)+\bar{N}\left(r,\frac{1}{g}\right)+\bar{N}(r,f)+\bar{N}(r,g)+\bar{N}\left(r,\frac{1}{(f-1)^m}\right)\\&\quad+\bar{N}\left(r,\frac{1}{(g-1)^m}\right)+\bar{N}\left(r,\frac{1}{f'}\right)+\bar{N}\left(r,\frac{1}{g'}\right)+2\log r+S(r,f),\end{aligned} \tag{20}$$

and

$$T(r,F)+m\left(r,\frac{1}{f'}\right)=T\left(r,\frac{f^n(f-1)^m f'}{z}\right)+m\left(r,\frac{1}{f'}\right)\ge T\left(r,f^n(f-1)^m\right)-\log r. \tag{21}$$

Similarly, we have

$$T(r,G)+m\left(r,\frac{1}{g'}\right)\ge T\left(r,g^n(g-1)^m\right)-\log r. \tag{22}$$

From (19) to (22) and applying Lemma 2.5, we get

$$F \equiv G \quad \text{or} \quad FG \equiv 1.$$

We discuss the following cases.

Case (i) Suppose that $FG \equiv 1$.

As in the proof of Theorem 1, in [5] we arrive at a contradiction.

Case (ii) $F \equiv G$, thus $F^* \equiv G^*$, that is,

$$\frac{1}{n+m+1} f^{n+m+1} - \frac{{}^mC_1}{n+m} f^{n+m} + \frac{{}^mC_2}{n+m-1} f^{n+m-1} + \cdots + (-1)^p \frac{1}{n+1} f^{n+1}$$
$$= \frac{1}{n+m+1} g^{n+m+1} - \frac{{}^mC_1}{n+m} g^{n+m} + \frac{{}^mC_2}{n+m-1} g^{n+m-1} + \cdots + (-1)^p \frac{1}{n+1} g^{n+1}.$$

Set $h = \frac{f}{g}$, we substitute $f = hg$ in the above, it follows that

$$(n+m)(n+m-1)\cdots(n+1) g^m \left(h^{n+m+1}-1\right) - {}^mC_1 (n+m+1)(n+m-1)\cdots (n+1) g^{m-1}\left(h^{n+m}-1\right) + \cdots + (-1)^p (n+m+1)(n+m)\cdots(n)\left(h^{n+1}-1\right) = 0. \tag{23}$$

If h is not a constant, using Lemma 2.6 and (23), we conclude that

$$\left[(n+m)(n+m-1)\cdots(n+1) g\left(h^{n+m+1}-1\right) - (n+m+1)(n+m-1)\cdots(n+1)\left(h^{n+m}-1\right)\right]^m$$
$$= (n+m)^m (n+m-1)^m \cdots (n+1)^m \left(h^{n+m+1}-1\right)^m g^m - {}^mC_1 (n+m)^{m-1}(n+m-1)^{m-1}\cdots$$
$$(n+1)^{m-1}(n+m+1)(n+m-1)\cdots(n+1)\left(h^{n+m+1}-1\right)^{m-1}\left(h^{n+m}-1\right) g^{m-1}$$
$$+ \cdots + (-1)^p (n+m+1)^m (n+m-1)^m \cdots (n+1)^m \left(h^{n+m}-1\right)^m$$
$$= (n+m)^{m-1}(n+m-1)^{m-1}\cdots(n+1)^{m-1}\left(h^{n+m+1}-1\right)^{m-1}\Big\{(n+m)(n+m-1)\cdots$$
$$(n+1) g^m \left(h^{n+m+1}-1\right) - {}^mC_1 (n+m+1)(n+m-1)\cdots(n+1) g^{m-1}\left(h^{n+m}-1\right) + \cdots\Big\}$$
$$+ (-1)^p (n+m+1)^m (n+m-1)^m \cdots (n+1)^m \left(h^{n+m}-1\right)^m$$

By (23), we get

$$= (n+m)^{m-1}(n+m-1)^{m-1}\cdots(n+1)^{m-1}\left(h^{n+m+1}-1\right)^{m-1}\Big\{-(n+m+1)(n+m)\cdots$$
$$\left(h^{n+1}-1\right)\Big\} + (n+m+1)^m (n+m-1)^m \cdots (n+1)^m \left(h^{n+m}-1\right)^m$$
$$= (n+m+1)(n+m-1)^{m-1}\cdots(n+1)^{m-1}\Big\{(n+m)^m \left(h^{n+m+1}-1\right)^{m-1}\left(h^{n+1}-1\right)$$
$$-(n+m+1)^{m-1}(n+m-1)\cdots(n+1)^m \left(h^{n+m}-1\right)^m\Big\}$$
$$= (n+m+1)(n+m-1)^{m-1}\cdots(n+1)^{m-1} Q(h),$$

where

$$Q(h) = (n+m)^m \left(h^{n+m+1}-1\right)^{m-1}\left(h^{n+1}-1\right) - (n+m+1)^{m-1}(n+m-1)\cdots(n+1)^m \left(h^{n+m}-1\right)^m.$$

Using Lemma 2.6, we get

$$Qh = (h-1)^4 (h-\beta_1)(h-\beta_2)\cdots(h-\beta_{2n+2m-4}).$$

where $\beta_j \in C \setminus \{0,1\}$, $(j = 1,2,\cdots,2n+2m-4)$ which are pairwise distinct.

This implies that every zero of $h-\beta_j$ $(j=1,2,\cdots,2n)$ has a multiplicity of at least n. By the Second Fundamental Theorem, we obtain that $n \le m$, which is again a contradiction.

Therefore h is a constant. We have from (23) that $h^{n+1}-1=0,\ h^{n+m}-1=0$, which imply $h=1$, and hence $f \equiv g$.

Proof of Theorem 1.1. Let F and G be given by (1) and (2). Suppose H is given as in Lemma 2.3, and $H \ne 0$. Proceeding as in the proof of Theorem 1.2 we can obtain (3) to (10). Since F and G share 1 *CM*, by Lemma 2.3, we have

$$T(r,f) \le N_2(r,F)+N_2(r,G)+N_2\left(r,\frac{1}{F}\right)+N_2\left(r,\frac{1}{G}\right).$$

Hence from (3) to (10) and (12) to (14), we get

$$(n-m-10)T(r,f)+(n-m-10)T(r,g) \le S(r,f)+S(r,g).$$

Hence $n \le m+10$, which contradicts that $n \ge m+11$.

Proof of Theorem 1.4. Let F and G be given by (1) and (2). Suppose H is given as in Lemma 2.3, and $H \ne 0$. Proceeding as in the proof of Theorem 1.2 we can obtain (3) to (10) and (11). Since F and G share 1 IM, by Lemma 2.3, obviously we have,

$$N_2(r,F)+N_2\left(r,\frac{1}{F}\right) \le 2N\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{(f-1)^m}\right)+N\left(r,\frac{1}{f'}\right),$$

$$N_2(r,G)+N_2\left(r,\frac{1}{G}\right) \le 2N\left(r,\frac{1}{g}\right)+N\left(r,\frac{1}{(g-1)^m}\right)+N\left(r,\frac{1}{g'}\right).$$

therefore (14) reduces to

$$(n-m-14)T(r,f)+(n-m-14)T(r,g) \le S(r,f)+S(r,g).$$

Hence $n \le m+14$, which contradicts that $n \ge m+15$. Proceeding in the same way as in Theorem 1.2 we get $f \equiv g$.

Proof of Theorem 1.3. Let F and G be given by (1) and (2). Suppose H is given as in Lemma 2.3, and $H \ne 0$. Proceeding as in the proof of Theorem 1.2 we can obtain (3) to (10). Since F and G share 1 *CM*, by Lemma 2.3, we have

$$T(r,F) \le N_2(r,F)+N_2\left(r,\frac{1}{F}\right)+N_2(r,G)+N_2\left(r,\frac{1}{G}\right).$$

Hence from (3) to (10) and (12) to (14), we get

$$(n-m-5)T(r,f)+(n-m-5)T(r,g) \le S(r,f)+S(r,g).$$

Hence $n \le m+5$, which contradicts that $n \ge m+6$. Proceeding in the same way as in Theorem 1.2, we get $f \equiv g$.

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