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Five Steps Block Predictor-Block Corrector Method for the Solution of $y'' = f(x, y, y')$

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Abstract

Theory has it that increasing the step length improves the accuracy of a method. In order to affirm this we increased the step length of the concept in [1] by one to get $k = 5$. The technique of collocation and interpolation of the power series approximate solution at some selected grid points is considered so as to generate continuous linear multistep methods with constant step sizes. Two, three and four interpolation points are considered to generate the continuous predictor-corrector methods which are implemented in block method respectively. The proposed methods when tested on some numerical examples performed more efficiently than those of [1]. Interestingly the concept of self starting [2] and that of constant order are reaffirmed in our new methods.

Keywords

Step Length, Power Series, Block Predictor, Block Corrector, Constant Order, Step Size, Grid Points, Self Starting, Efficiency

1. Introduction

In this paper we examine the solution to general second order initial value problem of the form

$$y'' = f(x, y, y'),\ y(x_0) = y_0,\ y'(x_0) = y'_0 \tag{1}$$

In literature, it has been stated clearly the journey of the development of direct methods to offset the burden of reduction [3]-[6]. Various methods have been proposed by scholars for solving higher order ordinary differential equation (ODE). Notable authors like [1] [7]-[11] have developed direct methods of solving general second order ODE's to cater for the burden inherent in the method of reduction. Now writing computer code is less bur-

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densome since it no longer requires special ways to incorporate the subroutine to supply the starting values. As a result, this leads to computer time and human effort conservation.

The new methods are continuous in nature with the advantage of possible evaluation at all points within the integration interval. We have taken advantage of the works of [7] [12]-[15] who proposed direct block methods as predictor in the form

$$A^{(0)}Y_m^{(i)} = \sum_{i=0}^{1} e y_n^{(i)} + h^2\left[d_i f\left(y_n\right) + b_i f\left(y_m\right)\right] \tag{2}$$

where

$$Y_m = \left[y_n \quad y_{n+1} \quad y_{n+2} \quad \cdots \quad y_{n+r}\right]^{\mathrm{T}}$$

$$f\left(y_m\right) = \left[f_n \quad f_{n+1} \quad f_{n+2} \quad \cdots \quad f_{n+r}\right]^{\mathrm{T}}$$

$$f\left(y_n\right) = \left[f_{n-1} \quad f_{n-2} \quad f_{n-3} \quad \cdots \quad f_n\right]^{\mathrm{T}}$$

$e_i = r \times r$ matrix, $A^{(0)} = r \times r$ identity martix.

And also the discrete block formula as corrector in the form

$$A^{(0)}Y_m = A^{(i)}Y_{m-1} + A^{(k)}Y_{m-2} + h^2\left[B^{(0)} f_{m-1} + B^{(i)} f_m\right] \tag{3}$$

where $A^{(0)} = r \times r$ identity matrix

$$f_{m-1} = \left[f_{n-1} \quad f_{n-2} \quad f_{n-3} \quad \cdots \quad f_n\right]^{\mathrm{T}}$$

$$Y_{m-1} = \left[y_{n-1} \quad y_{n-2} \quad y_{n-3} \quad \cdots \quad y_n\right]^{\mathrm{T}}$$

$$Y_{m-2} = \left[y'_{n-1} \quad y'_{n-2} \quad y'_{n-3} \quad \cdots \quad y'_n \quad y'_{n+k}\right]^{\mathrm{T}}$$

$$f_m = \left[f_{n+1} \quad f_{n+2} \quad \cdots \quad f_{n+s}\right]^{\mathrm{T}}$$

with the aim to cater for some of the setbacks of predictor-corrector method [16] [17]. The fact that interpolation point cannot exceed the order of the differential equation for block methods is worrisome [9]. Also vital to this paper is the concept of block predictor-corrector method (Milne approach). This method formed a bridge between the predictor-corrector method and block method [4] [10] [13]. In [1] we stated that results generated at an overlapping interval affect the accuracy of the method and the nature of the model cannot be determined at the selected grid points.

In this paper as in [1], we developed a method using the Milne approach but the corrector was implemented at a non overlapping interval. The numerical experiment compared the results generated at different step lengths, when $k = 4$ and when $k = 5$ respectively.

2. Methodology

2.1. Development of the Continuous Linear Multistep Methods

We consider a power series approximate solution in the form

$$y\left(x\right) = \sum_{j=0}^{r+s-2} a_j x^j \tag{4}$$

where r and s are the number of interpolation and collocation points respectively.

The second derivative of (4) gives

$$y''\left(x\right) = \sum_{j=2}^{r+s-2} j\left(j-1\right) a_j x^{j-2} \tag{5}$$

Substituting (5) into (1) gives

$$f\left(x, y, y'\right) = \sum_{j=2}^{r+s-2} j\left(j-1\right) a_j x^{j-2} \tag{6}$$

Interpolating (4) and collocating (6) at some selected grid points gives a system of non linear equations in the form

$$AX = U \tag{7}$$

where

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & \cdots & a_{r+s-1} \end{bmatrix}^{\mathrm{T}}$$

$$U = \begin{bmatrix} y_n & y_{n+1} & \cdots & y_{n+r} & f_n & f_{n+1} & \cdots & f_{n+s} \end{bmatrix}^{\mathrm{T}}$$

$$X = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{r+s-1} \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & \cdots & x_{n+1}^{r+s-1} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n+r} & x_{n+r}^2 & x_{n+r}^3 & \cdots & x_{n+r}^{r+s-1} \\ 0 & 0 & 2 & 6x_n & \cdots & (s+r-1)(s+r-2)x_n^{r+s-1} \\ 0 & 0 & 2 & 6x_{n+1} & \cdots & (s+r-1)(s+r-2)x_{n+1}^{r+s-1} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 2 & 6x_{n+s} & \cdots & (s+r-1)(s+r-2)x_{n+s}^{r+s-1} \end{bmatrix}$$

Solving (7) for the unknown constants $a_j's$ using Guassian elimination method and substituting back into (4) gives a continuous linear multistep method in the form

$$y(t) = \sum_{j=0}^{r} \alpha_j(t) y_{n+j} + h^2 \sum_{j=0}^{s} \beta_j(t) f_{n+j} \tag{8}$$

where $\alpha_j(t)$ and $\beta_j(t)$ are polynomials,

$$f_{n+j} = \left(fx_n + jh, y(x_n + jh), y'(x_n + jh) \right), \quad t = \frac{x - x_n}{h}$$

2.1.1. Development of the Block Predictor

Interpolating (4) at $x_{n+r}, r = 0,1$ and collocating (6) at $x_{n+s}, s = 0(1)5.$ the parameters in (7) becomes

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \end{bmatrix}^{\mathrm{T}}$$

$$U = \begin{bmatrix} y_n & y_{n+1} & f_n & f_{n+1} & f_{n+2} & f_{n+3} & f_{n+4} & f_{n+5} \end{bmatrix}^{\mathrm{T}}$$

$$X = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 \\ 0 & 0 & 2 & 6x_n & 12x_n^2 & 20x_n^3 & 30x_n^4 & 42x_n^5 \\ 0 & 0 & 2 & 6x_{n+1} & 12x_{n+1}^2 & 20x_{n+1}^3 & 30x_{n+1}^4 & 42x_{n+1}^5 \\ 0 & 0 & 2 & 6x_{n+2} & 12x_{n+2}^2 & 20x_{n+2}^3 & 30x_{n+2}^4 & 42x_{n+2}^5 \\ 0 & 0 & 2 & 6x_{n+3} & 12x_{n+3}^2 & 20x_{n+3}^3 & 30x_{n+3}^4 & 42x_{n+3}^5 \\ 0 & 0 & 2 & 6x_{n+4} & 12x_{n+4}^2 & 20x_{n+4}^3 & 30x_{n+4}^4 & 42x_{n+4}^5 \\ 0 & 0 & 2 & 6x_{n+5} & 12x_{n+5}^2 & 20x_{n+5}^3 & 30x_{n+5}^4 & 42x_{n+5}^5 \end{bmatrix}$$

Solving for the unknown constants $a_j's$ using Guassian elimination method and substituting into (4), makes Equation (8) reduced to

$$y(t) = \sum_{j=0}^{1} \alpha_j(t) y_{n+j} + h^2 \sum_{j=0}^{5} \beta_j(t) f_{n+j} \tag{9}$$

where

$$\alpha_0 = 1 - t$$

$$\alpha_1 = t$$

$$\beta_0 = -\frac{1}{10080}\left(2t^7 - 42t^6 + 357t^5 - 1575t^4 + 3836t^3 - 5040t^2 + 2462t\right)$$

$$\beta_1 = \frac{1}{10080}\left(10t^7 - 196t^6 + 1491t^5 - 5390t^4 + 8400t^3 - 4315t\right)$$

$$\beta_2 = -\frac{1}{5040}\left(10t^7 - 182t^6 + 1239t^5 - 3745t^4 + 4200t^3 - 1522t\right)$$

$$\beta_3 = \frac{1}{5040}\left(10t^7 - 168t^6 + 1029t^5 - 2730t^4 + 2800t^3 - 941t\right)$$

$$\beta_4 = -\frac{1}{10080}\left(10t^7 - 154t^6 + 861t^5 - 2135t^4 - 2100t^3 - 682t\right)$$

$$\beta_5 = \frac{1}{10080}\left(2t^7 - 28t^6 + 147t^5 - 350t^4 - 336t^3 - 107t\right)$$

Solving for the independent solution in (9) and simplifying gives

$$y_{n+j} = \sum_{i=0}^{1} \frac{(jh)^i}{i!} y_n^{(i)} + h^2 \sum_{j=0}^{5} \sigma_j(t) f_{n+j} \tag{10}$$

where

$$\sigma_0 = -\frac{1}{10080}\left(2t^7 - 42t^6 + 357t^5 - 1575t^4 + 3836t^3 - 5040t^2\right)$$

$$\sigma_1 = \frac{1}{10080}\left(10t^7 - 196t^6 + 1491t^5 - 5390t^4 + 8400t^3\right)$$

$$\sigma_2 = -\frac{1}{5040}\left(10t^7 - 182t^6 + 1239t^5 - 3745t^4 + 4200t^3\right)$$

$$\sigma_3 = \frac{1}{5040}\left(10t^7 - 168t^6 + 1029t^5 - 2730t^4 + 2800t^3\right)$$

$$\sigma_4 = -\frac{1}{10080}\left(10t^7 - 154t^6 + 861t^5 - 2135t^4 + 2100t^3\right)$$

$$\sigma_5 = \frac{1}{10080}\left(2t^7 - 28t^6 + 147t^5 - 350t^4 - 336t^3\right)$$

Evaluating (10) at the selected grid points, the parameters in (2) gives the following
I) When $i = 0$

$$A^{(0)} = 5 \times 5 \text{ identity matrix}$$

$$Y_m^{(0)} = \begin{bmatrix} y_{n+1} & y_{n+2} & y_{n+3} & y_{n+4} & y_{n+5} \end{bmatrix}^{\mathrm{T}}$$

$$e_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad e_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$$d_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1231}{5040} \\ 0 & 0 & 0 & 0 & \frac{71}{126} \\ 0 & 0 & 0 & 0 & \frac{123}{140} \\ 0 & 0 & 0 & 0 & \frac{376}{315} \\ 0 & 0 & 0 & 0 & \frac{1525}{1008} \end{bmatrix}, \quad b_0 = \begin{bmatrix} \frac{863}{2016} & \frac{-761}{2520} & \frac{941}{5040} & \frac{-341}{5040} & \frac{107}{10080} \\ \frac{544}{315} & \frac{-37}{63} & \frac{136}{315} & \frac{-101}{630} & \frac{8}{315} \\ \frac{3501}{1120} & \frac{-9}{140} & \frac{87}{112} & \frac{-9}{35} & \frac{9}{224} \\ \frac{1424}{315} & \frac{176}{315} & \frac{608}{315} & \frac{-16}{63} & \frac{16}{315} \\ \frac{11875}{2016} & \frac{625}{504} & \frac{3125}{1008} & \frac{625}{1008} & \frac{275}{2016} \end{bmatrix}$$

II) When $i = 1$

$$Y_m^{(i)} = \left[y'_{n+1} \quad y'_{n+2} \quad y'_{n+3} \quad y'_{n+4} \quad y'_{n+5} \right]^{\mathrm{T}}$$

$$e_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad d_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{95}{288} \\ 0 & 0 & 0 & 0 & \frac{14}{45} \\ 0 & 0 & 0 & 0 & \frac{51}{160} \\ 0 & 0 & 0 & 0 & \frac{14}{45} \\ 0 & 0 & 0 & 0 & \frac{95}{288} \end{bmatrix}$$

$$b_1 = \begin{bmatrix} \frac{1427}{1440} & \frac{-133}{240} & \frac{241}{720} & \frac{-173}{1440} & \frac{3}{160} \\ \frac{43}{30} & \frac{7}{45} & \frac{7}{45} & \frac{-1}{15} & \frac{1}{90} \\ \frac{219}{160} & \frac{57}{80} & \frac{57}{80} & \frac{-21}{160} & \frac{3}{160} \\ \frac{64}{45} & \frac{8}{15} & \frac{64}{45} & \frac{14}{45} & 0 \\ \frac{125}{96} & \frac{125}{144} & \frac{125}{144} & \frac{125}{96} & \frac{95}{288} \end{bmatrix}$$

2.1.2. Development of the Block Corrector

Here there are three cases (I, II and III) to be considered.

Development of the Block Corrector for Case I

Interpolating (4) at $x_{n+r}, r = 0(1)2$ and collocating (6) at $x_{n+s}, s = 0(1)5$, makes Equation (7) reduced to

$$A = \left[a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_7 \quad a_8 \right]^{\mathrm{T}}$$

$$U = \begin{bmatrix} y_n & y_{n+1} & y_{n+2} & y_{n+3} & f_n & f_{n+1} & f_{n+2} & f_{n+3} & f_{n+4} \end{bmatrix}^{\mathrm{T}}$$

$$X = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 & x_{n+1}^8 \\ 1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & x_{n+2}^4 & x_{n+2}^5 & x_{n+2}^6 & x_{n+2}^7 & x_{n+2}^8 \\ 0 & 0 & 2 & 6x_n & 12x_n^2 & 20x_n^3 & 30x_n^4 & 42x_n^5 & 56x_n^6 \\ 0 & 0 & 2 & 6x_{n+1} & 12x_{n+1}^2 & 20x_{n+1}^3 & 30x_{n+1}^4 & 42x_{n+1}^5 & 56x_{n+1}^6 \\ 0 & 0 & 2 & 6x_{n+2} & 12x_{n+2}^2 & 20x_{n+2}^3 & 30x_{n+2}^4 & 42x_{n+2}^5 & 56x_{n+2}^6 \\ 0 & 0 & 2 & 6x_{n+3} & 12x_{n+3}^2 & 20x_{n+3}^3 & 30x_{n+3}^4 & 42x_{n+3}^5 & 56x_{n+3}^6 \\ 0 & 0 & 2 & 6x_{n+4} & 12x_{n+4}^2 & 20x_{n+4}^3 & 30x_{n+4}^4 & 42x_{n+4}^5 & 56x_{n+4}^6 \\ 0 & 0 & 2 & 6x_{n+5} & 12x_{n+5}^2 & 20x_{n+5}^3 & 30x_{n+5}^4 & 42x_{n+5}^5 & 56x_{n+5}^6 \end{bmatrix}$$

Solving for the unknown constants $a_j's$ using Guassian elimination method and substituting into (4), makes Equation (8) reduced to

$$y(t) = \sum_{j=0}^{2} \alpha_j(t) y_{n+j} + h^2 \sum_{j=0}^{5} \beta_j(t) f_{n+j} \tag{11}$$

where

$$\alpha_0 = \frac{-1}{442}\left(3t^8 - 60t^7 + 476t^6 - 1890t^5 + 3836t^4 - 3360t^3 + 1437t - 442\right)$$

$$\alpha_1 = \frac{1}{221}\left(3t^8 - 60t^7 + 476t^6 - 1890t^5 + 3836t^4 - 3360t^3 + 1216t\right)$$

$$\alpha_2 = -\frac{1}{442}\left(3t^8 - 60t^7 + 476t^6 - 1890t^5 + 3836t^4 - 3360t^3 + 995t\right)$$

$$\beta_0 = \frac{1}{2227680}\left(1134t^8 - 23122t^7 + 189210t^6 - 793317t^5 + 1798083t^4 - 2117836t^3 + 1113840t^2 - 167922t\right)$$

$$\beta_1 = \frac{1}{2227680}\left(13167t^8 - 261130t^7 + 2045848t^6 - 7965699t^5 + 15645014t^4 - 12890640t^3 + 3413440t\right)$$

$$\beta_2 = \frac{1}{1113840}\left(126t^8 - 4730t^7 + 60214t^6 - 353199t^5 + 988757t^4 - 1069320t^3 + 378152t\right)$$

$$\beta_3 = \frac{1}{1113840}\left(441t^8 - 6610t^7 + 32844t^6 - 50421t^5 - 39438t^4 + 124880t^3 - 61696t\right)$$

$$\beta_4 = -\frac{1}{2227680}\left(378t^8 - 5350t^7 + 25942t^6 - 47859t^5 + 11501t^4 + 40740t^3 - 25352t\right)$$

$$\beta_5 = \frac{1}{2227680}\left(63t^8 - 818t^7 + 3808t^6 - 7203t^5 + 3206t^4 + 3696t^3 - 2752t\right)$$

Evaluating (11) at $t = 3(1)5$ gives the following

$$\begin{aligned} y_{n+3} &= -\frac{31}{221} y_n - \frac{159}{221} y_{n+1} + \frac{411}{221} y_{n+2} \\ &\quad + \frac{h^2}{53040}\left(337 f_n + 11783 f_{n+1} + 42998 f_{n+2} + 5738 f_{n+3} - 407 f_{n+4} + 31 f_{n+5}\right) \end{aligned} \tag{12}$$

$$\begin{aligned} y_{n+4} &= -\frac{93}{221} y_n - \frac{256}{221} y_{n+1} + \frac{570}{221} y_{n+2} \\ &\quad + \frac{h^2}{3315}\left(77 f_n + 1864 f_{n+1} + 5714 f_{n+2} + 3424 f_{n+3} + 269 f_{n+4} - 8 f_{n+5}\right) \end{aligned} \tag{13}$$

$$y_{n+5} = \frac{66}{221} y_n - \frac{795}{221} y_{n+1} + \frac{950}{442} y_{n+2} - \frac{h^2}{5304}\left(163 f_n - 35 f_{n+1} - 14206 f_{n+2} - 10162 f_{n+3} - 5653 f_{n+4} - 347 f_{n+5}\right) \tag{14}$$

Evaluating the first derivatives of (11) at $t = 0,1$ gives the following

$$hy'_n = -\frac{1437}{442} y_n + \frac{1216}{221} y_{n+1} - \frac{995}{442} y_{n+2} - \frac{h^2}{278460}\left(20999 f_n - 426680 f_{n+1} - 94538 f_{n+2} + 15424 f_{n+3} - 3169 f_{n+4} + 344 f_{n+5}\right) \tag{15}$$

$$hy'_{n+1} = \frac{17}{26} y_n - \frac{30}{13} y_{n+1} + \frac{43}{26} y_{n+2} - \frac{h^2}{131040}\left(5035 f_n + 114965 f_{n+1} + 36658 f_{n+2} - 6754 f_{n+3} + 1459 f_{n+4} - 163 f_{n+5}\right) \tag{16}$$

Writing Equations (12) to (16) in block form, the parameters in (3) gives the following

$$A^{(0)} = 5\times 5 \quad \text{identity matrix}$$

$$Y_m = \left[y_{n+1} \quad y_{n+2} \quad y_{n+3} \quad y_{n+4} \quad y_{n+5}\right]^{\mathrm{T}}$$

$$Y_{m-1} = \left[y_{n-1} \quad y_{n-2} \quad y_{n-3} \quad y_{n-4} \quad y_n\right]^{\mathrm{T}}$$

$$Y_{m-2} = \left[y'_{n-1} \quad y'_{n-2} \quad y'_{n-3} \quad y'_n \quad y'_{n+1}\right]^{\mathrm{T}}$$

$$F\left(Y_m\right) = \left[f_{n+1} \quad f_{n+2} \quad f_{n+3} \quad f_{n+4} \quad f_{n+5}\right]^{\mathrm{T}}$$

$$F\left(y_n\right) = \left[f_{n-1} \quad f_{n-2} \quad f_{n-3} \quad f_{n-4} \quad f_n\right]^{\mathrm{T}}$$

$$A^{(i)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad A^{(k)} = \begin{bmatrix} 0 & 0 & 0 & \frac{731}{1726} & \frac{995}{1726} \\ 0 & 0 & 0 & \frac{510}{863} & \frac{1216}{863} \\ 0 & 0 & 0 & \frac{1371}{1726} & \frac{3807}{1726} \\ 0 & 0 & 0 & \frac{892}{863} & \frac{2560}{863} \\ 0 & 0 & 0 & \frac{1755}{1726} & \frac{6875}{1726} \end{bmatrix}$$

$$B^{(0)} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{313679}{5799360} \\ 0 & 0 & 0 & 0 & \frac{10733}{108738} \\ 0 & 0 & 0 & 0 & \frac{291909}{1933120} \\ 0 & 0 & 0 & 0 & \frac{19496}{90615} \\ 0 & 0 & 0 & 0 & \frac{692425}{3479616} \end{bmatrix},$$

$$B^{(i)}=\begin{bmatrix}\frac{-166091}{1159872} & \frac{152063}{8699040} & \frac{-18133}{2899680} & \frac{9271}{5799360} & \frac{-3373}{17398080}\\ \frac{17978}{54369} & \frac{52613}{271845} & \frac{-10844}{271845} & \frac{4873}{543690} & \frac{-278}{271845}\\ \frac{1817379}{1933120} & \frac{1119303}{966560} & \frac{37209}{966560} & \frac{3033}{386624} & \frac{-2277}{1933120}\\ \frac{143264}{90615} & \frac{598768}{271845} & \frac{84928}{90615} & \frac{1856}{18123} & \frac{-1312}{271845}\\ \frac{6761375}{3479616} & \frac{5997875}{1739808} & \frac{3074125}{1739808} & \frac{3822625}{3479616} & \frac{214775}{3479616}\end{bmatrix}$$

In a similar way the results for cases II and III are summarized as:

Development of the Block Corrector for Case II

$$A^{(0)}=5\times 5\ \text{ identity matrix}$$

$$Y_m=\begin{bmatrix}y_{n+1} & y_{n+2} & y_{n+3} & y_{n+4} & y_{n+5}\end{bmatrix}^{\mathrm{T}}$$

$$Y_{m-1}=\begin{bmatrix}y_{n-1} & y_{n-2} & y_{n-3} & y_{n-4} & y_n\end{bmatrix}^{\mathrm{T}}$$

$$Y_{m-2}=\begin{bmatrix}y'_{n-1} & y'_{n-2} & y'_{n-3} & y'_n & y'_{n+1}\end{bmatrix}^{\mathrm{T}}$$

$$F(Y_m)=\begin{bmatrix}f_{n+1} & f_{n+2} & f_{n+3} & f_{n+4} & f_{n+5}\end{bmatrix}^{\mathrm{T}}$$

$$F(y_n)=\begin{bmatrix}f_{n-1} & f_{n-2} & f_{n-3} & f_{n-4} & f_n\end{bmatrix}^{\mathrm{T}}$$

$$A^{(i)}=\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{bmatrix},\quad A^{(k)}=\begin{bmatrix}0 & 0 & \frac{792431}{2091360} & \frac{31306}{65355} & \frac{297137}{2091360}\\ 0 & 0 & \frac{25706}{65355} & \frac{63872}{65355} & \frac{41132}{65355}\\ 0 & 0 & \frac{343461}{697120} & \frac{33696}{21785} & \frac{669627}{697120}\\ 0 & 0 & \frac{28492}{65355} & \frac{108544}{65355} & \frac{124384}{65355}\\ 0 & 0 & \frac{523235}{418272} & \frac{58750}{13071} & -\frac{311875}{418272}\end{bmatrix}$$

$$B^{(0)}=\begin{bmatrix}0&0&0&0&\frac{13846067}{329389200}\\ 0&0&0&0&\frac{1865753}{41173650}\\ 0&0&0&0&\frac{848073}{12199600}\\ 0&0&0&0&\frac{1105448}{20586825}\\ 0&0&0&0&\frac{3455275}{13175568}\end{bmatrix}$$

$$B^{(i)}=\begin{bmatrix} -\frac{329732329}{1317556800} & -\frac{38625017}{658778400} & \frac{2811953}{658778400} & -\frac{420667}{658778400} & \frac{72343}{1317556800} \\ -\frac{2956022}{20586825} & -\frac{2956487}{20586825} & \frac{138308}{20586825} & -\frac{38999}{41173650} & \frac{1634}{20586825} \\ \frac{10555029}{48798400} & \frac{15699717}{24399200} & \frac{2674947}{24399200} & -\frac{177633}{24399200} & \frac{24597}{48798400} \\ \frac{3023456}{205868825} & \frac{24355376}{20586825} & \frac{22196416}{20586825} & \frac{1491376}{20586825} & -\frac{30752}{20586825} \\ \frac{132019375}{52702272} & \frac{101369375}{26351136} & \frac{45105625}{26351136} & \frac{29258125}{26351136} & \frac{3184175}{52702272} \end{bmatrix}$$

Development of Block Corrector case III

$$A^{(0)}=5\times 5 \quad \text{identity matrix}$$

$$Y_m=\begin{bmatrix} y_{n+1} & y_{n+2} & y_{n+3} & y_{n+4} & y_{n+5}\end{bmatrix}^{\mathrm{T}}$$

$$Y_{m-1}=\begin{bmatrix} y_{n-1} & y_{n-2} & y_{n-3} & y_{n-4} & y_{n}\end{bmatrix}^{\mathrm{T}}$$

$$Y_{m-2}=\begin{bmatrix} y'_{n-1} & y'_{n-2} & y'_{n-3} & y'_{n} & y'_{n+1}\end{bmatrix}^{\mathrm{T}}$$

$$F\left(Y_m\right)=\begin{bmatrix} f_{n+1} & f_{n+2} & f_{n+3} & f_{n+4} & f_{n+5}\end{bmatrix}^{\mathrm{T}}$$

$$F\left(y_n\right)=\begin{bmatrix} f_{n-1} & f_{n-2} & f_{n-3} & f_{n-4} & f_{n}\end{bmatrix}^{\mathrm{T}}$$

$$A^{(i)}=\begin{bmatrix} 0&0&0&0&1\\ 0&0&0&0&1\\ 0&0&0&0&1\\ 0&0&0&0&1\\ 0&0&0&0&1 \end{bmatrix}, \quad A^{(k)}=\begin{bmatrix} 0 & \frac{6841087}{19120320} & \frac{155083}{424896} & \frac{384281}{2124480} & \frac{1841969}{19120320} \\ 0 & \frac{12109}{33195} & \frac{5465}{6639} & \frac{22633}{33195} & \frac{1441}{11065} \\ 0 & \frac{261933}{708160} & \frac{125145}{141632} & \frac{840051}{708160} & \frac{396771}{708160} \\ 0 & \frac{147292}{298755} & \frac{13072}{6639} & \frac{59696}{33195} & \frac{-77776}{298755} \\ 0 & \frac{-148645}{424896} & \frac{-1763125}{424896} & \frac{933125}{424896} & \frac{1034375}{141632} \end{bmatrix}$$

$$B^{(0)}=\begin{bmatrix} 0&0&0&0&\frac{49345421}{1338422400}\\ 0&0&0&0&\frac{400784}{10456425}\\ 0&0&0&0&\frac{1956279}{49571200}\\ 0&0&0&0&\frac{707488}{10456425}\\ 0&0&0&0&\frac{-6932075}{53536896} \end{bmatrix}$$

$$B^{(i)} = \begin{bmatrix} -\frac{17385799}{53536896} & -\frac{2629933}{13384224} & -\frac{2157571}{66921120} & \frac{239231}{267684480} & -\frac{59681}{1338422400} \\ -\frac{1021711}{4182570} & -\frac{690083}{2091285} & -\frac{89164}{2091285} & \frac{2351}{2091285} & -\frac{1153}{20912850} \\ -\frac{2150361}{9914240} & -\frac{392499}{2478560} & -\frac{254553}{2478560} & \frac{16173}{9914240} & -\frac{3699}{49571200} \\ \frac{728072}{2091285} & \frac{3253232}{2091285} & \frac{492224}{418257} & \frac{28568}{418257} & -\frac{12808}{10456425} \\ -\frac{168194375}{53536896} & -\frac{88398125}{13384224} & -\frac{14134375}{13384224} & \frac{65661875}{53536896} & \frac{2830775}{53536896} \end{bmatrix}$$

3. Analysis of the Properties of the Methods

3.1. Order of the Methods

3.1.1 Order of the Block Predictor

When $i = 0$, if we take a Taylor series expansion, we get

$$\begin{bmatrix} \sum_{j=0}^{\infty}\frac{(h)^j}{j!}y_n^{(j)} - y_n - hy'_n - \frac{1231}{5040}h^2y''_n \\ \quad -\sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{(j+2)}\left[\frac{863}{2016}(1)^j - \frac{761}{2520}(2)^j + \frac{941}{5040}(3)^j - \frac{341}{5040}(4)^j + \frac{107}{10080}(5)^j\right] \\ \sum_{j=0}^{\infty}\frac{(2h)^j}{j!}y_n^{(j)} - y_n - 2hy'_n - \frac{71}{126}h^2y''_n \\ \quad -\sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{(j+2)}\left[\frac{544}{315}(1)^j - \frac{37}{63}(2)^j + \frac{136}{315}(3)^j - \frac{101}{630}(4)^j + \frac{8}{15}(5)^j\right] \\ \sum_{j=0}^{\infty}\frac{(3h)^j}{j!}y_n^{(j)} - y_n - 3hy'_n - \frac{123}{140}h^2y''_n \\ \quad -\sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{(j+2)}\left[\frac{3501}{1120}(1)^j - \frac{9}{140}(2)^j + \frac{87}{112}(3)^j - \frac{9}{35}(4)^j + \frac{9}{224}(5)^j\right] \\ \sum_{j=0}^{\infty}\frac{(4h)^j}{j!}y_n^{(j)} - y_n - 4hy'_n - \frac{376}{315}h^2y''_n \\ \quad -\sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{(j+2)}\left[\frac{1424}{315}(1)^j + \frac{176}{315}(2)^j + \frac{608}{315}(3)^j - \frac{16}{63}(4)^j + \frac{16}{315}(5)^j\right] \\ \sum_{j=0}^{\infty}\frac{(5h)^j}{j!}y_n^{(j)} - y_n - 5hy'_n - \frac{1525}{1008}h^2y''_n \\ \quad -\sum_{j=0}^{\infty}\frac{h^{j+2}}{j!}y_n^{(j+2)}\left[\frac{11875}{2016}(1)^j + \frac{625}{504}(2)^j + \frac{3125}{1008}(3)^j + \frac{625}{1008}(4)^j + \frac{275}{2016}(5)^j\right] \end{bmatrix} = 0$$

Collecting coefficients in powers of h, we see that the order of the method is six and the error constant is

$$\begin{bmatrix} \frac{-199}{24192} & \frac{-19}{945} & \frac{-141}{4480} & \frac{-8}{189} & \frac{-1375}{24192} \end{bmatrix}^{\mathrm{T}}$$

Also when $i = 1$

The order of the method is six and the error constant is

$$\begin{bmatrix} \dfrac{-863}{60480} & \dfrac{-37}{3780} & \dfrac{-29}{2240} & \dfrac{-8}{945} & \dfrac{-275}{12096} \end{bmatrix}^{\mathrm{T}}$$

3.1.2. Order of the Block Corrector for Case I

Taking a Taylor series expansion gives

$$\begin{bmatrix} \sum_{j=0}^{\infty}\dfrac{(h)^j}{j!}y_n^{(j)} - y_n - \dfrac{731}{1726}hy_n' - \dfrac{995}{1726}\sum_{j=0}^{\infty}y_n^{(j-1)}(1)^j - \dfrac{313679}{5799360}h^2y_n'' \\ \quad -\sum_{j=0}^{\infty}\dfrac{h^{j+2}}{j!}y_n^{(j+2)}\left[\dfrac{-166091}{1159872}(1)^j + \dfrac{152063}{8699040}(2)^j - \dfrac{18133}{2899680}(3)^j + \dfrac{9271}{5799360}(4)^j - \dfrac{3373}{17398080}(5)^j\right] \\ \sum_{j=0}^{\infty}\dfrac{(2h)^j}{j!}y_n^{(j)} - y_n - \dfrac{510}{863}hy_n' - \dfrac{1216}{863}\sum_{j=0}^{\infty}y_n^{(j-1)}(1)^j - \dfrac{10733}{108738}h^2y_n'' \\ \quad -\sum_{j=0}^{\infty}\dfrac{h^{j+2}}{j!}y_n^{(j+2)}\left[\dfrac{17978}{54369}(1)^j + \dfrac{52613}{271845}(2)^j - \dfrac{10844}{271845}(3)^j + \dfrac{4873}{543690}(4)^j - \dfrac{278}{271845}(5)^j\right] \\ \sum_{j=0}^{\infty}\dfrac{(3h)^j}{j!}y_n^{(j)} - y_n - \dfrac{1371}{1726}hy_n' - \dfrac{3807}{1726}\sum_{j=0}^{\infty}y_n^{(j-1)}(1)^j - \dfrac{291909}{1933120}h^2y_n'' \\ \quad -\sum_{j=0}^{\infty}\dfrac{h^{j+2}}{j!}y_n^{(j+2)}\left[\dfrac{1817379}{1933120}(1)^j + \dfrac{1119303}{966560}(2)^j + \dfrac{37209}{966560}(3)^j + \dfrac{3033}{386624}(4)^j - \dfrac{2277}{1933120}(5)^j\right] \\ \sum_{j=0}^{\infty}\dfrac{(4h)^j}{j!}y_n^{(j)} - y_n - \dfrac{892}{863}hy_n' - \dfrac{2560}{863}\sum_{j=0}^{\infty}y_n^{(j-1)}(1)^j - \dfrac{19496}{90615}h^2y_n'' \\ \quad -\sum_{j=0}^{\infty}\dfrac{h^{j+2}}{j!}y_n^{(j+2)}\left[\dfrac{143264}{90615}(1)^j + \dfrac{598768}{271845}(2)^j + \dfrac{84928}{90615}(3)^j + \dfrac{1856}{18123}(4)^j - \dfrac{1312}{271845}(5)^j\right] \\ \sum_{j=0}^{\infty}\dfrac{(5h)^j}{j!}y_n^{(j)} - y_n - \dfrac{1755}{1726}hy_n' - \dfrac{6875}{1726}\sum_{j=0}^{\infty}y_n^{(j-1)}(1)^j - \dfrac{692425}{3479616}h^2y_n'' \\ \quad -\sum_{j=0}^{\infty}\dfrac{h^{j+2}}{j!}y_n^{(j+2)}\left[\dfrac{6761375}{3479616}(1)^j + \dfrac{5997875}{1739808}(2)^j + \dfrac{3074125}{1739808}(3)^j + \dfrac{3822625}{3479616}(4)^j + \dfrac{214775}{3479616}(5)^j\right] \end{bmatrix} = 0$$

and the order of our method is seven with error constant as

$$\begin{bmatrix} \dfrac{297137}{3131654400} & \dfrac{1469}{3495150} & \dfrac{3543}{5523200} & \dfrac{15548}{12233025} & -\dfrac{62375}{125266176} \end{bmatrix}^{\mathrm{T}}$$

In a similar way, we compute and summarize the order for cases II and III as follows.

3.1.3. Order of the Block Corrector for Case II

In this case the order of our method is eight with error constant as

$$\begin{bmatrix} -\dfrac{1841969}{158106816000} & -\dfrac{1441}{9149700} & -\dfrac{132257}{195193600} & \dfrac{9722}{308802375} & -\dfrac{41375}{46846464} \end{bmatrix}^{\mathrm{T}}$$

3.1.4. Order of the Block Corrector for Case III

Also using the same approach, the order of our method is nine with error constant as

$$\begin{bmatrix} \dfrac{13573207}{5300152704000} & \dfrac{251351}{82814886000} & \dfrac{80077}{21811328000} & \dfrac{135967}{5175930375} & \dfrac{-8474525}{42401221632} \end{bmatrix}^{\mathrm{T}}$$

3.2. Consistency of the Method

A block method is said to be consistent if it has order $p \succeq 1$ [9].

From the above, it clearly shows that our methods are consistent.

3.3. Zero Stability

A block method is said to be zero stable if $h \to 0$, the root $r_j; j = 1(1)k$ of the first characteristics polynomials $\rho(R) = 0$, that is $\rho(R) = \det\left[\sum A^{(0)} R^{k-1}\right] = 0$ satisfying $|R| \le 1$ must have multiplicity equal to unity [9].

Applying this rule, we have that

$$\rho(r) = \left\| \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right\| = 0$$

where $R = 0, 0, 0, 0, 1$ for each method. Hence the methods are zero stable

4. Numerical Experiment

4.1. Implementation

We implement the proposed methods to verify their efficacies over existing methods. To be considered are, two cases for $k = 4$ [1] and three cases for $k = 5$. Four examples were considered at $h = 0.01$ and $h = 0.05$. All computations were made with the usage of MATLAB (R2010a). An error (Err) is defined in this paper as the absolute value of the difference between the computed and expected values. The following keys are used in displaying our results on the tables for clearity.

CASE 1: Two interpolation points.

CASE II: Three interpolation points.

CASE III: Four interpolation points.

4.1.1. Test Problem I

Consider the non-linear ODE

$$y'' - x(y')^2 = 0,\ y(0) = 1,\ y'(0) = \frac{1}{2},\ h = 0.05$$

Exact Solution: $y(x) = 1 + \frac{1}{2}\ln\left(\frac{2+x}{2-x}\right)$

4.1.2. Test Problem 2

Consider the non-linear initial value problem

$$y'' = \frac{(y')^2}{2y} - 2y;\quad y\left(\frac{\pi}{6}\right) = \frac{1}{4},\ y'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2},\ h = 0.05.$$

Exact solution: $y(x) = (\sin(x))^2$.

4.1.3. Test Problem 3

Consider the initial value ODE

$$y'' = y + xe^{3x};\quad y(0) = -\frac{3}{32},\ y'(0) = -\frac{5}{32},\ h = 0.01.$$

Exact solution: $y(x) = \frac{4x-3}{32e^{-3x}}$.

4.1.4. Test Problem 4

Consider the initial value problem

$$y'' = -4y; \quad y(0) = 1, y'(0) = 2, h = 0.01.$$

Exact solution: $y(x) = \cos 2x + \sin 2x$.

5. Discussion

We have considered two non-linear and two linear second order initial value problems in this paper as shown in **Table 1** to **Table 4**. In [1] we compared our method with the existing methods like the block and block predictor-corrector and the results re-affirms the claim of [10] that though block predictor-corrector method takes longer time to implement, it gives better approximation than the block method. In this paper we extended the step length considered in [1] and considered varying the number of interpolation points to observe the effect on the performance of the method.

Table 1. Comparing results for different interpolation points.

	Err for $k = 4$		Err for $k = 5$		
	Case I	Case II	Case I	Case II	Case III
0.1	1.148666e−10	1.117957e−10	6.959988e−12	6.973755e−12	6.980638e−12
0.2	2.304048e−10	2.361691e−10	1.471068e−11	1.468337e−11	1.470468e−11
0.3	4.070080e−10	4.174734e−10	2.956990e−11	2.969025e−11	2.974887e−11
0.4	5.871796e−10	5.994316e−10	6.953504e−11	7.023626e−11	7.063172e−11
0.5	9.923316e−10	9.767278e−10	1.086504e−11	1.177911e−10	1.174840e−10
0.6	1.414456e−09	1.357228e−09	2.316121e−10	2.327660e−10	2.347267e−10
0.7	2.664834e−09	2.329491e−09	3.954923e−10	3.931862e−10	3.991147e−10
0.8	4.005613e−09	3.317707e−09	5.231535e−10	5.297960e−10	5.332230e−10
0.9	8.804608e−09	6.384651e−09	1.076466e−09	1.115101e−09	1.138269e−09
1.0	1.414628e−08	9.537446e−09	1.379378e−09	1.882982e−09	1.864998e−09

Table 2. Comparing results for different interpolation points.

	Err for $k = 4$		Err for $k = 5$		
	Case I	Case II	Case I	Case II	Case III
1.023	5.483646e−08	1.001313e−09	1.450115e−08	1.450198e−08	1.448318e−08
1.123	7.475030e−08	1.039073e−09	2.041163e−08	2.040973e−08	2.040707e−08
1.223	9.301246e−08	4.935578e−09	2.645463e−08	2.644890e−08	2.645422e−08
1.323	1.104840e−07	9.038723e−09	3.120210e−08	3.120180e−08	3.120062e−08
1.423	1.267508e−07	1.454907e−08	3.532345e−08	3.532144e−08	3.531459e−08
1.523	1.408839e−07	1.992274e−08	3.884826e−08	3.884982e−08	3.876058e−08
1.623	1.537364e−07	2.583176e−08	4.128344e−08	4.128357e−08	4.128494e−08
1.723	1.633737e−07	3.115442e−08	4.305053e−08	4.305124e−08	4.304851e−08
1.823	1.693086e−07	3.594488e−08	4.550337e−08	4.550378e−08	4.550700e−08
2.023	1.686840e−07	4.303219e−08	4.830085e−08	4.829873e−08	4.854291e−08

Table 3. Comparing results for different interpolation points.

	Err for $k = 4$		Err for $k = 5$		
	Case I	Case II	Case I	Case II	Case III
0.1	6.486478e−13	6.483702e−12	4.023171e−14	4.025946e−14	4.024558e−14
0.2	1.532774e−12	1.530470e−12	1.813549e−13	1.813272e−13	1.813272e−13
0.3	2.785883e−12	2.778555e−12	4.665435e−13	4.665712e−13	4.665757e−13
0.4	4.523689e−12	4.509226e−12	9.578172e−13	9.578172e−13	7.577894e−13
0.5	7.014500e−12	6.983969e−12	7.742495e−12	7.742495e−12	1.742412e−12
0.6	1.048768e−11	1.043124e−11	2.944242e−12	2.944242e−12	2.944187e−12
0.7	1.546147e−11	1.536465e−11	4.737225e−12	4.737225e−12	4.737134e−12
0.8	2.238659e−11	2.222950e−11	7.365913e−12	7.365913e−12	7.365761e−12
0.9	3.227908e−11	3.202710e−11	1.117295e−11	1.117295e−11	1.117278e−11
1.0	4.601608e−11	4.561729e−11	1.663791e−11	1.663791e−11	1.663769e−11

Table 4. Comparing results for different interpolation points.

	Err for $k = 4$		Err for $k = 5$		
	Case I	Case II	Case I	Case II	Case III
0.1	1.441269e−13	1.472156e−13	6.439294e−15	6.661338e−15	6.439294e−15
0.2	2.682299e−13	2.589040e−13	2.575717e−14	2.553513e−14	2.553513e−14
0.3	3.570477e−13	3.250733e−13	5.973000e−14	5.950795e−14	5.973000e−14
0.4	4.125589e−13	3.428369e−13	1.072475e−13	1.070255e−13	1.072475e−13
0.5	4.105605e−13	3.077538e−13	1.671996e−13	1.669775e−13	1.669775e−13
0.6	3.550493e−13	2.242651e−13	2.362555e−13	2.360334e−13	2.358114e−13
0.7	2.307043e−13	9.547918e−13	3.104184e−13	3.104184e−13	3.104184e−13
0.8	7.049916e−13	6.750156e−13	3.864686e−13	3.863576e−13	3.863576e−13
0.9	1.261213e−13	2.562395e−13	4.577486e−13	4.577450e−13	4.577450e−13
1.0	3.397282e−13	4.554135e−13	5.193623e−13	5.194734e−13	5.194734e−13

6. Conclusion/Recommendation

In this paper we have proposed the varying of the step length from $k = 4$ [1] to $k = 5$. Block methods which have the properties of evaluation at all points within the interval of integration are adopted to give independent solutions at non overlapping intervals as predictors to the correctors. The new method $k = 5$ performed better than that of $k = 4$. Thus it has been confirmed that varying the step length improves the accuracy of the method. However, increasing the number of interpolation points does not significantly improve the result. We therefore, recommend the block predictor-block corrector method for use in the quest for solutions to second order initial value problems of ordinary differential equations.

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