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Mass Transfer in a Centrifugal Turbine Agitator-Pump

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Abstract

This article is a continuation of the research, centering on a vacuum-filtration system, which is designed to reduce the concentration of calcium in water; a process is also known as—water softening. The problem of solving the concentration distribution of the initial (embryonic) particles of $CaCO_3$-particles, which were introduced into the limited volume of the apparatus with a turbine agitator-pump, is addressed through the use of diffusion and deterministic-stochastic models of mass transfer. The solution of the extreme problem allows determining the most important process parameters, such as time of dispersions homogenization and the dispersion mass flow rate to the surface of a special filter. For these parameters a comparative analysis of the adequacy of the theory was found through experiments, performed in the study. We found that uniform distribution of concentrations along the height of the apparatus is achieved by the angular velocity of the rotation 400 rpm for the turbine with 6 - 7 blades at the time of homogenization 14s. In this case, the dispersion mass flow to the surface of the cylindrical filter is ≥ 50 mg/s at an average concentration of the introduced $CaCO_3$ particles, which is equal to 10 g/L. We determined that the accuracy of the results depends on: the coordinates of the material input in the apparatus volume, the surface shape of the filter and the volumetric flow rate of the liquid (water), being discarded by the turbine blades in the normal direction to their surface.

Keywords

Calcium Removal, Dispersion, Mass Transfer, Modeling, Adequacy

1. Introduction

A vacuum-filtration system, designed to reduce the concentration of calcium in the water during the water sof-

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tening process was developed at the Applied Research Institute of Ben-Gurion University of the Negev (Israel) (**Figure 1**) [1]. The most important part of this system is centrifugal turbine agitator-pump, sucking and discarding flows of liquid (water) with suspended particles of $CaCO_3$ (dispersed flow, dispersion, suspension) to the surface of a special filter. Hydrodynamics and mass transfer in such a turbine agitator-pump were studied and presented in [2] [3]. We calculated the main hydrodynamic parameters, which characterize the dynamic interaction of turbine blades with the flows of a viscous liquid. Based on the empirical Equation: $\mathrm{Sh} = 2 + \alpha \mathrm{Re}_T^{1/6} \cdot \mathrm{Pe}^{1/3}$, the coefficients of mass transfer k and diffusion (dispersion) coefficients D_* of the $CaCO_3$ substance by mass transfer from liquid (water) to the surface of the initial (embryonic) spherical particles $CaCO_3$, introduced in the apparatus with turbine agitator-pump, were calculated.

The purpose of this paper is:

- To present a solution to problem of distribution of $CaCO_3$ particles in a limited volume of the apparatus with a turbine agitator-pump;
- To determine the most important technological process parameters, such as time of dispersion homogenization t_h and the dispersion mass flow rate to the filter surface q_f;
- To use experimentation to conduct an analysis of the adequacy of the theory.

2. Models of Mass Transfer

2.1. Diffusion Model

To determine the function of the concentration distribution W for dispersed particles of $CaCO_3$ in a limited volume of the apparatus with a turbine agitator-pump, in the absence of mass exchange, we use the following three-dimensional Equation:

$$\frac{\partial W}{\partial t} + \left(\overline{V}_i \cdot \nabla\right) W = \sum_{i=1}^{3} D_i \frac{\partial^2 W}{\partial x_i^2} \quad (1)$$

with initial and boundary conditions in the following form:

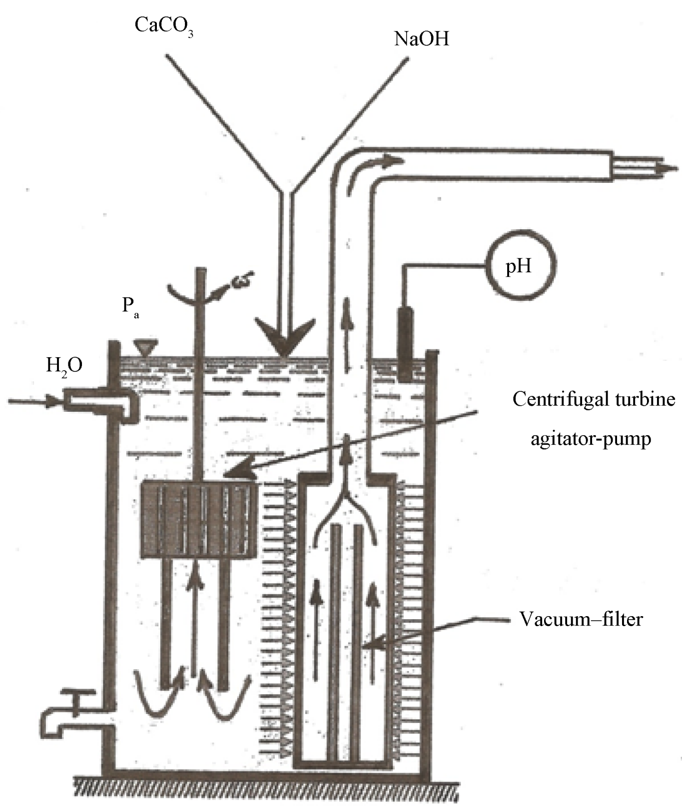

Figure 1. Vacuum-filtration system.

$$W_{|t=0} = \frac{M}{V_L} \prod_{i=1}^{3} \delta(x_i - x_{i0}),$$

$$\int_{x_{i0}-\varepsilon}^{x_{i0}+\varepsilon} \frac{M}{V_L} \prod_{i=1}^{3} \delta(x_i - x_{io}) \mathrm{d}x_i = \left(\frac{M}{V_L}\right)_{x_i = x_{i0}} = c_{av}, \varepsilon > 0,$$

$$k_i = \left(V_i \cdot W - D_i \frac{\partial W}{\partial x_i}\right)_{|s_i},$$

where $\overline{V_i} = \overline{V_p} - \overline{u_L}$ represents the relative velocity of the particle; D_i, which is the diffusion (dispersion) coefficients; x_i, which are cylindrical coordinates ($i = 1,2,3$ and match r, φ, z). The z axis is directed vertically upward from the center of the apparatus bottom. M is the mass of all $CaCO_3$ particles, introduced into the apparatus volume V_L at point $\overline{r_0}(x_{i0})$ in order to be distributed. k—represents specific dispersion fluxes to the bounding surface $s_i\left(\mathrm{kg/m^2 \cdot s}\right)$; δ—being the Dirac delta function [4]. The function $W = W\left(\overline{R}, t\right)$ in (1) is the probability of finding the particles group, moved and do not interacting with each other at time t at point $\overline{R} = \sum \overline{r_k}$, where $\overline{r_k}$ represents the movements of individual particles. W serves as a diffusing component (particles) concentration $\left(\mathrm{kg/m^3}\right)$ at point $\overline{R}$ in the time t.

The conclusion and solution of the equation (1) in the case of wandering particles with constant speeds V_i^* and coefficient D_i^* in the infinite volume are given, for example, by Chandrasekhar [5]. A similar approach to the description of the diffusion in the liquid suspended particles was used by Landau and Lifshitz [6]. Thus, the solution of the Equation (1) with the above mentioned initial conditions is represented in form [7]:

$$W = \frac{M \cdot \exp\left[-\sum_{i=1}^{3} \frac{\left(x_i - x_{i0} - V_i^*\right)^2}{4 D_i^* \cdot t}\right]}{2 V_L \sqrt{\pi \left(\sum_{i=1}^{3} \frac{V_i^{*2}}{4 D_i^*}\right) \cdot t}} \tag{2}$$

The dimension of the distribution function of the concentration W is $\left(\mathrm{kg/m^3}\right)$. The parameter V_i^* and D_i^* averaged velocities V_i and diffusion coefficients D_i in the relative (to the liquid) motion of the particles. They are determined as:

$$V_i = n \cdot \langle x_i \rangle, \quad D_i = \frac{1}{2} n \cdot \langle x_i^2 \rangle \tag{3}$$

Here n is the number of displacements of particles per unit of time; $\langle x_i \rangle, \langle x_i^2 \rangle$—respectively, the average displacements and mean square displacements of the diffusing particles. It is known, that they represent the first and second initial moments of the regulatory function $\overline{r_k}$ for displacement of individual particles.

2.2. Deterministic-Stochastic Model

The problem of determining such a function is reduced to the solution of the stochastic differential Langevin Equation [8] for the motion of dispersed particles in an external force field:

$$\frac{\mathrm{d}V_i}{\mathrm{d}t} = \left[\frac{1}{m}\left(\sum_{j=1}^{3} \overline{F_j} + \overline{\Phi_e} + \overline{\Phi_c}\right) + \overline{A(t)}\right] \cdot \overline{\eta}, \tag{4}$$

This is derived from the most general Meshcherskij Equation [9]. Here $\overline{F_j}$ are active forces, acting on the particles; $\overline{\Phi_e}, \overline{\Phi_c}$, which are the inertial forces in the portable motion of particles with the liquid and Coriolis force of inertia, respectively; $\overline{A(t)}$—is the perturbing acceleration, which is characteristic of the random effects on the particle by the surrounding liquid. The function $\overline{A(t)}$ has the following statistical properties: it does not depend on the relative velocity $\overline{V_i}$ and it changes quite rapidly, when compared with the $\overline{V_i}$. $\overline{\eta}$—change correction vector, which theoretically depends on the geometry of the apparatus with the turbine, the kinematical and the physical parameters of the process. $\overline{F_1} + \overline{F_2} = m\overline{g}\left(1 - \rho_i/\rho_p\right)$—represents particle weight and the lifting Archimedes force; $\overline{F_3} = -m\beta\left(\overline{V_p} - \overline{u_L}\right)$—describes resistance force, involved in the relative motion of the particles. $\left(\beta = c_R d_p \mu_L / m = 6 c_R \mu_L / \pi \rho_p d_p^2\right), 1/\mathrm{s}$; c_R—being the resistance coefficient, which depends on the concentration of the dispersed phase (particles of $CaCO_3$) and the mass transfer from the liquid to the surface of the particles (for the Stokes resistance force $c_R = 3\pi$). $\overline{\Phi_e} = -m\overline{a_e} = -m\left(\overline{a_e^n} + \overline{a_e^\tau}\right)$,

where $\overline{a_e^n}, \overline{a_e^\tau}$—represents the centripetal acceleration and rotation acceleration of the carrier medium respectively; $\overline{a_e^n} = -\omega_e^2 \overline{r}$ by $\overline{\omega_e} = \overline{\omega} = \overline{\text{const}}$, $\overline{a_e^\tau} = 0$, $\overline{a_c}$—Coriolis acceleration; $\overline{\Phi_c} = -m\overline{a_c} = -2\left(\overline{\omega_e} \times \overline{V_i}\right)$. Equation (4) models the Brownian motion of dispersed particles in the presence of external force fields as a Markov process in phase velocity space $\overline{V_i}$. We write further the Equation (4) in the projection on the axis of the moving coordinate system r, φ, z showing it to be rigidly attached to a rotating turbine and neglecting the Coriolis force, as shown below:

$$\frac{\mathrm{d}^2 x_i}{\mathrm{d}t^2} + (\beta \cdot \eta_i)\frac{\mathrm{d}x_i}{\mathrm{d}t} + (b_i \cdot \eta_i) x_i = A_i(t), \tag{5}$$

$\mathrm{d}x_i/\mathrm{d}t = V_{pi} - u_{Li}$ the relative velocity of the particle, projected on the axis r, φ, z. $\overline{\omega} \parallel z$.

$$b_1 = b_2 = \omega^2,\ b_3 = b = g\left(1 - \rho_L/\rho_p\right),\ x_3 = 1.$$

The first two Equations in (5) are identical in appearance. From these equations we must find the probability distributions $W\left(x_i, t; x_{i0}, V_{pi_0} - u_{Li_0}\right)$ and $W\left(V_{pi} - u_{Li}, t; x_{i0}, V_{pi_0} - u_{Li_0}\right)$, which respectively mean the probability of finding the relative displacement x_i and the relative velocity $\left(V_{pi} - u_{Li}\right)$ of the particle at a time t, if initially at $t = 0$ the particle was in position x_{i0} and had an initial speed $\left(V_{pi_0} - u_{Li_0}\right)$. To get this distribution we must first find a formal solution of the Equation (5). This can be done, for example, by the method of variation of parameters [5], considering Equation (5) as an ordinary differential equation. We can then bring the solution to the following form: $\overline{R} = \int_0^t \Psi(\eta) \cdot \overline{A}(\eta)\mathrm{d}\eta$, where $\overline{A}(\eta)$ has the same properties as $\overline{A(t)}$ in (4) and $\Psi(\eta)$—is the function, that is specified by a decision (5). Then, according to (5), the probability distribution of the vector $\overline{R}$ is given by:

$$W\left(\overline{R}\right) = \frac{\left[4\pi q \int_0^t \Psi^2(\eta)\mathrm{d}\eta\right]^{-3/2}}{\exp\left[\left|\overline{R}\right|^2 \Big/ 4q\int_0^t \Psi^2(\eta)\mathrm{d}\eta\right]} \tag{6}$$

where $q = \beta kT/m$, $k = 1.38 \times 10^{-23}\left(\mathrm{N \cdot m}/^\circ\mathrm{K}\right)$—is the Boltzmann constant [8]. Using the above findings and omitting the intermediate calculations, we write the expression for the displacement of the individual particles $\langle x_i \rangle$ and their relative velocities $\langle V_{pi} - u_{Li} \rangle$ in the form [7]:

$$\langle x_k \rangle = \frac{x_{k0}\left[ch\left(\frac{\beta_* t}{2}\right) + \frac{\eta_k \beta}{\beta_*} sh\left(\frac{\beta_* t}{2}\right)\right]}{\exp(\beta\eta_k)t/2} + \frac{2\left(V_{pk_0} - u_{Lk_0}\right) sh\left(\frac{\beta_* t}{2}\right)}{\beta_* \exp(\beta\eta_k)t/2}, \tag{7}$$

$$\langle z \rangle = z_0 - \frac{bt}{\beta} + \left[\frac{\beta\left(V_{pz_o} - u_{Lz_0}\right) + b}{\eta_z \beta^2}\right]\left(1 - \mathrm{e}^{-\eta_z \beta t}\right),$$

$$\langle x_k^2 \rangle = \langle x_k \rangle^2 + \frac{\eta_k kT}{m\omega^2}\left\{1 - \frac{\mathrm{e}^{-\beta\eta_k t}}{\beta_*^2}\left[2(\beta\eta_k)^2 sh^2\left(\frac{\beta_* t}{2}\right) + \beta_*(\beta\eta_k) sh(\beta_* t) + \beta_*^2\right]\right\}$$

$$\langle z^2 \rangle = \langle z \rangle^2 + \frac{2kT}{m\beta}\left[t - \frac{2}{\eta_z \beta}\left(1 - \mathrm{e}^{-\eta_z \beta t}\right) + \frac{1}{2\eta_z \beta}\left(1 - \mathrm{e}^{-2\eta_z \beta t}\right)\right]$$

$$\langle V_{pk} - u_{Lk} \rangle = \frac{\left(V_{pk_0} - u_{Lk_0}\right)}{\mathrm{e}^{\beta\eta_k t/2}}\left[ch\left(\frac{\beta_* t}{2}\right) - \frac{\beta\eta_k}{\beta_*} sh\left(\frac{\beta_* t}{2}\right)\right] - 2\frac{x_{k0}\omega^2 sh\left(\frac{\beta_* t}{2}\right)}{\beta_* \mathrm{e}^{\beta\eta_k t/2}},$$

$$\langle V_{pz} - u_{Lz} \rangle = \left(V_{pz_0} - u_{Lz_0}\right)\mathrm{e}^{-\beta\eta_z t} - \frac{b}{\beta}\left(1 - \mathrm{e}^{-\beta\eta_z t}\right).$$

There $k=1,2$ and corresponds to the direction of the axes r,φ. $\beta_*=\sqrt{(\beta\eta_k)^2-4\omega^2\eta_k}>0$; $\eta_i\,(i=1,2,3)$ —are correction vector projections on axes of cylindrical coordinate system r,φ,z.

2.3. Calculation of Diffusion Coefficients. Relative Velocities of Particles

We first determine the number of positive displacements of particles per unit of time n_i along the axes r,φ,z as follows: $n_i=\left|\dfrac{\langle V_{pi}-u_{Li}\rangle}{\langle x_i\rangle}\right|$. Using Equations (3) and (7), for the diffusion (dispersion) coefficient D_i we have:

$$D_k=\frac{n_k}{2}\left\{\langle x_k\rangle^2+\frac{D'\eta_k\beta}{\omega^2}\left[1-\frac{2(\beta\eta_k)^2 sh^2(\beta_*t/2)}{\beta_*^2 e^{\beta\eta_k t}}-\frac{\beta_*(\beta\eta_k)sh(\beta_*t)+\beta_*^2}{\beta_*^2 e^{\beta\eta_k t}}\right]\right\} \tag{8}$$

$$D_z=\frac{n_z}{2}\left\{\langle z\rangle^2+2D'\left[t-\frac{2}{\beta\eta_z}\left(1-e^{-\beta\eta_z t}\right)+\frac{1}{2\beta\eta_z}\left(1-e^{-2\beta\eta_k t}\right)\right]\right\}$$

$D'=\dfrac{kT}{m\beta}$—is the coefficient of Brownian diffusion, having, for example, the value: $D'=1.6\times10^{-14}\ \mathrm{m^2/s}$ for parameters of [3]:

$$d_{p_{av}}=2.73\times10^{-5}\ \mathrm{m},\quad \rho_p=\rho_{\mathrm{CaCO_3}}=2710\ \mathrm{kg/m^3},$$
$$\mu_L=1.06\times10^{-3}\ \mathrm{kg/m\cdot s},\quad T=293\ ^\circ\mathrm{K},$$
$$\beta=9658\ \mathrm{s^{-1}},\quad m=2.8\times10^{-11}\,\mathrm{kg}$$

Due to the smallness of the D', second summands in (8) can be neglected. Then the formulas D_i will take the form of:

$$D_i=\frac{1}{2}n_i\langle x_i\rangle^2 \tag{9}$$

The combination of Equation (9) with Equation (7) shows, that, if the initial velocities V_{pi_0} of the particles coincide with the carrier medium velocities u_{Li_0} (in the case $x_{i0}=0$, $b=0$) the diffusion coefficient $D_i=0$. This is consistent with the physical meaning of the process. Formula (9) are valid for the values of the coefficients $\beta_*>0$, that is, when $(\beta\eta_k)^2>4\omega^2\eta_k\ (k=1,2)$. If $\beta_*=0$, then $\beta^2\eta_k=4\omega^2$ and for the diameter of the diffusing particles $CaCO_3$ we have the relations:

$$\begin{aligned}\beta_*=0\to d_p=3\eta_k^{1/4}\sqrt{\frac{\mu_L}{\rho_p\omega}},\\ \beta_*>0\to d_p<3\eta_k^{1/4}\sqrt{\frac{\mu_L}{\rho_p\omega}}\end{aligned} \tag{10}$$

Therefore for a water softening process the value of the particle diameter d_p and the correction factor η_k can be estimated. For the data from [3] we have: $d_{p_{av}}=2.73\times10^{-5}$ m, $\omega=2\pi(\mathrm{rad/s})$; hence, the approximate value of the coefficient η_k at $(\beta_*=0)$ is: $\eta_k=1.77\times10^{-6}$. Because the distribution function W of Equation (2) consists of the relative velocity with the diffusion coefficients having constant value, the expression obtained for $(V_{pi}-u_{Li})$ from Equation (7) and for D_i from Equation (9) should be averaged over time. We take as an averaging time interval the time of dispersion homogenization t_h, that is, the time to reach an average concentration c_{av} of the diffusing component throughout the apparatus volume, while:

$$W_{|t=t_h}=c_{av}=\frac{M}{V_L},\quad \eta_c=\frac{\Delta c}{c_{av}}=\frac{|W-c_{av}|}{c_{av}}=1-\frac{W}{c_{av}}\to1,$$

where η_c—is the coefficient of heterogeneity of the concentration of the component in the volume of the apparatus. We have:

$$D_i^*=\frac{1}{t_h}\int_0^{t_h}D_i\mathrm{d}t,\quad V_i^*=\frac{1}{t_h}\int_0^{t_h}\left(V_{pi}-u_{Li}\right)\mathrm{d}t \tag{11}$$

Replace the exponential terms in Equations (7) and (9) at their approximate values [9]:

$$\mathrm{e}^{-\beta\eta_z t}\to 0,\quad \mathrm{e}^{-(\beta_*+\beta\eta_k)t/2}\to 0,$$
$$\mathrm{e}^{-\beta\eta_k t/2}\cdot ch(\beta_* t/2)\simeq \mathrm{e}^{-\beta\eta_k t/2}\cdot sh(\beta_* t/2)\simeq\left[0.5+0.25(\beta_*-\beta\eta_k)t\right]$$

Then at the point of mass M input $(x_{i0}=0)$ for average D_i^*, V_i^* from Equation (11) we obtain:

$$\begin{aligned}
D_k^* &= \frac{\left(V_{pk_0}-u_{Lk_0}\right)^2}{4\beta_*^2}\times\left|(\beta_*-\beta\eta_k)\left[\frac{(\beta_*-\beta\eta_k)^2}{12}t_h^2+\frac{(\beta_*-\beta\eta_k)}{2}t_h+1\right]\right| \\
D_z^* &= \frac{1}{2\beta^2}\left|\frac{b^2}{2}t_h-\frac{b}{\beta}\left[\frac{\beta\left(V_{pz_0}-u_{Lz_0}\right)+b}{\eta_z}\right]\right|, \\
V_k^* &= \left|\frac{\left(V_{pk_0}-u_{Lk_0}\right)(\beta_*-\beta\eta_k)}{\beta_*}\left[\frac{1}{2}+\frac{(\beta_*-\beta\eta_k)}{8}t_h\right]\right|, \\
V_z^* &= \frac{b}{\beta}=\frac{g(\rho_p-\rho_L)d_p^2}{18\mu_L}
\end{aligned}\tag{12}$$

A further task is to determine the time of dispersion homogenization t_h.

3. Process Parameters and Adequacy

3.1. Time Homogenization of Dispersion

Consider the following extreme problem: $\min\left(\Delta c=|W-c_{av}|\right)\to\frac{\partial}{\partial t}\left(|W-c_{av}|\right)_{|t=t_h}=0$. Substituting the function W from Equation (2), for the dispersion homogenization time t_h we obtain the following expression:

$$t_h=\frac{1}{\sum_{i=1}^{3}\left(V_i^{*2}\cdot D_i^*\right)}\times\left[-\left(\sum_{i=1}^{3}D_i^{*2}\right)+\sqrt{\left(\sum_{i=1}^{3}D_i^{*2}\right)^2+\left(\sum_{i=1}^{3}V_i^{*2}\cdot D_i^*\right)\cdot\left(\sum_{i=1}^{3}(x_i-x_{i0})^2\cdot D_i^*\right)}\right]\tag{13}$$

The diffusion coefficient D_i^* and the relative velocities V_i^* in Equation (13) also contain unknown parameter t_h. Therefore, the calculation of t_h is required in order to carry out an iterative method. As a first approximation we can use parameter values from [3] and the dimensionless Equation for the time of homogenization t'_h, obtained by study of the suspensions homogenization process in apparatuses with stirrers [7], as follows:

$$t_h'=\eta\cdot\mathrm{Re}_T^m=\left[\frac{(p_s-p_a)H_T^2}{\omega_h\rho_s D_*^2}\right]\cdot\mathrm{Re}_T^m\tag{14}$$

Here $D_*=\sqrt[3]{\prod_{i=1}^{3}D_i^*}$ is the average value of the diffusion (dispersion) coefficient; $\mathrm{Re}=\frac{\omega d_T^2}{\upsilon_s}$ is the Reynolds number for turbine; $\upsilon_s=\mu_s/\rho_s$, $\mu_s=\mu_L\left(1+2.5\varepsilon_p+7.35\varepsilon_p^2\right)$, $\rho_s=\varepsilon_p\rho_p+(1-\varepsilon_p)\rho_L$, $m=(-0.4)-(-0.8)$ Due to component $CaCO_3$ concentration W at the time of homogenization t_h tends to its average value c_{av} throughout the apparatus volume and does not depend on the coordinates shown in (**Figure 2**). The exponential term in Equation (2) can be neglected and for controlled precision of calculation W we can use the following simple formula:

$$W_{|t=t_h}=\frac{M}{V_L\sqrt{\frac{\pi t_h}{D_*}\sum_{i=1}^{3}V_i^{*2}}}\simeq c_{av}\tag{15}$$

Table 1, we present the values of homogenization time t'_h, t_h and the average concentration $W_{|t=t_h}$ which

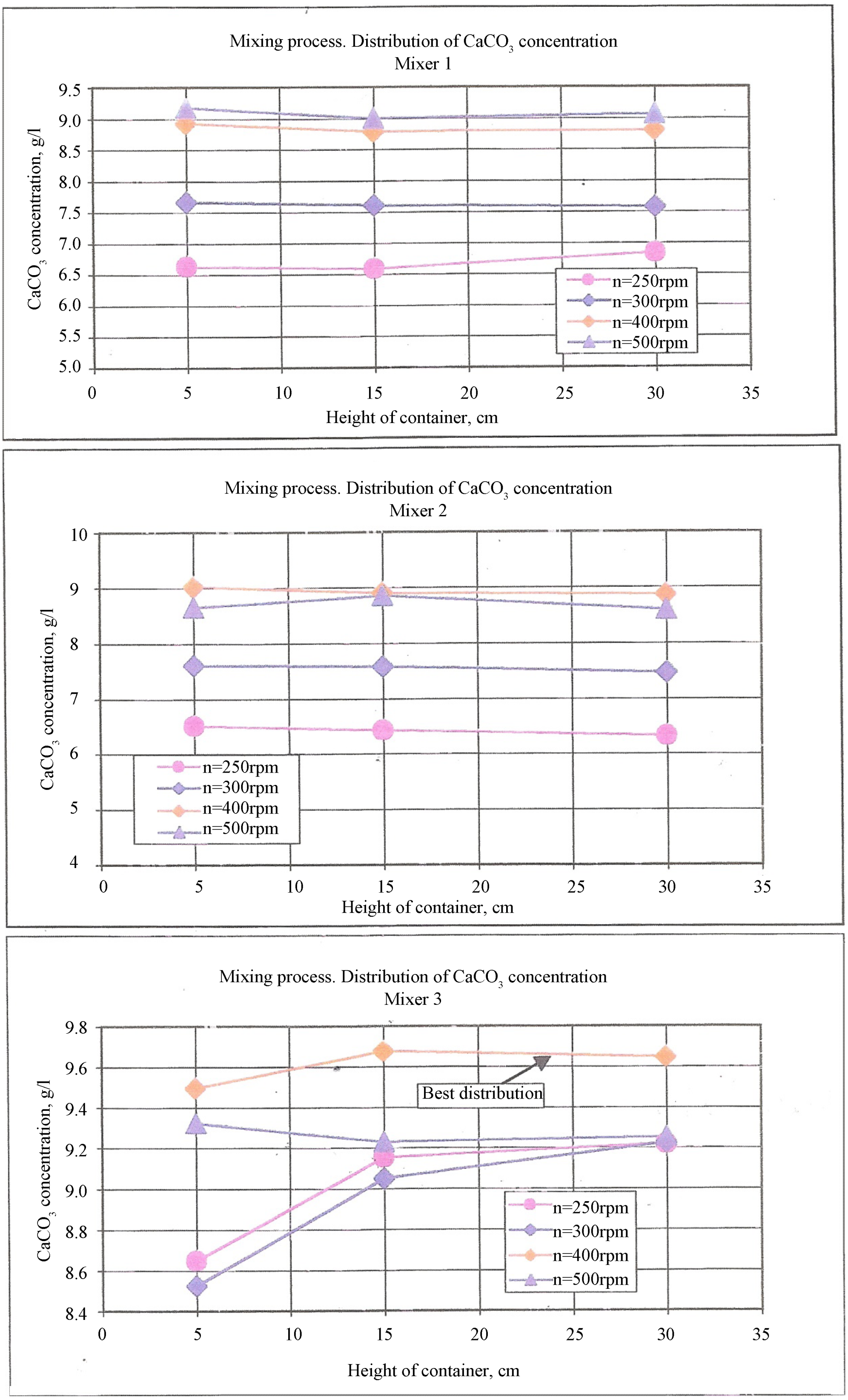

Figure 2. Distribution of $CaCO_3$ concentration.

Table 1. Time of homogenization and average concentration.

Original data	Calculated data	Technological parameters
$R_a = 0.1375\ \text{m},\ H_a = 0.4\ \text{m},$ $V_L = 18.5\times10^{-3}\ \text{m}^3,$ $d_T = 2R_T = 0.11\ \text{m},$ $H_T = 0.062\ \text{m},$ $d_{p_{av}} = 2.73\times10^{-5}\ \text{m},\ M = 0.185\ \text{kg},$ $\rho_p = \rho_{CaCO_3} = 2710\ \text{kg/m}^3,$ $\varepsilon_p = 0.01\ \text{kg/kg}\ (1\%CaCO_3),$ $\rho_L = 10^3\ \text{kg/m}^3,$ $\mu_L = 1.06\times10^{-3}\ \text{kg/m}\cdot\text{s},$ $\rho_s = 1017\ \text{kg/m}^3,$ $\mu_s = 1.087\times10^{-3}\ \text{kg/m}\cdot\text{s},$ $\upsilon_s = 1.068\times10^{-6}\ \text{m}^2/\text{s},$ $(p_s - p_a) = 150\ \text{N/m}^2,$ $(x_k - x_{k0}) = R_T = 0.055\ \text{m},$ $(z - z_0) = 0.$	$b = 6.177\ \text{m/s},\quad \beta = 9658\ \text{s}^{-1}.$ $(\omega_1 = 60\ \text{rpm},\ \text{Re}_{T1} = 71119) \to \beta_* = 3.565\times10^{-3}\ \text{s}^{-1},$ $D_* = 2.95\times10^{-5}\ \text{m}^2/\text{s},\quad \eta_k > 1.77\times10^{-6},$ $t'_{h1} = 28.5\ \text{s}(m = -0.73,\ \eta = 103688\ \text{s}),$ $(V_{pk_0} - u_{Lk_0}) \le 3.5\cdot10^{-4}\ \text{m/s},$ $V_k^* \le 0.63\times10^{-3}\ \text{m/s},\ V_z^* = 0.64\times10^{-3}\ \text{m/s}.$ $(\omega_2 = 400\ \text{rpm},\ \text{Re}_{T2} = 474129) \to \beta_* = 0.158\ \text{s}^{-1},$ $D_* = 4.48\times10^{-5}\ \text{m}^2/\text{s},\ \eta_k > 7.86\times10^{-5},$ $t'_{h2} = 14.5\ \text{s}\ (m = -0.47, \eta = 6744\ \text{s}),$ $(V_{pk_0} - u_{Lk_0}) \le 1.58\times10^{-3}\ \text{m/s},$ $V_k^* = 0.63\times10^{-3}\ \text{m/s},\ V_z = 0.64\times10^{-3}\ \text{m/s}.$	**Time of homogenization** $t_{h\vert_{\omega_1}} = 28.61\ \text{s} = t'_{h1},$ $t_{h\vert_{\omega_2}} = 13.76\ \text{s} \simeq t'_{h2},$ $\Delta t_h = 5\%.$ **The average concentration** $W_{\vert\omega_2, t_{h2}} = 9.28\ \text{kg/m}^3 = 9.28\ \text{g/l}$ $\simeq c_{av} = 10\ \text{g/l},$ $\Delta W = 7.2\%.$

are calculated by the empirical data of [2] [3], Equations (14) and (15) and the analytical formulas shown in Equations (12) and (13).

As seen from **Table 1**, using the formulas in Equations (13) and (14) for calculating the homogenization time t_h, gives almost perfectly matched results for the above presented values of the geometrical and phy-sical parameters. An important role is played by the coordinates $(x_i - x_{i0})$ of the substance input point in the volume of the apparatus. Calculation results for average concentration $\tilde{n}_{av}$, according to formula in Equation (15), are consistent with the experimental data, presented in **Figure 2**, for turbine agitator-pumps numbered 1 $(n = 6)$ and 2 $(n = 7)$ at angular velocities of rotation $\omega = (400\text{-}500)$ rpm. Deviation values c_{av} of about 4% correspond to a turbine agitator-pump number 3 with the number of blades shown as $(n = 12)$.

3.2. Mass Flows of Dispersion

We describe in Equation (16) the dynamic boundary conditions for the dispersed flow of particles, flowing through the turbine agitator-pump, with the achievement of the homogenization time t_h:

$$\int_{s_i} k_{i|t=t_h} \cdot \mathrm{d}s_i = q_i = \xi_i \cdot c_{av} \cdot Q_i \tag{16}$$

Here $(i = 1,2,3)$, $s_1 = s_2 = s_T = \pi d_T H_T$—is the lateral surface of the turbine; $s_3 = s_{in} = \pi d_{in}^2/4$—is the cross-section area of the turbine suction tube; $\xi_3 = \xi_z = 1$, $Q_1 = Q_2 = Q_n$—is the volumetric flow of liquid (water) discharged by the turbine blades in the normal direction to their surface, (m^3/s); $Q_3 = Q_{in}$—is the flow rate at the inlet to the turbine suction pipe; and ξ_i—represents proportionality factors. Expressions in the Equation (16) are dispersion mass flows in (kg/s) in the inlet (q_z) and outlet (q_r, q_φ) of the turbine agitator-pump. At the same time should be valid the law of mass conservation, then: $q_z = q_r + q_\varphi \to \xi_r + \xi_\varphi = 1$.

Using the results of the experiments, presented in [2] [3], we can determine the mass flow rate of the dispersion to the filter surface q_f with use of the formula, shown in the Equation (17):

$$q_f = c_{av} \cdot Q_{Wf} = c_{av}\left(\xi_f \cdot Q_n \cdot \frac{s_f}{s_b}\right), \tag{17}$$

where Q_{Wf}—is the volumetric flow rate of water, cleaned from $CaCO_3$—particles, passing through the filter surface and ξ_f represents the proportionality factor for the filter; $s_f = \pi d_f H_f \varepsilon_f$—represents lateral surface of the filter; $s_b = b_T H_T \cdot n$—represents the surface of n turbine blades and b_T—represents the distance between the centers of macro vortices, formed in the rear region after screws [3]. **Table 2**, below, is an example of calculating the mass flows of dispersion q_r, q_φ, q_z, q_f.

Table 2. Mass flows of dispersion: example of calculation.

Original data:	Calculated data: mass flows of dispersion
$d_{in}=0.044\text{ m}$, $d_T=0.11\text{ m}$, $d_f=0.014\text{ m}$, $H_T=0.062\text{ m}$, $b_T=0.022\text{ m}$, $H_f=0.15\text{ m}$, $\varepsilon_f=0.46$, $n=6$, $s_f=3.035\times10^{-3}\text{ m}^2$, $s_T=0.0214\text{ m}^2$, $s_{in}=1.52\times10^{-3}\text{ m}^2$, $s_b=8.184\times10^{-3}\text{ m}^2$, $\omega_h=60\text{ rpm}$, $t_h=28.5\text{ s}$, $V_{in}=0.3485\text{ m/s}$, $Q_{in}=Q_n=5.3\times10^{-4}\text{ m}^3/\text{s}$, $Q_{Wf}=5\times10^{-6}\text{ m}^3/\text{s}$, $\xi_r=0.4$, $\xi_\varphi=0.6$, $c_{av}=10\text{ kg/m}^3=10\text{ g/l}$.	$q_r=2.12\times10^{-3}\text{ kg/s}$, $q_\varphi=3.18\times10^{-3}\text{ kg/s}$, $q_z=5.3\times10^{-3}\text{ kg/s}$, $q_{f_{n=6}}=5\times10^{-5}\text{ kg/s}$, $\xi_f=0.025$.

From **Table 2** and Equation (17) we see, that in order to increase the mass flow rate of dispersion to the surface of the filter q_f, we must first increase its surface s_f and increase the normal flow rate of liquid Q_n. The boundary conditions, expressed through Equation (16), can be used for analytical determination of the correction $\eta_r,\eta_\varphi,\eta_z$. For that to take effect, we need to substitute into Equation (16) the distribution function of the concentration W from Equation (2) and the relative velocity V_i^* and the diffusion coefficient D_i^* from Equation (12). However, the expressions for $\eta_r,\eta_\varphi,\eta_z$ are too cumbersome. Therefore, the calculation of W should be guided by the values of the coefficients in **Table 1**.

4. Conclusions

The result of this research shows that the diffusion and deterministic-stochastic models of the substance transfer (particles of $CaCO_3$) in a limited volume of apparatus with the turbine agitator-pump can be used as an attachment to the issue of the removal of calcium from water, a process often called water softening. The main results of the use of the developed mathematical and physical models are:

- Optimal distribution of $CaCO_3$ particles concentrations within the volume of the apparatus is achieved by the angular velocity of rotation $\omega=400\text{ rpm}$ for the turbine with 6 - 7 blades and homogenization time of $t_h\simeq14\text{ s}$. At that dispersion, mass flows to the surface of the cylindrical filter are $q_f\geq50\text{ mg/s}$ at an average concentration of $(\text{CaCO}_3)c_{av}=10\text{ g/l}$.
- The accuracy of the calculation process parameters for homogenization time t_h and mass flow rate q_f affects the coordinates of the input point (x_i-x_{i0}), the shape and surface of the filter s_f and the volumetric flow of liquid (water) Q_n, as discarded by turbine blades in the normal direction to their surface. Some hydrodynamic parameters involved in the calculation, such as the speed of the carrier medium (water) u_L, volumetric flow rates Q_{in},Q_n were calculated on the basis of the vortex hydrodynamic model, described in [2]. Mass transfer parameters, described as the average diffusion (dispersion) coefficients D_* and the average diameters of the diffusing particles d_{av}, used in the calculations are taken from [3], preceding to this study.
- Some formulas, such as Equations (13)-(15), can be used in calculations of homogenization time for suspensions. They can also be used to determine average concentrations of diffusing components in various types of mixing devices and dispersion homogenizers.

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Notations

a—accelerating the carrier medium (water), m/s^2; c—concentration, kg/m^3; d—the diameter of the particle, the turbine, m; D—the diffusion (dispersion) coefficient, m^2/s; $\bar{F},\bar{\Phi}$—the force, N; H—the height, m; k—the specific flow of the dispersion, $\text{kg/m}^2\cdot\text{s}$; the mass transfer coefficient, m/s; M—mass, kg; m—mass, kg; index; n—the number of turbine blades; p—pressure, N/m^2; Q—volumetric flow rate of liquid (water), m^3/s; q -mass flow rate, kg/s; R—radius, m; s—surface, m^2; t—time, s; T—temperature, $^\circ\text{K}$; u—speed of the carrier medium (water), m/s; V—the particle velocity, m/s; the volume of the apparatus, m^3; W—the concentration of particles $CaCO_3$, kg/m^3; r, φ, z—coordinates, m.

Greek symbols: ε—the concentration of the initial (embryonic) particles of $CaCO_3$, introduced into the volume of the apparatus, kg/kg; μ—the dynamic viscosity coefficient, $kg/m \cdot s$; ν—the coefficient of kinematical viscosity, m^2/s; ρ—density, kg/m^3; ω—angular velocity of the turbine, rpm.

Indices: a—apparatus; av—the average value; in—the entrance to a suction tube of the turbine agitator-pump; b—lateral surface; c—Coriolis force of inertia; e—portable motion together with the turbine; f—filter; h—homogenization; L—liquid (water); n—normal direction; p—particle of $CaCO_3$ R—resistance; s—surface; T—turbine; W—water; r, φ, z—radial, tangential and axial directions; 0—the initial value; *—the averaged value.

Criteria

$\mathrm{Re} = \dfrac{\omega \cdot d_T^2}{\nu_s}$—Reynolds number;

$Sh = \dfrac{k \cdot d_p}{D_*}$—Sherwood number;

$Pe = \dfrac{u_{L\varphi} \cdot d_p}{D_*}$—Peclet number.

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