

# Optimal Capacity Expansion Policy with a Deductible Reservation Contract

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## ABSTRACT

*This paper investigates an optimal capacity expansion policy for innovative product in a context of one supplier and one retailer. With a fully deductible contract, we employ the Stackelberg game model to examine the negotiation process of capacity expansion in a single period. We first derive the retailer's optimal reservation strategy and then characterize the optimal capacity expansion policy for the supplier. We also investigate the impacts of reservation price on the optimal strategy of capacity reservation and expansion as well as the supplier's expected profits.*

**Keywords:** Supply Chain, Capacity Expansion, Deductible Reservation Contract, Stackelberg Game

## 1. Introduction

This paper is concerned with capacity expansion policy for innovative product in a setting of one supplier and one retailer. Evidently, capacity management is an important issue for innovative product, which is often characterized by volatile demand, short life cycle and long lead time. In fact, due to the highly volatile demand, the supplier often suffers from capacity shortage with the adoption of exact capacity expansion policy. Therefore, the retailer also loses revenues and his market reputation is damaged. Despite the need for higher revenue and improved service levels, the supplier may not be ready to expand capacity proactively because of financial risks due to higher capacity cost, long (capacity) lead time and high demand volatility. However, if the retailer agrees to share the financial risks by forward reservation, then the supplier may be motivated to expand capacity more aggressively. In this paper, the retailer reserves a capacity prior to demand realization, and in exchange, the supplier commits to have the "excess" capacity in addition to the reservation amount. This kind of capacity expansion policy provides a win-win situation for both the supplier and the retailer.

In the paper, we employ the fully deductible contract: the retailer pays a fee upfront for each unit of capacity reserved. When the retailer actually utilizes the reserved

capacity (*i.e.*, placing a firm order), the reservation fee is deductible from the order payment. However, if the reserved capacity is not fully utilized within the specified time period, the reservation fee associated with unused capacity is not refundable. Interestingly, supplier's announcement of excess capacity is a unique feature of the deductible reservation (DR) contract.

We consider a two-level supply chain in which a supplier offers an innovative product to one retailer facing a stochastic demand. Throughout the paper, we assume that the reservation price of the DR contract is exogenously determined. Obviously, the negotiation process for capacity expansion policy can be described as a Stackelberg game in which the supplier is the leader and the retailer is the follower. The objective of the current paper is to design an appropriate capacity expansion policy that allows both the supplier and the retailer to optimize their expected profits. Specifically, with an exogenously given reservation contract, we firstly analyze the retailer's optimal strategy, and then study the supplier's optimal capacity expansion policy. Finally, we illustrate the impacts of reservation price on the optimal capacity expansion policy and provide with some managerial insights.

The literature on capacity reservation is fairly abundant. There are some earlier literature related to capacity reservation mainly discuss the retailer's optimal strategy.

Sample references are [1-3]. Moreover, the work in this filed can be divided into two main categories in terms of retailer's motivations to reserve capacity. The first category considers the case in which the retailer reserves a certain portion of the future capacity to achieve potential cost reduction, for the sample references we refer readers to [4-10].

The second category, including [11-13], studies the problem that the retailer is motivated to offer early commitment on the future capacity so as to ensure a certain level of production availability. Moreover, Cachon and Lariviere [14] investigate capacity contracting in the context of supplier-buyer forecast coordination. Murat and Wu [15] show that, by fully deductible reservation contracts, the supplier has the incentive to expand the capacity proactively. They conclude that as the buyer's revenue margin decreases, the supplier faces a sequence of four profit scenarios with decreasing desirability. Jin and Wu [16] propose a capacity expansion policy that the supplier will have excess capacity in addition to reservation amount from the buyer. With a deductible reservation contract, they show that supply chain coordination can be achieved and both players benefit from supply chain coordination.

Evidently, our work on capacity expansion policy for innovative product mainly differs from earlier work in four aspects. First of all, the papers reviewed above mainly discuss the retailer's decision-making behavior, with little concern on supplier's perspective. We investigate the supplier's optimal strategy on capacity expansion policy in addition to the retailer's optimal strategy. Secondly, most existing papers consider endogenous wholesale price. However, in this paper we assume that the wholesale price is determined exogenously by the market or by earlier negotiations. Thirdly, the papers reviewed above assume that the supplier does not build any capacity without retailer's upfront commitment. However, with knowledge of market demand information, the supplier has the incentive to build capacity even without retailer's commitment. Finally, different from the perspective of supply chain coordination, we pay our attention on the players' interactions through modeling the process as a Stackelberg Game to derive optimal capacity expansion policy.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 discusses the retailer's optimal reservation strategy with an exogenously given reservation contract, and then describes the optimal capacity expansion policy for the supplier. Section 4 investigates the impacts of reservation price on the optimal capacity reservation and expansion policy. Finally, Section 5 concludes the paper.

## 2. Model Description

We consider a two-echelon supply chain in a single period, in which a supplier (called her) sells an innovative product to a retailer (called him). The retailer faces a stochastic demand  $D$ , with the probability density function  $f(\cdot)$  and cumulative distribution function  $F(\cdot)$ . Assume that the two parties hold symmetrical information about the market demand and the cost structure. The initial capacity level of the supplier is assumed to be zero. In the model, we propose a capacity reservation contract with fully deductible payment and assume that the reservation price  $r$  is an exogenously given constant parameter. To encourage the retailer to reserve capacity more readily, we let  $r \leq w$ , where  $w$  is the unit purchasing price charged by the supplier.

The sequence of the events is as follows:

1) At stage 0, the supplier announces the excess capacity  $E$ , which is the amount of capacity the supplier prepares to have in addition to (and regardless of) the retailer's reservation amount  $R$ .

2) Based on the excess capacity  $E$  and the demand forecasting information, the retailer decides the reservation amount  $R$  and pays  $rR$  to the supplier.

3) After receiving  $R$ , the supplier expands her capacity to  $R + E$  with marginal cost  $c$ .

4) At stage 1, the demand  $D$  is realized, then the retailer places an order  $\min(D, R + E)$ , with the unit purchasing cost  $w$ . The selling price for each product is  $p$  and any unmet demand will be lost.

5) The supplier deducts the amount of  $r \min(D, R)$  from the retailer's purchasing cost, but keeps the amount  $r \max(R - D, 0)$ .

6) The supplier salvages the residual capacity with unit salvage value  $s$ .

Obviously, the above negotiation process for capacity expansion policy can be modeled as a Stackelberg game, in which the supplier is the leader and the retailer is the follower. The supplier has complete visibility to the retailer's decision-making process. Suppose the two parties in the supply chain are risk-neutral. The aim of this paper is to characterize the optimal capacity expansion policy that allows both the supplier and the retailer to maximize their respective expected profits. In order to avoid trivial cases, we assume that  $s < c < w < p$ . As salvaging residual capacity will incur additional logistical and processing costs, we assume that the salvage value  $s$  is strictly less than the capacity expansion cost  $c$ .

## 3. The Optimal Capacity Expansion Policy

### 3.1. The Optimal Strategy for the Retailer

As reservation price  $r$  of the fully deductible reservation

contract is an exogenous given constant parameter, the retailer offers an early commitment on a certain portion of future capacity just before the supplier expands capacity. When the stochastic demand is realized, the retailer places an order and the reservation cost can be deducted from the purchasing cost. In this situation, at stage 0 with the excess capacity  $E$  offered by the supplier, the retailer determines the optimal reservation amount  $\hat{R}(E)$  that maximizes his expected profit.

$$\begin{aligned} \hat{\Pi}_M(R) &= E\{(p-w)\min(D, R+E) - rR + r\min(D, R)\} \\ &= (p-w)(R+E) - \int_0^R (R-x)dF(x) \\ &\quad - (p-w)\int_0^{R+E} (R+E-x)dF(x). \end{aligned} \tag{1}$$

The first term on the right-hand side of Equation (1) denotes the retailer’s profit from selling the innovative product, the second and the third term represents the retailer’s effective reservation cost paid to the supplier so as to ensure a certain level of availability.

Evidently, the more the effective reservation cost is, the higher the available capacity level will be in future.

**Lemma 1** Given the supplier’s excess capacity  $E$ , there exists a unique optimal reservation amount  $\hat{R}(E)$  to maximize the retailer’s expected profit, which is determined by

$$rF(\hat{R}(E)) = (p-w)[1 - F(\hat{R}(E)+E)]. \tag{2}$$

**Proof.** For any given  $E$ , taking the first and second derivatives of  $\hat{\Pi}_M(R)$  with respect to  $R$ , we get that

$$\begin{aligned} \frac{\partial \hat{\Pi}_M(R)}{\partial R} &= (p-w)[1 - F(R+E)] - rF(R), \\ \frac{\partial^2 \hat{\Pi}_M(R)}{\partial R^2} &= -(p-w)f(R+E) - rf(R) < 0. \end{aligned}$$

This implies that  $\hat{\Pi}_M(R)$  is strictly concave in  $R$ , and hence the optimal reservation amount  $\hat{R}(E)$  is uniquely determined by the first order condition given in Equation (2).

Lemma 1 shows that, for a given  $E$ , the retailer’s optimal strategy is to reserve  $\hat{R}(E)$ , which is uniquely determined by Equation (2). It follows from Lemma 1 that the optimal reservation amount  $\hat{R}(E)$  is monotonically decreasing in  $E$ . To see this, differentiating Equation (2) on both sides with respect to  $E$  and rearranging items, we get that

$$\begin{aligned} [rf(\hat{R}(E)) + (p-w)f(\hat{R}(E)+E)] \cdot \partial \hat{R}(E) / \partial E &= -(p-w) \cdot f(\hat{R}(E)+E) < 0, \text{ which implies that} \\ \partial \hat{R}(E) / \partial E &< 0. \end{aligned}$$

Therefore, for a certain level of future

demand, the larger the excess capacity is, the smaller the possibility of disruptions in future supply will be. Since the supplier prepares to set a higher level of excess capacity (*i.e.*, increasing  $E$ ), the retailer will reserve less. In this scenario, the supplier undertakes more financial risks in contrast with the retailer. Furthermore, we can see that  $F(\hat{R}(E)) = 0$  (and hence  $\hat{R}(E) = 0$ ) as  $E \rightarrow 0$ ; and  $F(\hat{R}(E)) = (p-w)/(p-w+r)$  as  $E \rightarrow \infty$ .

### 3.2. The Optimal Capacity Expansion Policy for the Supplier

In anticipation of the retailer’s optimal response behavior for any given  $E$ , we proceed to investigate the supplier’s optimal capacity expansion policy that maximizes her expected profit. By taking the retailer’s response function  $\hat{R}(E)$  into account, the supplier’s expected profit can be expressed as

$$\begin{aligned} \hat{\Pi}_s(E) &= E\left[ w\min(D, \hat{R}(E)+E) \right. \\ &\quad \left. + s(\hat{R}(E)+E-D)^+ + r\hat{R}(E) \right. \\ &\quad \left. - r\min(D, \hat{R}(E)) - c(\hat{R}(E)+E) \right]. \end{aligned} \tag{3}$$

where  $\hat{R}(E)$  is an implicit function of  $E$  given in Equation (2). On the right hand side of Equation (3), the first term is the supplier’s revenues from delivering the innovative product to the retailer; the second term denotes the supplier’s revenues from salvaging the residual capacity; the third and the fourth terms represent the retailer’s effective reservation cost paid to the supplier and the last term is the cost of expanding capacity.

To derive an explicit expression of  $\hat{R}(E)$  and make future analysis easier, throughout the paper we mainly discuss the scenario that the customer demand is uniformly distributed (other distributions can be analyzed similarly).

We assume that the customer demand  $D$  is uniformly distributed over the interval  $[0, \lambda]$  with  $\lambda > 0$ . Note that the assumption of uniform distribution is a simplification of reality, but it is sufficient to capture the main features of capacity reservation policy and derive managerial insights in practice. Specifically, from Lemma 1 we get that,

$$\hat{R}(E) = \frac{(p-w)(\lambda-E)}{p-w+r}, \tag{4}$$

and the total capacity of the supplier after capacity expansion is

$$\hat{R}(E) + E = \frac{(p-w)\lambda + rE}{p-w+r}. \tag{5}$$

Different values of  $E$  represent different capacity ex-

pansion strategies.  $E = 0$  means exact capacity expansion policy;  $E > 0$  represents aggressive capacity expansion policy with potentially higher gains and  $E < 0$  represents overbooking. However, credibility with the retailer is crucial for the supplier in the industry, so overbooking is not considered as an acceptable business. We will only consider the case where  $E \geq 0$ . Obviously, the total capacity should be no more than the maximum possible demand  $\lambda$ , i.e.,  $\hat{R}(E) + E \leq \lambda$ , which turns out to be  $E \leq \lambda$ . Therefore, we will confine our analysis on  $E \in [0, \lambda]$  in the rest of the paper.

By Equation (2), the supplier's objective function can be reformulated as:

$$\hat{\Pi}_s(E) = (w - c)(\hat{R}(E) + E) - (w - s) \frac{(\hat{R}(E) + E)^2}{2\lambda} + r \frac{\hat{R}(E)^2}{2\lambda} \quad (6)$$

Now, we characterize the properties of supplier's expected profit function with an exogenously given reservation price  $r$  ( $r \in [0, w]$ ), which are stated in the following lemma.

**Lemma 2** Let  $\alpha_1 = p/(2p - s)$ ,

$\theta_1 = (p - w)/(w - s)$ , then we have the following results.

1) If  $w \leq \alpha_1 p$ , then  $\hat{\Pi}_s(E)$  is convex in  $E$  for any  $r \in [0, w]$ ;

2) If  $w > \alpha_1 p$ , then  $\hat{\Pi}_s(E)$  is convex in  $E$  for any  $r \in [0, \theta_1(p - w)]$ , and  $\hat{\Pi}_s(E)$  is concave in  $E$  for any  $r \in [\theta_1(p - w), w]$ .

**Proof.** Since  $\partial \hat{R}(E)/\partial E = -(p - w)/(p - w + r)$  and  $\partial(\hat{R}(E) + E)/\partial E = r/(p - w + r)$  by Equation (4), we get from Equation (6) that the first and the second derivatives of  $\hat{\Pi}_s(E)$  with respect to  $E$  are

$$\begin{aligned} \frac{\partial \hat{\Pi}_s(E)}{\partial E} &= (w - c) \frac{\partial(\hat{R}(E) + E)}{\partial E} + r \frac{\hat{R}(E)}{\lambda} \frac{\partial \hat{R}(E)}{\partial E} \\ &\quad - (w - s) \frac{\hat{R}(E) + E}{\lambda} \frac{\partial(\hat{R}(E) + E)}{\partial E} \quad (7) \\ &= \frac{r(s - c)}{p - w + r} - \frac{r(\lambda - E)}{\lambda(p - w + r)^2} \\ &\quad \times [(p - w)^2 - (w - s)r] \end{aligned}$$

and

$$\frac{\partial^2 \hat{\Pi}_s(E)}{\partial E^2} = \frac{r(w - s)[(p - w)^2/(w - s) - r]}{\lambda(p - w + r)^2}$$

Obviously, we know that if  $r > (p - w)^2/(w - s) =$

$\theta_1(p - w)$ , then we have  $\partial^2 \hat{\Pi}_s(E)/\partial E^2 < 0$ ; and if  $r \leq \theta_1(p - w)$ , then  $\partial^2 \hat{\Pi}_s(E)/\partial E^2 \geq 0$ .

Since  $0 \leq r \leq w$ , we know that  $\theta_1(p - w) = w$  at  $w = \alpha_1 p$ . Therefore, we get that 1)  $\theta_1(p - w) \geq w$  when  $w \leq \alpha_1 p$ , and hence  $\hat{\Pi}_s(E)$  is convex in  $E$  for any  $r \in [0, w]$ ; 2)  $\theta_1(p - w) < w$  when  $w > \alpha_1 p$ , and hence  $\hat{\Pi}_s(E)$  is convex in  $E$  for any given  $r \in [0, \theta_1(p - w)]$  and  $\hat{\Pi}_s(E)$  is concave in  $E$  for any given  $r \in [\theta_1(p - w), w]$ .

Lemma 2 indicates that if the purchasing price is no more than  $\alpha_1 p$ , then with any exogenously given  $r$ , the supplier's expected profit is decreasing in  $E$  as the excess capacity is smaller than a critical point while increasing in  $E$  as the excess capacity exceeds the critical point. On the other hand, if the purchasing price is larger than  $\alpha_1 p$ , then when  $r$  is not greater than a threshold  $\theta_1(p - w)$ , the supplier's expected profit is decreasing in  $E$  as the excess capacity is smaller than a critical point while increasing in  $E$  as excess capacity exceeds the critical point; when the reservation price exceeds the threshold, the supplier's expected profit is increasing in  $E$  as the excess capacity is smaller than a critical point while decreasing in  $E$  as excess capacity exceeds the critical point.

Following from Lemma 2, we can obtain the supplier's optimal level of excess capacity with any given reservation price  $r \in [0, w]$ .

**Proposition 1** Let  $\alpha_2 = (c - s)/(2p - s)$  and  $\theta_2 = (p - w + c - s)/(w - c)$ .

1) If  $w \leq (\alpha_1 + \alpha_2)p$ , then the supplier will set the optimal excess capacity  $\hat{E} = 0$ ;

2) If  $w > (\alpha_1 + \alpha_2)p$ , then

$$\hat{E} = \begin{cases} 0, & \text{if } r \in [0, \theta_2(p - w)], \\ \lambda \frac{(w - c)[r - \theta_2(p - w)]}{(w - s)[r - \theta_1(p - w)]}, & \text{if } r \in [\theta_2(p - w), w]. \end{cases}$$

**Proof.** From Equation (7), we have that

$$\begin{aligned} \left. \frac{\partial \hat{\Pi}_s(E)}{\partial E} \right|_{E=0} &= \frac{r(s - c)}{p - w + r} - \frac{r}{(p - w + r)^2} \\ &\quad \times [(p - w)^2 - (w - s)r] \\ &= \frac{r(w - c)[r - \theta_2(p - w)]}{(p - w + r)^2}, \end{aligned}$$

$$\left. \frac{\partial \hat{\Pi}_s(E)}{\partial E} \right|_{E=\lambda} = \frac{r(s - c)}{p - w + r} < 0.$$

We consider the following cases by noting that  $\theta_1 < \theta_2$ .

1)  $w \leq \alpha_1 p$ .

For this case, we get that  $w \leq \theta_1(p-w)$  and hence  $\hat{\Pi}_s(E)$  is convex in  $E$  for any given  $r \in [0, w]$  (by Lemma 2). Moreover,  $w \leq \alpha_1 p$  implies that  $(p-w)^2 - (w-s)r > 0$ . Then, it follows from Equation (7) and  $s < c$  that  $\partial \hat{\Pi}_s(E) / \partial E \leq 0$  in  $E \in [0, \lambda]$ . Therefore, the supplier obtains her maximum profit at  $\hat{E} = 0$ .

2)  $\alpha_1 p < w \leq (\alpha_1 + \alpha_2) p$ .

For this case, we get that  $\theta_1(p-w) < w \leq \theta_2(p-w)$ . From Lemma 2, we know that  $\hat{\Pi}_s(E)$  is convex in  $E$  with a given  $r \in [0, \theta_1(p-w)]$ . Similar to case (a), we get that the optimal excess capacity is  $\hat{E} = 0$ .

For any given  $r \in [\theta_1(p-w), w]$ ,  $\hat{\Pi}_s(E)$  is concave in  $E$  (Lemma 2(2)). Since  $r \leq w \leq \theta_2(p-w)$ , we know that  $\partial \hat{\Pi}_s(E) / \partial E|_{E=0} \leq 0$ , which implies that the optimal excess capacity is  $\hat{E} = 0$ .

3)  $w > (\alpha_1 + \alpha_2) p$ .

$w > (\alpha_1 + \alpha_2) p$  implies that  $w > \theta_2(p-w) > \theta_1(p-w)$ . Following from Lemma 2, we know that  $\hat{\Pi}_s(E)$  is convex in  $E$  when  $r \in [0, \theta_1(p-w)]$  and concave in  $E$  when  $r \in [\theta_1(p-w), \theta_2(p-w)]$ . For both cases, since  $\hat{\Pi}_s(E)$  is decreasing over  $E \in [0, \lambda]$  by noting that  $\partial \hat{\Pi}_s(E) / \partial E|_{E=0} \leq 0$ , we can conclude that the optimal excess capacity is  $\hat{E} = 0$ .

When  $r \in [\theta_2(p-w), w]$ ,  $\hat{\Pi}_s(E)$  is concave in  $E$ , and  $\partial \hat{\Pi}_s(E) / \partial E|_{E=0} > 0$ . Therefore, the optimal excess capacity  $\hat{E}$  can be derived from the first order condition determined by Equation (7), which is

$$\hat{E} = \lambda \frac{(w-c)[r - \theta_2(p-w)]}{(w-s)[r - \theta_1(p-w)]}$$

In conclusion, 1) when  $0 < w \leq (\alpha_1 + \alpha_2) p$ , the optimal excess capacity is  $\hat{E} = 0$  for any exogenously given  $r \in [0, w]$ ; and 2) when  $w > (\alpha_1 + \alpha_2) p$ , we have

$$\hat{E} = \begin{cases} 0, & \text{if } r \in [0, \theta_2(p-w)], \\ \lambda \frac{(w-c)[r - \theta_2(p-w)]}{(w-s)[r - \theta_1(p-w)]}, & \text{if } r \in [\theta_2(p-w), w]. \end{cases}$$

The proof is completed.

Obviously, Proposition 1 clearly implies that the optimal capacity expansion policy for the supplier is to adopt the exact capacity expansion policy if  $w \leq (\alpha_1 + \alpha_2) p$ . Moreover, the intuition underlying Proposition 1 is clear. If the purchasing price is no more than a threshold--- $(\alpha_1 + \alpha_2) p$ , which means that the retailer's marginal

profit by selling one unit of innovative product is larger than a critical value--- $(1 - \alpha_1 - \alpha_2) p$ , then the retailer has an incentive to reserve a larger amount capacity because the revenue loss due to capacity shortage is very big, and the retailer is willing to undertake more financial risk for capacity expansion to ensure a higher level of capacity availability. By observing this, the supplier believes that the retailer's reservation amount is large enough to meet the future demand and thus take the exact capacity expansion policy with  $\hat{E} = 0$ .

On the other hand, if the purchasing price exceeds the threshold  $(\alpha_1 + \alpha_2) p$ , which means the retailer's marginal profit is smaller than the critical value, then the retailer is encouraged to reserve more to ensure a higher level of capacity availability in future with a smaller reservation price  $r \leq \theta_2(p-w)$ . In this situation, the supplier will also adopt exact capacity expansion policy with  $\hat{E} = 0$ . However, when the reservation price is larger than  $\theta_2(p-w)$ , the retailer will reserve less. To avoid future capacity shortage, the supplier will expand the capacity aggressively with  $\hat{E} > 0$ . Therefore, the supplier and the retailer's optimal strategies with any exogenous constant  $r \in [0, w]$  can be summarized as the following proposition.

**Proposition 2** 1) If  $w \leq (\alpha_1 + \alpha_2) p$ , then  $\hat{E} = 0$  and  $\hat{R} = \hat{R} + \hat{E} = \lambda(p-w)/(p-w+r)$ .

2) If  $w > (\alpha_1 + \alpha_2) p$ , then

$$\hat{R} = \begin{cases} \frac{\lambda(p-w)}{p-w+r}, & \text{if } r \in [0, \theta_2(p-w)], \\ \frac{\lambda(p-w)(c-s)}{(w-s)r - (p-w)^2}, & \text{if } r \in [\theta_2(p-w), w]. \end{cases}$$

$$\hat{E} = \begin{cases} 0, & \text{if } r \in [0, \theta_2(p-w)], \\ \lambda \frac{(w-c)[r - \theta_2(p-w)]}{(w-s)[r - \theta_1(p-w)]}, & \text{if } r \in [\theta_2(p-w), w]. \end{cases}$$

Hence, the supplier's total capacity is  $\hat{R} + \hat{E} = \lambda(p-w)/(p-w+r)$  if  $r \in [0, \theta_2(p-w)]$ ; and  $\hat{R} + \hat{E} = \lambda[(w-c)r - (p-w)]^2 / [(w-s)r - (p-w)]^2$  if  $r \in [\theta_2(p-w), w]$ .

**Proof.** The results follow directly from Proposition 1 and Equation (4).

#### 4. The Impact of Reservation Price

In this subsection, we investigate the impacts of reservation price  $r$  on the optimal capacity reservation policy. Specially, we would like to show how  $r$  affects the optimal excess capacity  $\hat{E}$ , the retailer's optimal reservation amount  $\hat{R}$  and the supplier's capacity level  $\hat{R} + \hat{E}$ .

**4.1. The Case  $w > (\alpha_1 + \alpha_2)p$**

The following proposition presents the results of comparative statics of reservation price.

**Proposition 3** If  $w > (\alpha_1 + \alpha_2)p$ , then 1)  $\hat{R}$  is decreasing in  $r$  over  $r \in [0, \theta_2(p-w)]$ ; and 2)  $\hat{R}$  is decreasing in  $r$  and  $\hat{E}$  and  $\hat{R} + \hat{E}$  are both increasing in  $r$  over  $r \in [\theta_2(p-w), w]$ .

**Proof.** For part 1), we know that  $\hat{E} = 0$  and it is easy to see that  $\hat{R} = \hat{R} + \hat{E} = \lambda(p-w)/(p-w+r)$  is decreasing in  $r$ . We now consider part 2). For  $r \in [\theta_2(p-w), w]$ , we know that

$$\hat{R} = \lambda(p-w)(c-s)/[(w-s)r - (p-w)^2]$$

$$= \lambda\theta_1(c-s)/[r - \theta_1(p-w)] \quad (\text{Proposition 2}). \text{ Then,}$$

$$\frac{d\hat{R}}{dr} = -\frac{\lambda\theta_1(c-s)}{[r - \theta_1(p-w)]^2} < 0. \quad (8)$$

By noticing that  $\theta_2 > \theta_1$ , it follows from Proposition 2 that for  $r \in [\theta_2(p-w), w]$

$$\frac{d\hat{E}}{dr} = \lambda \frac{(w-c)}{(w-s)} \cdot \frac{(\theta_2 - \theta_1)(p-w)}{[r - \theta_1(p-w)]^2} > 0. \quad (9)$$

which indicates that  $\hat{E}$  is increasing in  $r$  over  $r \in [\theta_2(p-w), w]$ . Combining the above two equations and rearranging items, we get that

$$\frac{d(\hat{R} + \hat{E})}{dr} = \frac{\lambda(p-w)^2(c-s)}{(w-s)^2[r - \theta_1(p-w)]^2} > 0. \quad (10)$$

Hence,  $\hat{R} + \hat{E}$  is creasing in  $r$  for the case  $w > (\alpha_1 + \alpha_2)p$  and  $r \in [\theta_2(p-w), w]$ .

Proposition 3 shows that, under the condition that the purchasing price  $w$  is larger than  $(\alpha_1 + \alpha_2)p$  and the reservation price  $r$  exceeds  $\theta_2(p-w)$ , the supplier's optimal excess capacity increases while the retailer's optimal reservation amount decreases as  $r$  increases. This is because the retailer takes more risks of over reservation and the supplier benefits more from the capacity reservation. Even though, the supplier's optimal capacity level  $\hat{R} + \hat{E}$  is still increasing in  $r$  since the decreasing rate of  $\hat{R}$  is smaller than the increasing rate of  $\hat{E}$ .

**Corollary 1** If  $w > (\alpha_1 + \alpha_2)p$ , then for  $r \in [\theta_2(p-w), w]$ , the supplier's optimal expected profit  $\Pi_s(\hat{E})$  is decreasing in  $r$ .

**Proof** It follows from Proposition 2 and Equation (10) that

$$\left[ (w-c) - \frac{(w-s)(\hat{R} + \hat{E})}{\lambda} \right] \frac{d(\hat{R} + \hat{E})}{dr}$$

$$= \left[ (w-c) - (w-s) \frac{(w-c)r - (p-w)^2}{(w-s)r - (p-w)^2} \right]$$

$$\cdot \frac{\lambda(p-w)^2(c-s)}{(w-s)^2[r - \theta_1(p-w)]^2}$$

$$= \frac{(p-w)^2(c-s)}{(w-s)[r - \theta_1(p-w)]} \cdot \frac{\lambda(p-w)^2(c-s)}{(w-s)^2[r - \theta_1(p-w)]^2}$$

$$= \frac{\lambda(p-w)^4(c-s)^2}{(w-s)^3[r - \theta_1(p-w)]^3}.$$

From Equation (8) and Proposition 2, we have

$$\frac{r\hat{R}}{\lambda} \frac{d\hat{R}}{dr} = -\frac{r(p-w)(c-s)}{(w-s)r - (p-w)^2} \times \frac{\lambda\theta_1(c-s)}{[r - \theta_1(p-w)]^2}$$

$$= -\frac{\lambda r(p-w)^2(c-s)^2}{(w-s)^2[r - \theta_1(p-w)]^3}.$$

Therefore, we get that

$$\frac{d\Pi_s(\hat{E})}{dr} = \left[ (w-c) - \frac{(w-s)(\hat{R} + \hat{E})}{\lambda} \right] \cdot \frac{d(\hat{R} + \hat{E})}{dr}$$

$$+ \frac{r\hat{R}}{\lambda} \frac{d\hat{R}}{dr} + \frac{\hat{R}^2}{2\lambda}$$

$$= \frac{\lambda(p-w)^4(c-s)^2}{(w-s)^3[r - \theta_1(p-w)]^3}$$

$$- \frac{\lambda r(p-w)^2(c-s)^2}{(w-s)^2[r - \theta_1(p-w)]^3}$$

$$+ \frac{\lambda(p-w)^2(c-s)^2}{2(w-s)^2[r - \theta_1(p-w)]^2}$$

$$= -\frac{\lambda(p-w)^2(c-s)^2}{2(w-s)^2[r - \theta_1(p-w)]^2} < 0.$$

Hence, when  $w > (\alpha_1 + \alpha_2)p$ ,  $\Pi_s(\hat{E})$  is decreasing in  $r$  for  $r \in [\theta_2(p-w), w]$ .

From Corollary 1 we get that the retailer's optimal expected profit  $\Pi_M(\hat{R})$  is increasing in  $r$  by noting that the total supply chain profit is unrelated to  $r$ . In this situation, the high reservation price results in much low reservation amount of capacity. Consequently, the supplier needs to build excess capacity to match the demand

in the future. However, the increasing costs of building up the capacity have a negative effect on the supplier's optimal expected profit, that is, the supplier's optimal expected profit is decreasing in  $r$ . Although, the expected profit for the entire supply chain can be increased since it will alleviate the effects of double marginalization. In view of this point, it is advised that the supplier should not to choose a higher reservation price in a decentralized supply chain.

If we choose the reservation price  $r$  in the interval  $[0, \theta_2(p-w)]$ , then it follows from Proposition 2 that  $\hat{E} = 0$ . Taking derivative of  $\Pi_s(\hat{E})$  with respect to  $r$ , we get that

$$\begin{aligned} \frac{d\Pi_s(\hat{E})}{dr} &= \left[ (w-c) - \frac{(w-s-r)\hat{R}}{\lambda} \right] \cdot \frac{d\hat{R}}{dr} + \frac{\hat{R}^2}{2\lambda} \\ &= \left[ -(w-c) + \frac{(w-s-r)(p-w)}{(p-w+r)} \right] \\ &\quad \cdot \frac{\lambda(p-w)}{(p-w+r)^2} + \frac{\lambda(p-w)^2}{2(p-w+r)^2} \\ &= \frac{\lambda(p-w)}{2(p-w+r)^3} \\ &\quad \cdot [-(p+w-2c)r + (p-w)(p-w+2c-2s)]. \end{aligned} \tag{11}$$

Let  $\theta_3 = (p-w+2c-2s)/(p+w-2c)$ . It can be verified that  $\theta_3 < \theta_2$ , therefore, for this case,  $\Pi_s(0)$  is increasing for smaller  $r$  ( $r < \theta_3(p-w)$ ) and decreasing for larger  $r$  ( $\theta_3(p-w) < r < \theta_2(p-w)$ ). For smaller  $r$ , the supplier can benefit from the reservation since the retailer decreases his amount of reservation capacity and undertakes some risk of higher demand as  $r$  increases. However, for larger  $r$ , the supplier may lose some profit when  $r$  is increased since the reservation capacity is decreased too much.

In summary, the optimal profit for the supplier is unimodal in  $r$  when  $w > (\alpha_1 + \alpha_2)p$  and the optimal reservation price can be set to  $r = \theta_3(p-w)$ .

#### 4.2. The Case $w < (\alpha_1 + \alpha_2)p$

When  $w < (\alpha_1 + \alpha_2)p$ , the supplier will not expand her capacity, *i.e.*,  $\hat{E} = 0$ . For the retailer, the optimal reservation amount is  $\hat{R} = \lambda(p-w)/(p-w+r)$ . If the reservation price approaches zero, the retailer would set the reservation amount at the highest possible demand  $\lambda$ . However, as  $r$  increases, the retailer's optimal reservation amount will decrease.

**Proposition 4** If  $w \leq (\alpha_1 + \alpha_2)p$ , then  $\hat{R}$  (and  $\hat{R} + \hat{E}$ ) is decreasing in  $r$ . And the supplier's expected

profit  $\Pi_s(0)$  is unimodal.

**Proof.** Since  $\hat{E} = 0$ , it is easy to see that  $\hat{R} = \hat{R} + \hat{E} = \lambda(p-w)/(p-w+r)$  is decreasing in  $r$ . The derivative of  $\Pi_s(0)$  with respect to  $r$  is the same as Equation (11). By noting that  $\theta_2 > \theta_3$  and  $w < \theta_2(p-w)$ , it follows from Equation (11) that, if  $w < \theta_3(p-w)$ , then  $\Pi_s(0)$  is increasing in  $r$  over  $[0, w]$ ; and if  $\theta_2(p-w) > w > \theta_3(p-w)$ , then  $\Pi_s(0)$  is increasing in  $r$  over  $[0, \theta_3(p-w)]$  and decreasing in  $r$  over  $[\theta_3(p-w), w]$ .

For the case  $w < (\alpha_1 + \alpha_2)p$ ,  $\Pi_s(0)$  is increasing for smaller  $r$  ( $r < \theta_3(p-w)$ ) and decreasing for larger  $r$  ( $r > \theta_3(p-w)$ ). For smaller  $r$ , the supplier can benefit from the reservation since the retailer decreases his amount of reservation capacity and undertakes some risk of higher demand as  $r$  increases. However, for larger  $r$ , the supplier may lose some profit when  $r$  is increased since the reservation capacity is decreased too much.

In the previous analysis, we implicitly assume that the supplier always accept the retailer's capacity reservation. However, the deductible reservation contract can be conducted only if the supplier could earn some profits. Now we identify the condition under which the supplier has an incentive to accept the retailer's capacity reservation. For this case, if the supplier accepts the retailer's reservation, then the supplier's optimal expected profit is

$$\begin{aligned} \Pi_s(0) &= (w-c)\hat{R} - (w-s-r)\frac{\hat{R}^2}{2\lambda} \\ &= \hat{R} \left[ (w-c) - \frac{(w-s-r)(p-w)}{2(p-w+r)} \right] \\ &= \frac{\hat{R}}{2(p-w+r)} \\ &\quad \cdot [(p+w-2c)r + (w-2c+s)(p-w)]. \end{aligned}$$

Therefore, the supplier accepts the retailer's reservation only if  $(p+w-2c)r + (w-2c+s)(p-w) \geq 0$ , *i.e.*,  $r \geq (2c-w-s)(p-w)/(p+w-2c)$ . This condition holds if the purchasing cost  $w$  is high relative to the capacity building cost  $c$ . Then the supplier has an incentive to accept the retailer's capacity reservation; otherwise, the supplier will raise the reservation price  $r$  or unit purchase price  $w$  so that she can obtain some profits.

#### 5. Concluding Remarks

Capacity management plays a significant role on innovative product. In the paper, the capacity expansion policy not only provides with a risk-sharing mechanism for both the supplier and the retailer, but also improves the retailer's potential revenue. Specifically, we propose a fully deductible contract where the retailer reserves future capacity with a fee that can be deducted from the

purchasing price. Additionally, the supplier's ex ante announcement of "excess" capacity is a unique feature of the deductible reservation contract. Given the reservation contract, we figure out the optimal capacity expansion policy and also study the effects of reservation price on the optimal strategy as well as the supplier's optimal profit. Finally, we address the issues of how to set the reservation price from the perspective of the supplier in different situations.

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## REFERENCES

- [1] K. Jain and E. A. Silver, "The Single Period Procurement Problem Where Dedicated Supplier Capacity Can be Reserved," *Naval Research Logistics*, Vol. 42, No. 6, 1995, pp. 915-934. [doi:10.1002/1520-6750\(199509\)42:6<915::AID-NAV3220420605>3.0.CO;2-M](https://doi.org/10.1002/1520-6750(199509)42:6<915::AID-NAV3220420605>3.0.CO;2-M)
- [2] A. Brown and H. Lee, "Optimal Pay-to-Delay Capacity Reservation with Application to the Semiconductor Industry," *Working Paper*, Vanderbilt University, Nashville, Tennessee, 1998.
- [3] G. P. Cachon, "Supply Chain Coordination with Contracts," S. Graves and T. de Kok Eds., *The Handbook of Operations Research and Management Science, Supply Chain Management*, Amsterdam, The Netherlands, 2003.
- [4] J. S. Bonser and S. D. Wu, "Procurement Planning to Maintain Both Short-Term Adaptiveness and Long-Term Perspective," *Management Science*, Vol. 47, No. 6, 2001, pp. 769-786. [doi:10.1287/mnsc.47.6.769.9814](https://doi.org/10.1287/mnsc.47.6.769.9814)
- [5] D. A. Serel, M. Dada and H. Moskowitz, "Sourcing Decisions with Capacity Reservation Contracts," *European Journal of Operational Research*, Vol. 131, No. 3, 2001, pp. 635-648. [doi:10.1016/S0377-2217\(00\)00106-5](https://doi.org/10.1016/S0377-2217(00)00106-5)
- [6] R. W. Seifert, U. W. Thonemann and W. H. Hausman, "Optimal Procurement Strategies for Online Spot Markets," *European Journal of Operational Research*, Vol. 152, No. 3, 2004, pp.781-799. [doi:10.1016/S0377-2217\(02\)00754-3](https://doi.org/10.1016/S0377-2217(02)00754-3)
- [7] D. J. Wu, P. R. Kleindorfer and J. E. Zhang, "Optimal Bidding and Contracting Strategies for Capital-Intensive Goods," *European Journal of Operational Research*, Vol. 137, No. 3, 2002, pp. 657-676. [doi:10.1016/S0377-2217\(01\)00093-5](https://doi.org/10.1016/S0377-2217(01)00093-5)
- [8] V. Martinez-de-Albeniz and D. Simchi-Levi, "A Portfolio Approach to Procurement Contracts," *Production and Operations Management*, Vol. 14, No. 1, 2005, pp. 90-114. [doi:10.1111/j.1937-5956.2005.tb00012.x](https://doi.org/10.1111/j.1937-5956.2005.tb00012.x)
- [9] O. Ozer and W. Wei, "Strategic Commitments for an Optimal Capacity Decision under Asymmetric Forecast Information," *Management Science*, Vol. 52, No. 8, 2006, pp. 1238-1257. [doi:10.1287/mnsc.1060.0521](https://doi.org/10.1287/mnsc.1060.0521)
- [10] Q. Fu, C. Y. Lee and C. P. Teo, "Procurement Management Using Option Contracts: Random Spot Price and the Portfolio Effect," *IIE Transactions*, Vol. 42, No. 11, 2010, pp. 793-811. [doi:10.1080/07408171003670983](https://doi.org/10.1080/07408171003670983)
- [11] G. Eppen and A. Iyer, "Backup Agreements in Fashion Buying - the Value of Upstream Flexibility," *Management Science*, Vol. 43, No. 11, 1997, pp. 1469-1484. [doi:10.1287/mnsc.43.11.1469](https://doi.org/10.1287/mnsc.43.11.1469)
- [12] D. Barnes-Schuster, Y. Bassok and R. Anupindi, "Coordination and Flexibility in Supply Contracts with Options," *Manufacturing & Service Operations Management*, Vol. 4, No. 3, 2002, pp. 171-207. [doi:10.1287/msom.4.3.171.7754](https://doi.org/10.1287/msom.4.3.171.7754)
- [13] X. Gan, S. P. Sethi and H. Yan, "Coordination of Supply Chains with Risk-Averse Agents," *Production and Operations Management*, Vol. 13, No. 2, 2004, pp.135-149. [doi:10.1111/j.1937-5956.2004.tb00150.x](https://doi.org/10.1111/j.1937-5956.2004.tb00150.x)
- [14] G. P. Cachon and M. A. Lariviere, "Supply Chain Coordination with Revenue Sharing Contracts: Strengths and Limitations," *Management Science*, Vol. 51, No. 1, 2005, pp. 30-44. [doi:10.1287/mnsc.1040.0215](https://doi.org/10.1287/mnsc.1040.0215)
- [15] M. Erkoc and S. D. Wu, "Managing High-Tech Capacity Expansion via Reservation Contracts," *Production and Operations Management*, Vol. 14, No. 2, 2005, pp. 232-251. [doi:10.1111/j.1937-5956.2005.tb00021.x](https://doi.org/10.1111/j.1937-5956.2005.tb00021.x)
- [16] M. Jin and S. D. Wu, "Capacity Reservation Contracts for High-Tech Industry," *European Journal of Operational Research*, Vol. 176, No. 3, 2007, pp. 1659-1677. [doi:10.1016/j.ejor.2005.11.008](https://doi.org/10.1016/j.ejor.2005.11.008)